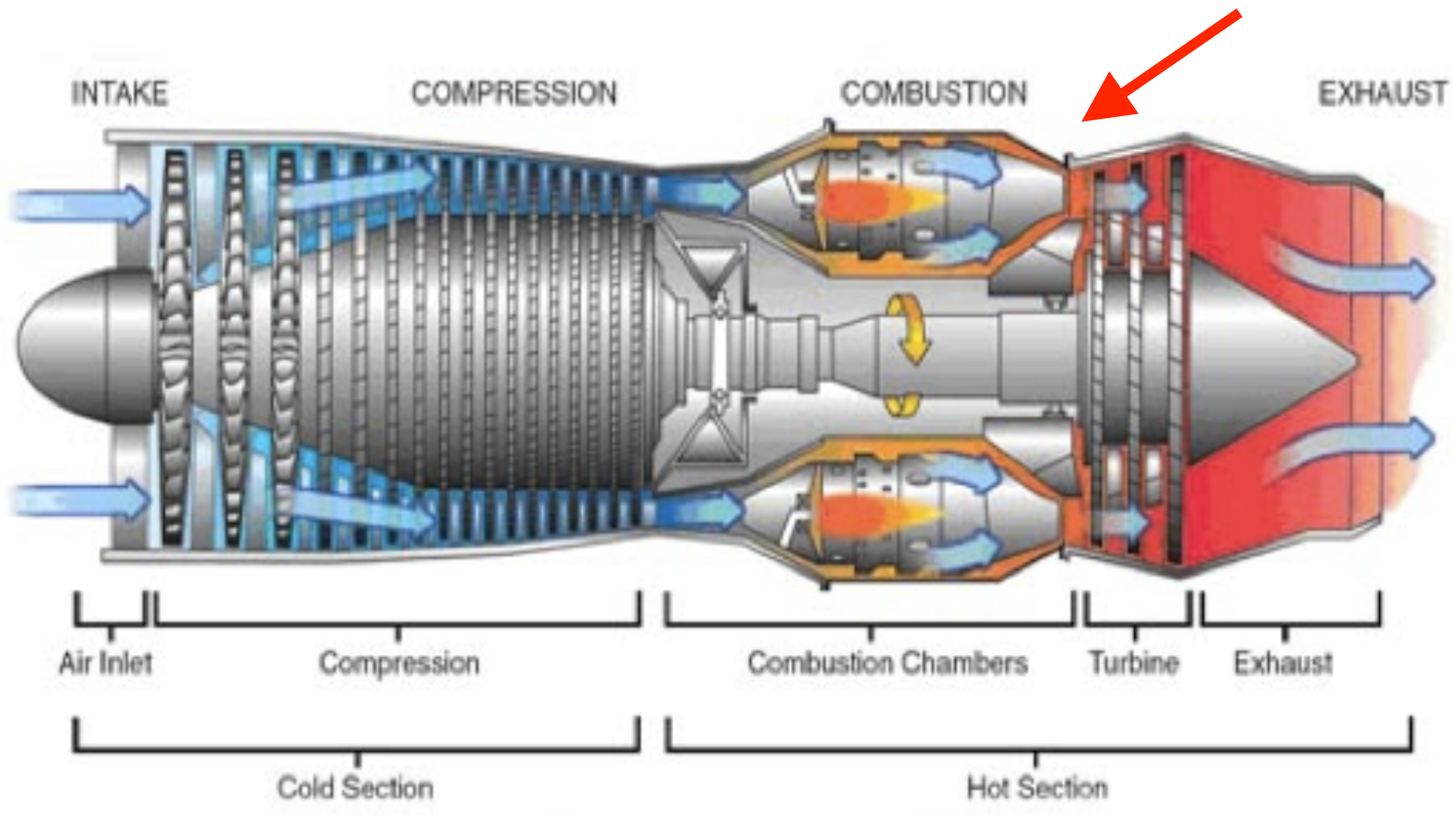
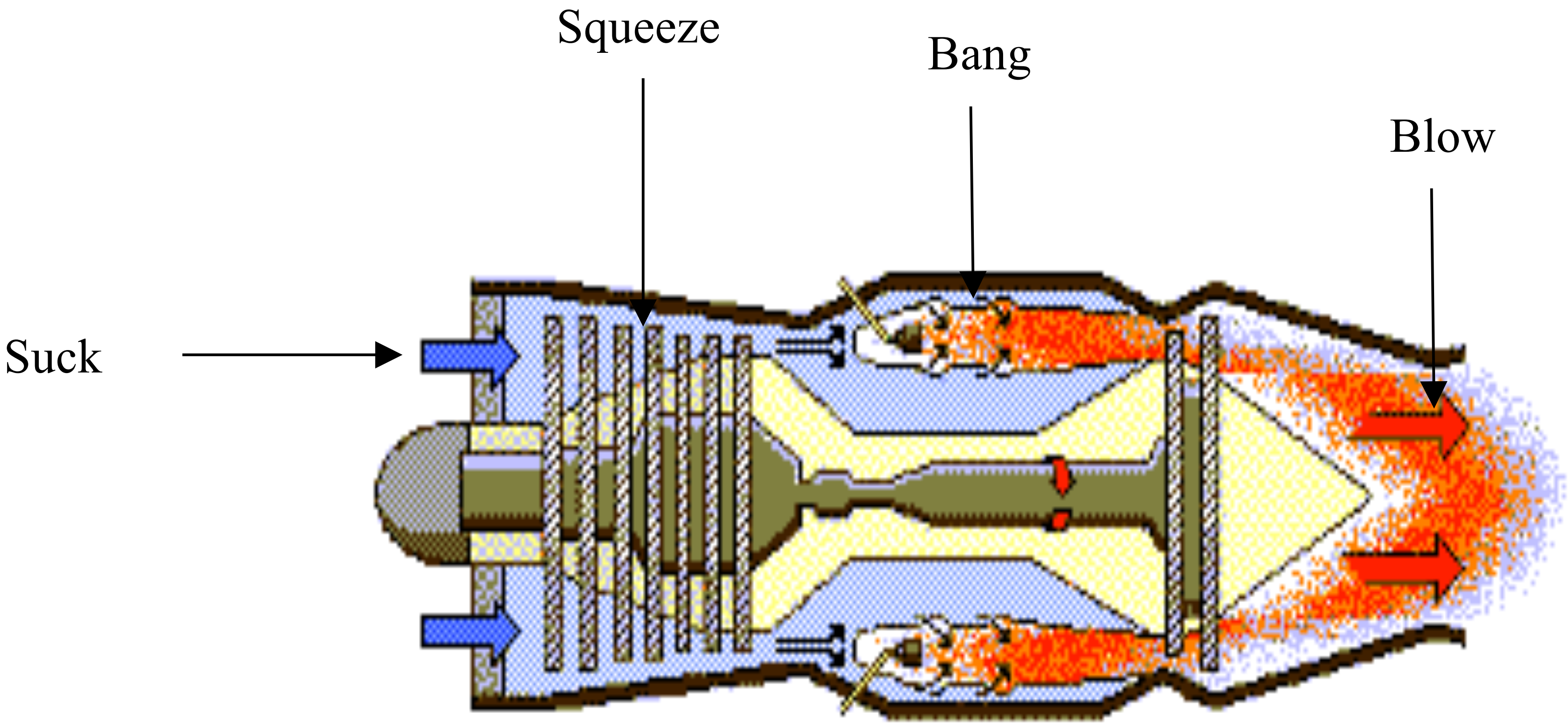
Section 4.1: Introduction to Jet Propulsion

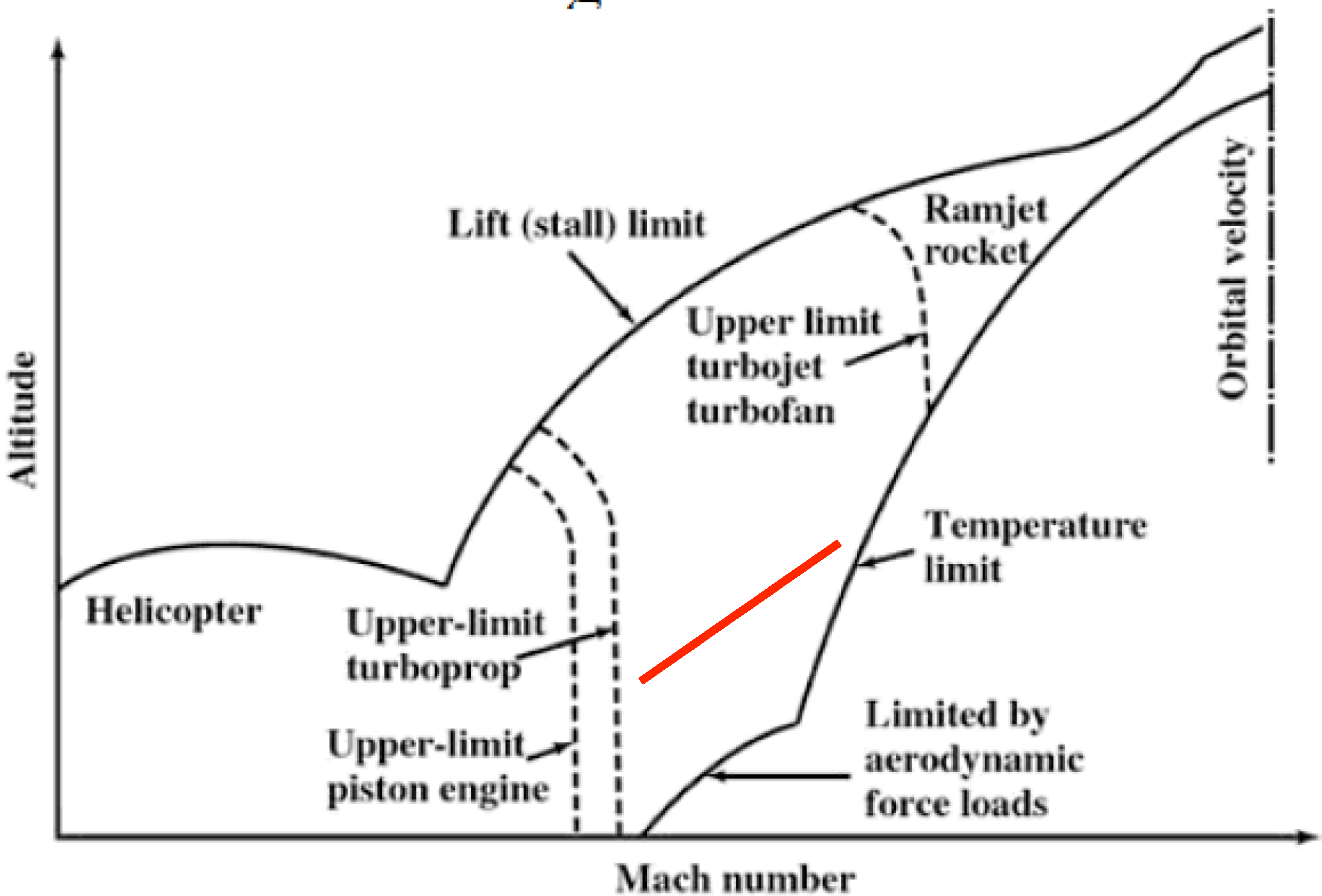


Jet Propulsion Basics

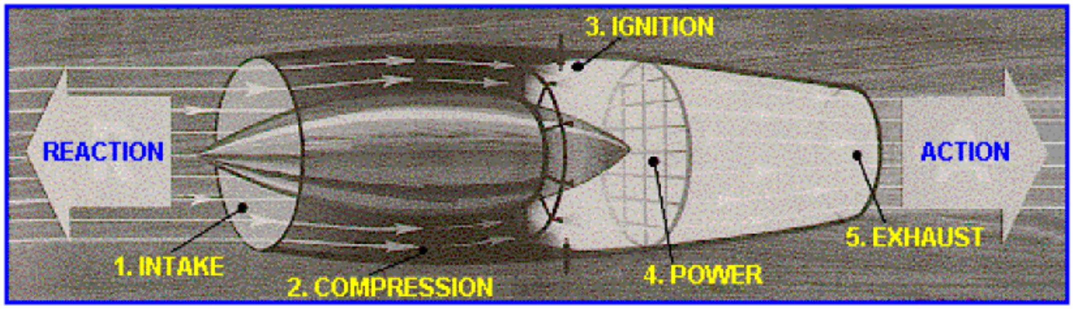


Credit: USAF Test Pilot School

Operational Flight Envelope for Various Flight Vehicles

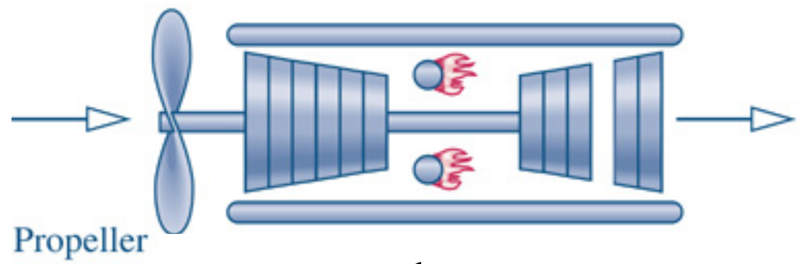


Basic Types of Jet Engines



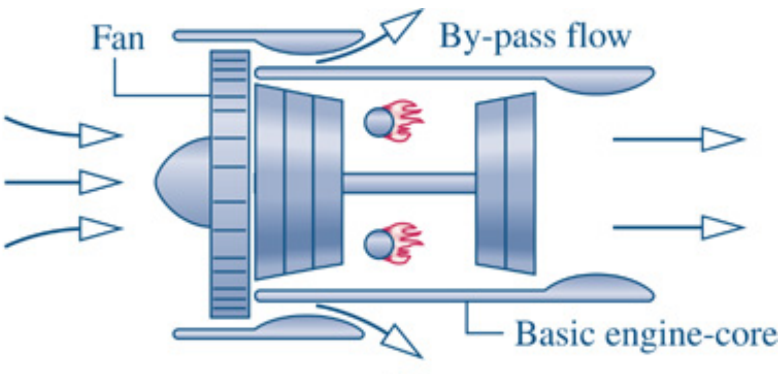
Ramjet

High Speed, Supersonic Propulsion, Passive Compression/Expansion



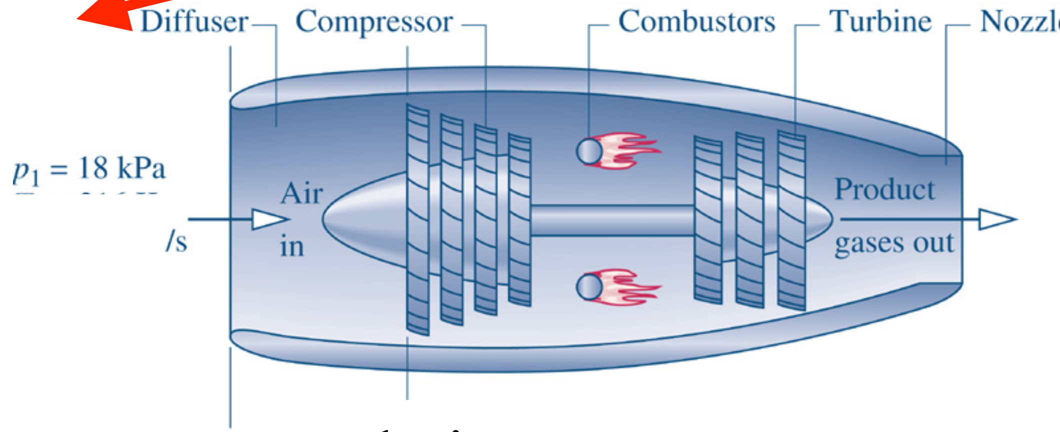
Turboprop

Low to Intermediate Subsonic
Small Commuter Planes



Turbofan

Larger Passenger Airliners
Intermediate Speeds, Subsonic Operation



Turbojet

High Speeds Supersonic or Subsonic Operation

Basic Types of Jet Engines (2)

- **Thrust produced by increasing the kinetic energy of the air in the opposite direction of flight**

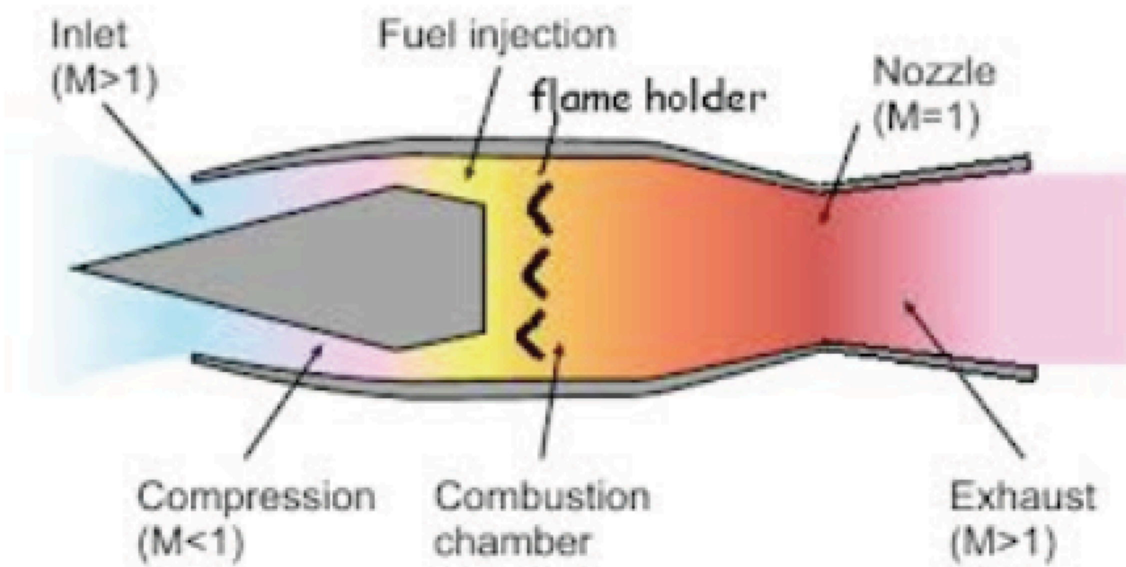
- **Slight acceleration of a large mass of air**
  → Engine driving a propeller

- **Large acceleration of a small mass of air**
  → Turbojet or turbofan engine

- **Combination of both**
  → Turboprop engine

Basic Types of Jet Engines (3)

- Ramjet Engine



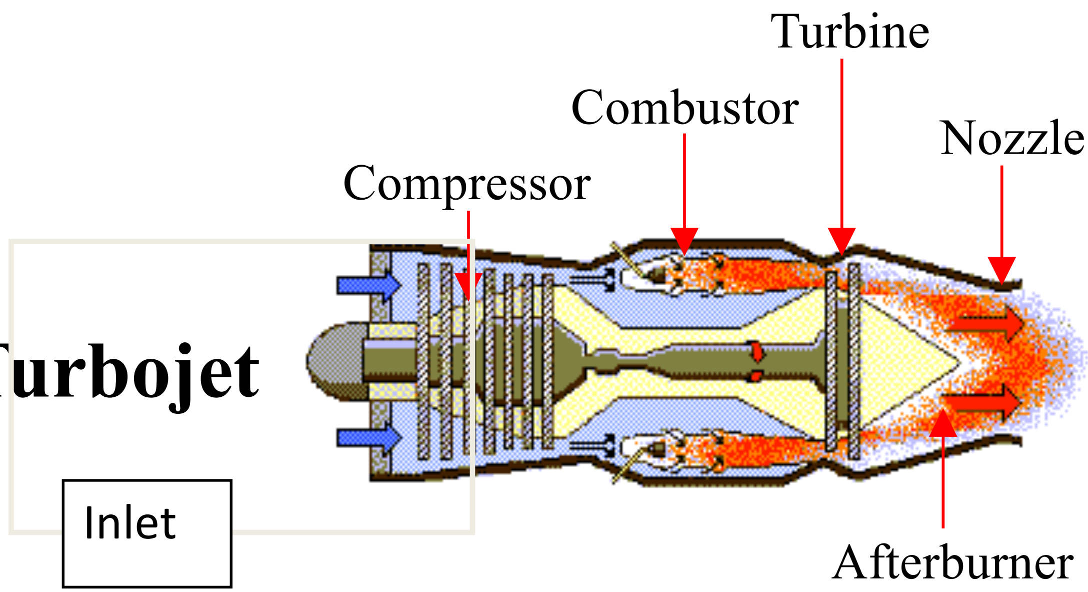
Ramjets cannot produce thrust at zero airspeed; they cannot move an aircraft from a standstill. A ramjet powered vehicle, therefore, requires an assisted take-off like a rocket assist to accelerate it to a speed where it begins to produce thrust.

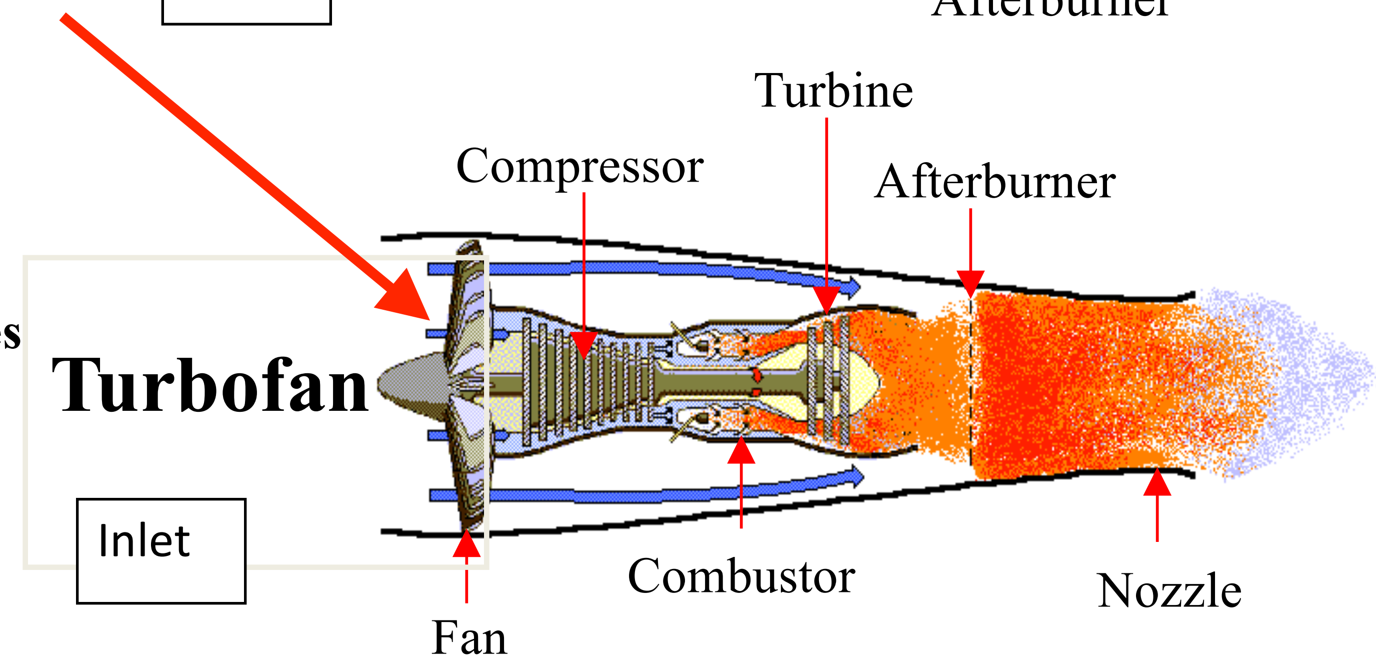
Basic Types of Jet Engines (4)

- **Turbojet/Turbo fan engines Combustion Cycle steps**

- **Compression and Power extraction steps use Turbo-machinery To augment cycle**

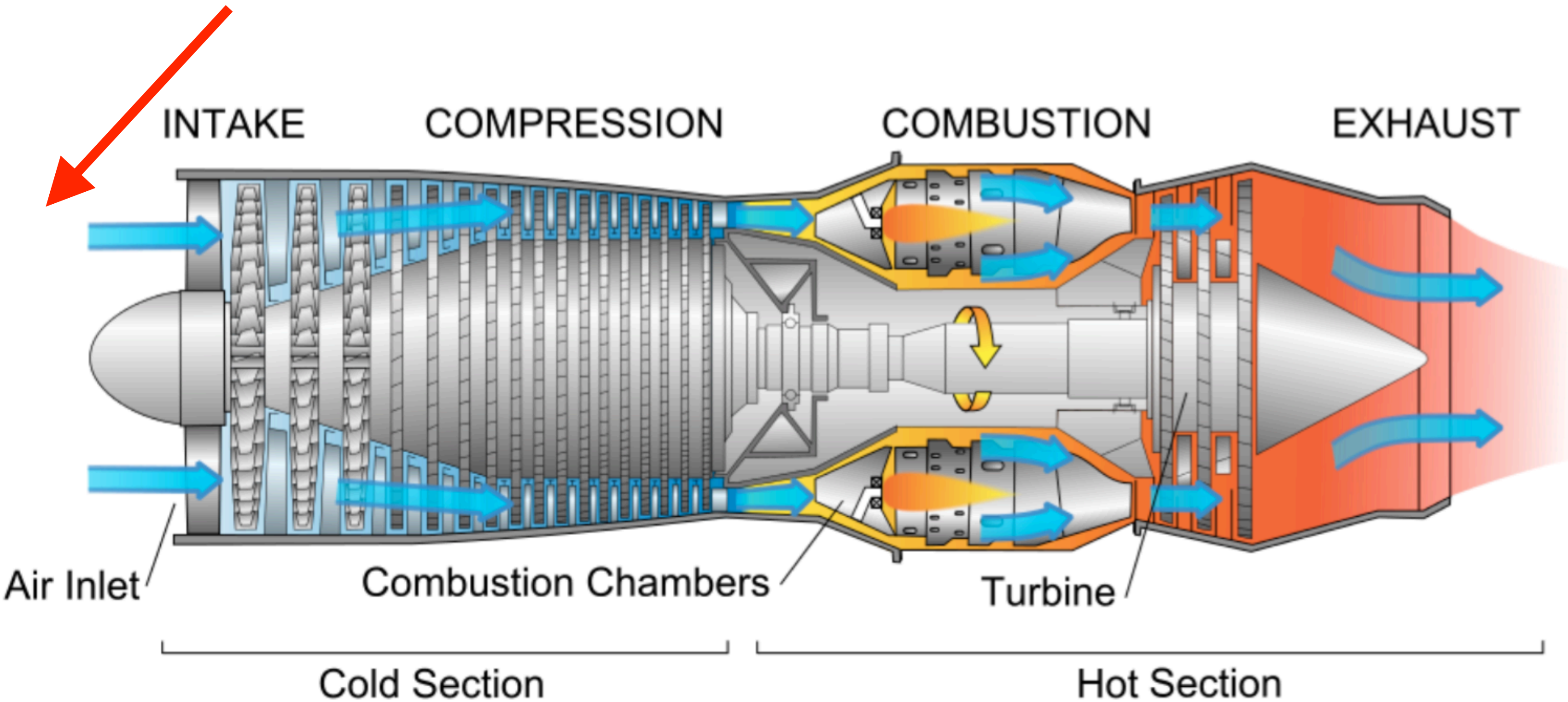
- **Ramjet Achieves Compression Across Inlet shockwaves**





Basic Types of Jet Engines (5)

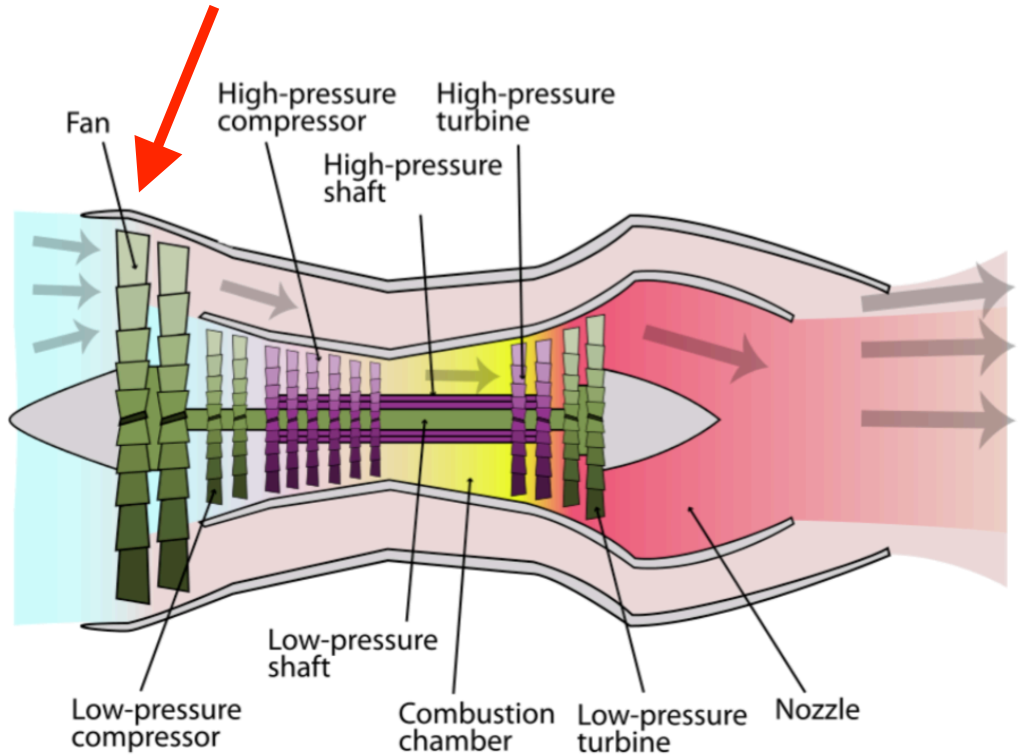
turbojet. Simple turbine engine that produces all of its thrust from the exhaust from the turbine section. However, because all of the air is passing through the whole turbine, all of it must burn fuel.



Basic Types of Jet Engines (6)

Turbofan. Turbine primarily drives a fan at the front of the engine. Most engines drive the fan directly from the turbine. Part of the air enters the turbine section of the engine, and the rest is bypassed around the engine. In high-bypass engines, most of the air only goes through the fan and bypasses the rest of the engine and providing most of the thrust.

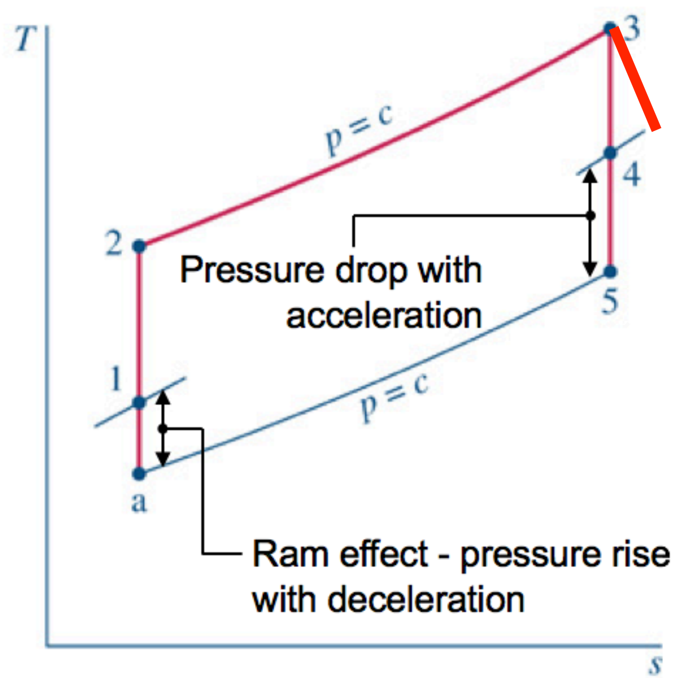
A turbofan thus can be thought of as a turbojet being used to drive a ducted fan, with both of those contributing to the thrust. The ratio of the mass-flow of air bypassing the engine core compared to the mass-flow of air passing through the core is referred to as the bypass ratio.

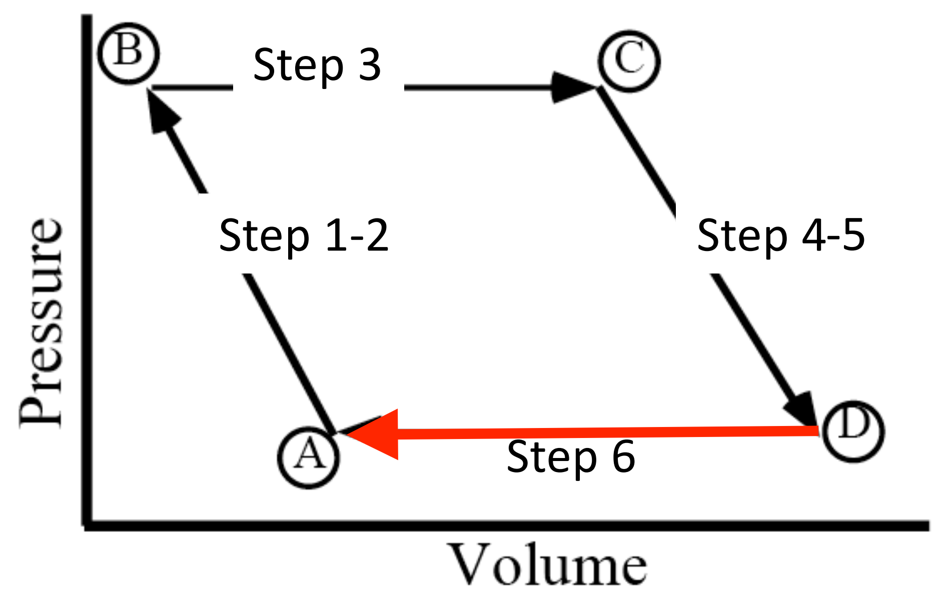


Brayton Cycle for Jet Propulsion

Step	Process
1) Intake *(suck)*	Isentropic Compression
2) Compress the Air *(squeeze)*	Adiabatic Compression
3) Add heat *(bang)*	Constant Pressure Combustion
4) Extract work *(blow)*	Isentropic Expansion in Nozzle
5) Exhaust	Heat extraction by surroundings

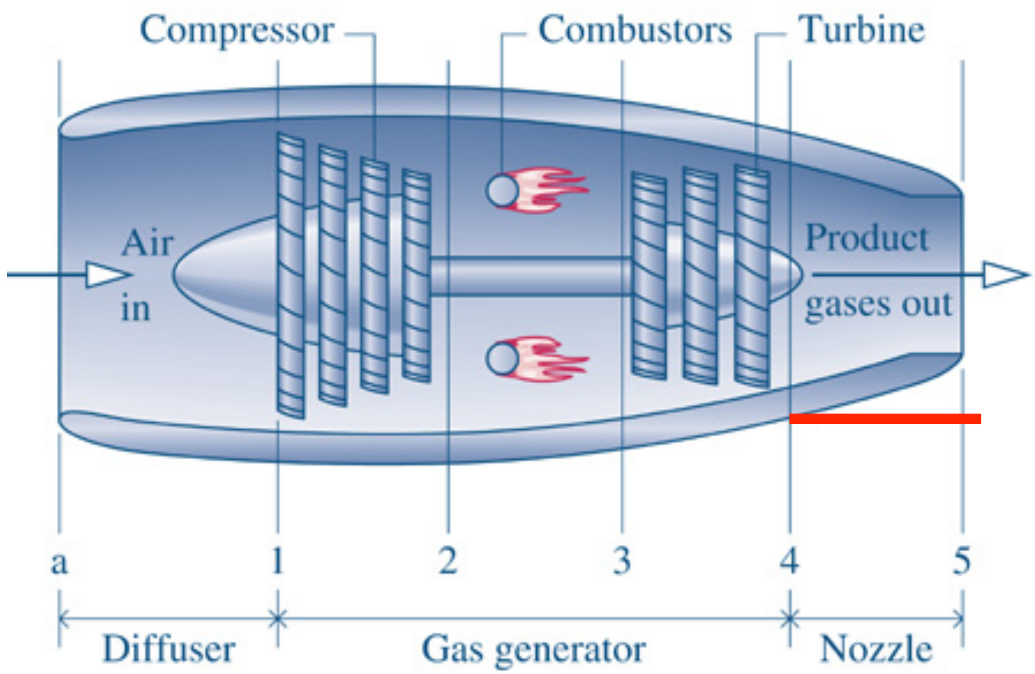
… step 5 above happens In the exhaust plume and has minimal Effect on engine performance

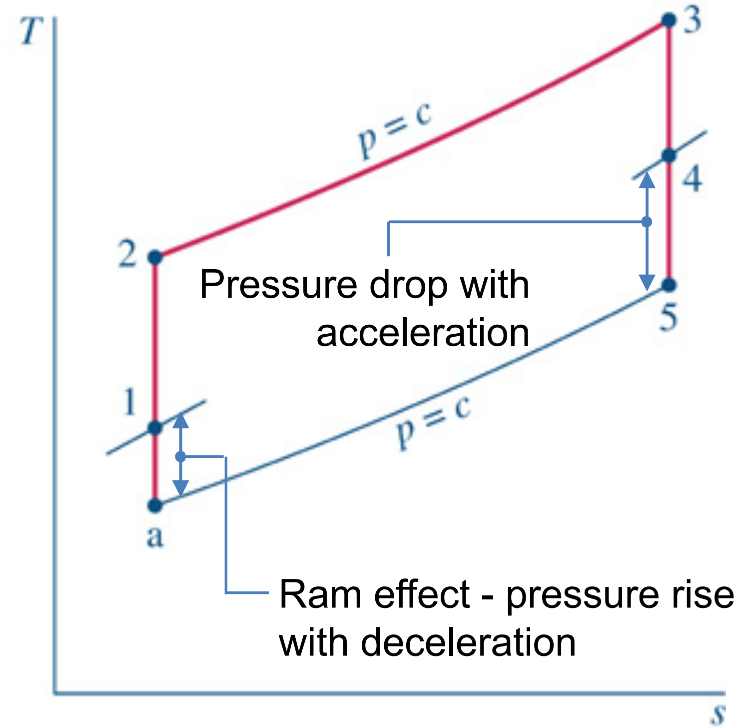




(Credit Narayanan Komerath, Georgia Tech)

Ideal TurboJet Cycle Analysis
Very Similar to Brayton Cycle

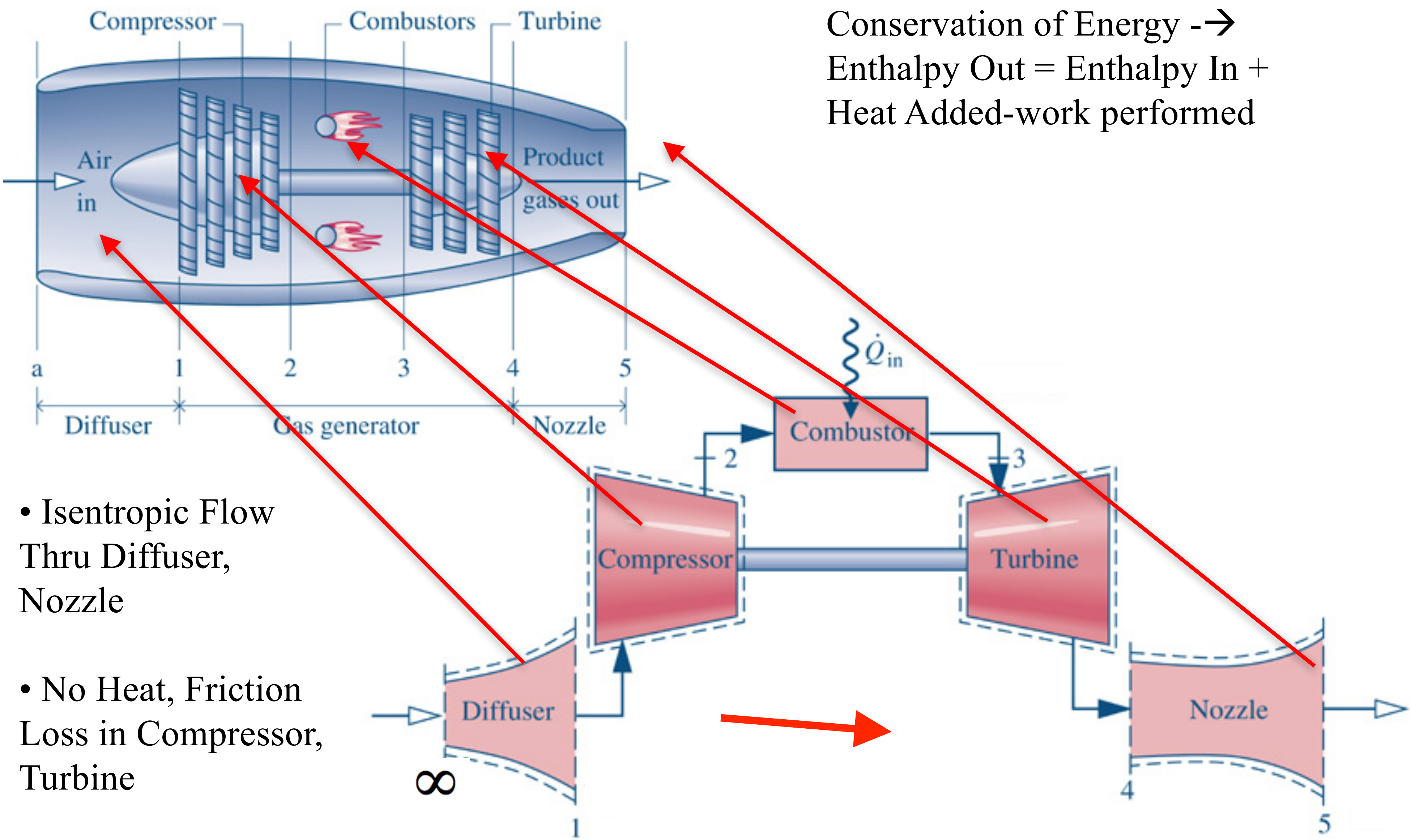




a-1 Isentropic increase in pressure (diffuser)
1-2 Isentropic compression (compressor)
2-3 Isobaric heat addition (combustion chamber)
3-4 Isentropic expansion (turbine)
4-5 Isentropic decrease in pressure with an increase in fluid velocity (nozzle)

Idealized Thermodynamic Model

Conservation of Energy -→
Enthalpy Out = Enthalpy In +
Heat Added-work performed



- Isentropic Flow Thru Diffuser, Nozzle
- No Heat, Friction Loss in Compressor, Turbine

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Idealized Thermodynamic Model (2)

Combustor Heat Input Rate:

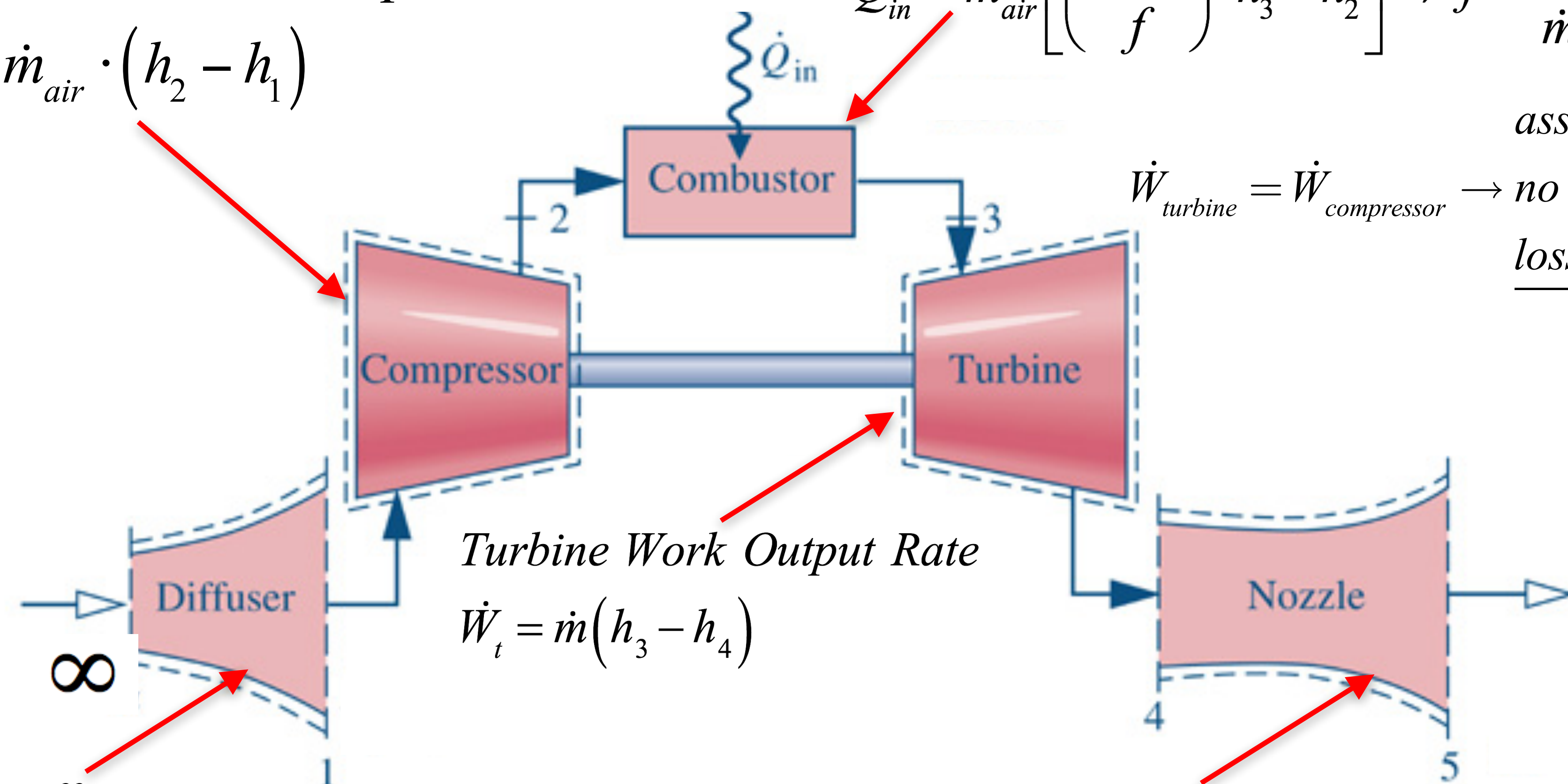
$$\dot{Q}_{in} = \dot{m}_{air}\left[\left(\frac{f+1}{f}\right)\cdot h_3 - h_2\right] \rightarrow f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

Compressor Work Input Rate:

$$\dot{W}_c = \dot{m}_{air}\cdot\left(h_2 - h_1\right)$$

$$\dot{W}_{turbine} = \dot{W}_{compressor} \rightarrow \text{no losses}$$

assume no losses



Turbine Work Output Rate

$$\dot{W}_t = \dot{m}\left(h_3 - h_4\right)$$

Across Diffuser:

$$h_1 + \frac{1}{2}V_1^2 = h_\infty + \frac{1}{2}V_\infty^2$$

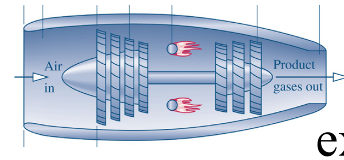
Across Nozzle

$$h_4 + \frac{V_4^2}{2} = h_5 + \frac{V_5^2}{2}$$

Idealized Thermodynamic Model (3)

- **Energy balance →** change in the stagnation enthalpy rate of the gas flow between the exit and entrance of the engine is equal to the added chemical enthalpy rate of the injected fuel flow.

in (∞) →



→ exit (5)

$$\left(\dot{m}_{air} + \dot{m}_{fuel}\right)\cdot h_{0_{exit}} = \dot{m}_{air}\cdot h_{0_{\infty}} + \dot{m}_{fuel}\cdot h_{fuel}$$

$$h_{0_{exit}} = h_{exit} + \frac{1}{2}V_{exit}^2 \;,\; h_{0_{\infty}} = h_{\infty} + \frac{1}{2}V_{\infty}^2$$

- Letting $f = \dot{m}_{air} / \dot{m}_{fuel} \rightarrow h = c_p \cdot T$

$$\left(\frac{f+1}{f}\right)\left(h_{exit} + \frac{1}{2}V_{exit}^2\right) = h_{\infty} + \frac{1}{2}V_{\infty}^2 + \frac{1}{f}\cdot h_{fuel} =$$

$$\boxed{\left(\frac{f+1}{f}\right)\left(c_{p_{exit}}T_{exit} + \frac{1}{2}V_{exit}^2\right) = c_{p_{\infty}}h_{\infty} + \frac{1}{2}V_{\infty}^2 + \frac{1}{f}\cdot h_{fuel}}$$

Idealized Thermodynamic Model (3)

- The high energy content of hydrocarbon fuels is remarkably large and allow extended powered flight to be possible.

A typical value of fuel enthalpy for JP-4 jet fuel is

$$h_f|_{JP-4} = 4.28 \times 10^7 \; J/kg.$$

As a comparison, the enthalpy of Air at sea level static conditions is

$$h|_{Airat288.15K} = C_p T_{SL} = 1005 \times 288.15 = 2.896 \times 10^5 \; J/kg.$$

The ratio is

$$\frac{h_f|_{JP-4}}{h|_{Airat288.15K}} = 148.$$

Credit: B. Cantwell Stanford

Jet Engine Performance Performance Parameters

- **Propulsive Force (Thrust)**
  - The force resulting from the velocity at the nozzle exit
- **Propulsive Power**
  - The equivalent power developed by the thrust of the engine
- **Propulsive Efficiency**
  - Relationship between propulsive power and the rate of kinetic energy production
- **Thermal Efficiency**
  - Relationship between kinetic energy rate of the system and the hat of heat Input the system

Propulsive and Thermal Efficiency of Cycle

Propulsive Efficiency =

Propulsive power

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)}$$

Kinetic energy production rate

Thermal Efficiency = →

Kinetic energy production rate

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Combustion Enthalpy of Fuel

Look a Product of Efficiencies

$$\eta_{propulsive} \times \eta_{propulsive} =$$

Thermal

$$\frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} \times \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\dot{W}_p}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Jet Engine Performance – *Propulsive and Thermal Efficiency*

Look a Product of Efficiencies

$$\eta_{propulsive} \times \eta_{\text{~~propulsive~~ thermal}} =$$

$$\frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} \times \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\dot{W}_p}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Overall Thermodynamic Cycle Efficiency =

Net Propulsion Power Output/Net Heat Input

$$\eta_{overall} = \eta_{thermal}\,\eta_{propulsive}$$

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Jet Engine Performance Efficiencies

Propulsive Efficiency

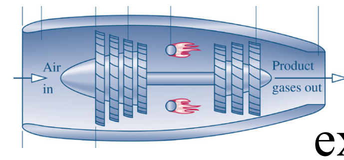
Ratio of Power Developed from Engine (desired beneficial output) Thrust to Change in Kinetic Energy of the Moving Airstream (cost of propulsion)

Thrust Equation:

$$F = \left(\dot{m}_{air} + \dot{m}_{fuel}\right)\cdot V_{exit} - \dot{m}_{air}\cdot V_{inlet} + \left(p_{exit} - p_{\infty}\right)\cdot A_{exit}$$

$$f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}} \rightarrow Optimal\ \ Nozzle \rightarrow p_{exit} = p_{\infty}$$

$$\rightarrow F \approx \dot{m}_{air}\cdot\left[\left(\frac{f+1}{f}\right)\cdot V_{exit} - V_{\infty}\right]$$

in (∞) →  → exit (5)

$$Optimal\ \ Nozzle \rightarrow p_{exit} = p_{\infty}$$

$$\dot{W}_{p} = F\cdot V_{aircraft} = \dot{m}_{air}\cdot\left(\left(\frac{f+1}{f}\right)V_{exit} - V_{\infty}\right)\cdot V_{\infty}$$

Propulsive Power

The power developed from the thrust of the engine

Jet Engine Performance Efficiencies (2)

Propulsive Efficiency

Ratio of Power Developed from Engine (desired beneficial output) Thrust to Change in Kinetic Energy of the Moving Airstream (cost of propulsion)

Propulsive power

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{\dot{m}_{air} \cdot \left(\left(\frac{f+1}{f}\right) V_{exit} - V_{\infty}\right) \cdot V_{\infty}}{\dot{m}_{air} \cdot \left(\frac{1}{2}\left(\frac{f+1}{f}\right) V^2_{exit} - \frac{1}{2} V^2_{\infty}\right)}$$

Kinetic energy production rate

assuming $\dot{m}_{air} >> \dot{m}_{fuel} \rightarrow f << 1$

$$\eta_{propulsive} = \frac{2 \cdot \left(V_{exit} - V_{\infty}\right) \cdot V_{\infty}}{\left(V_{exit} + V_{\infty}\right) \cdot \left(V_{exit} - V_{\infty}\right)} = \frac{2 \cdot V_{\infty}}{\left(V_{exit} + V_{\infty}\right)} = \frac{2}{\left(1 + V_{exit} / V_{\infty}\right)}$$

Maximum propulsive efficiency achieved by generating thrust moving as much air as possible with as little a change in velocity across the engine as possible.

Jet Engine Performance Efficiencies (3)

Thermal Efficiency

The thermal efficiency of a thermodynamic cycle compares work output from cycle to heat added…

Analogously, thermal efficiency of a propulsion cycle directly compares change in gas kinetic energy across engine to energy released through combustion.

$$\rightarrow \quad \eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} =$$

Kinetic energy production rate

Thermal power available from the fuel

$$1 - \frac{Heat\ Rejected\ During\ Cycle}{Heat\ Input\ During\ Cycle} = \frac{\left(\frac{1}{2}\left(\frac{f+1}{f}\right)V^2_{exit} - \frac{1}{2}V^2_{\infty}\right)}{\frac{1}{f} \cdot h_{fuel}}$$

Jet Engine Performance Efficiencies (4)

- **Rewriting the expression**

$$\eta_{thermal} = \frac{\left(\frac{1}{2}\left(\frac{f+1}{f}\right)V^2{}_{exit} - \frac{1}{2}V^2{}_{\infty}\right)}{\frac{1}{f}\cdot h_{fuel}} = 1 - \frac{\frac{1}{f}\cdot h_{fuel} + \frac{1}{2}V^2{}_{\infty} - \frac{1}{2}\left(\frac{f+1}{f}\right)V^2{}_{exit}}{\frac{1}{f}\cdot h_{fuel}}$$

- **Rewriting in terms of the gas enthalpies where** $\frac{1}{2}V^2 = h_0 - h$

$$\eta_{thermal} = 1 - \frac{\frac{1}{f}\cdot h_{fuel} + \left(h_{0_\infty} - h_\infty\right) - \left(\frac{f+1}{f}\right)\left(h_{0_{exit}} - h_{exit}\right)}{\frac{1}{f}\cdot h_{fuel}}$$

Jet Engine Performance Efficiencies (5)

- **From Energy Balance** $\frac{1}{f}\cdot h_{fuel} = h_\infty + \frac{1}{2}V_\infty^2 - \left(\frac{f+1}{f}\right)\left(h_{exit} + \frac{1}{2}V_{exit}^2\right)$

- **Substituting and Rearranging**

$$\eta_{thermal} = 1 - \frac{\left(\frac{f+1}{f}\right)(h_{exit}) - h_\infty - \left(\frac{f+1}{f}\right)h_\infty + \left(\frac{f+1}{f}\right)h_\infty}{\frac{1}{f}\cdot h_{fuel}} =$$

$$1 - \frac{\left(\frac{f+1}{f}\right)(h_{exit} - h_\infty) - \left[1 - \left(\frac{f+1}{f}\right)\right]h_\infty}{\frac{1}{f}\cdot h_{fuel}} = 1 - \frac{\left(\frac{f+1}{f}\right)(h_{exit} - h_\infty) - \frac{1}{f}\cdot h_\infty}{\frac{1}{f}\cdot h_{fuel}}$$

Jet Engine Performance Efficiencies (6)

$$\eta_{thermal} = 1 - \frac{\left(\frac{f+1}{f}\right)\left(h_{exit} - h_{\infty}\right) - \frac{1}{f}\cdot h_{\infty}}{\frac{1}{f}\cdot h_{fuel}}$$

$$f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

$$\rightarrow \left(\frac{f+1}{f}\right)\left(h_{exit} - h_{\infty}\right) - \frac{1}{f}\cdot h_{\infty} = \text{Heat Rejected During Cycle}$$

$$\frac{1}{f}\cdot h_{fuel} = \text{Heat Input During Cycle}$$

Jet Engine Performance Efficiencies (7)

- Strictly speaking engine is not closed system because of fuel mass addition across the burner.

- Heat rejected by exhaust consists of two distinct parts.

1. Heat rejected by conduction from nozzle flow to the surrounding atmosphere

2. Physical removal from the thermally equilibrated nozzle flow of a portion equal to the added fuel mass flow.

Fuel mass flow carries enthalpy into system by injection/combustion in burner and exhaust fuel mass flow carries ambient enthalpy out mixing with the surroundings.

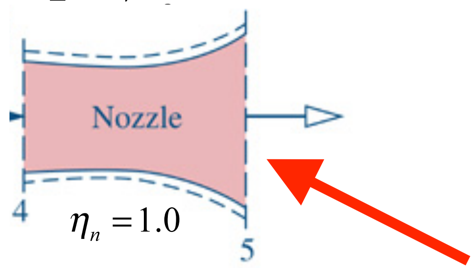
There is no net mass increase or decrease to the system.

Propulsive and Thermal Efficiency Revisited

@ Cruise Assumed Optimized Nozzle → $p_{exit} = p_\infty$ $\qquad T_{exit} = T_4 \cdot \left(\frac{P_4}{p_{exit}}\right)^{\frac{\gamma-1}{\gamma}}$

$$F = \dot{m}\left(V_{exit} - V_\infty\right) \qquad \dot{W}_p = F \cdot V_\infty$$

Nozzle Enthalpy Balance



$$\dot{m}\left(h_4 + \frac{V_4^2}{2}\right) = \dot{m}\left(h_5 + \frac{V_5^2}{2}\right) = \dot{m}\left(h_{exit} + \frac{V_{exit}^2}{2}\right) \to V_4 \approx 0 \to V_{exit} = \sqrt{2\left(h_4 - h_{exit}\right)}$$

$$\eta_{propulsive} = \frac{\dot{W}_p}{\dot{m}_{air}\left(K.E._{exit} - K.E._\infty\right)} \qquad \eta_{thermal} = \frac{\left(K.E._{exit} - K.E._\infty\right)}{\dot{m}_{fuel} \cdot h_{fuel}}$$

$$\eta_{total} = \eta_{prop} \cdot \eta_{thermal} = \frac{F \cdot V_\infty}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Propulsive and Thermal Efficiency Revisited (2)

$$\dot{Q}_{total} = \dot{m}\left(h_{0_3} - h_{02}\right) \qquad \dot{Q}_{\substack{out \\ excess}} = \dot{m}\left(h_{0_{exit}} - h_{0\infty}\right)$$

$$P_{prop} = F_{thrust} \cdot V_\infty = \left(\dot{m} \cdot V_{exit} - \dot{m} \cdot V_\infty\right) \cdot V_\infty = \frac{1}{2}\left(\dot{m} \cdot V^2{}_{exit}\right)\left(2\left(\frac{V_{exit}}{V_\infty}\right) - 2\left(\frac{V_{exit}}{V_\infty}\right)^2\right)$$

$$K.E._{net} = \frac{1}{2}\dot{m} \cdot \left(V^2{}_{exit} - V^2{}_\infty\right) = \frac{1}{2}\left(\dot{m} \cdot V^2{}_{exit}\right) \cdot \left(1 - \left(\frac{V_\infty}{V_{exit}}\right)^2\right)$$

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._\infty\right)} = \frac{\frac{1}{2}\left(\dot{m} \cdot V^2{}_{exit}\right)\left(2\left(\frac{V_{exit}}{V_\infty}\right) - 2\left(\frac{V_{exit}}{V_\infty}\right)^2\right)}{\frac{1}{2}\left(\dot{m} \cdot V^2{}_{exit}\right) \cdot \left(1 - \left(\frac{V_\infty}{V_{exit}}\right)^2\right)} = \frac{2\left(\left(\frac{V_{exit}}{V_\infty}\right) - \left(\frac{V_{exit}}{V_\infty}\right)^2\right)}{\left(1 - \left(\frac{V_\infty}{V_{exit}}\right)^2\right)}$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._\infty\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\left(\frac{1}{2}V^2{}_{exit}\right) \cdot \left(1 - \left(\frac{V_\infty}{V_{exit}}\right)^2\right)}{\left(h_{0_3} - h_{02}\right)}$$

Propulsive and Thermal Efficiency Revisited (3)

$$K.E._{\substack{out\\excess}} = K.E._{net} - P_{prop} =$$

$$\frac{1}{2}\left(\dot{m}\cdot V^2{}_{exit}\right)\cdot\left(1-\left(\frac{V_\infty}{V_{exit}}\right)^2\right)-\frac{1}{2}\left(\dot{m}\cdot V^2{}_{exit}\right)\left(2\left(\frac{V_{exit}}{V_\infty}\right)-2\left(\frac{V_{exit}}{V_\infty}\right)^2\right)=$$

$$\frac{1}{2}\dot{m}\cdot V^2{}_{exit}\cdot\left(1-\left(\frac{V_\infty}{V_{exit}}\right)^2-2\left(\frac{V_{exit}}{V_\infty}\right)+2\left(\frac{V_{exit}}{V_\infty}\right)^2\right)=$$

$$\frac{1}{2}\dot{m}\cdot V^2{}_{exit}\cdot\left(1-2\left(\frac{V_{exit}}{V_\infty}\right)+\left(\frac{V_{exit}}{V_\infty}\right)^2\right)=\frac{1}{2}\dot{m}\cdot V^2{}_{exit}\cdot\left(1-\left(\frac{V_{exit}}{V_\infty}\right)\right)$$

Propulsive and Thermal Efficiency Revisited (9)

Summary

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{2\left(\left(\frac{V_{exit}}{V_{\infty}}\right) - \left(\frac{V_{exit}}{V_{\infty}}\right)^2\right)}{\left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^2\right)}$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\left(\frac{1}{2}V^2{}_{exit}\right) \cdot \left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^2\right)}{\left(h_{0_3} - h_{02}\right)}$$

$$K.E._{\substack{out \\ excess}} = K.E._{net} - P_{prop} = \frac{1}{2}\dot{m} \cdot V^2{}_{exit} \cdot \left(1 - \left(\frac{V_{exit}}{V_{\infty}}\right)\right)$$

"Equivalence Ratio" and Engine Performance

- Combustion efficiency and stability limits are depending on several parameters : fuel, equivalence ratio, air stagnation pressure and temperature

- The ***equivalence ratio*** is used to characterize the mixture ratio Of airbreathing engines ... *analogous to O/F for rocket propulsion*

- The *equivalence ratio*,Φ , is defined as the ratio of the actual fuel-air ratio to the stoichiometric fuel-air ratio.

- For $\Phi = 1$, no oxygen is left in exhaust produc ... combustion is called *stoichiometric*

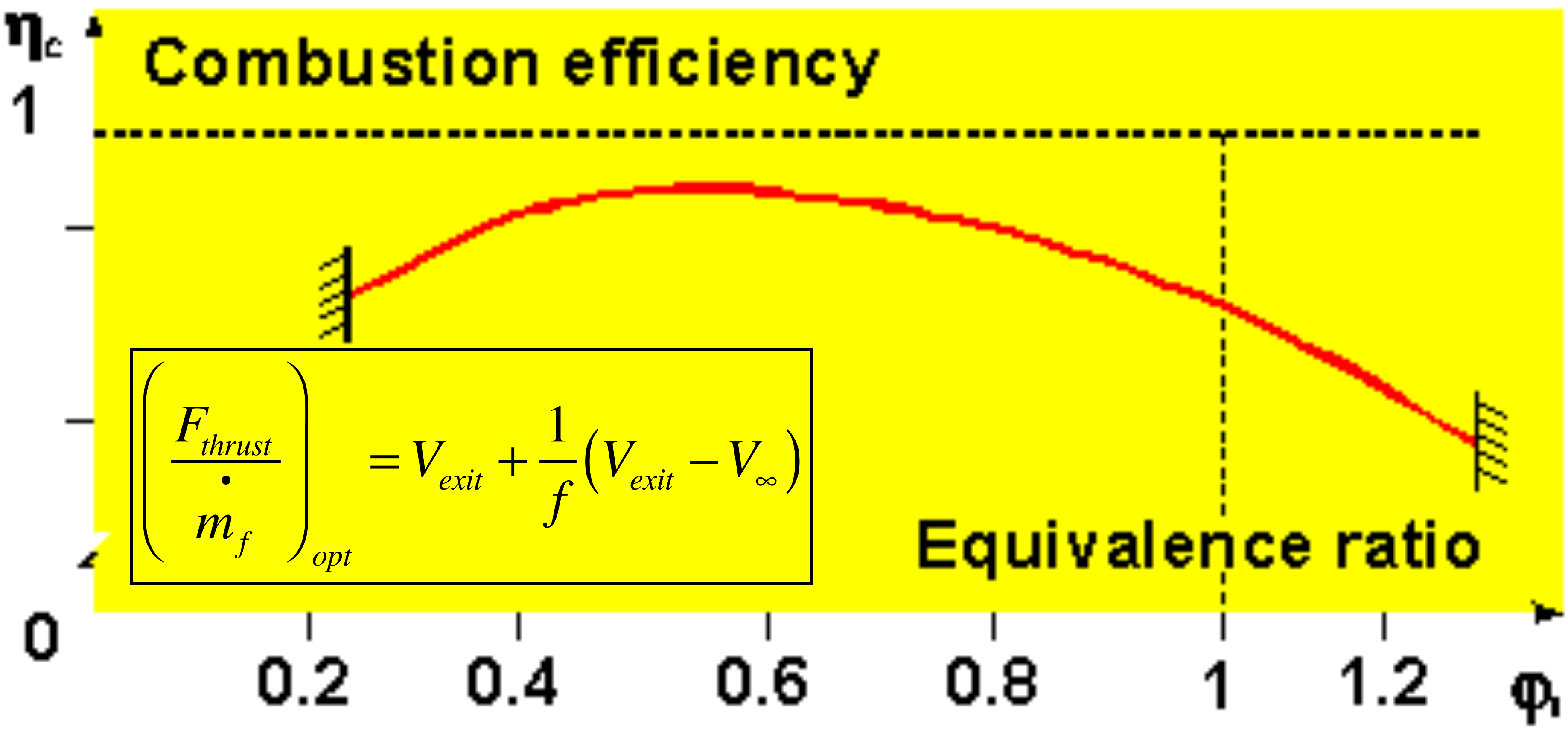
$$\Phi \equiv \frac{\left[\dfrac{\dot{m}_{fuel}}{\dot{m}_{air}}\right]_{actual}}{\left[\dfrac{\dot{m}_{fuel}}{\dot{m}_{air}}\right]_{stoich}} = \frac{f_{stoich}}{f_{actual}}$$

… $\Phi > 1$ ---> a rich mixture
… $\Phi < 1$ ---> lean mixture

"Equivalence Ratio" and Engine Performance (2)

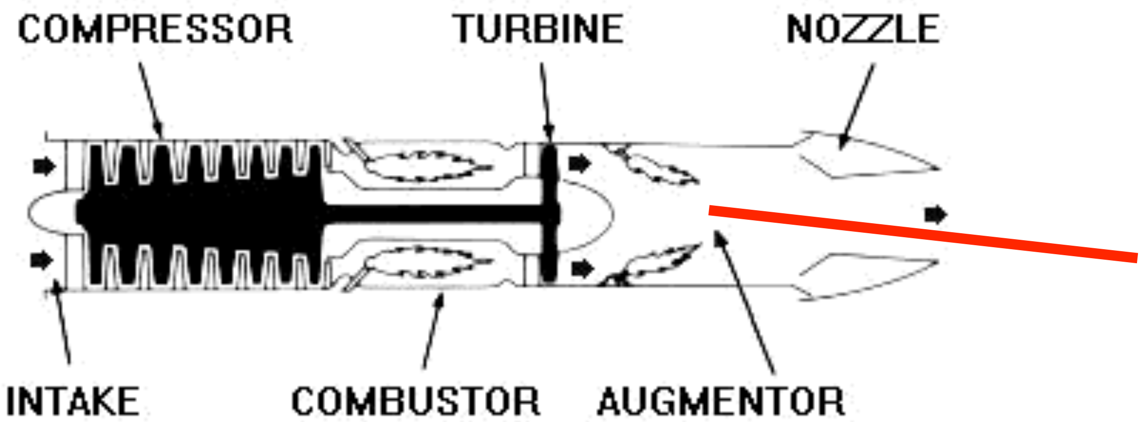
- Unlike Rockets .. Ramjets … and air breathing propulsion systems tend to be more efficient when engine runs leaner than *stoichiometric*
- *Also Thermal Capacity of Turbine Materials Limits Maximum Allowable Combustion Temperature, not Allowing Engine to Run Stoichiometric*

$$\eta_{thermal} = 1 - \frac{(f+1)(h_{exit} - h_{\infty}) - h_{\infty}}{\frac{1}{f} \cdot h_{fuel}}$$



$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = V_{exit} + \frac{1}{f}\left(V_{exit} - V_{\infty}\right)$$

"Equivalence Ratio" and Engine Performance (3)



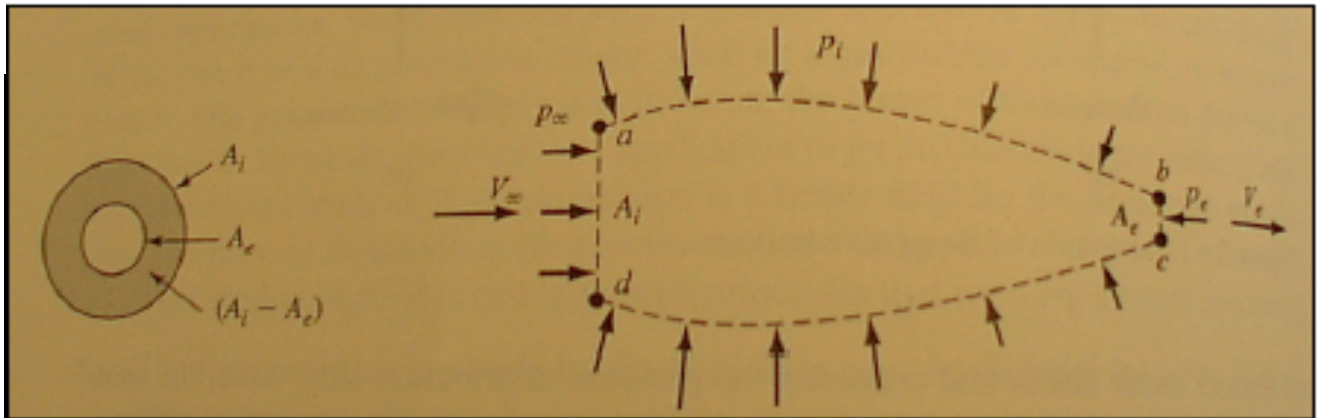
- … that is why afterburners work … left over O_2 after combustion

Additional fuel is introduced into the hot exhaust and burned using excess O_2 from main combustion

- The afterburner increases the temperature of the gas ahead of the nozzle Increases exit velocity

- The result of this increase in temperature is an increase of about 40 percent in thrust at takeoff and a much larger percentage at high speeds

Specific Thrust of Air Breathing Engine



$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{net}$$

Analogous to I_{sp}

Net thrust

$$F_{thrust} = \dot{m}_{exit} V_{exit} - \dot{m}_\infty V_\infty + \left(p_{exkit} - p_\infty\right) \cdot A_{exit} \rightarrow$$

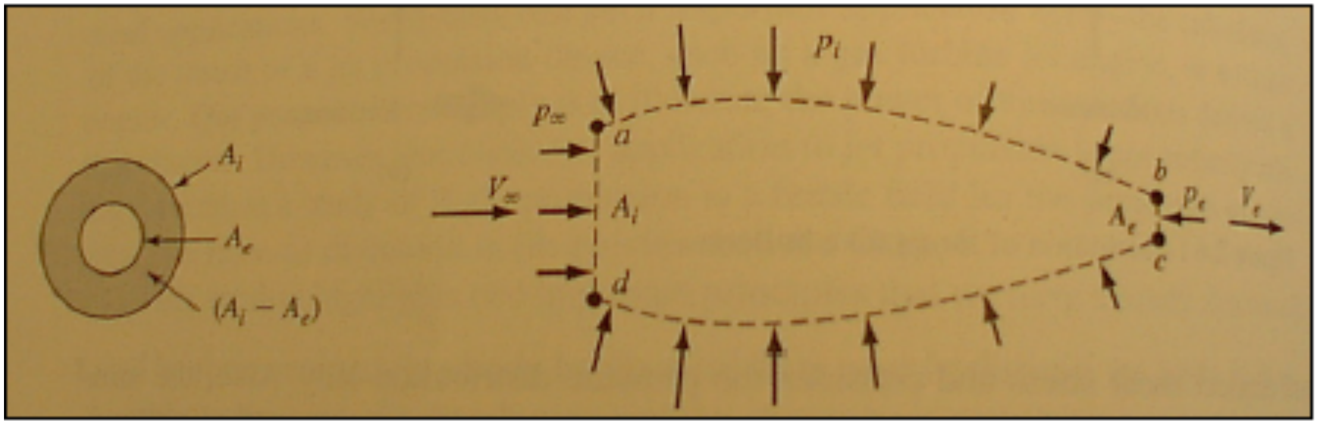
$$\dot{m}_\infty = \dot{m}_{air}$$

$$\dot{m}_{exit} = \dot{m}_{air} + \dot{m}_{fuel}$$

$$f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

$$F_{thrust} = \dot{m}_{air}\left[\left(\frac{\dot{m}_{air} + \dot{m}_{fuel}}{\dot{m}_{air}}\right) V_{exit} - V_\infty\right] + \left(p_{exit} - p_\infty\right) \cdot A_{exit} = \dot{m}_{air}\left[\left(\frac{1+f}{f}\right) V_e - V_i\right] + \left(p_e - p_\infty\right) \cdot A_e$$

Specific Thrust of Air Breathing Engine (2)



$$Thrust = \dot{m}_e V_e - \dot{m}_i V_i + \left(p_e A_e - p_\infty A_e\right)$$

Cruise design condition
When $p_e = p_\infty$

$$\left(\frac{F_{thrust}}{\dot{m}_f}\right)_{opt} = \frac{\left[\dot{m}_f + \dot{m}_{air}\right]V_{exit} - \dot{m}_{air} V_\infty}{\dot{m}_f} = [f+1]V_{exit} - f \cdot V_\infty = V_{exit} + f \cdot \left(V_{exit} - V_\infty\right)$$

%Ram Drag Reduced at lower air-fuel ratio "*f*"

$$f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

Jet Engine Fuel Efficiency Performance Measure

Thrust Specific Fuel Consumption (TSFC) → Inverse of Specific Thrust

$$TSFC = \frac{\dot{m}_f}{F_{thrust}} \approx \frac{1}{I_{sp} g_0}$$

- *Measure of fuel economy*
- *Analogous to specific impulse in Rocket Propulsion*

$$Typical\ Turbojet \approx TSFC = \left(2-4\right)_{\frac{lbm}{lbf-hr}}$$

$$SFC|_{JT9D-takeoff} \cong 0.35$$

$$SFC|_{JT9D-cruise} \cong 0.6$$

$$SFC|_{militaryengine} \cong 0.9to1.2$$

$$SFC|_{militaryenginewithafterburning} \cong 2.$$

TSFC generally goes up engine moves from takeoff to cruise, as energy required to produce a thrust goes up with increased percentage of stagnation pressure losses and with increased momentum of incoming air.

Breguet Aircraft Range Equation

- Aviation Analog of "Rocket Equation"
- Assumes Constant Lift-to-Drag (L/D) and Constant Overall Efficiency

$$\eta_{overall} = \eta_{propulsive} \cdot \eta_{\text{~~propulsive~~ thermal}} = \frac{\dot{W}_p}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{F_{thrust} \cdot V_\infty}{\dot{m}_{fuel} \cdot h_{fuel}}$$

For Fight Optimal Conditions

$$\rightarrow V_\infty = \frac{\eta_{overall} \cdot \dot{m}_{fuel} \cdot h_{fuel}}{F_{thrust}}$$

$$Total\ Range:$$

$$R = \int V_\infty dt = \int \left(\frac{\eta_{overall} \cdot \dot{m}_{fuel} \cdot h_{fuel}}{F_{thrust}} \right) \cdot dt$$

- Fuel mass flow is directly related to the change in aircraft weight

$$\dot{m}_{fuel} = -\frac{1}{g} \frac{dW}{dt}$$

Breguet Aircraft Range Equation (2)

• In equilibrium (cruise) flight Thrust equals drag and aircraft weight equals lift …

$$T = D = L / \left(\frac{L}{D}\right) = W / \left(\frac{L}{D}\right)$$

• Subbing into Range Equation

$$R = \int V_\infty dt = -\int \left(\frac{\eta_{overall} \cdot \frac{1}{g} \frac{dW}{dt} \cdot h_{fuel}}{W / \left(\frac{L}{D}\right)} \right) \cdot dt = -\eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \int \left(\frac{dW}{W}\right)$$

• Integration Gives

$$R = -\eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \left[\ln\left(W_{final}\right) - \ln\left(W_{initial}\right)\right] = \eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right)$$

$$\boxed{R = \eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right)}$$

Breguet Aircraft Range Equation (3)

$$R = \eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right)$$

• Result highlights the key role played by the engine overall efficiency in available aircraft range.

• Note that as the aircraft burns fuel it must increase altitude to maintain constant L/D , and the required thrust decreases.

Breguet Aircraft Range Equation (4)

- Compare to "Rocket Equation"

$$R = \eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right)$$

$$R = \eta_{overall} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right) = \frac{F_{thrust} \cdot V_{\infty}}{\dot{m}_{fuel} \cdot h_{fuel}} \cdot \frac{h_{fuel}}{g} \cdot \left(\frac{L}{D}\right) \cdot \ln\left(\frac{W_{initial}}{W_{final}}\right) =$$

$$\frac{F_{thrust}}{\dot{m}_{fuel} \cdot g} \cdot \left(\frac{L}{D} \cdot V_{\infty}\right) \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right) = I_{sp} \cdot \left(\frac{L}{D} \cdot V_{\infty}\right) \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)$$

$$\frac{R \cdot g_o}{V_{\infty}} = \left(\frac{L}{D}\right) \cdot g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)$$

Breguet Aircraft Range Equation (5)

- Breguet Range Equation, Scaled Range Velocity

$$\bar{V} \equiv \frac{R \cdot g_o}{V_\infty} = \left(\frac{L}{D}\right) \cdot g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)$$

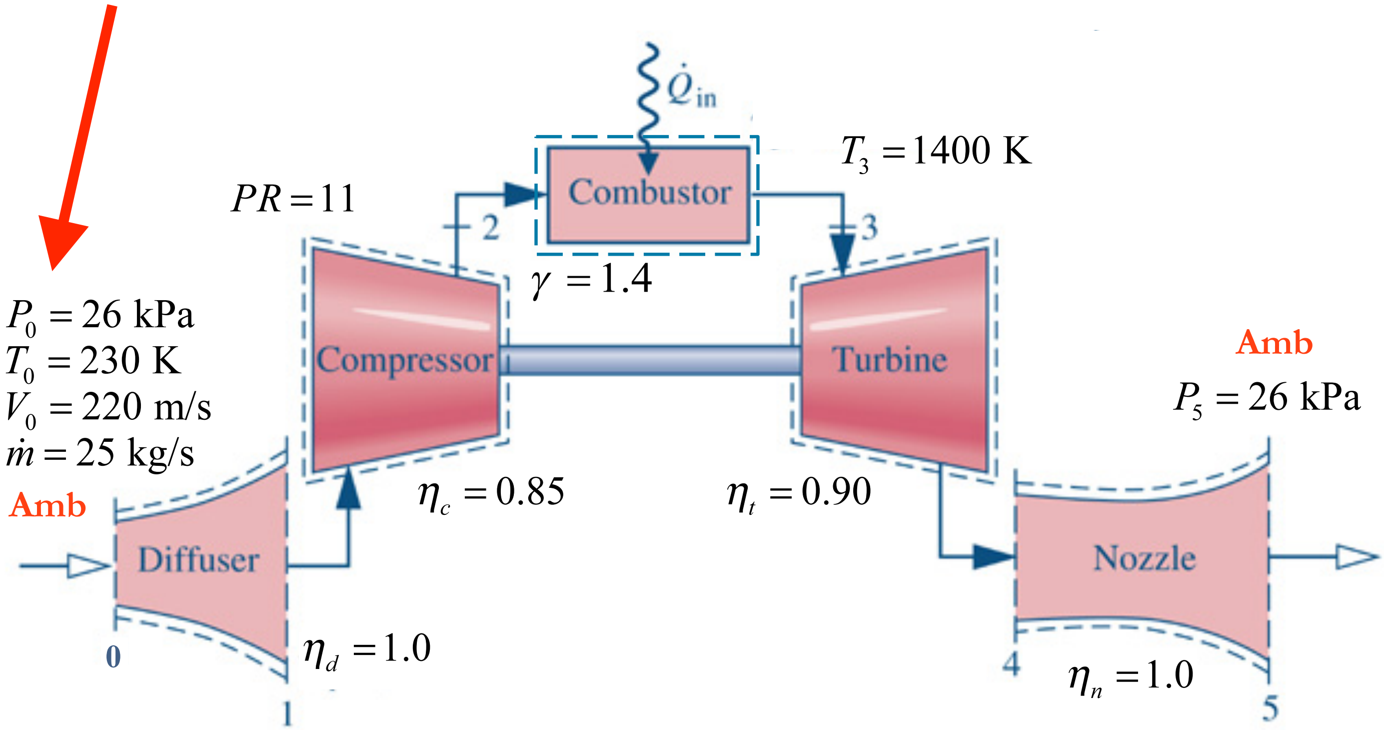
- Rocket Equation, Available Propulsion ΔV

$$\Delta V = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)$$

Same Basic Physics
Same Basic Solution!

Section 4.1 Homework

Given: A turbojet engine operating as shown below



Calculate:

(a) The properties at all the state points in the cycle
(b) The heat transfer rate in the combustion chamber (*kW*)
(c) The velocity at the nozzle exit (*m/s*)
(d) The propulsive force (*lbf*)
(e) The propulsive power developed (*kW*)
(f) Propulsive Efficiency
(g) Thermal Efficiency
(h) Total Efficiency
(i) Draw *T-s* diagram
(j) Draw p-v diagram

- Assume Isentropic Diffuser, Nozzle
- Compressible, Combustor Turbine NOT! Isentropic
- Assume Constant C_p, C_v across cycle
- Air massflow >> fuel massflow
- Assume $\gamma = 1.4$ throughout cycle

Section 4.1 Homework (2)

Given: A turbojet engine operating as shown below

Incoming Air to Turbojet (@ to station *3*)

- Molecular weight = 28.96443 *kg/kg-mole*
- γ = 1.40
- R_g = 287.058 *J/kg-K*
- T_∞ = 230 K
- p_∞ = 26 kPa
- V_∞ = 220 *m/sec*
- Universal Gas Constant: R_u = 8314.4612 *J/kg-K*

Calorically Perfect Gas

$$\gamma = \frac{c_p}{c_v}$$

$$R_g = c_p - c_v$$

$$c_p = \frac{\gamma}{\gamma - 1} \cdot R_g$$

$$c_v = \frac{1}{\gamma - 1} \cdot R_g$$

For …Isentropic Conditions →

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Ideal Gas

$$p = \rho \cdot R_g \cdot T$$

Section 4.1 Homework (3)

Given: Across Components

Isentropic Diffuser

Assume D_{inlet} = 60.96 cm (24 in.)
$D_{outlet} = 1.5\ x\ D_{inlet}$

$$h_{0_1} \equiv h_1 + \frac{V_1^2}{2} = h_\infty + \frac{V_\infty^2}{2}$$

ambient

$$h_{0_1} \approx C_{p1} \cdot T_{0_1}$$

$$P_{0_1} = P_{0_\infty} \cdot \left(\frac{T_{0_1}}{T_{0_\infty}} \right)^{\frac{\gamma-1}{\gamma}}$$

Compressor

ηc = isentropic power input / actual power input

$$\eta_c = \frac{h_{0_{2|s=0}} - h_{0_1}}{h_{0_2} - h_{0_1}} \rightarrow$$

$$h_{0_1} = C_{p_{air}} \cdot T_{0_1}$$
$$h_{0_2} = C_{p_{air}} \cdot T_{0_2\ actual}$$
$$h_{0_{2|s=0}} = C_{p_{air}} \cdot T_{0_2\ ideal}$$
$$\frac{\dot{w}_c}{\dot{m}} = h_{0_2} - h_{0_1}$$

$$s_2 - s_1 = C_p \ln\left(\frac{T_{2_{actual}}}{T_1} \right) - R_g \ln\left(\frac{p_2}{p_1} \right)$$

Assume compressor outlet Mach number is essentially zero

Section 4.1 Homework (4)

Given: Across Components

Combustor

$$contant\ pressure, \dot{m}_{air} >> \dot{m}_{air}$$

$$C_p, \gamma \sim const, \quad T_3 = T_{flame} = 1400K$$

$$s_3 - s_2 = C_p \ln\left(\frac{T_{flame}}{T_{2_{actual}}}\right)$$

Assume combustor outlet Mach number is essentially zero

Turbine

$$\eta_t = \frac{\text{actual power output}}{\text{isentropic power poutput}}$$

$$\eta_t = \frac{h_{0_3} - h_{0_4}}{h_{03} - h_{0_{4_{s=0}}}} \rightarrow \begin{array}{l} h_{0_3} = C_{p_{air}} \cdot T_{0_3} \\ h_{0_4} = C_{p_{air}} \cdot T_{0_{4actual}} \\ h_{0_{4_{s=0}}} = C_{p_{air}} \cdot T_{0_{4ideal}} \end{array}$$

$$Assume \rightarrow \frac{\dot{w}_t}{\dot{m}} = \frac{\dot{w}_c}{\dot{m}} = h_{0_3} - h_{0_4}$$

$$\frac{P_{0_4}}{P_{0_3}} = \left(\frac{h_{0_3} - \frac{1}{\eta_t}\frac{\dot{w}_c}{\dot{m}}}{h_{0_3}}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 - \frac{1}{\eta_t \cdot h_{0_3}}\frac{\dot{w}}{\dot{m}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$s_4 - s_3 = C_p \ln\left(\frac{T_{0_{4actual}}}{T_{0_3}}\right) - R_g \ln\left(\frac{P_{0_4}}{P_{0_3}}\right)$$


Questions??