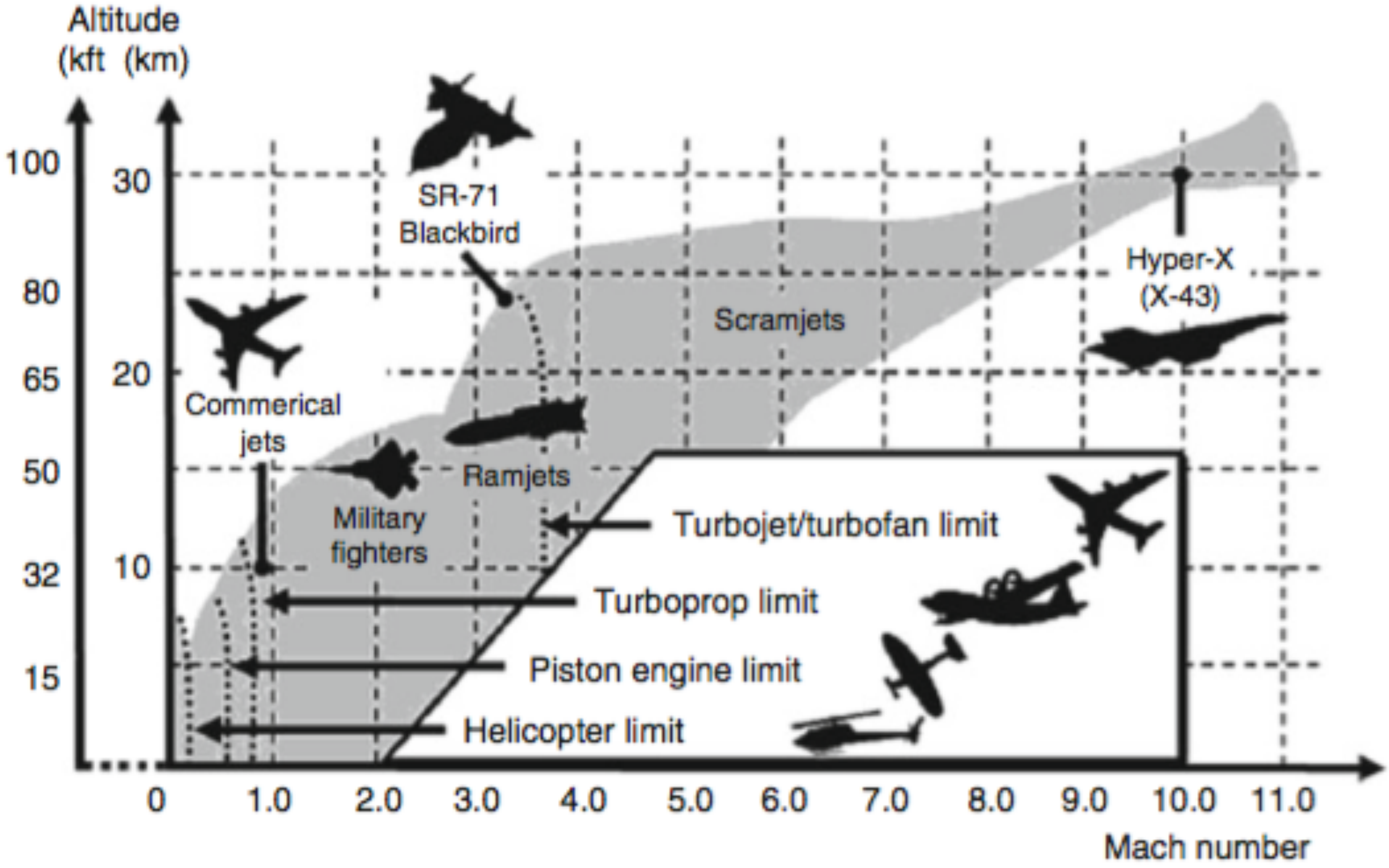
Section 4.2. The Standard Atmosphere



Atmospheric Model Background

- Earth's atmosphere is an extremely thin sheet of air extending from surface of Earth to edge of space.

- If Earth were size of a basketball, a tightly held pillowcase would represent thickness of atmosphere.

- Gravity holds atmosphere to Earth's surface. Within atmosphere, very complex chemical, therrmodynamic, and fluid dynamics effects occur.

- Atmosphere is not uniform; fluid properties are constantly changing with time and place. We call this change “weather”.

- Variations in air properties extend upward from surface of Earth. Sun heats surface of Earth, and some of this heat goes into warming air near surface, heated air is also diffused or convected up through atmosphere.

Atmospheric Model Background *(2)*

- Thus the air temperature (and the local speed of sound) is highest near the surface and decreases as altitude increases.

- Air pressure is proportional to the weight of the air over a given location. Tus air pressure decreases as we increase altitude.

- Air density depends on both the temperature and the pressure through the equation of state and also decreases with increasing altitude.

- Due to localized sun angles, terrain, gravitational changes with latitude, and coriolis forces due to earth's rotation, properties within the atmosphere change significantly with time of year, and global latitude/longitude.

- To help with establishing "mean designs" for air-vehicles, it is extremely useful to define "standard" conditions, and develop models of how the atmospheric properties vary as a function of altitude .. Thus the so-called Standard Day, and Standard Atmosphere."

Hydrostatic Model of the Atmosphere

- For a fluid in rest without shearing stresses, any elementary fluid element will be subjected to two types of forces, namely, surface forces due to the pressure and a body force equal to the weight of the element.

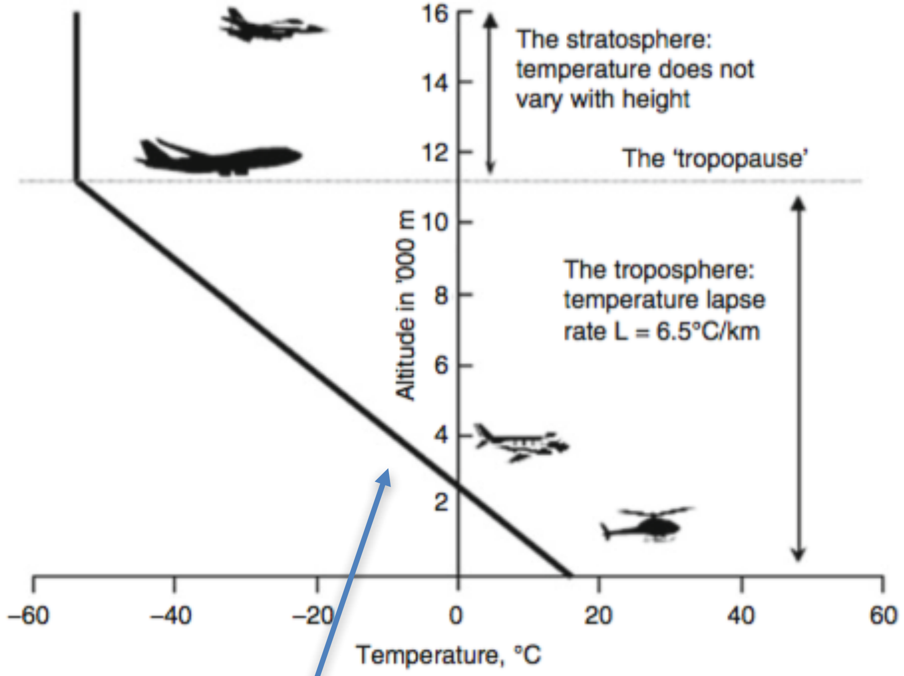
Hydrostatic Equation

$$\frac{\partial p}{\partial z} = -\rho \cdot g = -\frac{p}{R_g \cdot T} \cdot g$$

Integrating over a depth of atmosphere
Assuming constant gravity

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln\left(\frac{p_1}{p_1}\right) = -\frac{g}{R_g} \int_{z_1}^{z_2} \frac{d\xi}{T(\xi)}$$

$$T(z) = T_1 - \alpha_L \cdot z \rightarrow \alpha_L \equiv Lapse\ Rate$$



Need temperature profile
To complete integral

Temperature "Lapse Rate" Model

Hydrostatic Model of the Atmosphere (2)

$$\ln\left(\frac{p_2}{p_1}\right) = -\frac{g}{R_g}\int_{z_1}^{z_2}\frac{d\xi}{T(\xi)} = -\frac{g}{R_g}\int_{z_1}^{z_2}\frac{d\xi}{T_1 - \alpha_L \cdot \xi}$$

$$T(z) = T_1 - \alpha_L \cdot z \rightarrow \alpha$$

$$\ln\left(\frac{p_2}{p_1}\right) = -\frac{g}{R_g}\int_{z_1}^{z_2}\frac{d\xi}{T_1 - \alpha_L \cdot \xi} =$$

$$\frac{g}{\alpha_L R_g}\left[\ln\left(T_1 - \alpha_L \cdot z_2\right) - \ln\left(T_1 - \alpha_L \cdot z_1\right)\right] =$$

$$\frac{g}{\alpha_L R_g}\ln\left(\frac{T_1 - \alpha_L \cdot z_2}{T_1 - \alpha_L \cdot z_1}\right)$$

Hydrostatic Model of the Atmosphere (3)

$$Let \rightarrow z_1 = 0, T = T_{sl}$$

$$\rightarrow \ln\left(\frac{p_2}{p_{SL}}\right) = \frac{g}{\alpha_L R_g} \ln\left(\frac{T_1 - \alpha_L \cdot z_2}{T_1}\right) = \frac{g \cdot T_{sl}}{\alpha_L R_g} \ln\left(1 - \frac{\alpha_L}{T_{sl}} \cdot z_2\right)$$

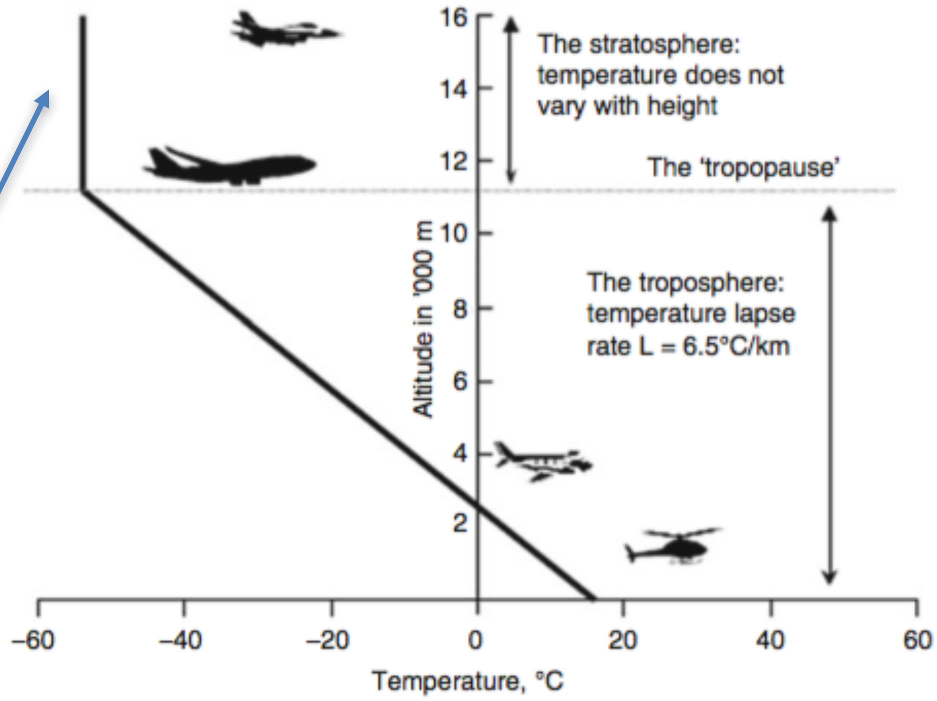
$$\boxed{p(z) = p_{sl} \cdot \left(1 - \frac{\alpha_L}{T_{sl}} \cdot z\right)^{\frac{g}{\alpha_L R_g}}}$$

Decay of Pressure with Altitude in Troposphere

Hydrostatic Model of the Atmosphere (4)

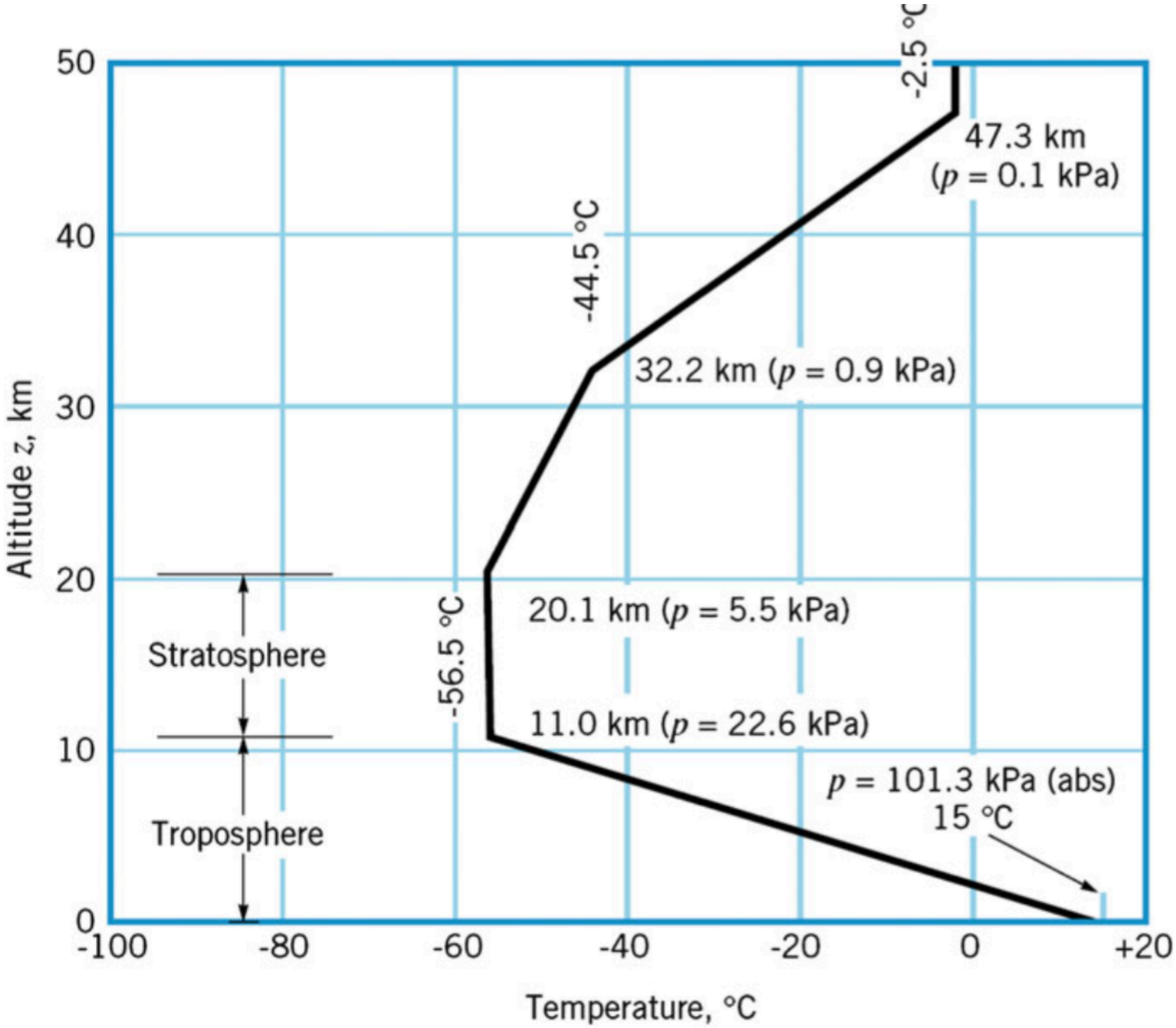
- For the stratosphere atmospheric layer (between 11.0 and 20.1 km), the temperature has a constant value (isothermal conditions) which is 56.5 C (or 69.7 F, 389.97 R, 216.65 K).

$$p(z) = p_s \cdot e^{-\frac{g}{R_g \cdot T_s}\left(z - z_s\right)}$$



Decay of Pressure with Altitude in Stratosphere

Extension to Mesosphere



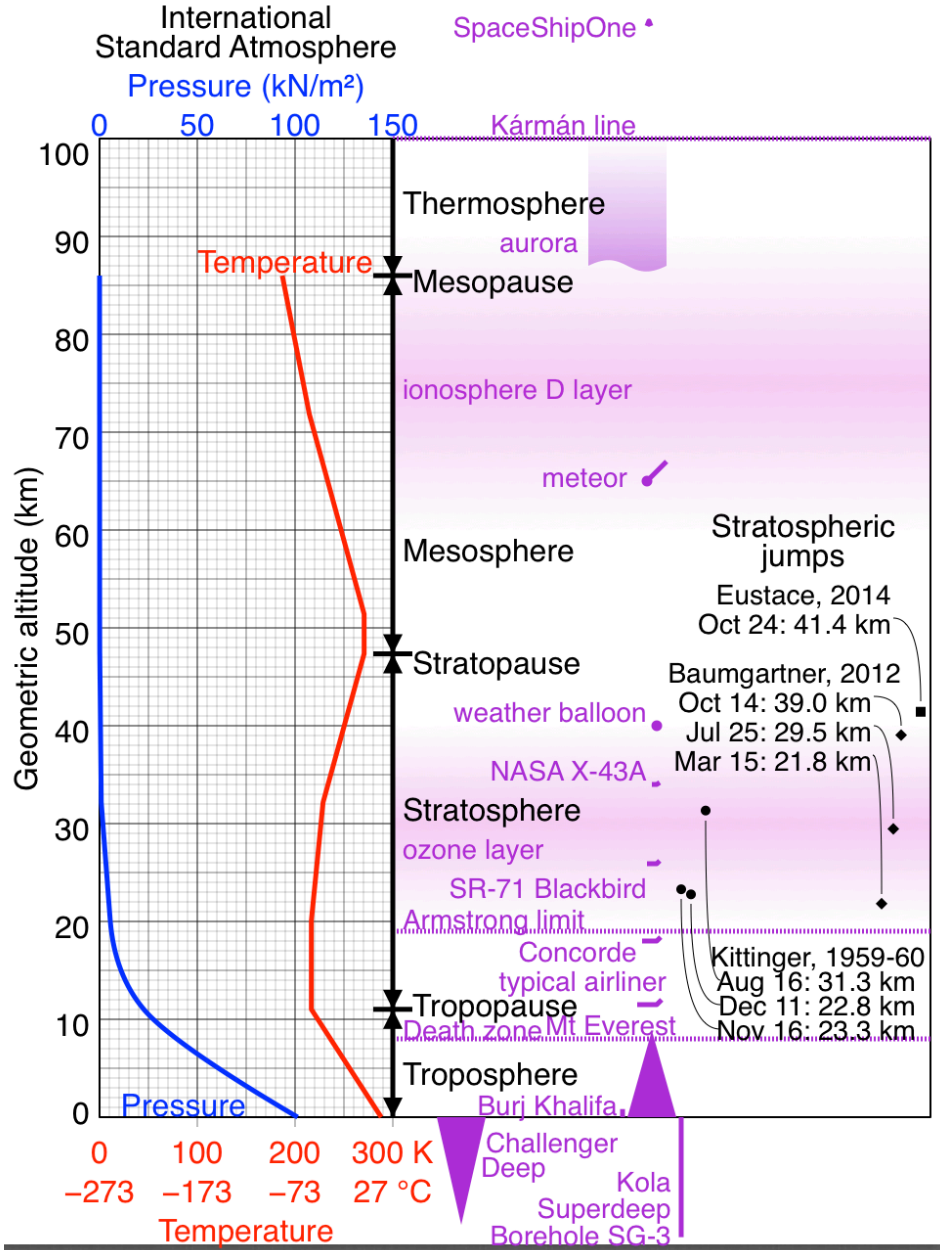
International Standard Atmospheric Model

- There are a myriad of atmospheres available .. Each serving a certain technical purpose …

see link on section 4 wep page .. http://www.spacewx.com/Docs/AIAA_G-003B-2004.pdf

- Basically, working models break into distinct altitude layers

1) Troposphere
   - → *$h < 11$ km (36,000 ft)*
2) Lower Stratosphere
   - → *$11\text{ km} \le h < 25$ km (82,000 ft)*
3) Upper Stratosphere
   - → *$25\text{ km} \le h < 50$ km*
4) Mesosphere
   - → *$h > 50$ km*



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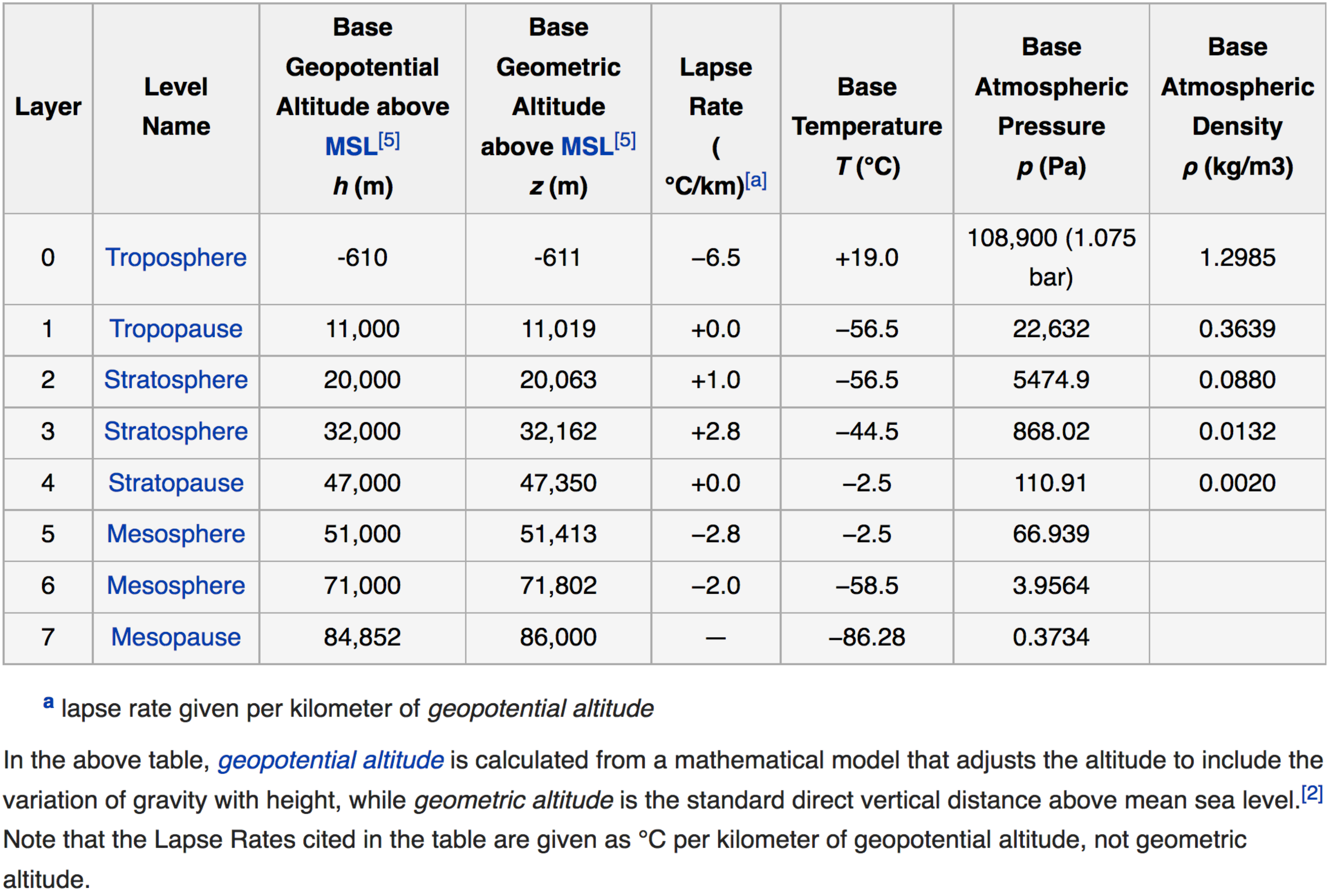
International Standard Atmospheric Model (2)

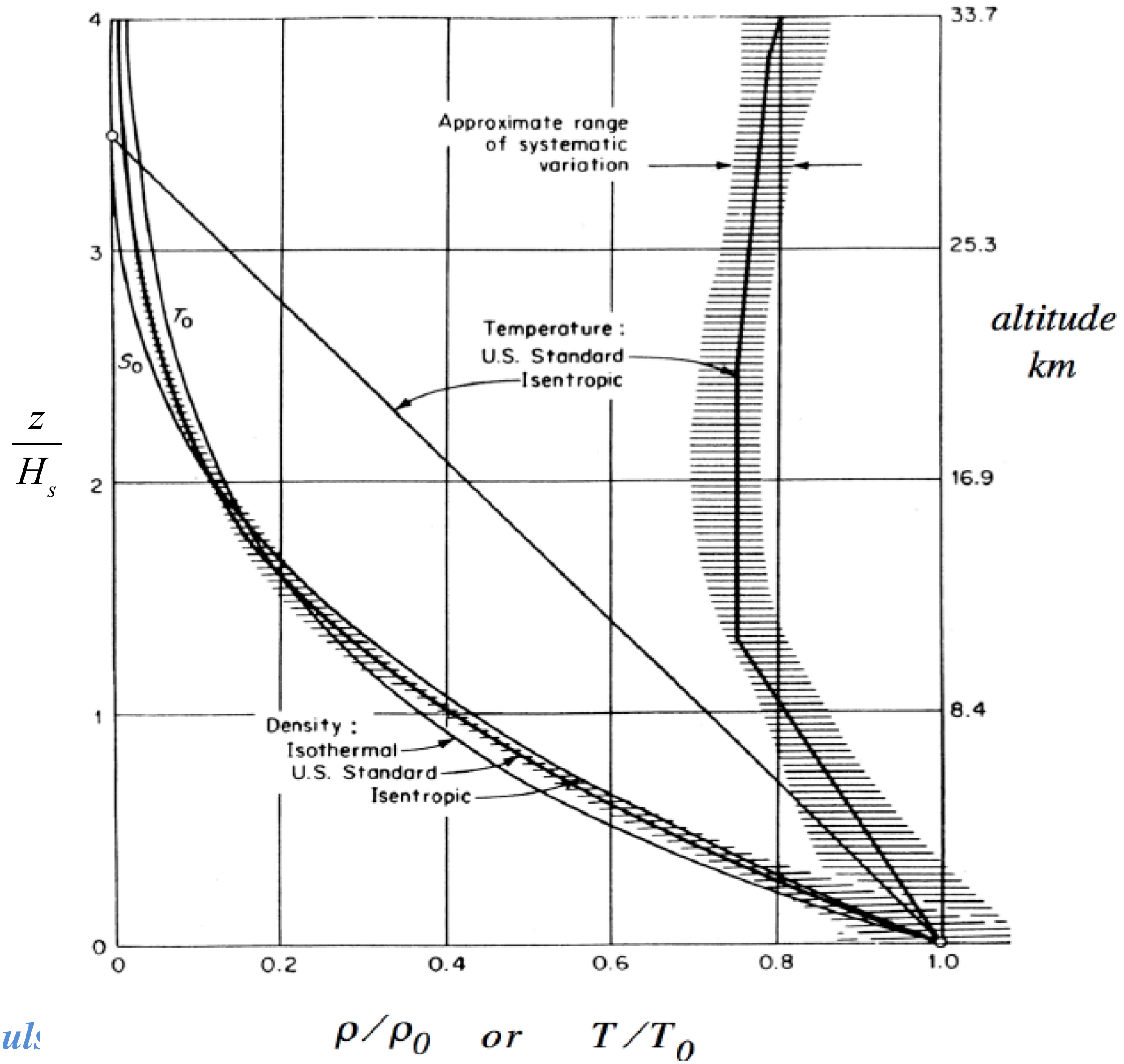


Layer	Level Name	Base Geopotential Altitude above MSL[5] h (m)	Base Geometric Altitude above MSL[5] z (m)	Lapse Rate (°C/km)[a]	Base Temperature T (°C)	Base Atmospheric Pressure p (Pa)	Base Atmospheric Density ρ (kg/m3)
0	Troposphere	-610	-611	–6.5	+19.0	108,900 (1.075 bar)	1.2985
1	Tropopause	11,000	11,019	+0.0	–56.5	22,632	0.3639
2	Stratosphere	20,000	20,063	+1.0	–56.5	5474.9	0.0880
3	Stratosphere	32,000	32,162	+2.8	–44.5	868.02	0.0132
4	Stratopause	47,000	47,350	+0.0	–2.5	110.91	0.0020
5	Mesosphere	51,000	51,413	–2.8	–2.5	66.939	
6	Mesosphere	71,000	71,802	–2.0	–58.5	3.9564	
7	Mesopause	84,852	86,000	—	–86.28	0.3734	

[a] lapse rate given per kilometer of *geopotential altitude*

In the above table, *geopotential altitude* is calculated from a mathematical model that adjusts the altitude to include the variation of gravity with height, while *geometric altitude* is the standard direct vertical distance above mean sea level.[2] Note that the Lapse Rates cited in the table are given as °C per kilometer of geopotential altitude, not geometric altitude.

1976 US Standard Atmospheric Model

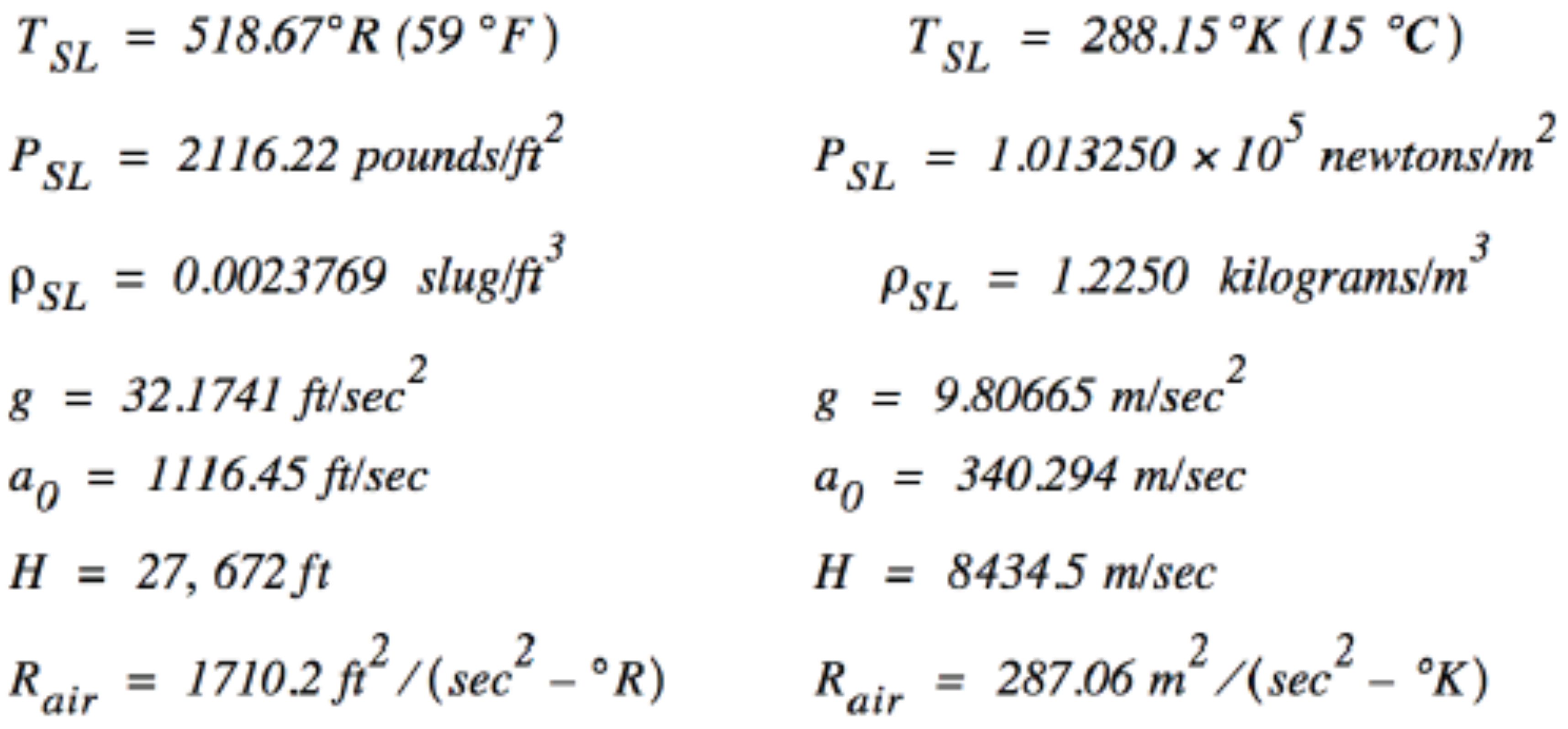


1976 US Standard Atmospheric Model *(2)*

$$\frac{z}{H_s} \rightarrow \quad \begin{aligned} z &= geometric\ altitude \\ H_s &= \text{"}scale\ height\text{"} \end{aligned}$$

1976 US Standard Atmosphere

$$H_s = \frac{c_0^2}{\gamma \cdot g_0} = \frac{R_g \cdot T_0}{g_0}$$



$T_{SL} = 518.67°R\ (59\ °F)$

$T_{SL} = 288.15\ °K\ (15\ °C)$

$P_{SL} = 2116.22\ pounds/ft^2$

$P_{SL} = 1.013250 \times 10^5\ newtons/m^2$

$\rho_{SL} = 0.0023769\ \ slug/ft^3$

$\rho_{SL} = 1.2250\ \ kilograms/m^3$

$g = 32.1741\ ft/sec^2$

$g = 9.80665\ m/sec^2$

$a_0 = 1116.45\ ft/sec$

$a_0 = 340.294\ m/sec$

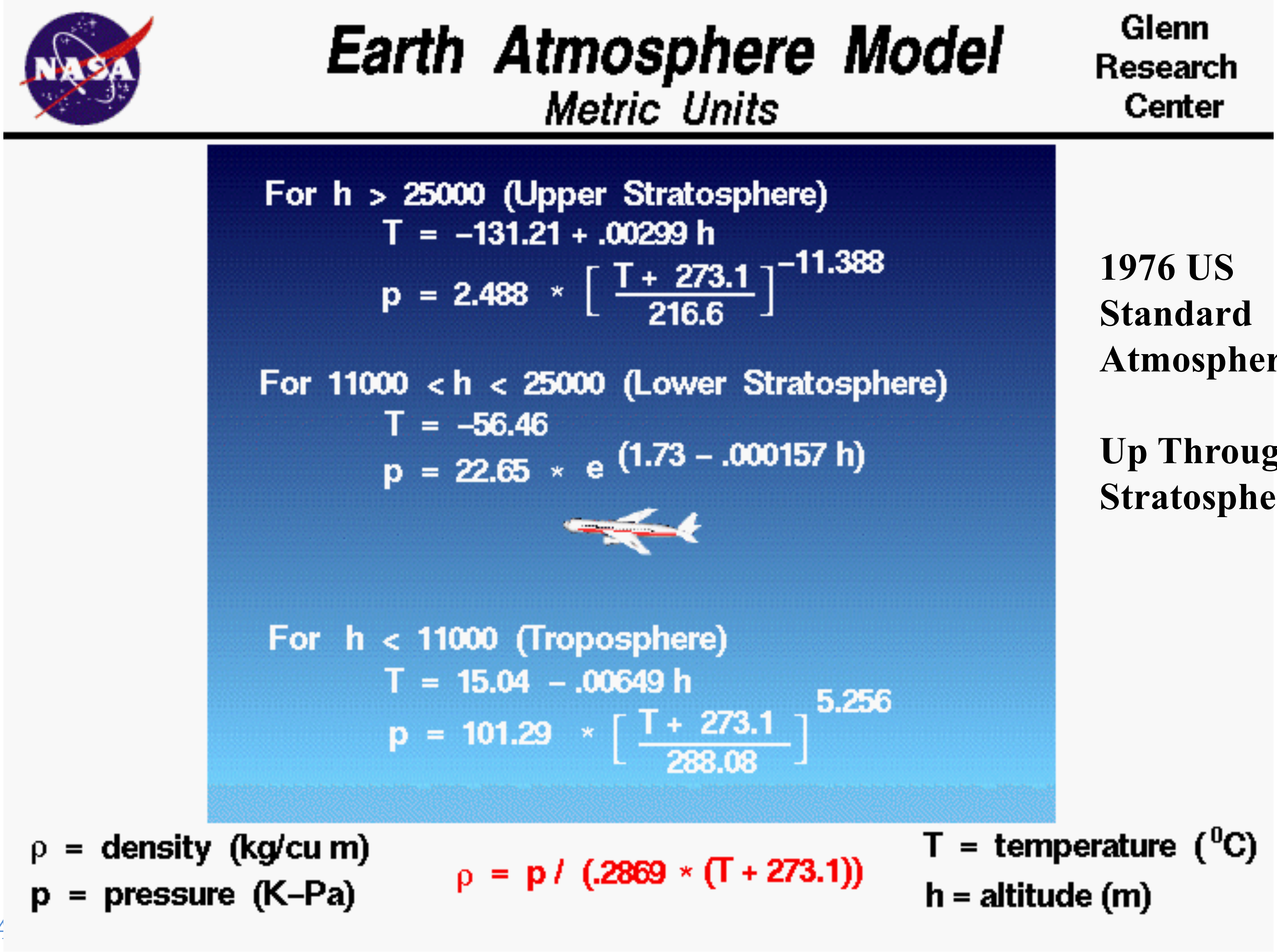
$H = 27,672\ ft$

$H = 8434.5\ m/sec$

$R_{air} = 1710.2\ ft^2/(sec^2 - °R)$

$R_{air} = 287.06\ m^2/(sec^2 - °K)$

1976 US Standard Atmospheric Model *(3)*



Earth Atmosphere Model

Metric Units

Glenn
Research
Center

For h > 25000 (Upper Stratosphere)

$$T = -131.21 + .00299\,h$$

$$p = 2.488 * \left[\frac{T + 273.1}{216.6}\right]^{-11.388}$$

For 11000 < h < 25000 (Lower Stratosphere)

$$T = -56.46$$

$$p = 22.65 * e^{(1.73 - .000157\,h)}$$

For h < 11000 (Troposphere)

$$T = 15.04 - .00649\,h$$

$$p = 101.29 * \left[\frac{T + 273.1}{288.08}\right]^{5.256}$$

ρ = density (kg/cu m)

p = pressure (K-Pa)

$$\rho = p / (.2869 * (T + 273.1))$$

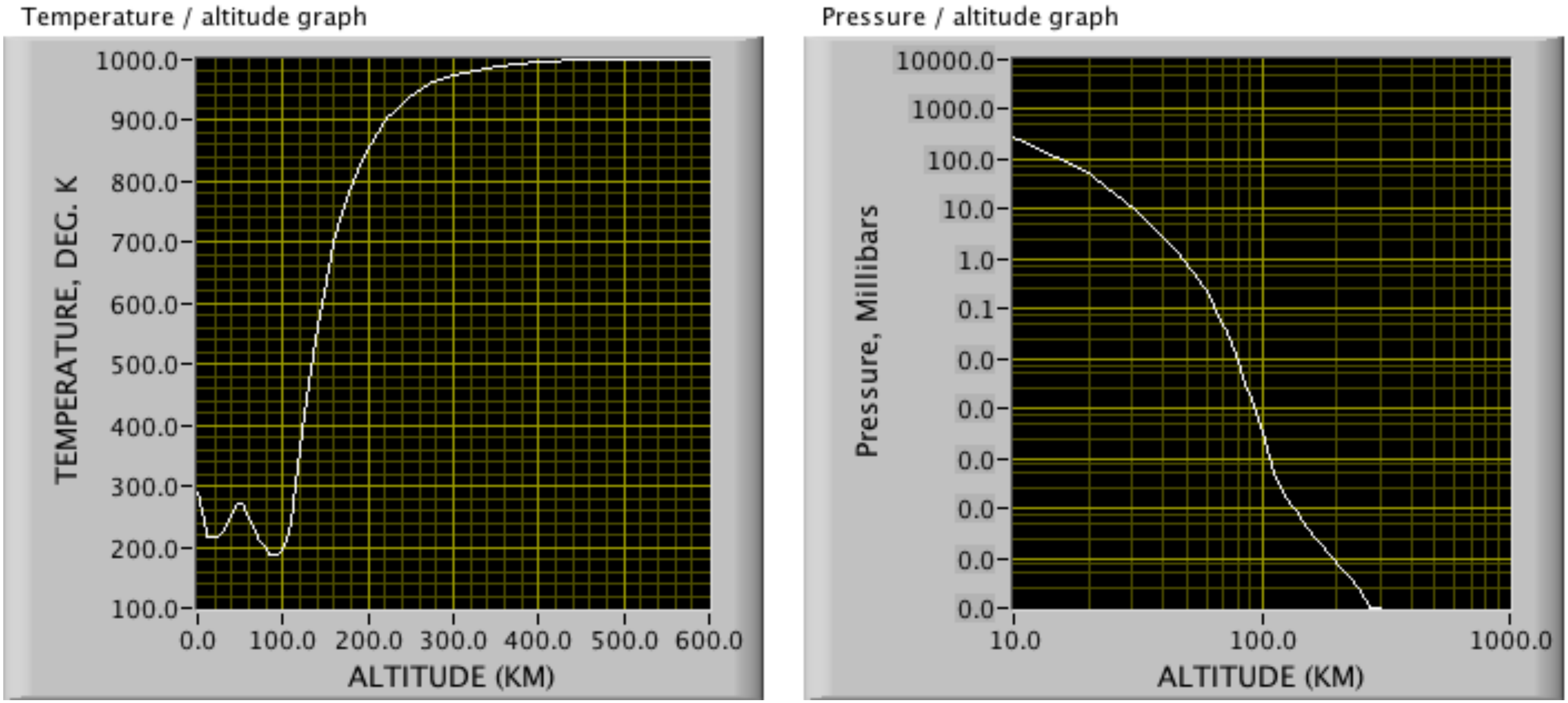
T = temperature (^{0}C)

h = altitude (m)

1976 US Standard Atmosphere

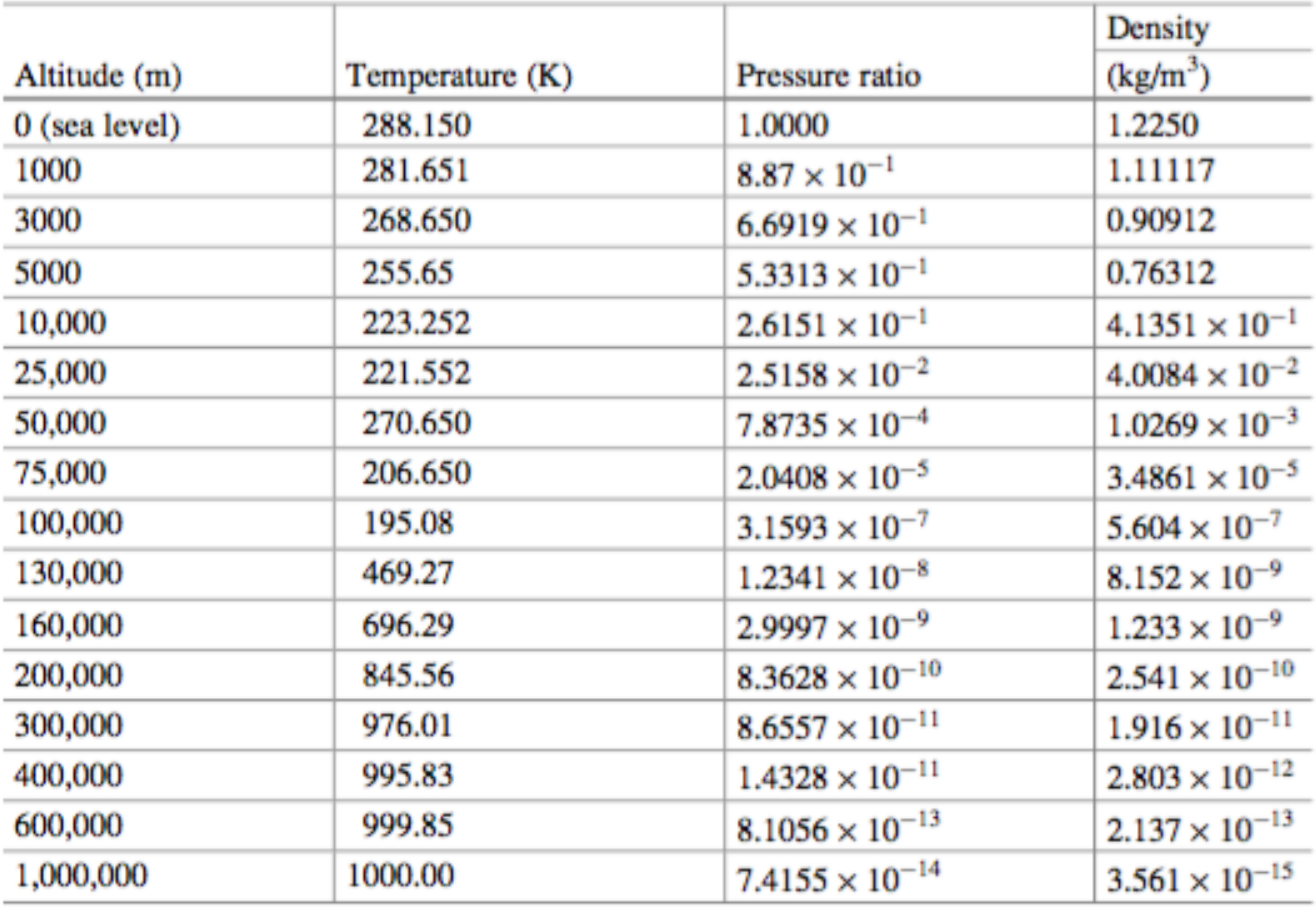
Up Through Stratosphere

1976 US Standard Atmospheric Model*(4)*



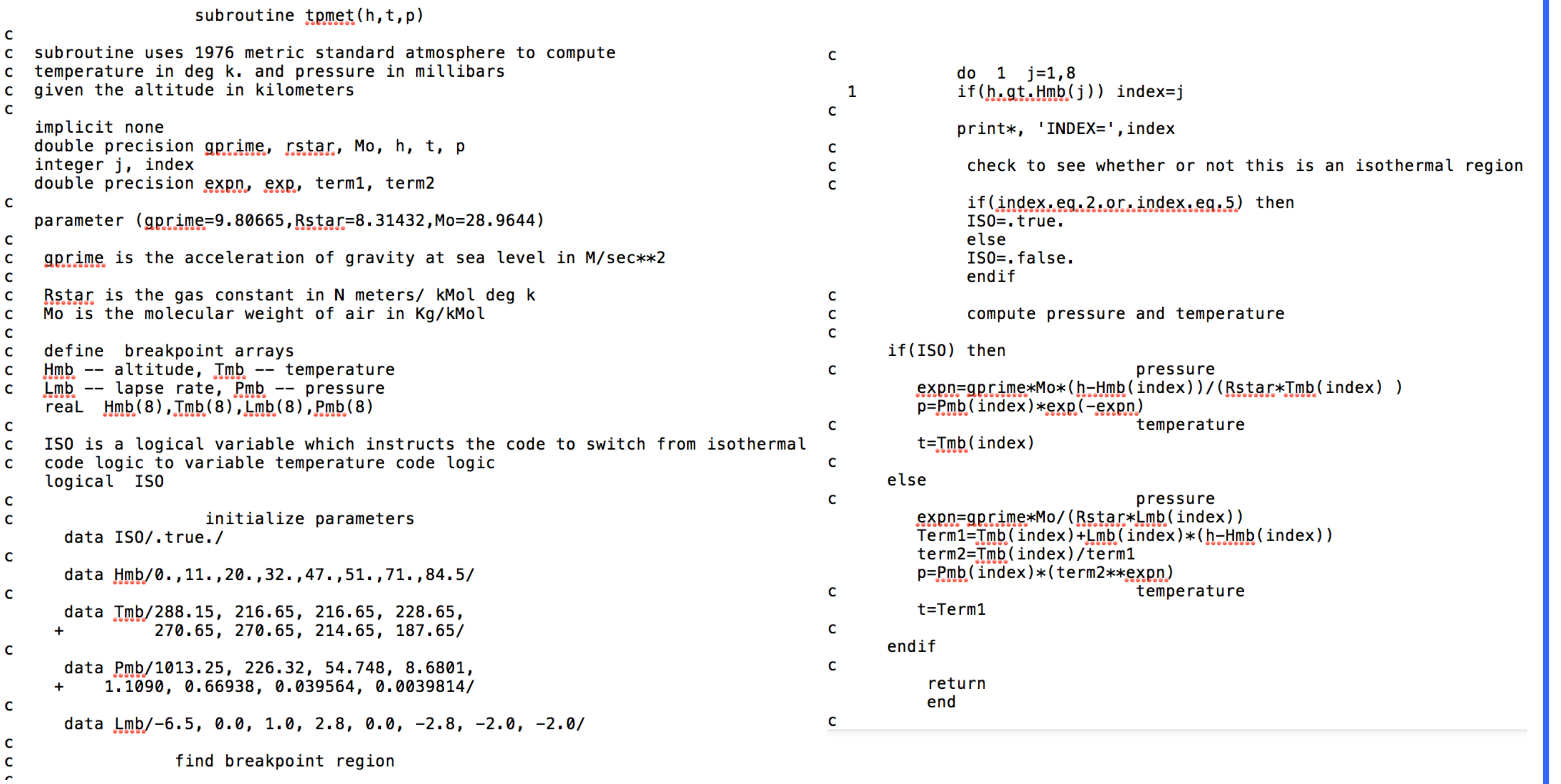
1976 US Standard Atmosphere up Through Mesosphere

1976 US Standard Atmospheric Model*(5)*



Altitude (m)	Temperature (K)	Pressure ratio	Density (kg/m^3)
0 (sea level)	288.150	1.0000	1.2250
1000	281.651	8.87×10^{-1}	1.11117
3000	268.650	6.6919×10^{-1}	0.90912
5000	255.65	5.3313×10^{-1}	0.76312
10,000	223.252	2.6151×10^{-1}	4.1351×10^{-1}
25,000	221.552	2.5158×10^{-2}	4.0084×10^{-2}
50,000	270.650	7.8735×10^{-4}	1.0269×10^{-3}
75,000	206.650	2.0408×10^{-5}	3.4861×10^{-5}
100,000	195.08	3.1593×10^{-7}	5.604×10^{-7}
130,000	469.27	1.2341×10^{-8}	8.152×10^{-9}
160,000	696.29	2.9997×10^{-9}	1.233×10^{-9}
200,000	845.56	8.3628×10^{-10}	2.541×10^{-10}
300,000	976.01	8.6557×10^{-11}	1.916×10^{-11}
400,000	995.83	1.4328×10^{-11}	2.803×10^{-12}
600,000	999.85	8.1056×10^{-13}	2.137×10^{-13}
1,000,000	1000.00	7.4155×10^{-14}	3.561×10^{-15}

1976 US Standard Atmosphere Source Code



```
                   subroutine tpmet(h,t,p)
c
c  subroutine uses 1976 metric standard atmosphere to compute
c  temperature in deg k. and pressure in millibars
c  given the altitude in kilometers
c
   implicit none
   double precision gprime, rstar, Mo, h, t, p
   integer j, index
   double precision expn, exp, term1, term2
c
   parameter (gprime=9.80665,Rstar=8.31432,Mo=28.9644)
c
c   gprime is the acceleration of gravity at sea level in M/sec**2
c
c   Rstar is the gas constant in N meters/ kMol deg k
c   Mo is the molecular weight of air in Kg/kMol
c
c   define  breakpoint arrays
c   Hmb -- altitude, Tmb -- temperature
c   Lmb -- lapse rate, Pmb -- pressure
    reaL  Hmb(8),Tmb(8),Lmb(8),Pmb(8)
c
c   ISO is a logical variable which instructs the code to switch from isothermal
c   code logic to variable temperature code logic
    logical  ISO
c
c                   initialize parameters
      data ISO/.true./
c
      data Hmb/0.,11.,20.,32.,47.,51.,71.,84.5/
c
      data Tmb/288.15, 216.65, 216.65, 228.65,
     +         270.65, 270.65, 214.65, 187.65/
c
      data Pmb/1013.25, 226.32, 54.748, 8.6801,
     +    1.1090, 0.66938, 0.039564, 0.0039814/
c
      data Lmb/-6.5, 0.0, 1.0, 2.8, 0.0, -2.8, -2.0, -2.0/
c
c                find breakpoint region
c
c
             do  1  j=1,8
  1          if(h.gt.Hmb(j)) index=j
c
             print*, 'INDEX=',index
c
c             check to see whether or not this is an isothermal region
c
              if(index.eq.2.or.index.eq.5) then
              ISO=.true.
              else
              ISO=.false.
              endif
c
c             compute pressure and temperature
c
      if(ISO) then
c                                  pressure
         expn=gprime*Mo*(h-Hmb(index))/(Rstar*Tmb(index) )
         p=Pmb(index)*exp(-expn)
c                                  temperature
         t=Tmb(index)
c
      else
c                                  pressure
         expn=gprime*Mo/(Rstar*Lmb(index))
         Term1=Tmb(index)+Lmb(index)*(h-Hmb(index))
         term2=Tmb(index)/term1
         p=Pmb(index)*(term2**expn)
c                                  temperature
         t=Term1
c
      endif
c
          return
          end
c
```

1976 US Standard Atmosphere up Through Mesosphere

Earth Global Reference Atmospheric Model (Earth-GRAM 2010)

GRAM ..

- Global model, using empirical database, giving density, temperature, pressure, winds, and selected atmospheric constituent concentrations, from the surface of the Earth to orbital altitudes, as a function of geographic position and time of year.

- Local perturbations about mean conditions are also included. Model is also suitable for use as a subroutine in a trajectory code or orbit propagator program or other programs used for simulations of in-flight or on-orbit atmospheric variability in density, temperature, or winds.

https://see.msfc.nasa.gov/model-gram

Homework 4.2

KGW-1 (later re-designated as LTV-N-2) was the US Navy's version of American flying bomb *JB-2 Loon*. It was developed to be carried on the aft deck of submarines in watertight containers. The first submarine to employ them was the SS-348 *Cusk* which successfully launched its first Loon on 12 February 1947 in Point Mugu, California. It has the following data:

- Static thrust 2200 N with air inlet speed of 180 m/s @ Sea Level
- Intake area 0.145 m^2
- Fuel is standard 80-octane gasoline having heating value $Q_R = 40$ MJ/kg
- Burner efficiency 0.90
- Typical flight duration is 1800 s
- Exhaust temperature 735 K

Assume Nozzle Optimized for Sea Level

Homework 4.2 (2)

Assume specific heat of air $Cp_a = 1.005 \frac{\text{kJ}}{\text{kgK}}$ and specific heat of hot gases $Cp_h = 1.12 \frac{\text{kJ}}{\text{kgK}}$

$$h_{fuel} = \eta_{combustor} \cdot Q_R$$

Calculate

1. Air mass flow rate into engine
2. Exhaust velocity
3. Maximum temperature inside the engine
4. Maximum pressure
5. Thrust specific fuel consumption (TSFC)
6. Average range *Launch Weight = 2,150 kg*

Assume Stagnation

7. Mean L/D for (Sea Level) Cruise Conditions

Questions??