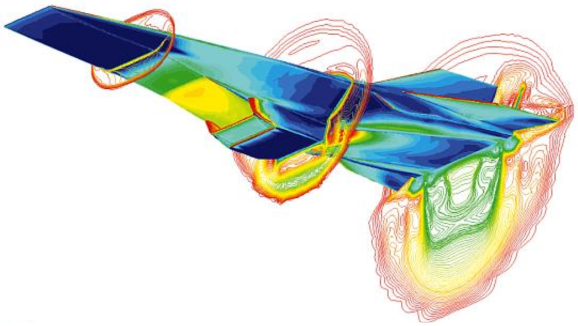
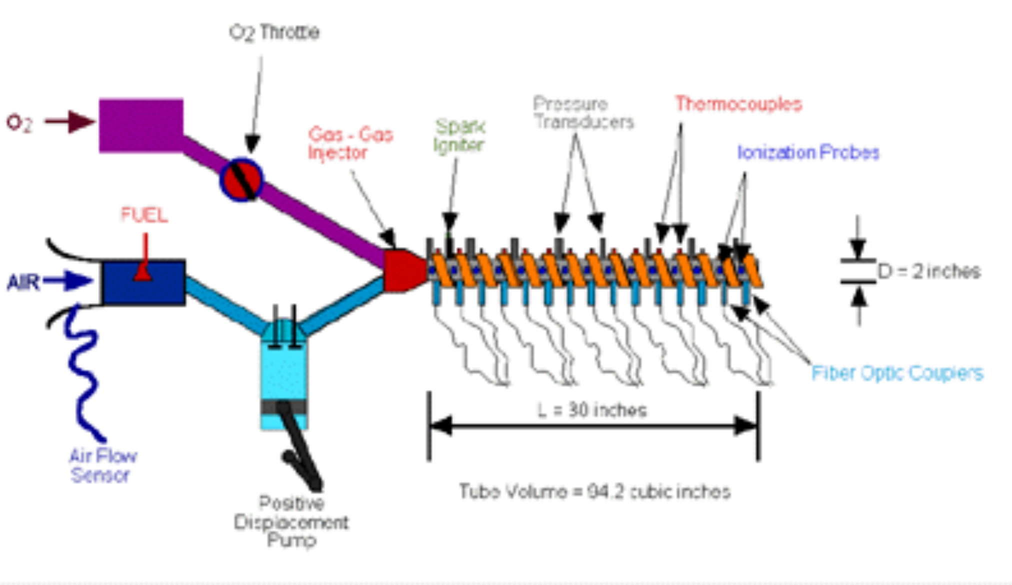
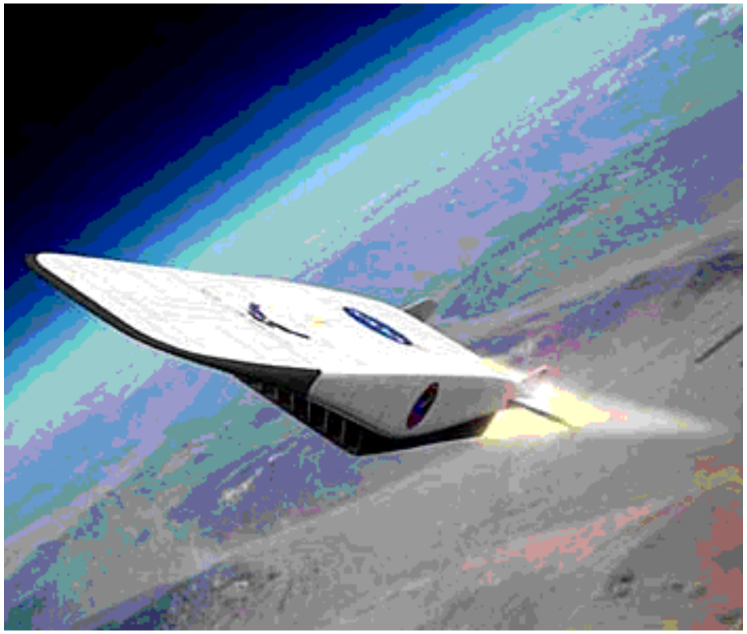
Section 4.4: Supersonic Combustion Ramjets (SCRAMjets) and Combined Cycle Engines

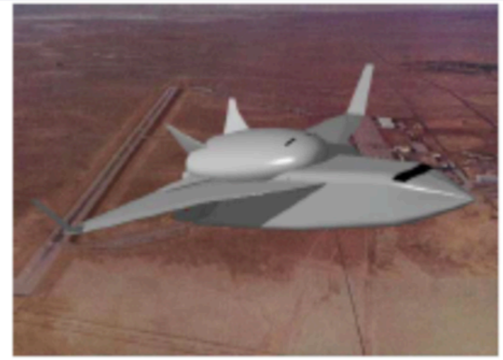






ScramJet Applications

Space Access



Weapons

RLV (Affordable, timely access to space)

Far-Term

Hypersonic Cruiser (Global Reach/Attack)

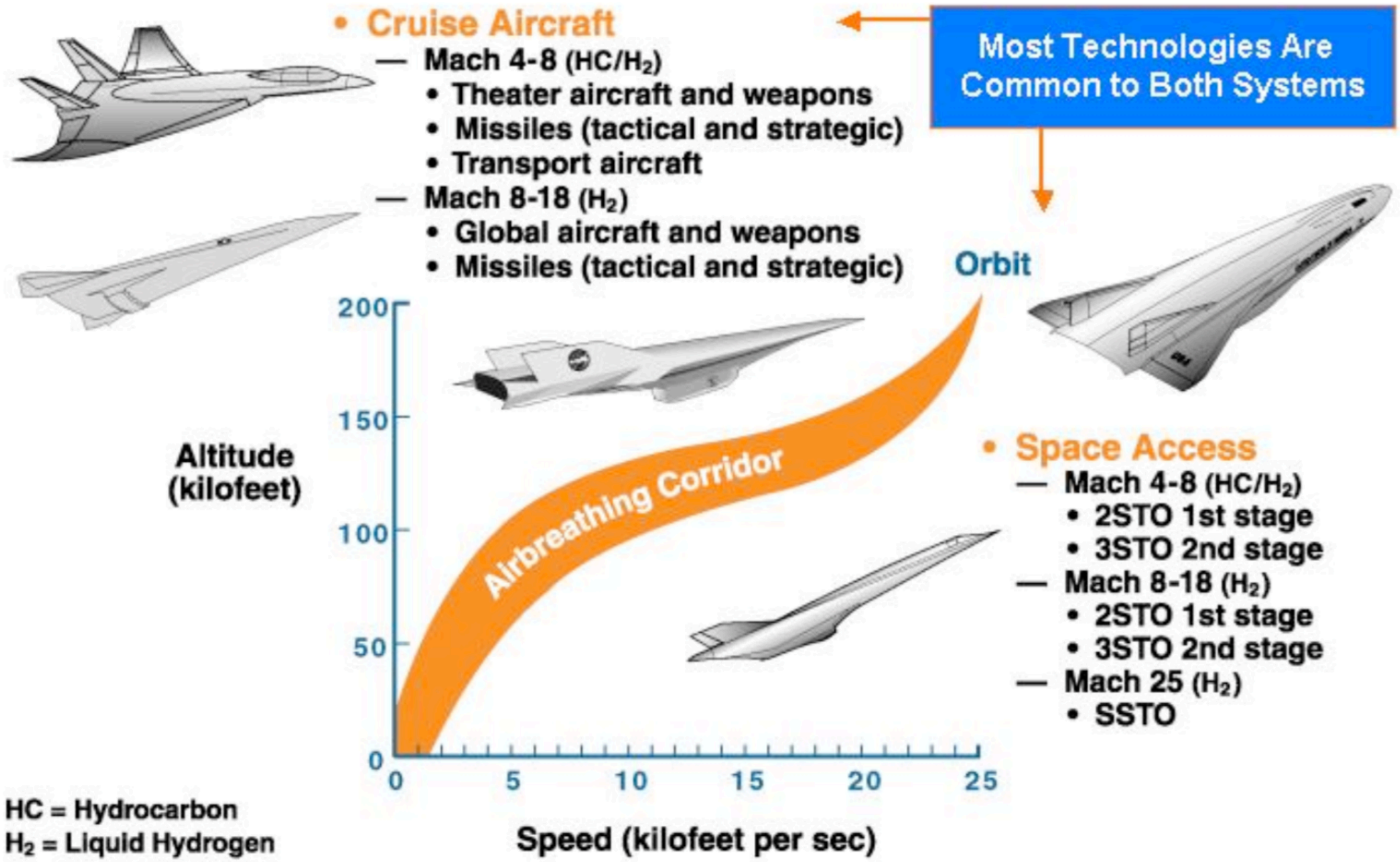
Mid-Term

Hypersonic Missile (Time-critical targets)

Near-Term

Pursue Stepping-Stone Approach

POTENTIAL AIR-BREATHING HYPERSONIC VEHICLE APPLICATIONS AND THEIR FLIGHT ENVELOPES



Operations Payoffs for Airbreathing Launch

- Decreased gross lift-off weight, resulting in smaller facilities and easier handling
- Wider range of emergency landing sites for intact abort
- Powered flyback/go-around & more margin at reduced power
- Self-ferry & taxi capabilities
- Greatly expanded launch windows (double or triple)
- Rapid orbital rendezvous (up to three times faster than rockets)
- Wider array of landing sites from orbit, with 2,000-mile cross range and increased range
- Reduced sensitivity to weight growth

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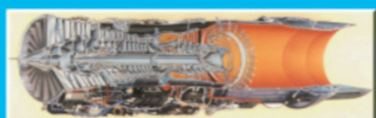
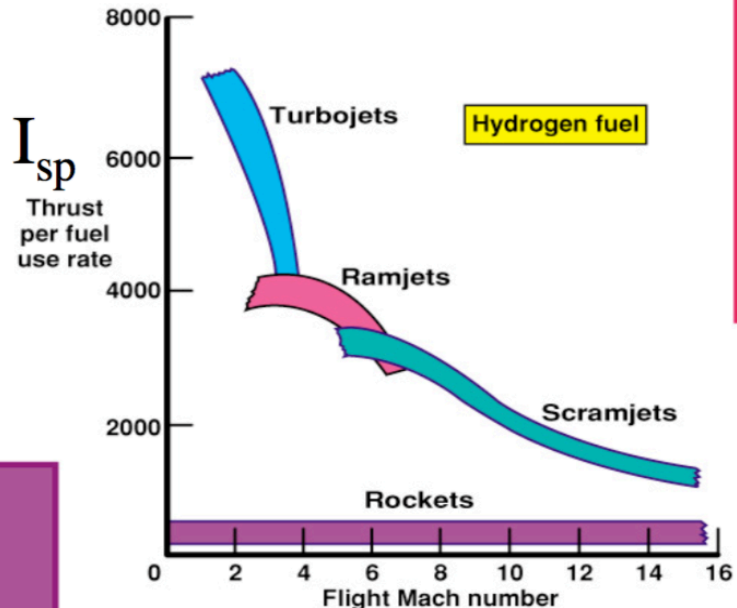
Fuel Efficiencies of Various High Speed Propulsion Systems

Turbojet



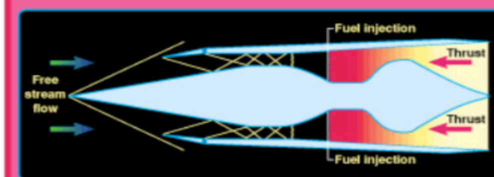
- Uses atmospheric air
- Complex turbomachinery
- Subsonic combustion
- Mach range: 0 to 3

Propulsion systems comparison



Ramjet

- Uses atmospheric air
- "Ram" compression → no turbomachinery
- Subsonic combustion
- Mach range: 2 to 6



Rocket

- Carries oxidizer
- No turbomachinery
- Subsonic combustion
- Mach range: 0 to 25

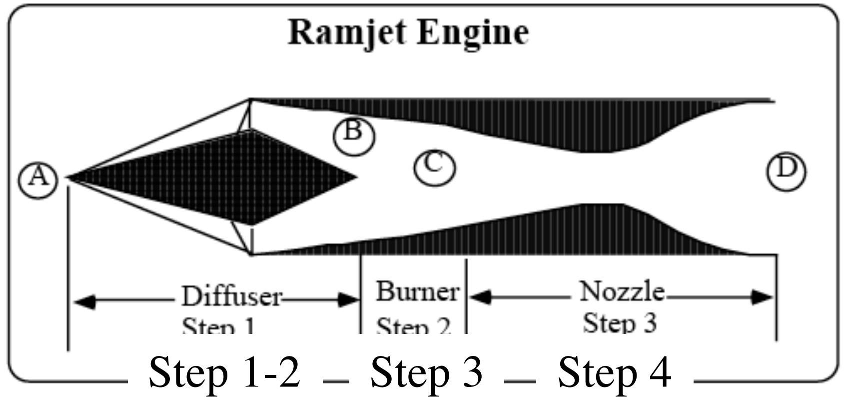
Minimal turbo-machinery

Scramjets have the highest efficiency above Mach 6

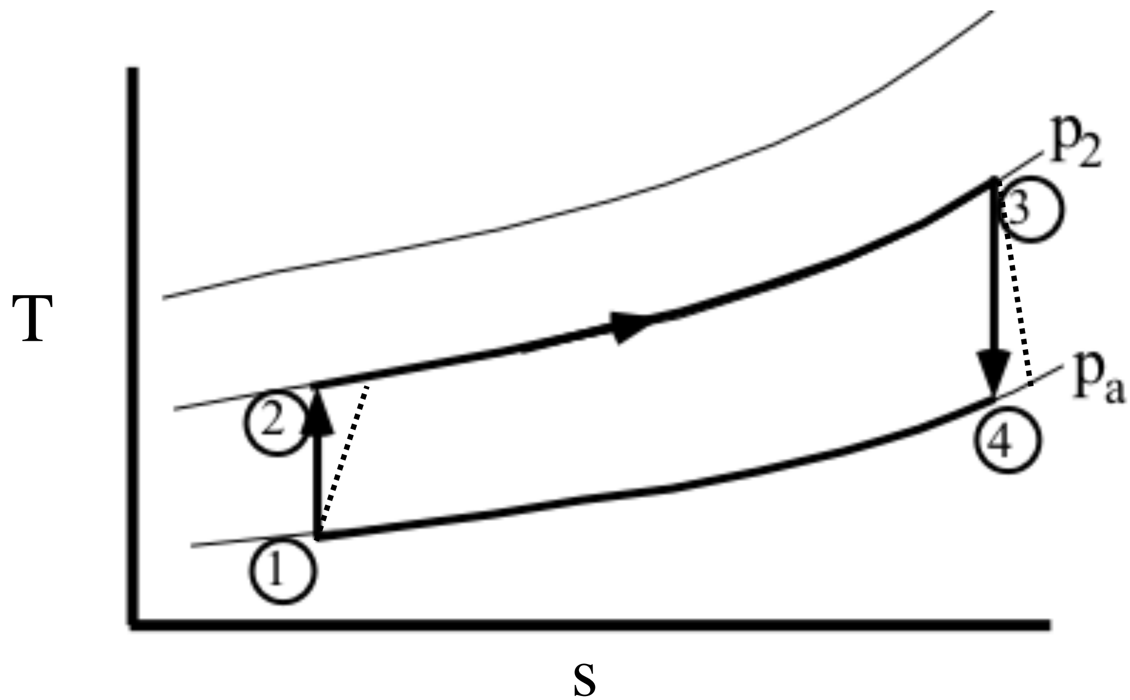
Scramjet

- Uses atmospheric air
- "Ram" compression → no turbomachinery
- Supersonic combustion
- Mach range: 5 to ~25

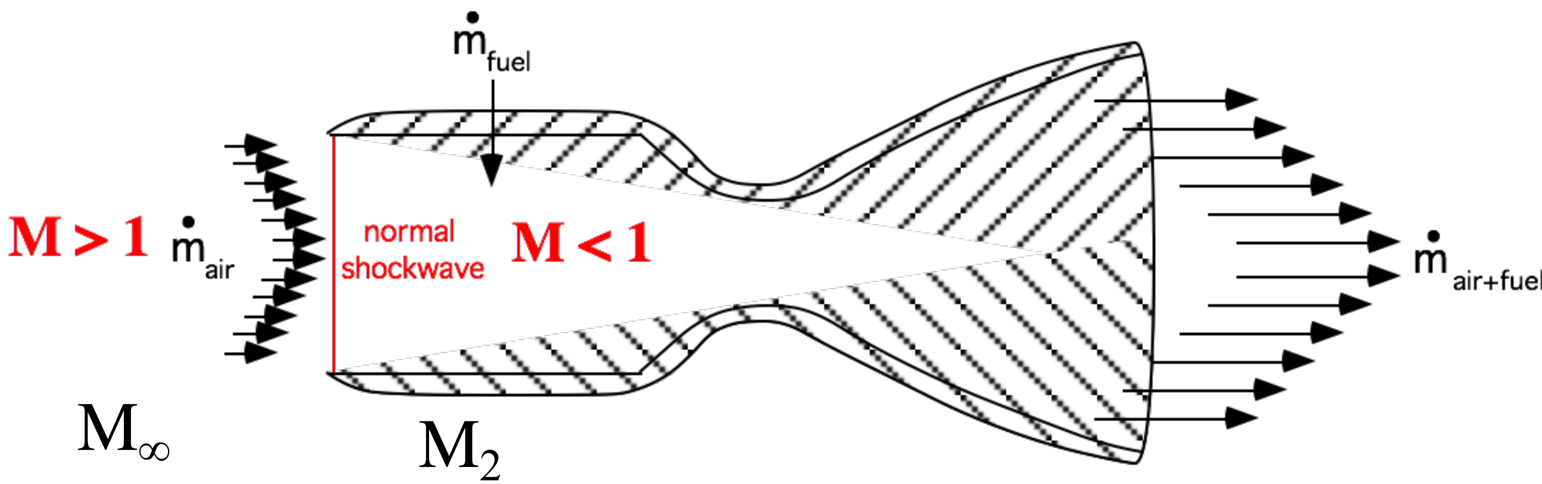
Ideal Ramjet Cycle Analysis Revisited



T-s Diagram



Ideal Ramjet: *Inlet and Diffuser with normal shock*

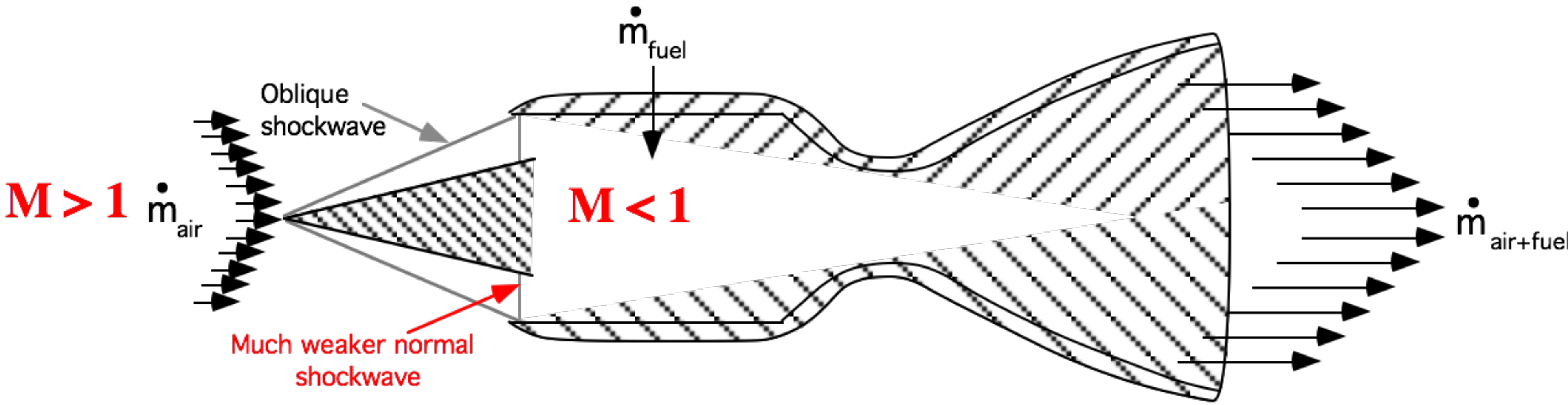


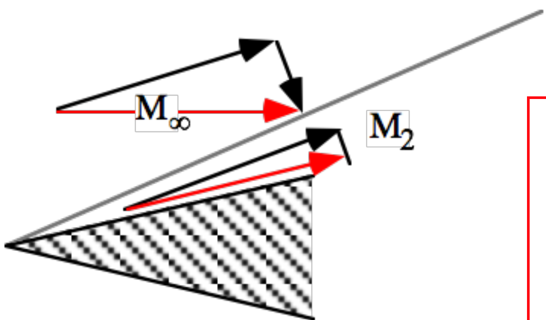
- Mechanical Energy is Dissipated into Heat

- Huge Loss in Momentum

Ideal Ramjet: *Inlet and Diffuser with Oblique shock*

- So … we put a spike in front of the inlet





- How does this spike Help?

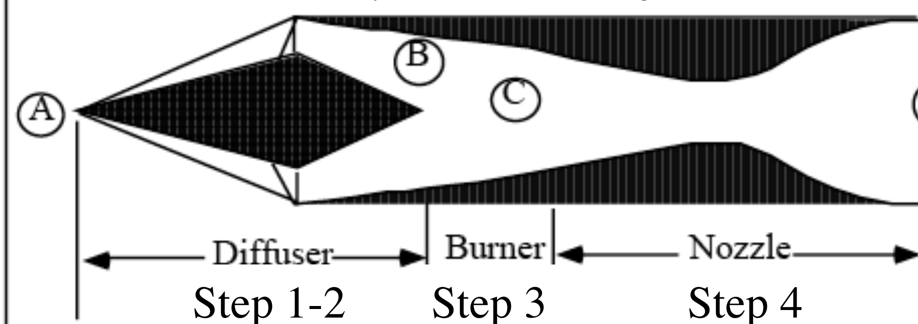
- By forming an Oblique Shock wave ahead of the inlet

Thermodynamic Efficiency of Ideal Ramjet, revisited

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)}$$

Ramjet Engine

(Credit Narayanan Komerath, Georgia Tech)



i) As engine pressure ratio, P_B/P_A, goes up … *η goes up*

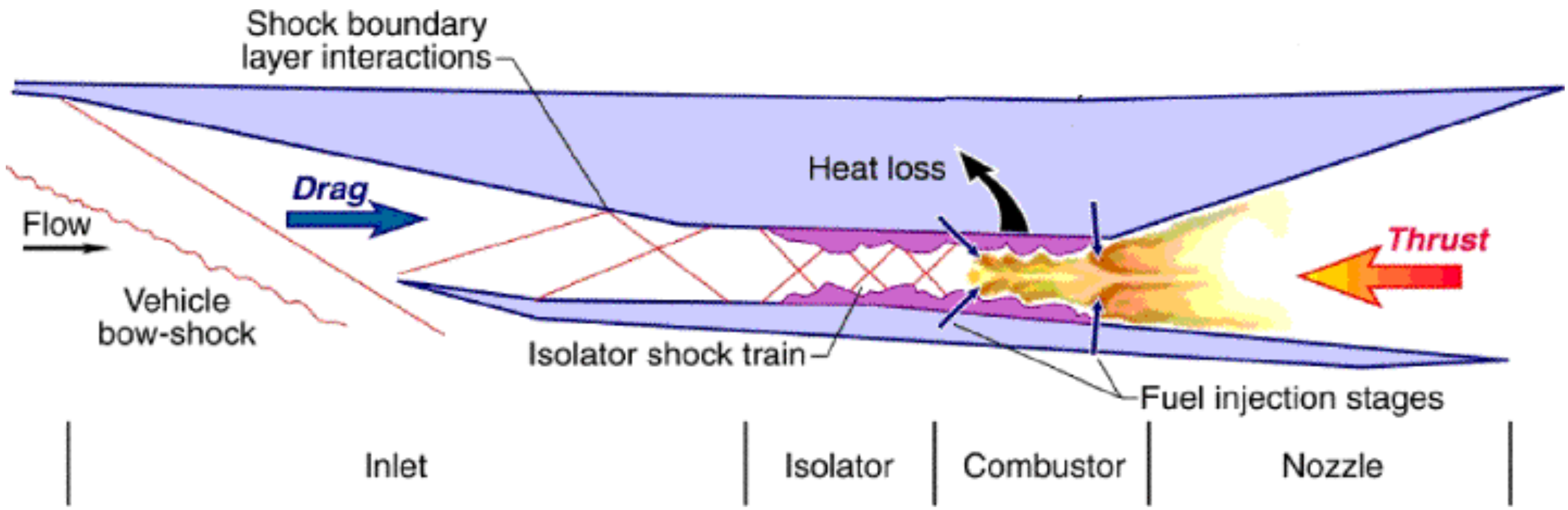
ii) As combustor temperature difference T_C-T_B goes up … *η goes up*

iii) As inlet total pressure ratio (P_{0_B}/P_{0_A}) goes down … (stagnation pressure loss goes up) … *η goes down*

ScrRamjet Design Issues, I

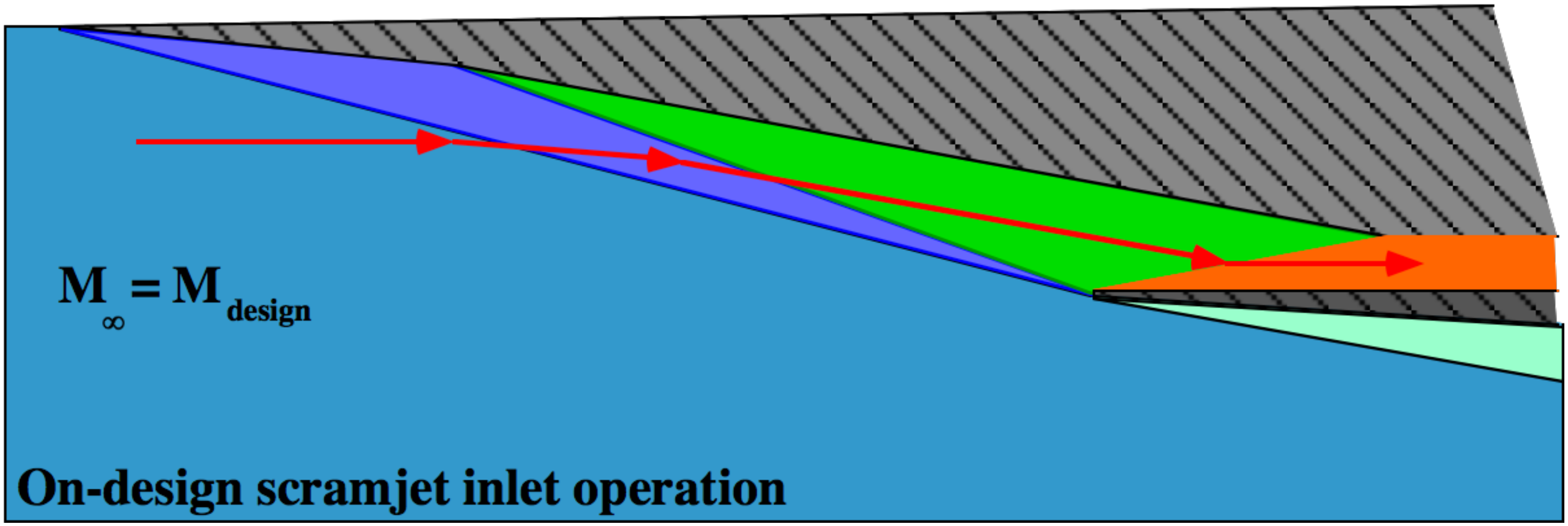
- What if we keep engine flow path supersonic to minimize stagnation pressure loss?

- How do we keep the Inflow supersonic?



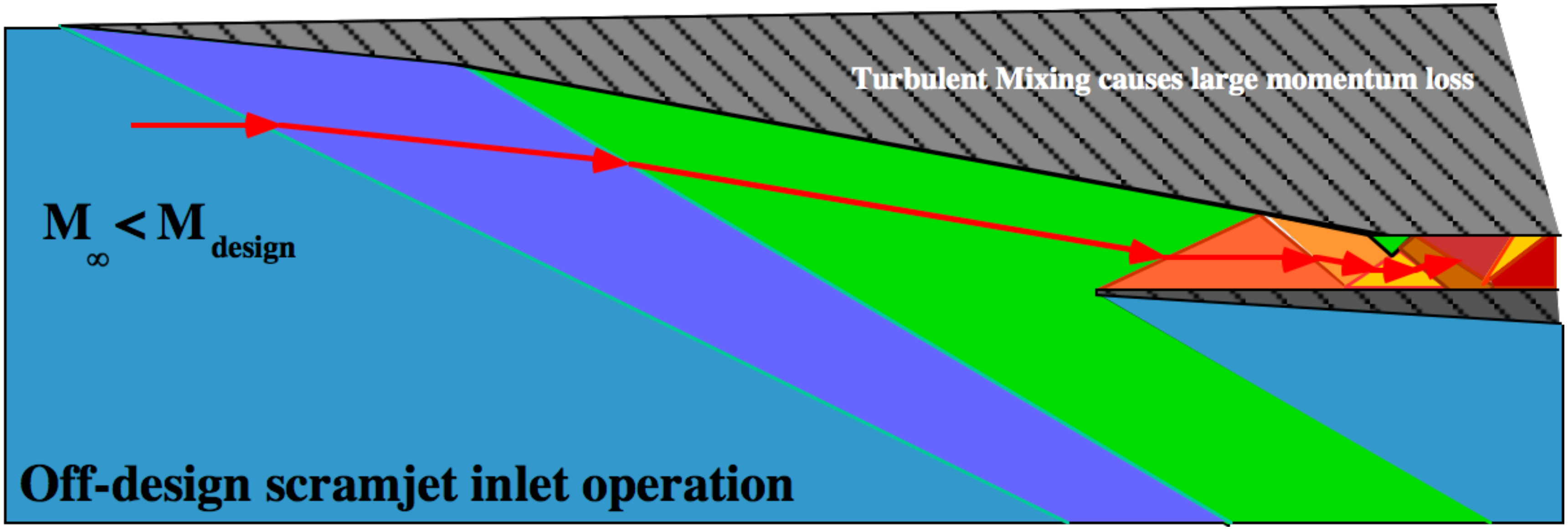
- Series of very weak (highly oblique) shockwaves and expansion shocks keep the flow supersonic throughout the engine

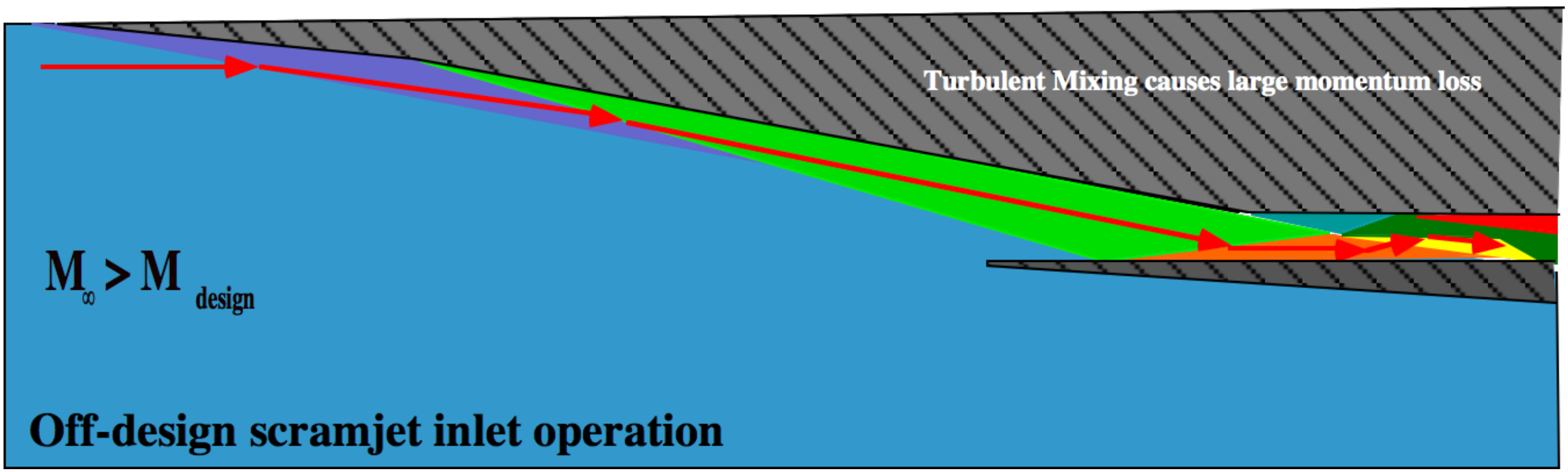
ScrRamjet Design Issues, I
Inlet "Point Design"



- SCRAMjets are *VERY* …
  Sensitive to inlet mach number

Inlet "Point Design"(cont'd)





X-43A Side by Side Comparison

- Subtle but important shape differences Mach 10 Inlet likely would not start at Mach 7

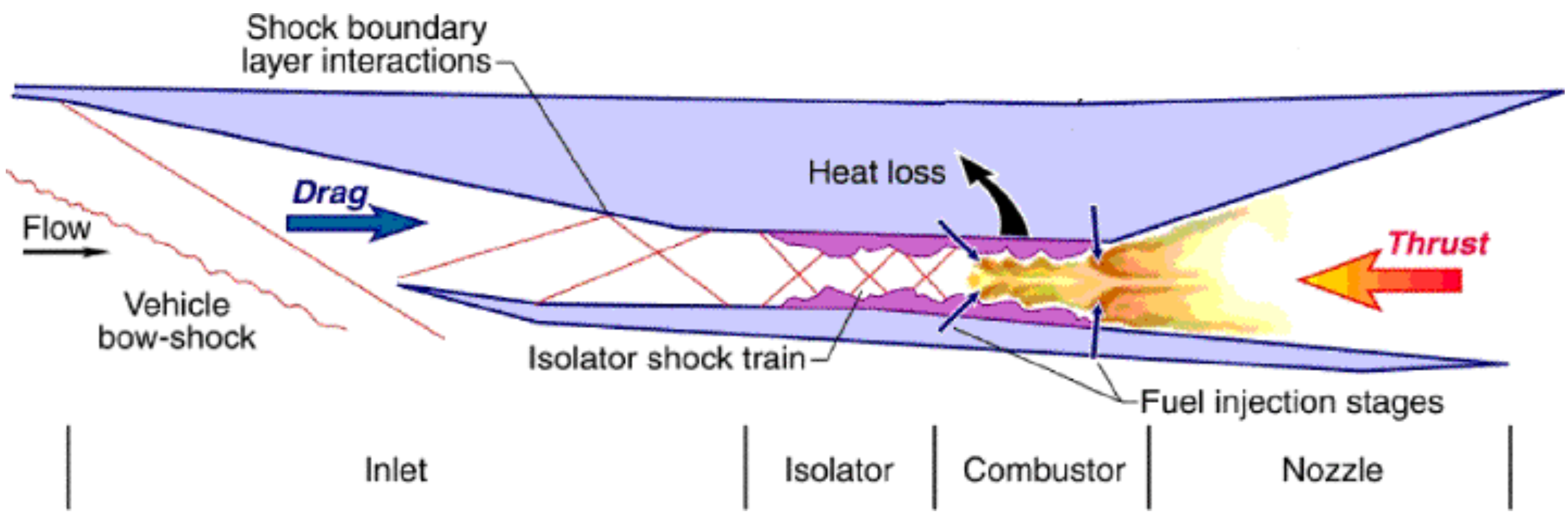
Mach 7 Vehicle

Mach 10 Vehicle

ScrRamjet Design Issues, I

- What if we keep engine flow path supersonic to minimize stagnation pressure loss?

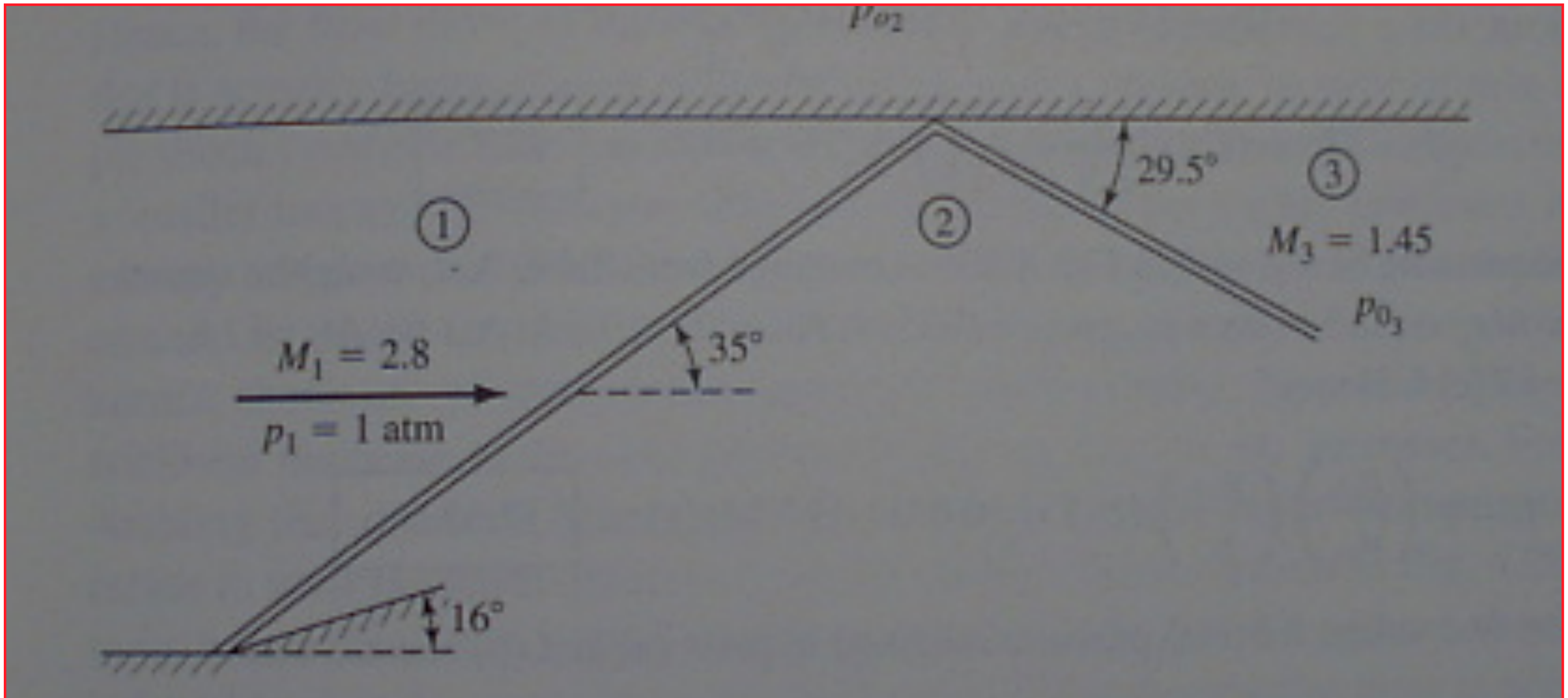
- How do we keep the Inflow supersonic?



- Series of very weak (highly oblique) shockwaves and expansion shocks keep the flow supersonic throughout the engine

Background on Supersonic Inlet Design

- Anderson,
  Chapter 4 pp. 152-164

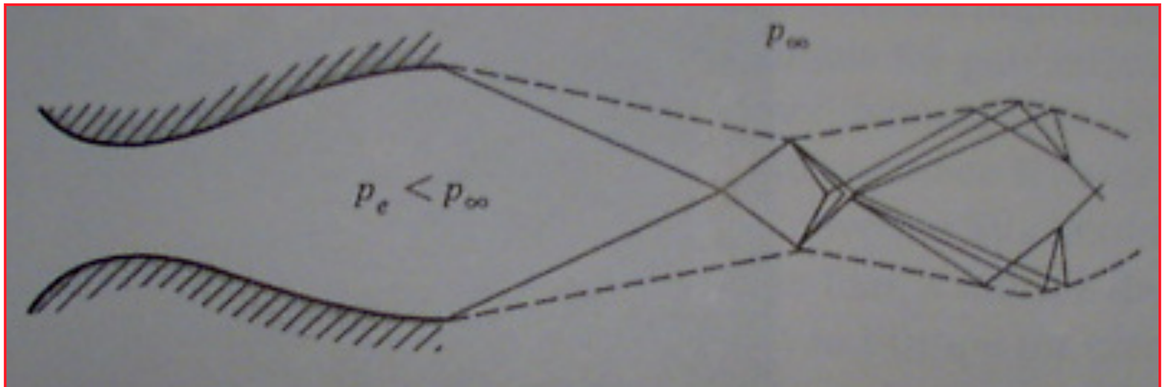


(a)

Rules for Shock Wave Reflections from Solid and Free Boundaries

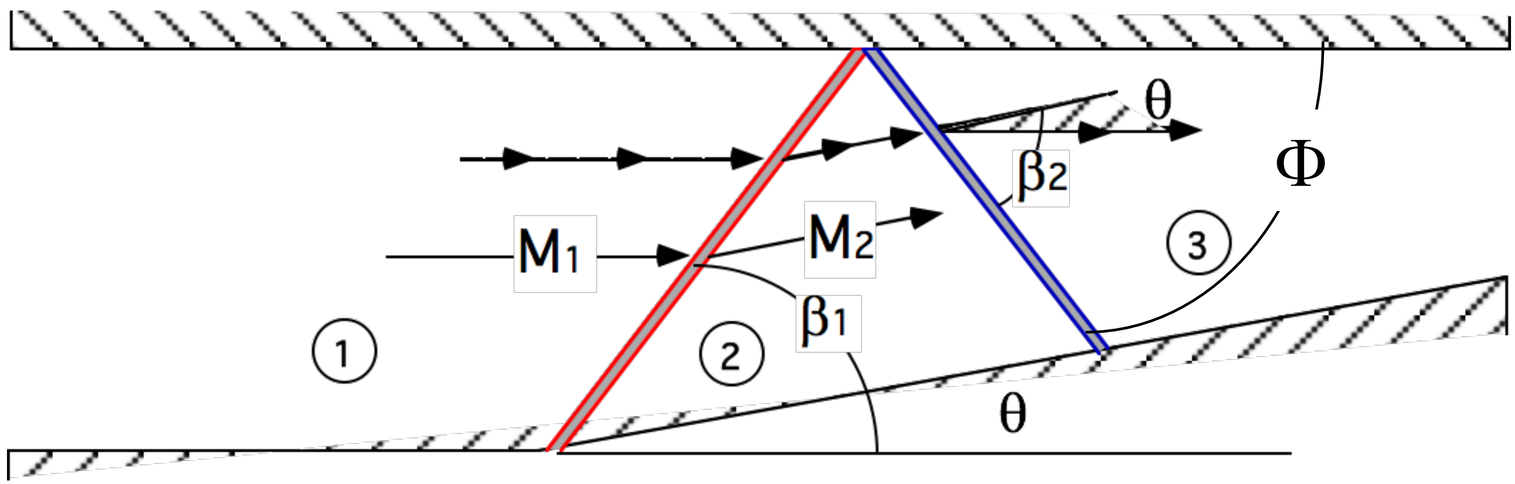
1. Waves Incident on a Solid Boundary reflect in a Like manner;
   Compression wave reflects as compression wave, expansion wave reflects as expansion wave

2. Waves Incident on a Free Boundary reflect in an Opposite manner;
   Compression wave reflects as expansion wave, expansion wave reflects as compression wave



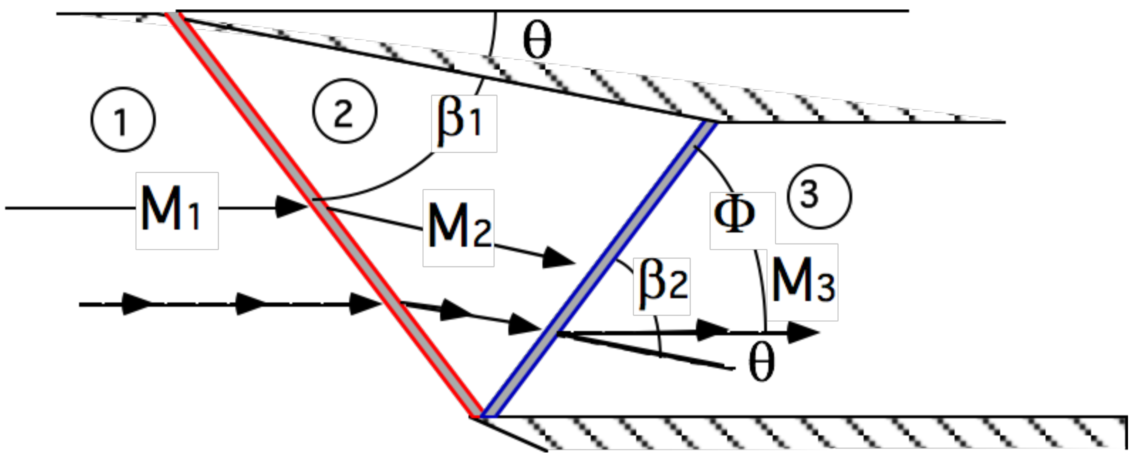
Schematic of the diamond wave pattern in the exhaust from a supersonic nozzle.

Geometry of a Shock Wave: Reflected From a Solid Boundary



$\beta_1 \neq \beta_2$ • Not Billiards … Why?

SCRAMJet Inlet Design Example



- Find β_2, Φ, M_3

- Example Calculation

$$M_1 = 3.6$$

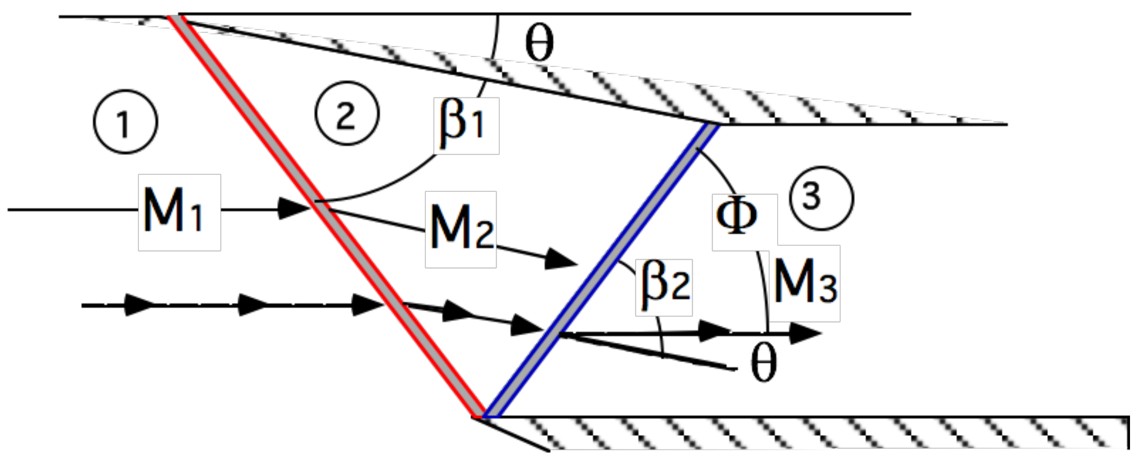
$$\theta = 20^o$$

$$\gamma = 1.4$$

$$\tan(\theta) = \frac{2\left\{M_1^{\,2}\sin^2(\beta) - 1\right\}}{\tan(\beta)\left[2 + M_1^{\,2}\left[\gamma + \cos(2\beta)\right]\right]} \rightarrow$$

$$\frac{180}{\pi}\,\mathrm{atan}\left(\frac{2\left(3.6^2\sin^2\left(\frac{\pi}{180}34.1102\right) - 1\right)}{\left(\tan\left(\frac{\pi}{180}34.1102\right)\right)\left(2 + 3.6^2\left(1.4 + \cos\left(\frac{\pi}{180}2\cdot 34.1102\right)\right)\right)}\right) = 20^\circ$$

SCRAMJet Inlet Design Example (cont'd)



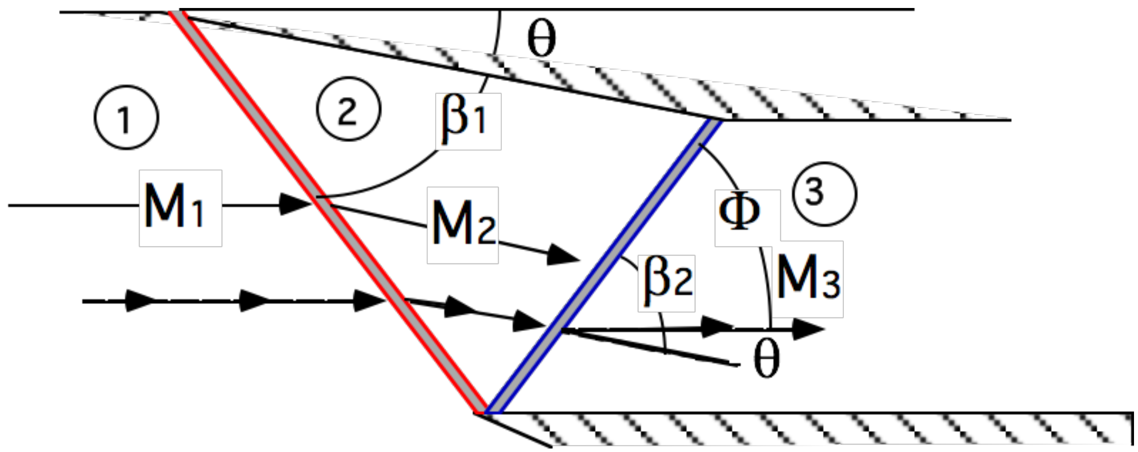
- Solve for β_1

$$\tan(\theta) = \frac{2\left\{M_1^{\,2}\sin^2(\beta)-1\right\}}{\tan(\beta)\left[2+M_1^{\,2}\left[\gamma+\cos(2\beta)\right]\right]} \rightarrow$$

$$\frac{180}{\pi}\operatorname{atan}\left(\frac{2\left(3.6^2\sin^2\left(\frac{\pi}{180}34.1102\right)-1\right)}{\left(\tan\left(\frac{\pi}{180}34.1102\right)\right)\left(2+3.6^2\left(1.4+\cos\left(\frac{\pi}{180}2\cdot 34.1102\right)\right)\right)}\right) = 20°$$

$$\boxed{\beta_1 = 34.1102^\circ}$$

SCRAMJet Inlet Design Example (cont'd)

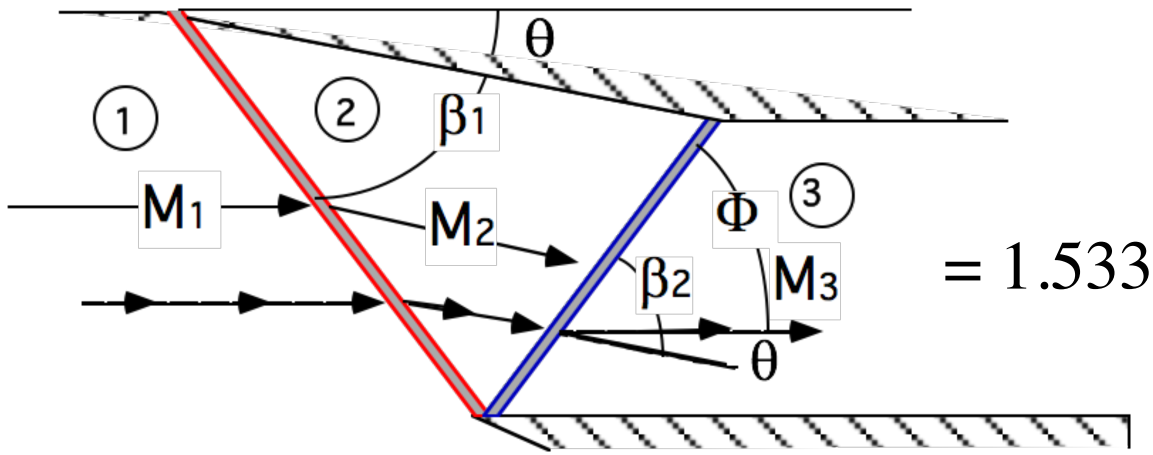


- $M_{1n} = M_1 \sin \beta_1 = 3.6 \sin\left(\frac{\pi}{180} 34.1102\right) = 2.0188$

- Normal Shock Solver-->

$$M_2 n = 0.574168 \rightarrow M_2 = \frac{M_2 n}{\sin(\beta_1 - \theta)} = \frac{0.574168}{\sin\left(\frac{\pi}{180}(34.1102 - 20)\right)} = 2.3552$$

SCRAMJet Inlet Design Example (cont'd)



$= 1.5333$

θ-β-M solver, for $M_2 = 2.3552$ and $\theta_2 = 20°$,

---> $\beta_2 = 45.0534°$ ——> $\Phi = 45.0534° – 20° = 25.0534°$

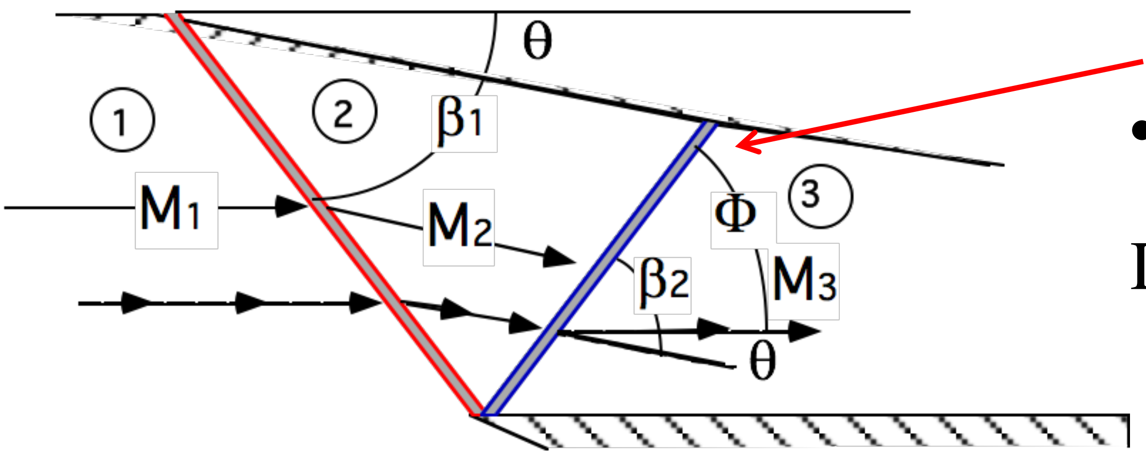
$$\left(M_2 n\right)_{\beta_2} = 2.3552 \sin(45.0534) = 0.649303$$

$$\rightarrow M_3 = \frac{\left(M_2 n\right)_{\beta_2}}{\sin(\beta_2 - \theta)} = \frac{0.649303}{\sin\left(\frac{\pi}{180}(45.0534 - 20)\right)} = 1.5333$$

So we have Gotten inside The duct and Still are supersonic!

Off Design Operation

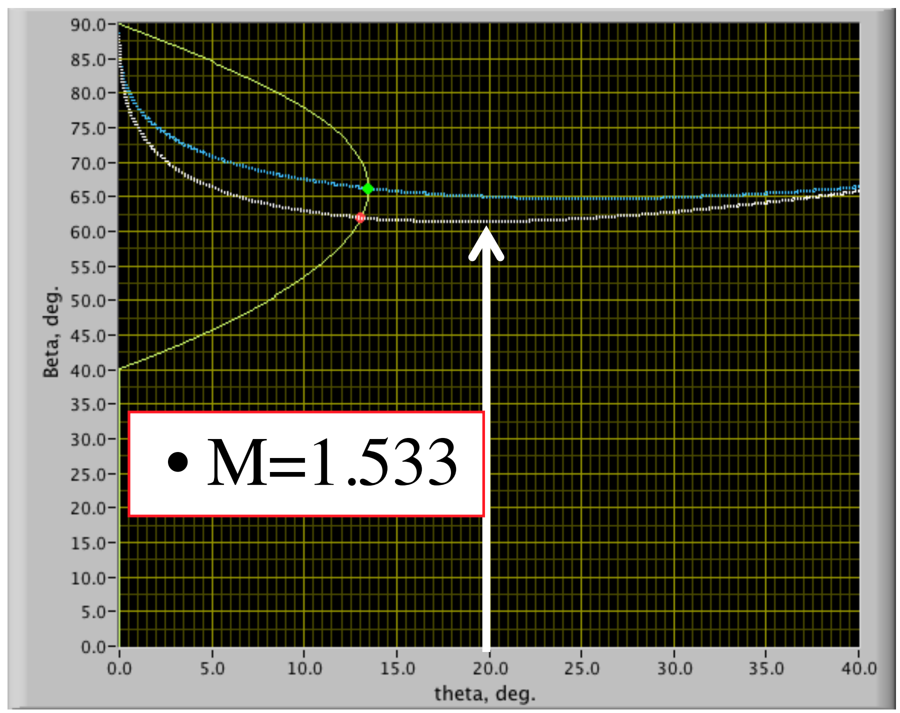
- What if we do out ramp geometry incorrectly for inlet mach?



- $20° > \theta_{max}$

Detached Shockwave!

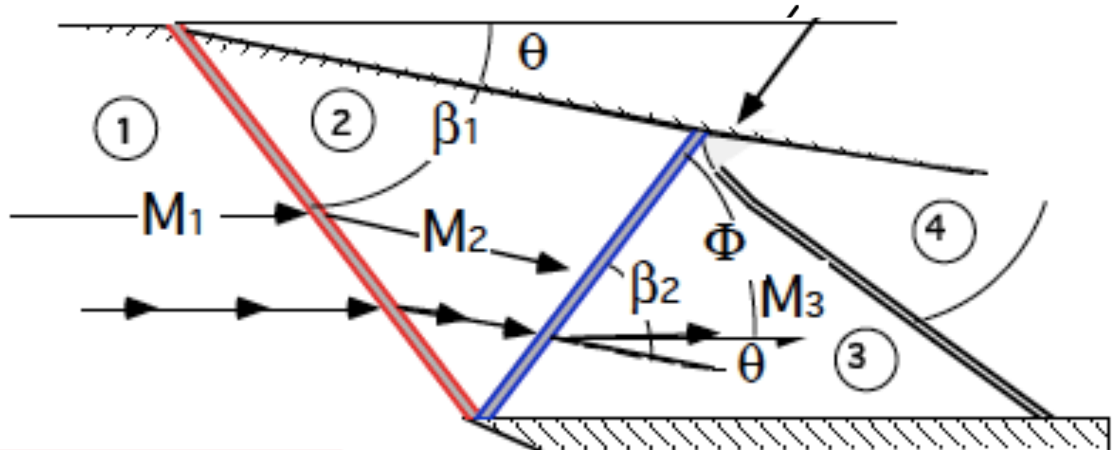
θ-β-M solver, for
$M_2 = 2.3552$ and $\theta_2 = 20°$,

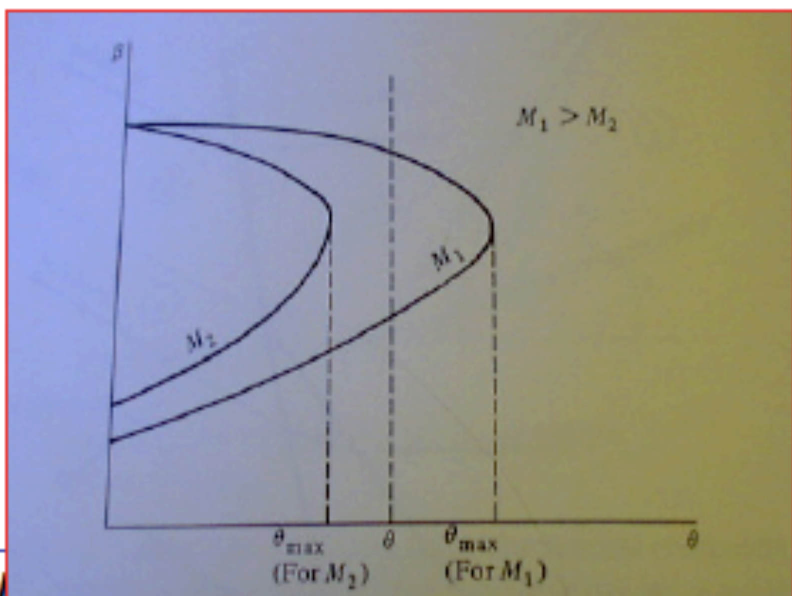


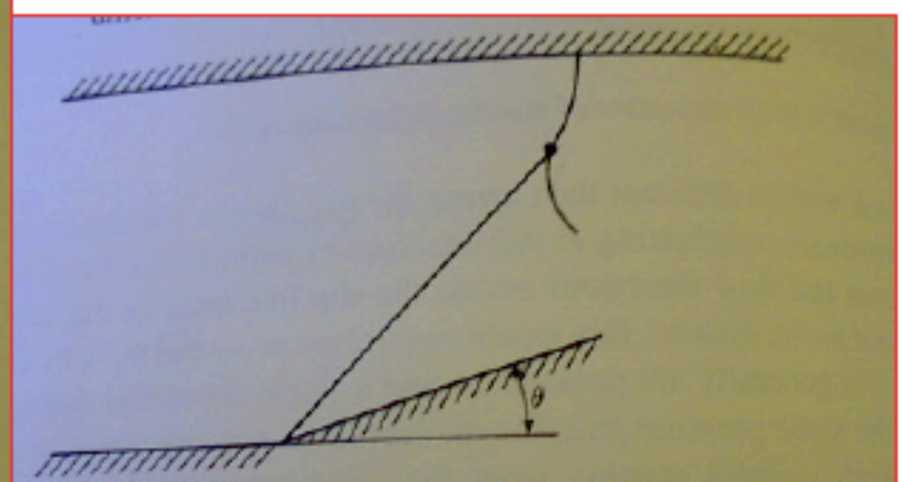
Off Design Operation

(cont'd)

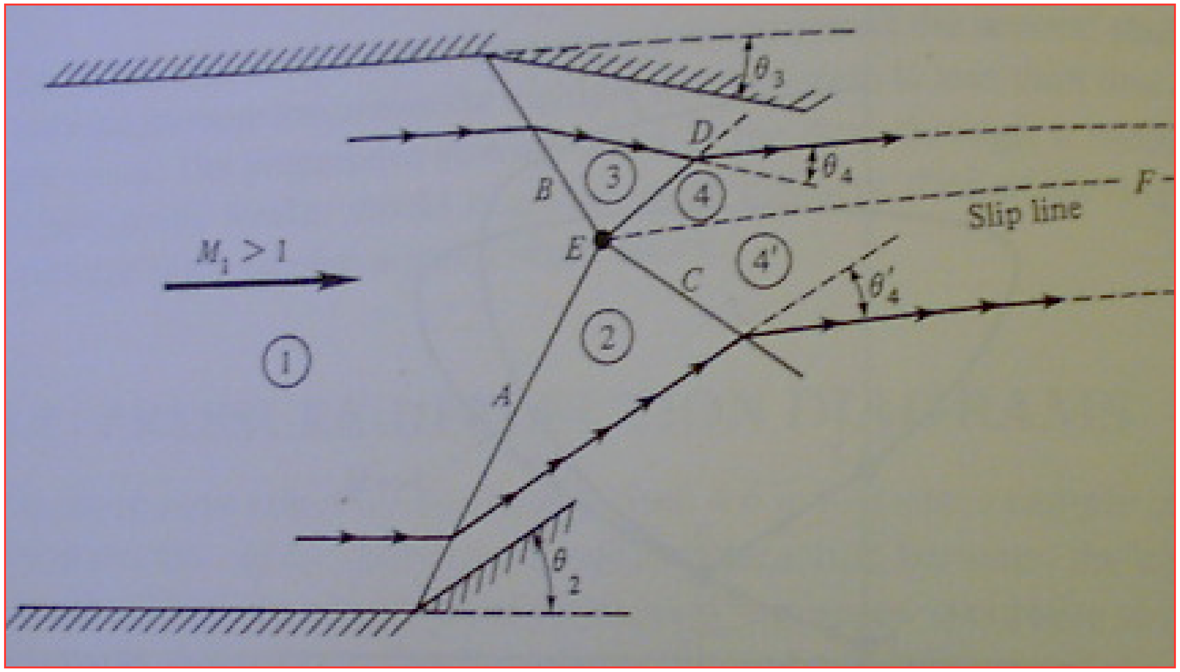
Mach reflection ... localized strong shockwave ... starts bad train of events leading to flow separation and possible unstart







Geometry of a Shock Wave: Reflected From another Shock wave

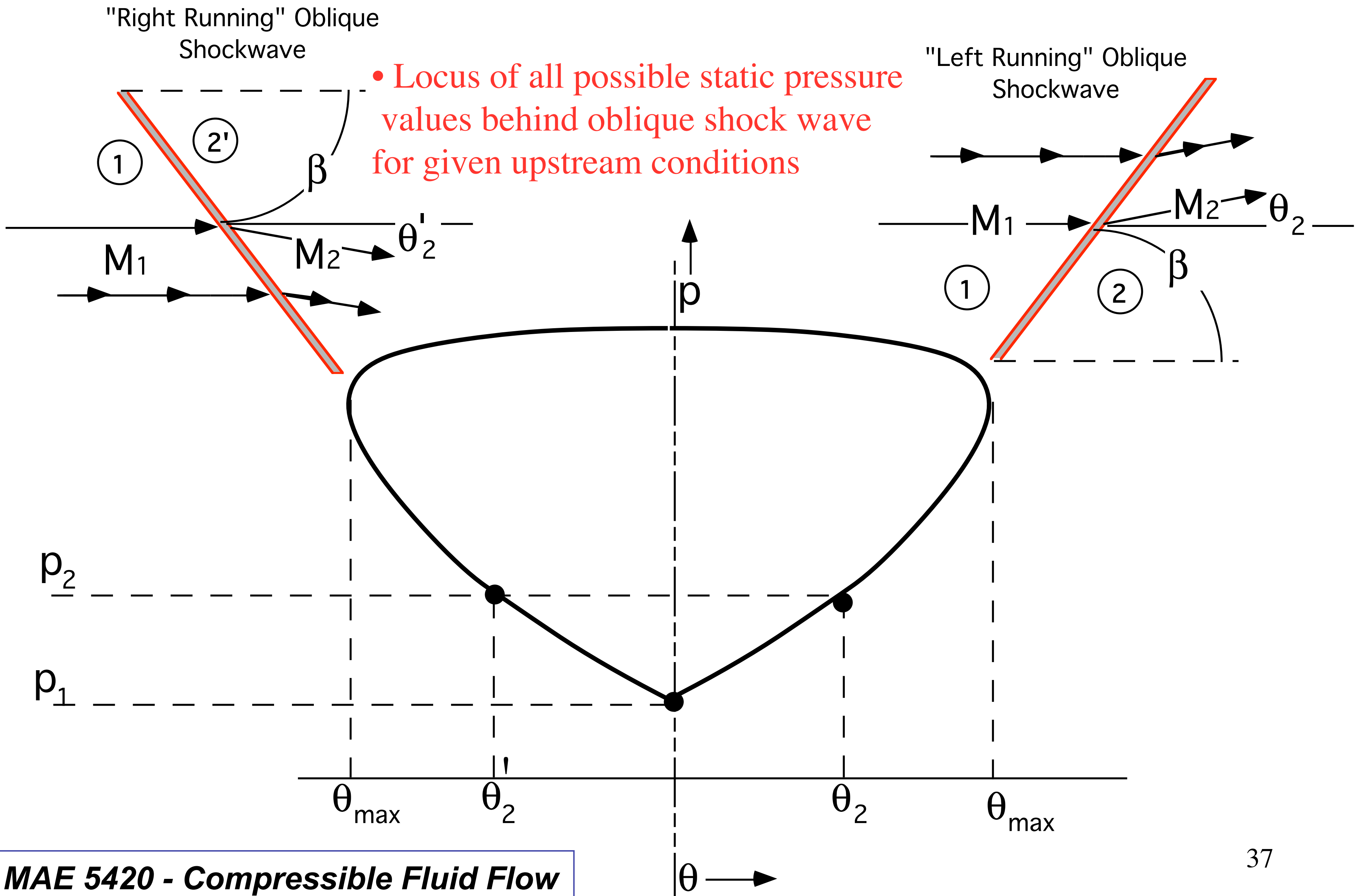


- Shock waves not only reflect from solid boundaries, they also reflect from each other

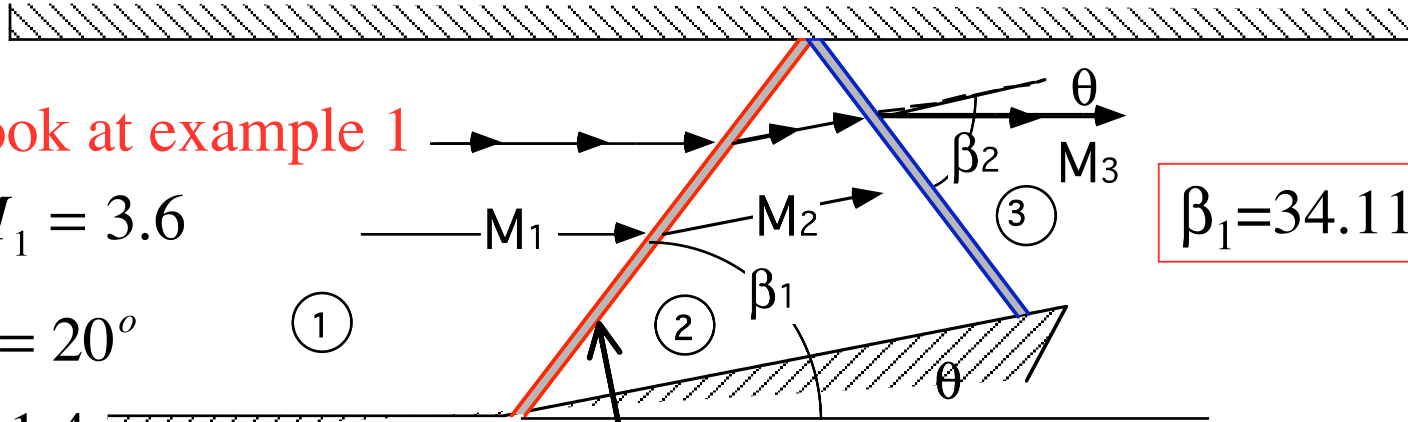
- *Need "more tools" to analyze problem completely*

Pressure-Deflection Diagrams

- Locus of all possible static pressure values behind oblique shock wave for given upstream conditions



Pressure-Deflection Diagrams (cont'd)



- Look at example 1

$M_1 = 3.6$

$\theta = 20^o$

γ=1.4

- Left running wave

$\boxed{\beta_1 = 34.1102^\circ}$

- $M_{1n} = M_1 \sin \beta_1 = 3.6 \sin\left(\frac{\pi}{180} 34.1102\right) = 2.0188$
- Normal Shock Solver-->

$$M_2 n = 0.574168 \rightarrow M_2 = \frac{M_2 n}{\sin(\beta_1 - \theta)} = \frac{0.574168}{\sin\left(\frac{\pi}{180}(34.1102 - 20)\right)} = 2.3552$$

$\boxed{p_2/p_1 = 4.588283}$

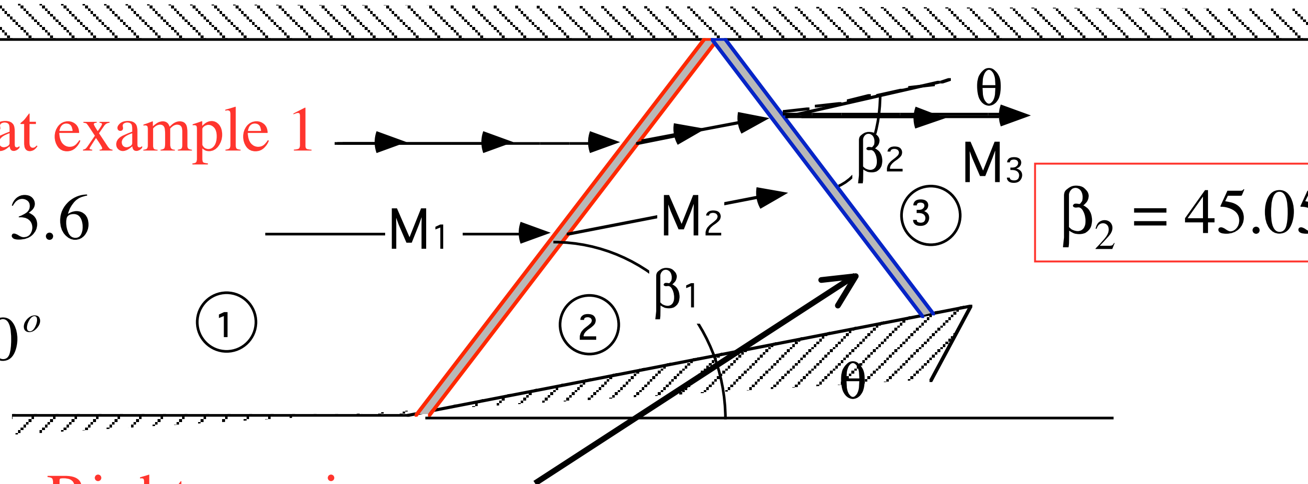
Pressure-Deflection Diagrams (cont'd)

- Look at example 1

$M_1 = 3.6$

$\theta = 20^o$

$\gamma = 1.4$



- Right running wave

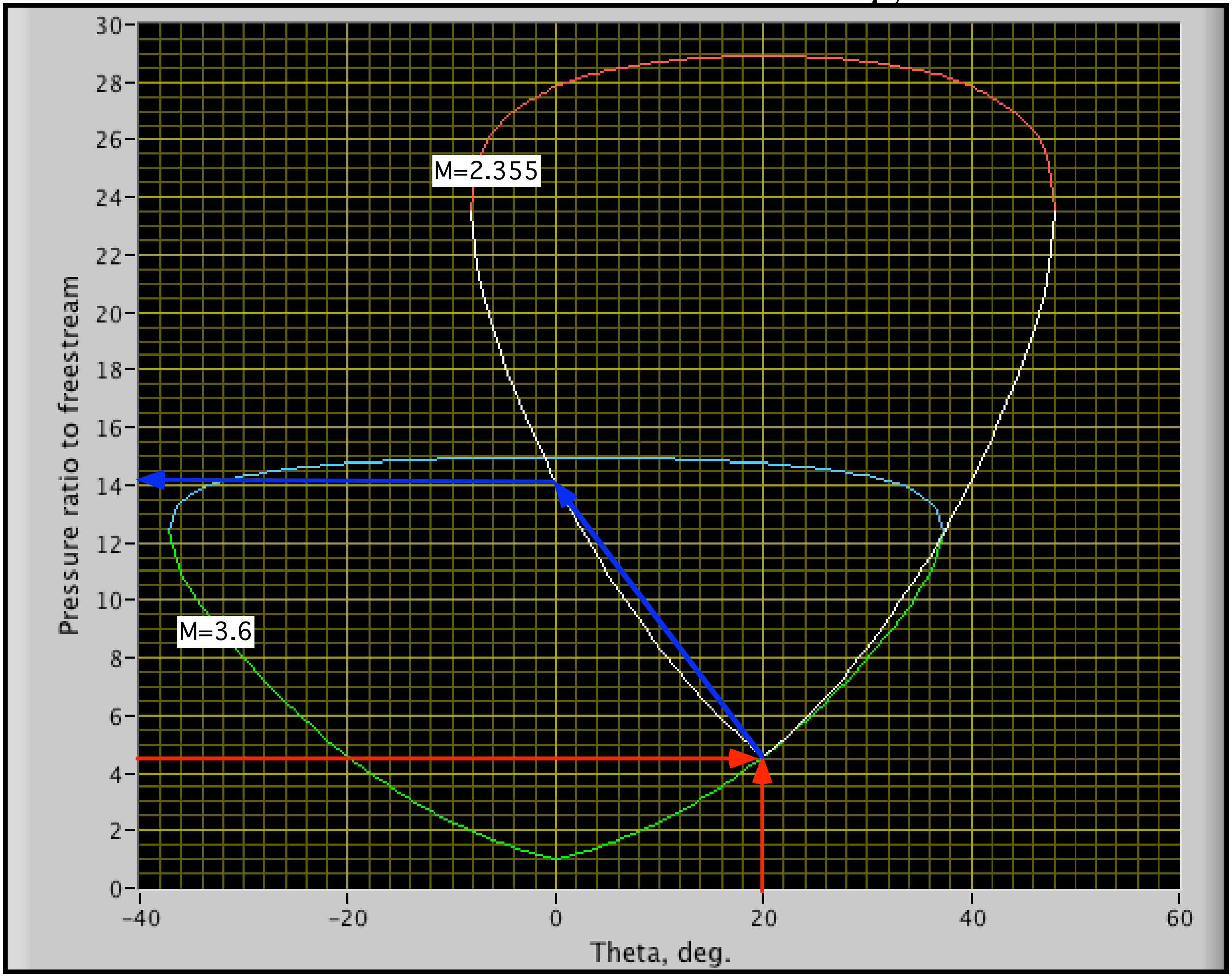
$\beta_2 = 45.0534°$

$$\left(M_2 n\right)_{\beta_2} = 2.3552 \sin(45.0534) = 0.649303$$

$$\rightarrow M_3 = \frac{\left(M_2 n\right)_{\beta_2}}{\sin(\beta_2 - \theta)} = \frac{0.649303}{\sin\left(\frac{\pi}{180}(45.0534 - 20)\right)} = 1.5333$$

$p_3/p_2 = 3.075094$ --->
$p_3/p_1 = (p_3/p_2)(p_2/p_1) = (3.075094)(4.588283) = 14.109$

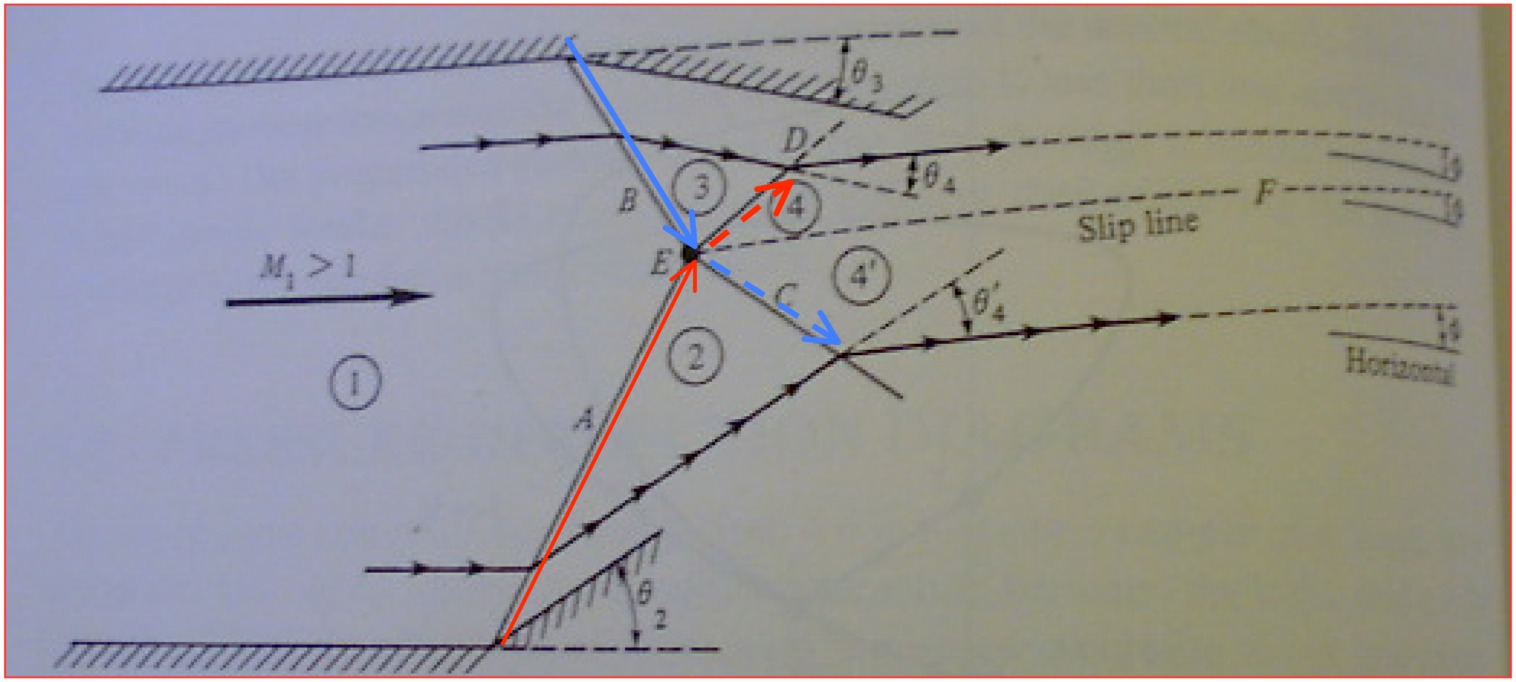
Pressure-Deflection Diagrams (cont'd)



Intersection of Shocks of Opposite Families

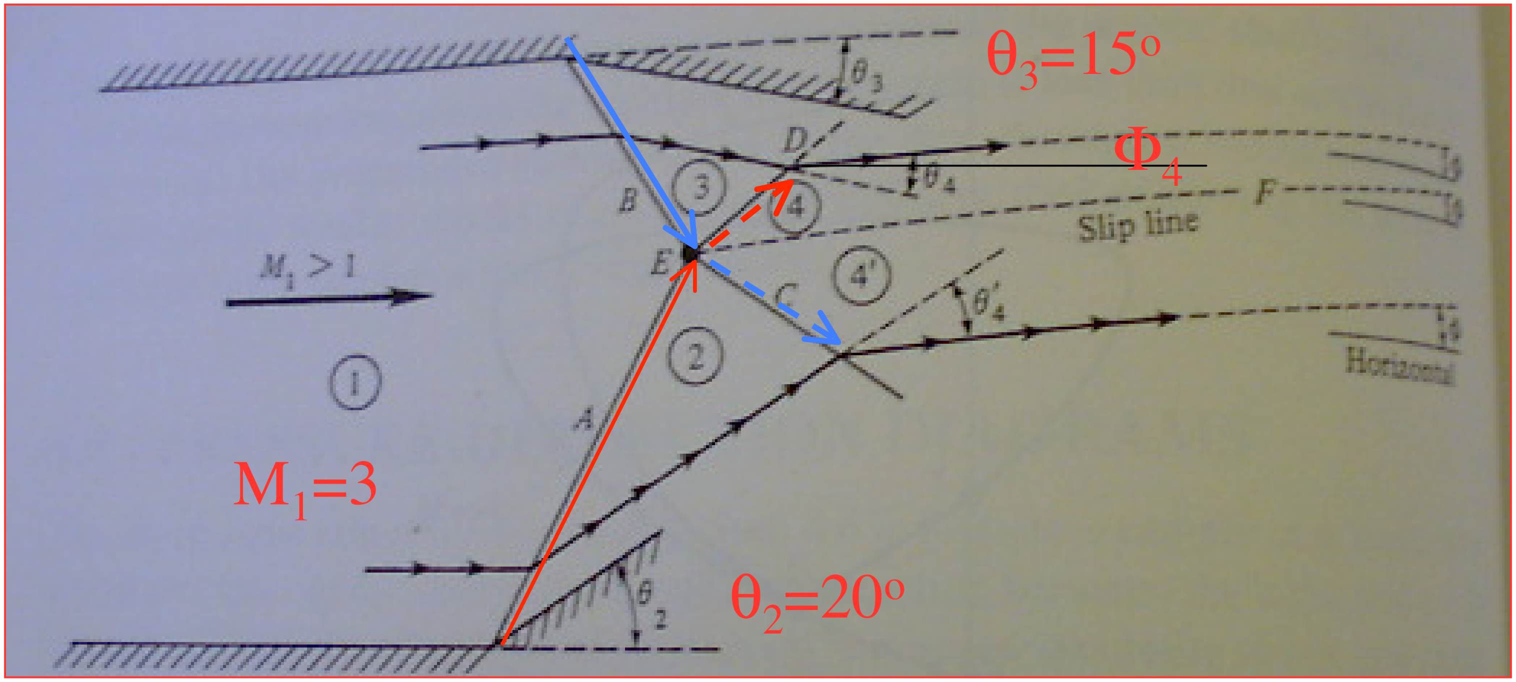
Intersection of “left-running” and “right running” Shock waves

Intersection of Shocks of Opposite Families (cont'd)



- Shock emanating from {A,B} intersect and continue as refracted shocks {C,D} .. At intersection point E, $p_4 = p'_4$

Intersection of Shocks of Opposite Families, Example



- Example: M_1=3, p_1=*1atm*, θ_2=20°, θ_3=15°
- Plot p,θ diagram, p_4, and flow direction Φ_4

Numerical Calculations for Example

From the θ-β-M solver: For $\theta_2 = 20°$ and $M_1 = 3$, $\beta = 37.8°$.

$$M_{n_1} = M_1 = \sin\beta = (3\ )\sin(37.8) = 1.839;\ \frac{p_2}{p_1} = 3.783$$

$$M_{n_2} = 0.6078;\ M_2 = \frac{M_{n_2}}{\mathrm{Sin}(\beta-\theta)} = \frac{0.6078}{\mathrm{Sin}(37.8-20)} = 1.99$$

For $\theta_3 = 15°$ and $M_1 = 3$, $\beta = 32.2$

$$M_{n_1} = M_1 \sin\beta = (3)\sin(32.2) = 1.60;\ \frac{p_3}{p_1} = 2.82$$

$$M_{n_3} = 0.6684;\ M_3 = \frac{M_{n_3}}{Sin(\beta-\theta)} = \frac{0.6684}{\mathrm{Sin}(32.2-15)} = 2.26$$

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Numerical Calculations for Example

(cont'd)

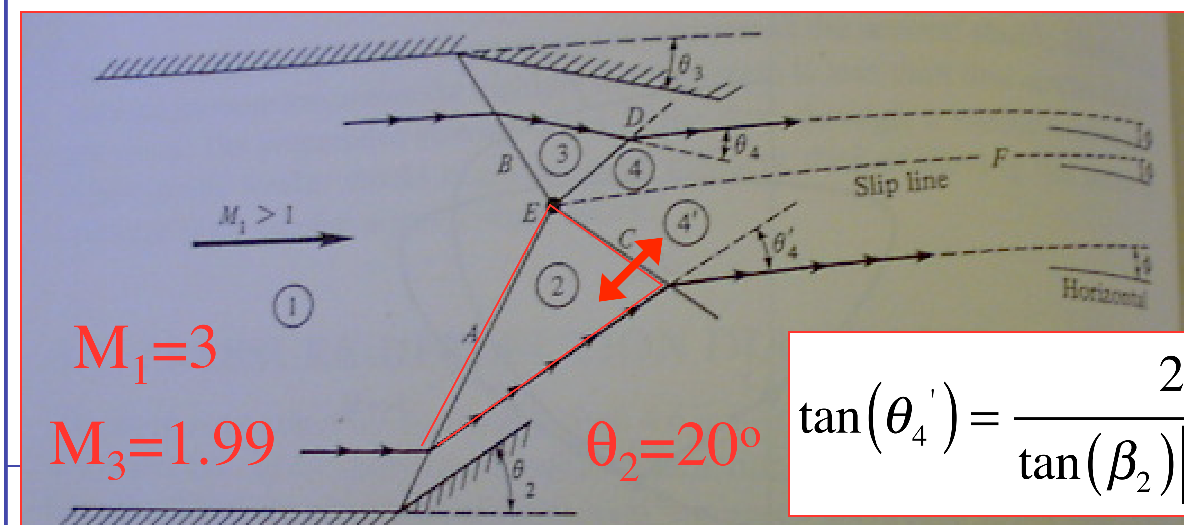
For the upstream flow represented by region 2, plot a pressure deflection diagram from the following calculations:

β_2	θ_4'	$M_{n_2} = M_2 \sin\beta$	$\frac{p_4'}{p_2}$	$\frac{p_4'}{p_1} = \frac{p_4'}{p_2}\frac{p_2}{p_1}$	$\Delta\theta = \theta_2 - \theta_4'$
30	0	1	1	3.783	20
33.3	4	1.09	1.219	4.61	16
37.2	8	1.20	1.513	5.72	12
41.6	12	1.32	1.866	7.06	8
46.7	16	1.45	2.286	8.65	4
53.5	20	1.60	2.820	10.67	0

Pick these values (for β_2)

calculate these values (for θ_4')

$$\frac{p_2}{p_1} = 3.783$$



$$\tan(\theta_4') = \frac{2\left\{M_2^2 \sin^2(\beta_2) - 1\right\}}{\tan(\beta_2)\left[2 + M_2^2\left[\gamma + \cos(2\beta_2)\right]\right]}$$

Numerical Calculations for Example

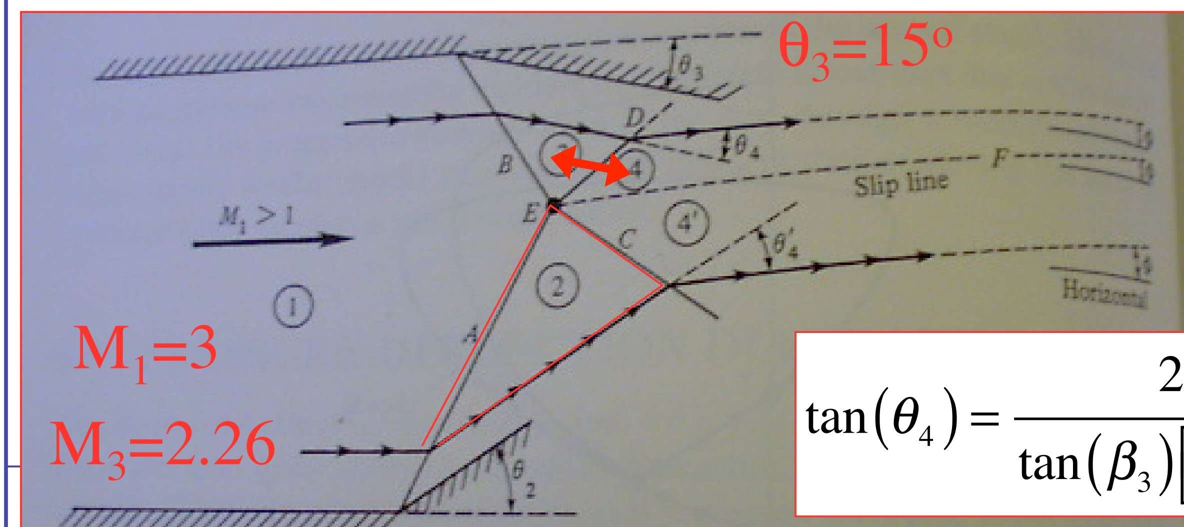
(cont'd)

For the upstream flow represented by region 3, plot a pressure deflection diagram from the following calculations:

β_3	θ_4	$M_{n_3} = M_3 \sin\beta$	$\frac{p_4}{p_2}$	$\frac{p_4}{p_1} = \frac{p_4}{p_3}\frac{p_3}{p_1}$	$\Delta\theta = \theta_4 - \theta_3$
26	0	1	1	2.82	-15
29	4	1.096	1.23	3.47	-11
33	8	1.23	1.598	4.51	- 7
36.8	12	1.35	1.96	5.53	- 3
41.5	16	1.50	2.458	6.93	1
46.8	20	1.65	3.01	8.49	5
53.7	24	1.82	3.698	10.43	9

Pick these values

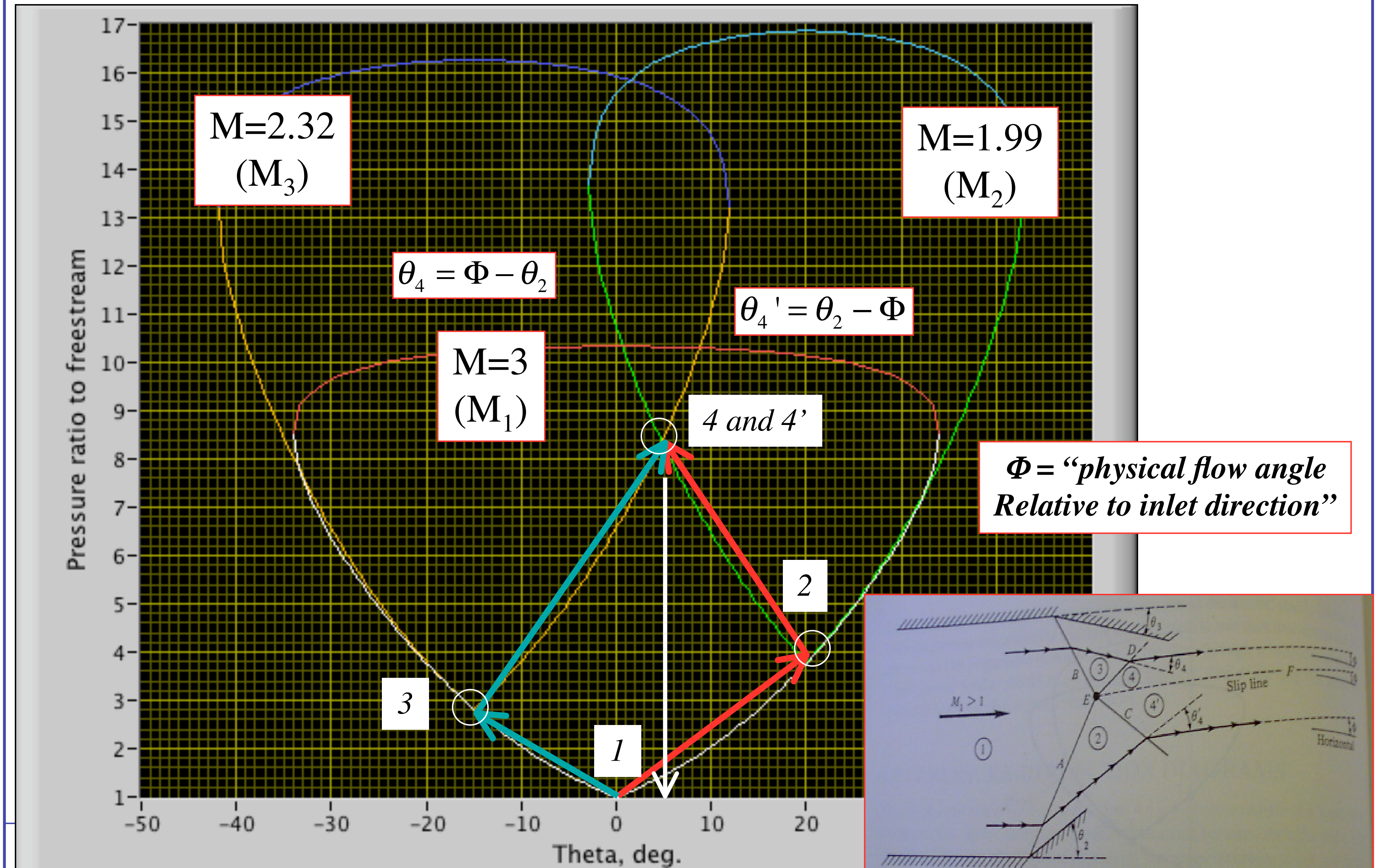
calculate these values



$$\frac{p_3}{p_1} = 2.82$$

$$\tan(\theta_4) = \frac{2\{M_3^2 \sin^2(\beta_3) - 1\}}{\tan(\beta_3)\left[2 + M_3^2\left[\gamma + \cos(2\beta_3)\right]\right]}$$

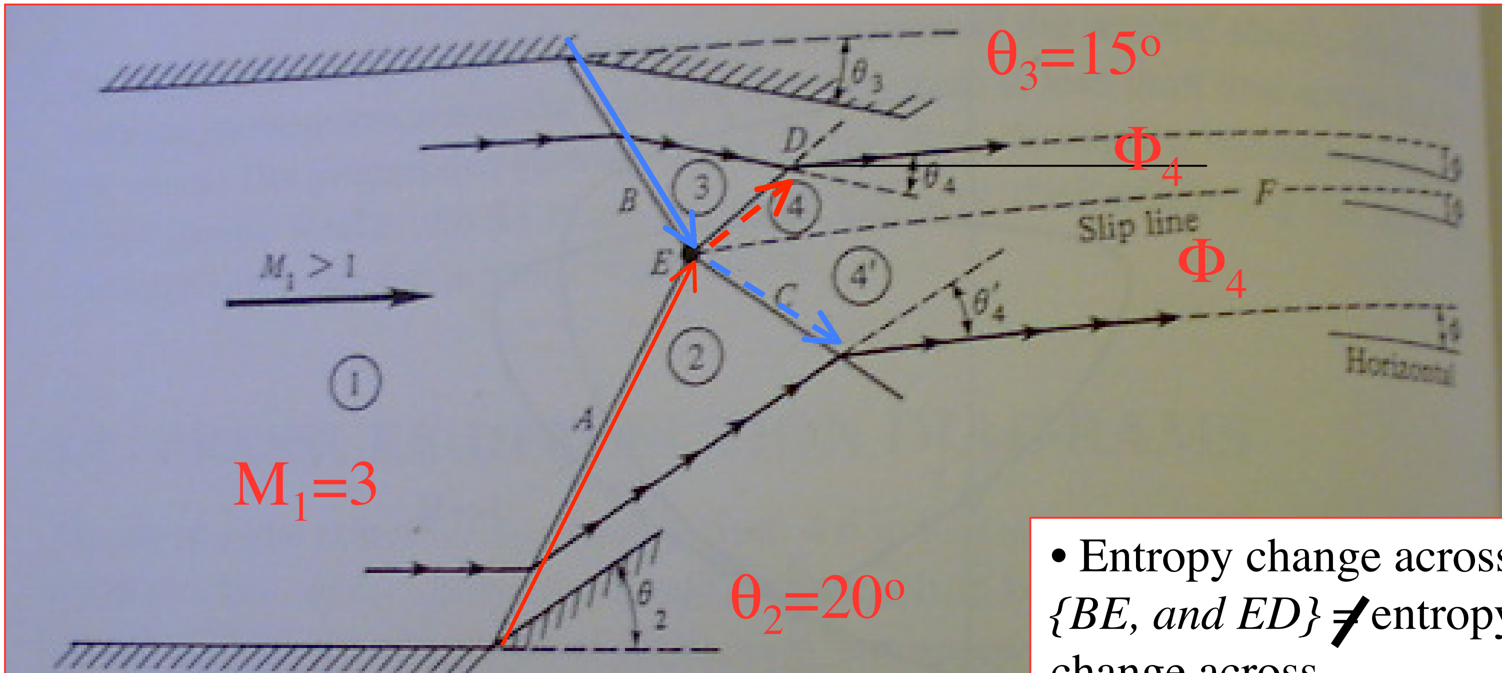
Pressure-deflection-diagram for Example



Φ = "physical flow angle Relative to inlet direction"

Numerical Calculations for Example

(cont'd)



From the graph on previous page

$$p_4 = p_4{}' = \frac{p_4}{p_1}\, p_1 = (8.3)(1) = \boxed{8.3\ \text{atm}}$$

$$\boxed{\Phi = 4.5^\circ}$$

• Entropy change across *{BE, and ED}* ≠ entropy change across Shocks *{AE, and EC}* ….

Creates "slip line" emanating From E between regions 4 and 4'

Intersection of Shocks of The Same Family

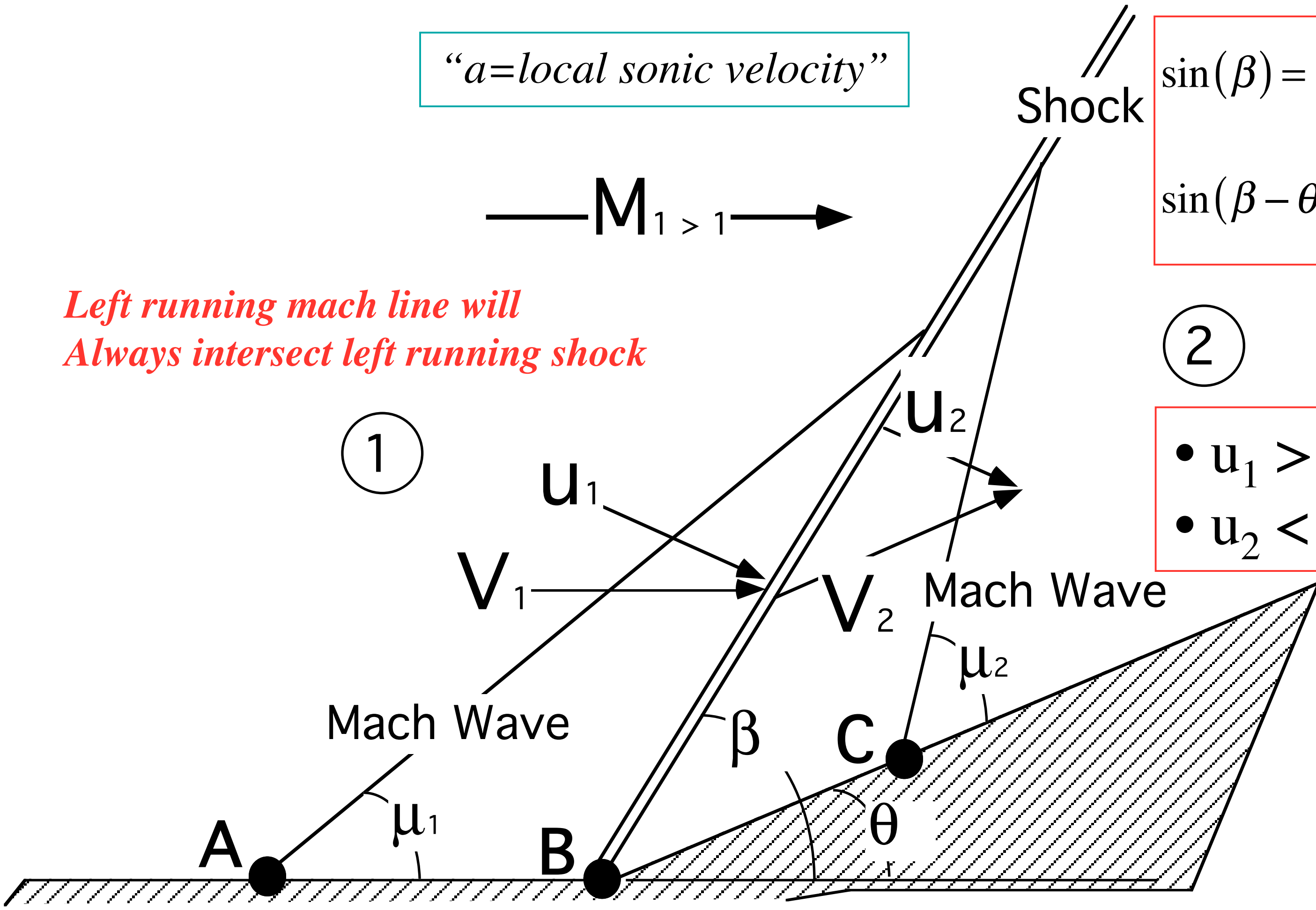
- Will Mach waves intersect shock wave?

"a=local sonic velocity"

$$\sin(\beta) = \frac{u_1}{V_1} \rightarrow \sin(\mu_1) = \frac{a_1}{V_1}$$

$$\sin(\beta - \theta) = \frac{u_2}{V_2} \rightarrow \sin(\mu_2) = \frac{a_2}{V_2}$$

Left running mach line will Always intersect left running shock



- $u_1 > a_1$ --> $\mu_1 < \beta$
- $u_2 < a_2$ --> $\mu_2 > \beta - \theta$

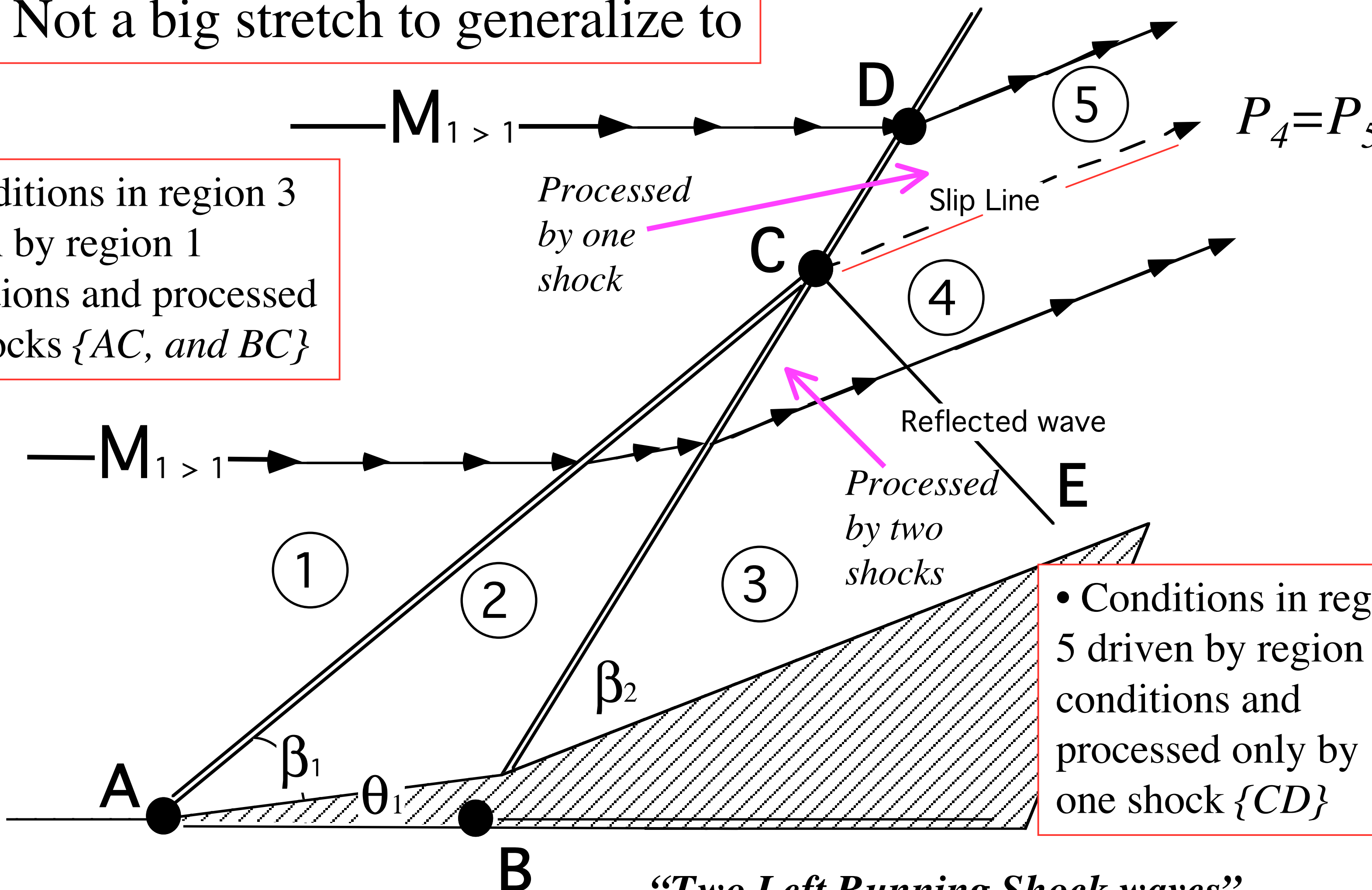
- **yes!**

Intersection of Shocks of The Same Family

(cont'd)

- Not a big stretch to generalize to

- Conditions in region 3 driven by region 1 conditions and processed by shocks *{AC, and BC}*



- Conditions in region 5 driven by region 1 conditions and processed only by one shock *{CD}*

"Two Left Running Shock waves"

Intersection of Shocks of The Same Family

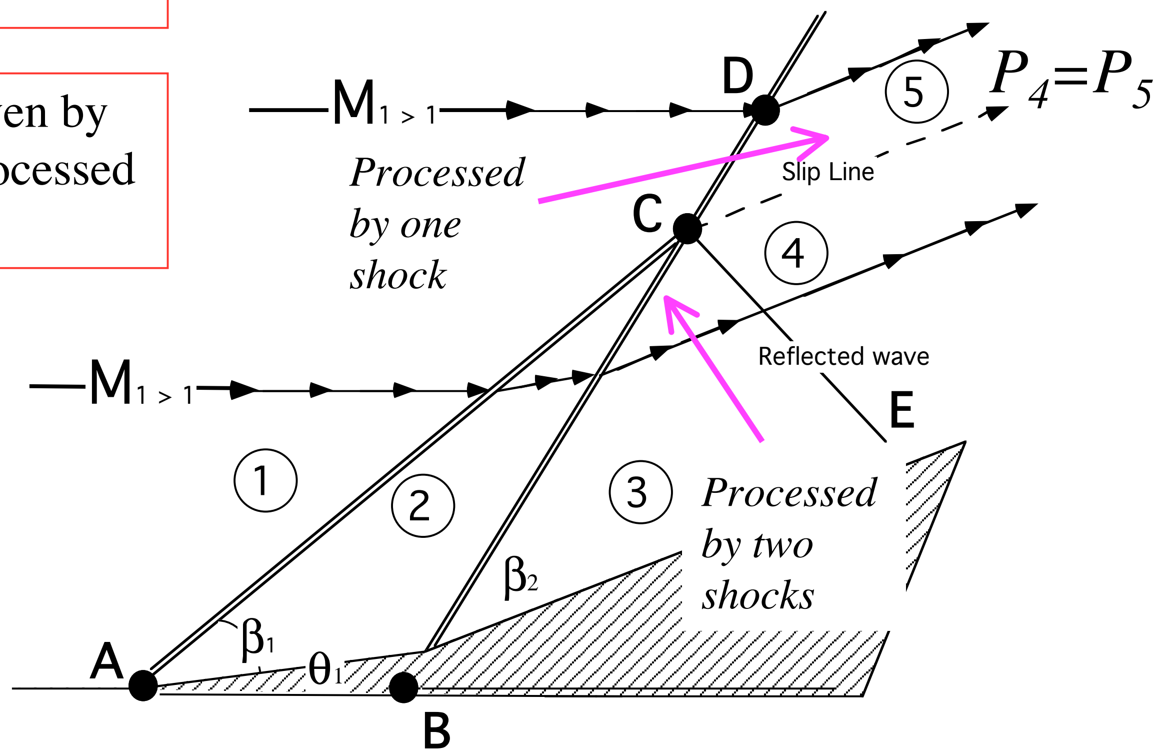
(cont'd)

- Conditions in region 3 driven by Upstream conditions and processed by shocks *{AC, and BC}*

- Conditions in region 5 driven by Upstream conditions and processed Only by shock *{CD}*

- Entropy change across *{CD}* ≠ entropy change across Shocks *{AC, and BC}* ….

  Creates "slip line" emanating From C between regions 4 and 5



"Two Left Running Shock waves"

Intersection of Shocks of The Same Family

(cont'd)

- within slip zone $P_4=P_5$ *and* $\theta_4 = \theta_5$,

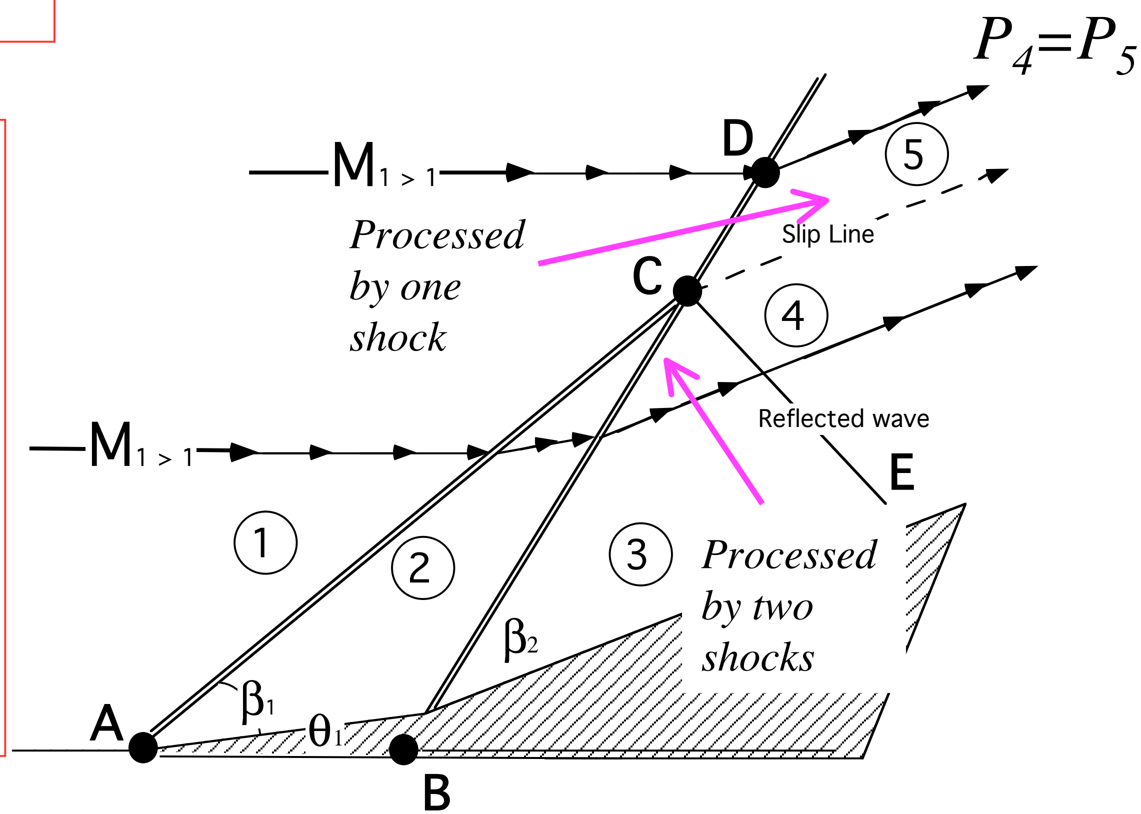
- in general for arbitrary

Shock waves … $P_3 \neq P_5$

But .. $P_4=P_5$

… How?…Reflected wave E...

… weak shock or
Expansion wave depending
On relationship of P_3 to P_5



"Two Left Running Shock waves"

Intersection of Shocks of The Same Family

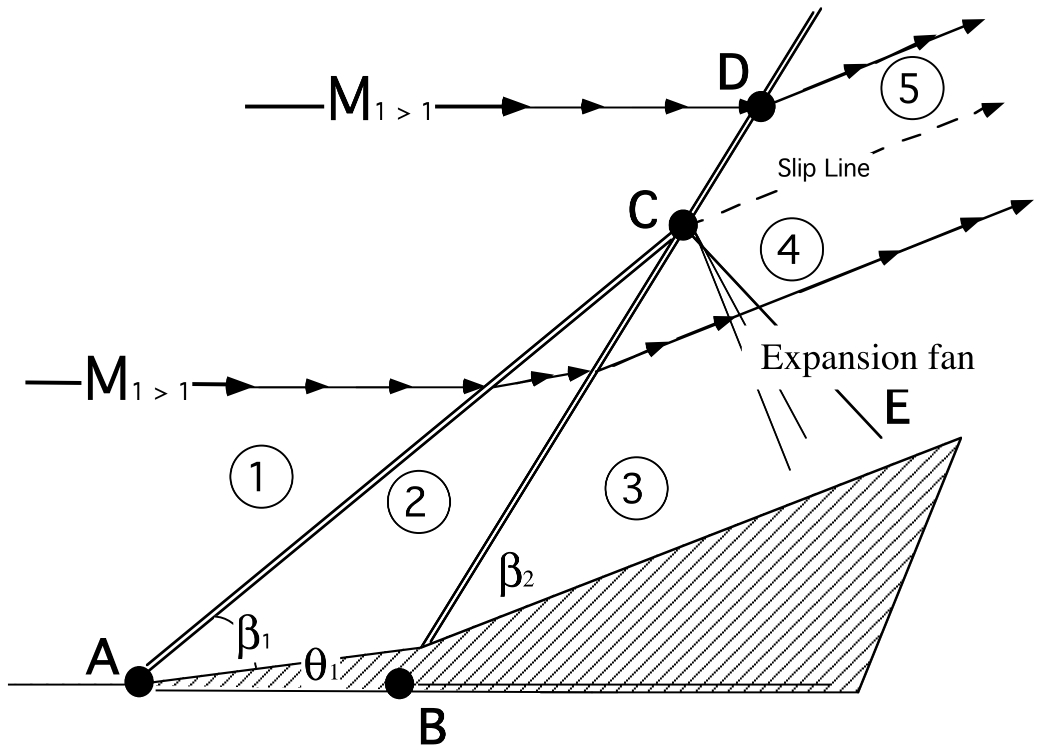
(cont'd)

- within slip zone $P_4=P_5$
  and $\theta_4 = \theta_{5,\ also}\ \theta_3 = \theta_5$

$P_3 > P_5$

… expansion fan drops

P_3 to P_3 (= P_5)



"Two Left Running Shock waves"

Intersection of Shocks of The Same Family

(cont'd)

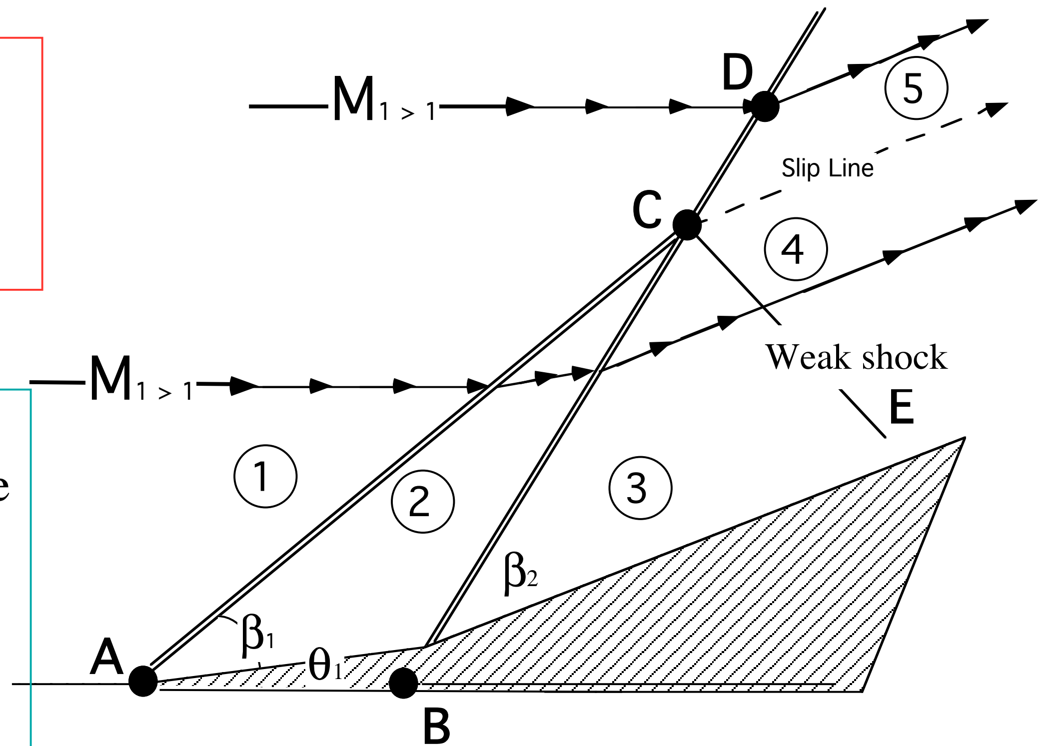
• within slip zone $P_4 = P_5$
and $\theta_4 = \theta_{5,\ also}\ \theta_3 = \theta_5$

$P_3 < P_5$

… weak shock wave
compresses P_3 to P_4 (= P_5)

• Model must be smart enough
To accommodate this difference

Must iteratively adjust
Strength of waves CD and CE
Until $P_4 = P_5$ on output side



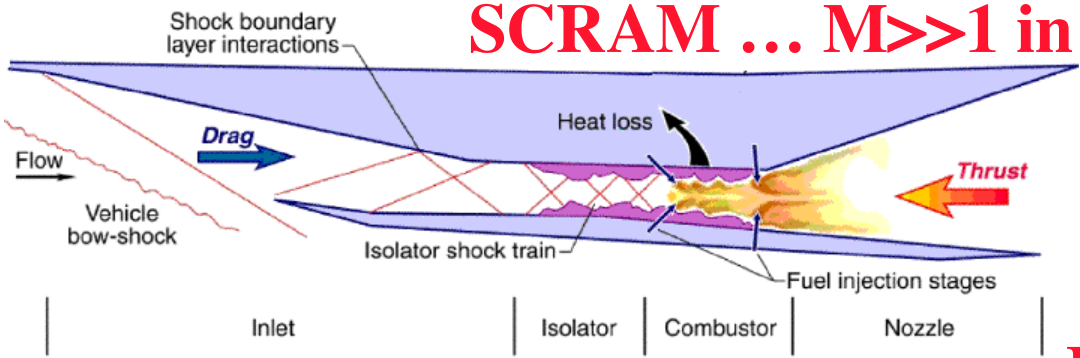
"Two Left Running Shock waves"

ScrRamjet Design Issues, II

• Supersonic flow makes flow control within the combustion chamber more difficult.

• Massflow entering combustion chamber must mix with fuel and have sufficient time for initiation and reaction, while traveling supersonically through combustion chamber, before the burned gas is expanded through the thrust nozzle.

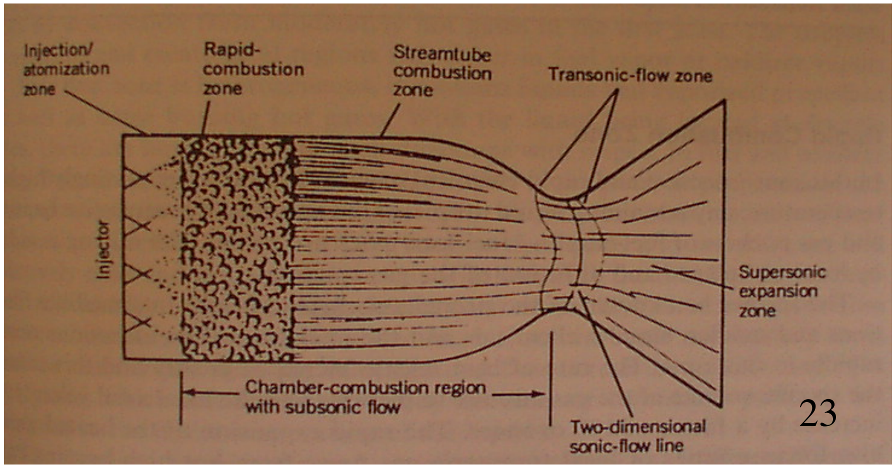
SCRAM … M>>1 in burner



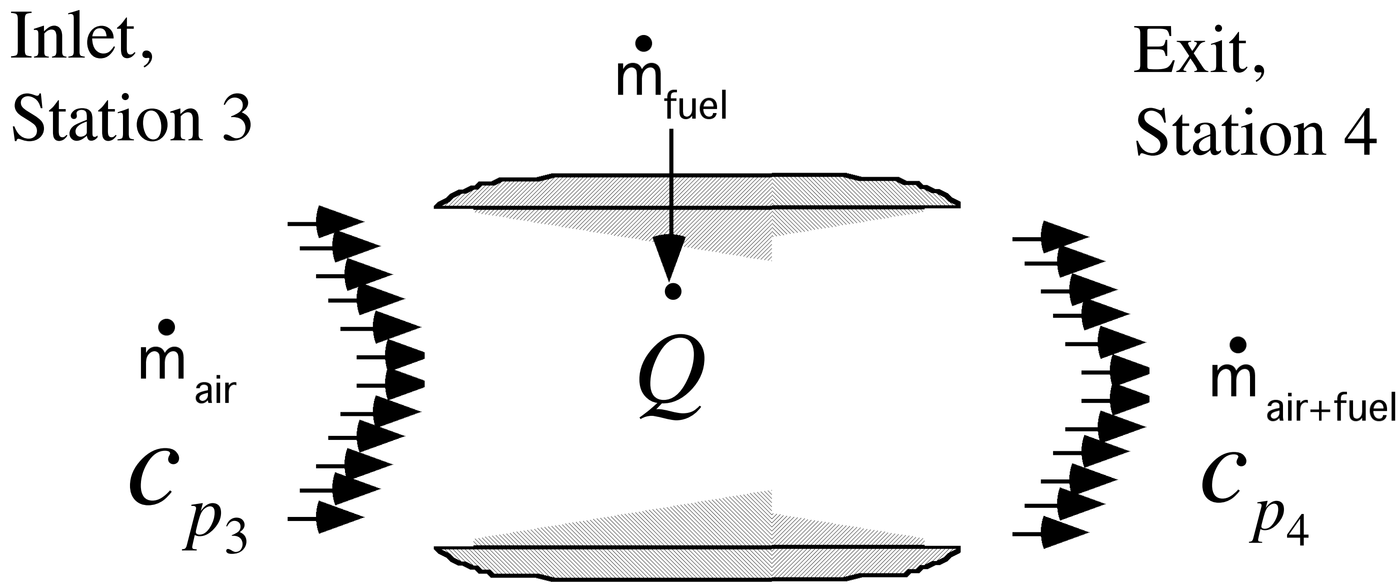
Rocket … M<<1 in burner

$$aA + bB \underset{k_b}{\overset{k_f}{\rightleftarrows}} cC + dD$$

Reaction times are Critically tuned to Flow path mach number



Review of 1-D Combustion Model



Consider generic combustor model

i) *Enthalpy Balance*

$$\dot{Q} = \dot{m}_{fuel} \cdot h_{fuel} \rightarrow \dot{Q} + \dot{m}_{air}\left(h_3 + \frac{V_3^{\,2}}{2}\right) = \left(\dot{m}_{air} + \dot{m}_{fuel}\right)\left(h_4 + \frac{V_4^{\,2}}{2}\right)$$

Review of 1-D Combustion Model (2)

Consider generic combustor model

$$\dot{Q} = \dot{m}_{fuel} \cdot h_{fuel} \rightarrow \dot{Q} + \dot{m}_{air}\left(h_3 + \frac{V_3^{\,2}}{2}\right) = \left(\dot{m}_{air} + \dot{m}_{fuel}\right)\left(h_4 + \frac{V_4^{\,2}}{2}\right)$$

$$\dot{m}_{fuel} \cdot h_{fuel} + \dot{m}_{air}\left(h_3 + \frac{V_i^{\,2}}{2}\right) = \left(\dot{m}_{air} + \dot{m}_{fuel}\right)\left(h_e + \frac{V_e^{\,2}}{2}\right)$$

$$\frac{\dot{m}_{fuel}}{\dot{m}_{air}} \cdot h_{fuel} + \left(h_3 + \frac{V_3^{\,2}}{2}\right) = \left(1 + \frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right)\left(h_4 + \frac{V_4^{\,2}}{2}\right) \rightarrow$$

$$\frac{1}{f} \cdot h_{fuel} + \left(h_3 + \frac{V_3^{\,2}}{2}\right) = \left(\frac{f+1}{f}\right)\left(h_4 + \frac{V_4^{\,2}}{2}\right) \rightarrow f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

Review of 1-D Combustion Model (3)

• Stagnation Pressure Ratio Across Combustor is Proportional to the ratio of the fuel enthalpy and the incoming air stagnation enthalpy

$$\frac{T_{0_4}}{T_{0_3}} = \left(\frac{f}{f+1}\right)\left(\frac{1}{f}\cdot\frac{h_{fuel}}{c_{p_4}T_{0_3}} + \frac{c_{p_3}}{c_{p_4}}\right) = \left(\frac{f}{f+1}\right)\left(\frac{c_{p_3}}{c_{p_4}}\right)\left(1+\frac{1}{f}\cdot\frac{h_{fuel}}{c_{p_3}T_{0_3}}\right)$$

• Resulting Mach Number Change Across Combustor

$$\boxed{\frac{M_4{}^2\left[1+\frac{\gamma_4-1}{2}M_4{}^2\right]}{\left[1+\gamma_4 M_4{}^2\right]^2} = \left(\frac{f+1}{f}\right)^2\left[\frac{T_{04}}{T_{0_3}}\right]\left[\frac{\gamma_3 R_{g_4}}{\gamma_4 R_{g_3}}\right]\frac{M_3{}^2\left[1+\frac{\gamma_3-1}{2}M_3{}^2\right]}{\left[1+\gamma_3 M_3{}^2\right]^2}}$$

Review of 1-D Combustion Model (4)

$$F(M_3) \equiv \left(\frac{f+1}{f}\right)^2 \left[\frac{T_{04}}{T_{0_3}}\right]\left[\frac{\gamma_3 R_{g_4}}{\gamma_4 R_{g_3}}\right] \frac{M_3^{\,2}\left[1+\frac{\gamma_3-1}{2}M_3^{\,2}\right]}{\left[1+\gamma_3 M_3^{\,2}\right]^2}$$

$$\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]M_{exit}^{\,4}+\left[1-F(M_{inlet})2\gamma\right]M_{exit}^{\,2}-F(M_{inlet})=0$$

$\rightarrow$ *use quadratic formula*

$$M_{exit}=\sqrt{\frac{-\left[1-F(M_{inlet})2\gamma\right]\pm\sqrt{\left[1-F(M_{inlet})2\gamma\right]^2+4\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]F(M_{inlet})}}{2\left[\frac{\gamma-1}{2}-\gamma^2 F(M_{inlet})\right]}}$$

On the Difference Between Subsonic and Supersonic Combustion

$$M_{exit} = \sqrt{\frac{-\left[1-F(M_{inlet})2\gamma\right] \pm \sqrt{\left[1-F(M_{inlet})2\gamma\right]^2 + 4\left[\frac{\gamma-1}{2} - \gamma^2 F(M_{inlet})\right]F(M_{inlet})}}{2\left[\frac{\gamma-1}{2} - \gamma^2 F(M_{inlet})\right]}}$$

$$\textit{two solutions...pick subsonic solution if } (M_{inlet} < 1,\ \Delta q > 0)$$

$$\textit{...pick supersonic solution if } (M_{inlet} > 1,\ \Delta q > 0)$$

Cannot cross over Mach = 1
Otherwise second law of thermodynamics is violated

On the Difference Between Subsonic and Supersonic Combustion (2)

- MAE 5320 Lecture 5.4 (Anderson Chapter 3), Rayleigh Equations

$$H_0^{(*)} = \frac{c_p \cdot T_0}{c_p \cdot T_0^{(*)}} = 2\left(1+\gamma\right) \cdot \frac{M^2\left[1+\frac{\gamma-1}{2}M^2\right]}{\left[1+\gamma M^2\right]^2}$$

$$H^{(*)} \equiv \left[\frac{c_p \cdot T}{c_p \cdot T^{(*)}}\right] = \left[\frac{\left(1+\gamma\right)\cdot M}{1+\gamma \cdot M^2}\right]^2$$

$$\frac{s-s^{(*)}}{c_p} = \ln\left[M^2 \cdot \left(\frac{1+\gamma}{\left[1+\gamma \cdot M^2\right]}\right)^{\frac{\gamma+1}{\gamma}}\right]$$

$$\frac{\Delta q^{(*)}}{c_p \cdot T_o} = \frac{T_0^*}{T_o} - 1 = \frac{1-H_0^*}{H_0^*}$$

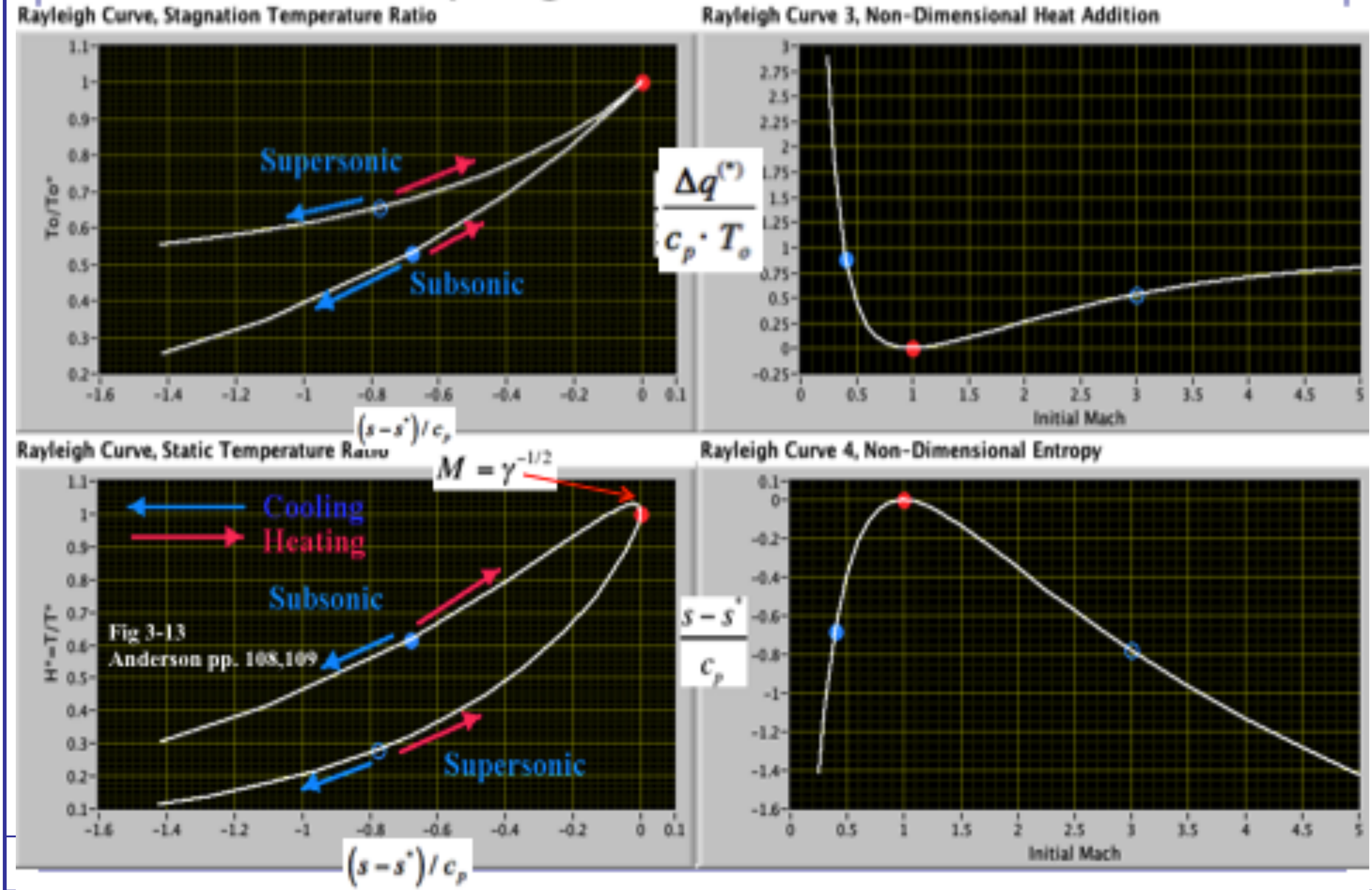
- Set of Parametric curves with property ratios as a function of Mach number

- Relates current conditions to those that occur for thermal choke point

- Entirely at function of Current Mach number

- Prescribed heat addition Necessary to thermally choke flow

Rayleigh Flow Curves (4)



On the Difference Between Subsonic and Supersonic Combustion (4)

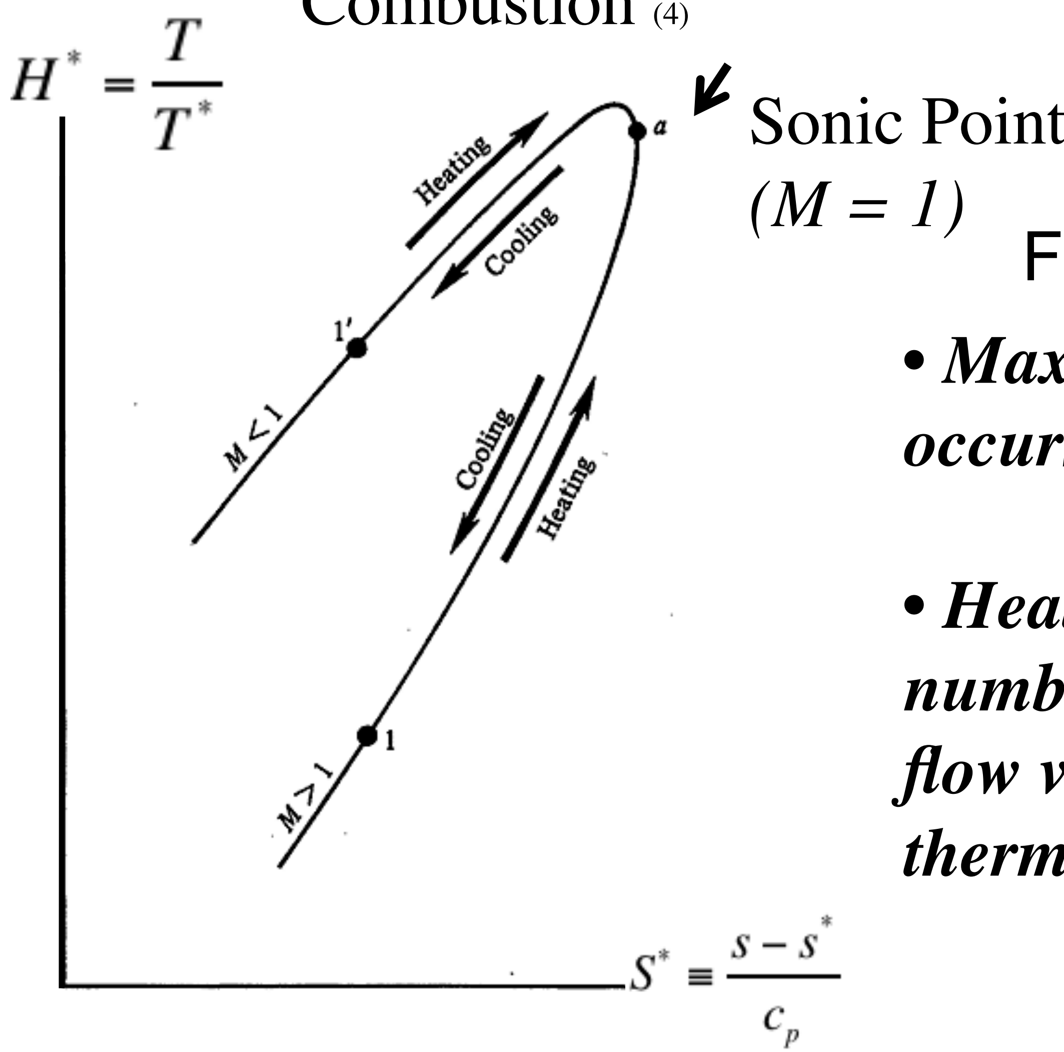


Figure 3.13 | The Rayleigh curve.

Figure 3-13 From Anderson

- ***Maximum entropy change occurs at sonic point***

- ***Heating to supersonic Mach numbers starting from subsonic flow violates second law of thermodynamics***

On the Difference Between Subsonic and Supersonic Combustion (5)

Rayleigh Curve (Anderson pp. 108,109)

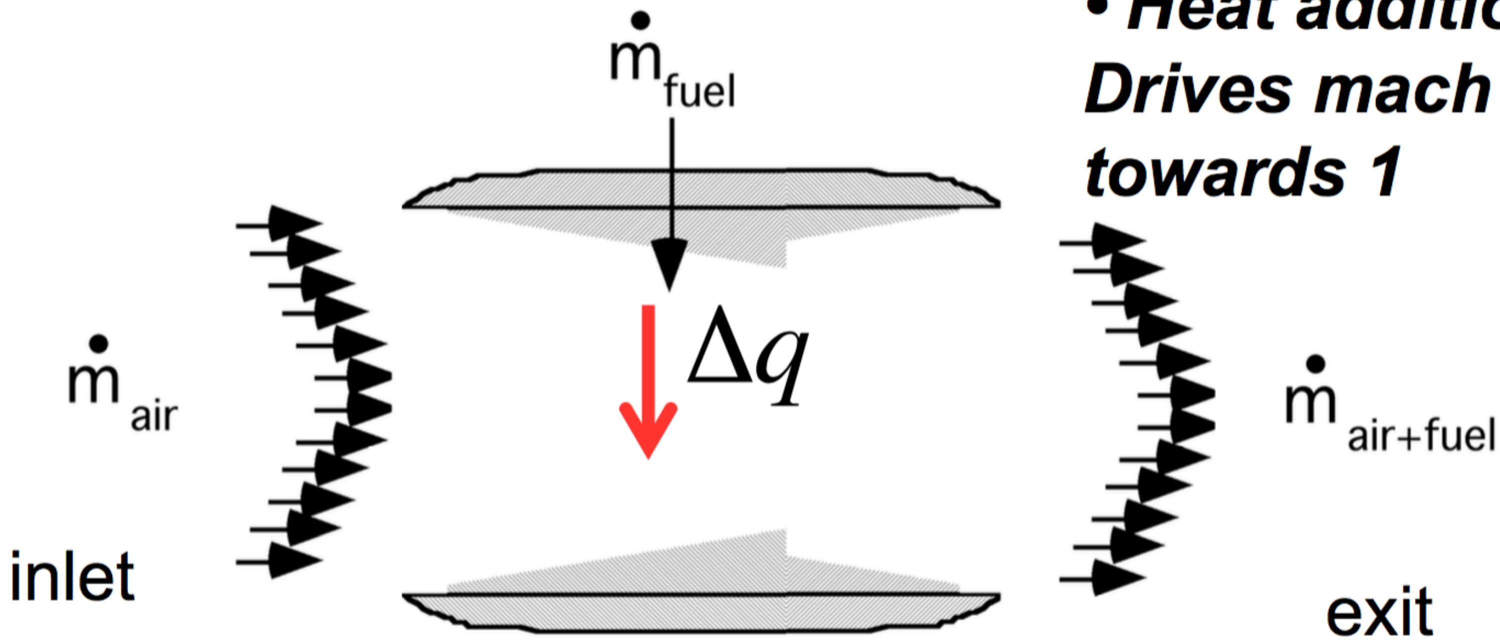
1. For *supersonic flow* in region 1, i.e., $M_1 > 1$, when heat is added

- *a.* Mach number decreases, $M_2 < M_1$
- *b.* Pressure increases, $p_2 > p_1$
- *c.* Temperature increases, $T_2 > T_1$
- *d.* Total temperature increases, $T_{o_2} > T_{o_1}$
- *e.* Total pressure decreases, $p_{o_2} < p_{o_1}$
- *f.* Velocity decreases, $u_2 < u_1$

2. For *subsonic* flow in region 1, i.e., $M_1 < 1$, when heat is added

- *a.* Mach number increases, $M_2 > M_1$
- *b.* Pressure decreases, $p_2 < p_1$
- *c.* Temperature increases for $M_1 < \gamma^{-1/2}$ and decreases for $M_1 > \gamma^{-1/2}$
- *d.* Total temperature increases, $T_{o_2} > T_{o_1}$
- *e.* Total pressure decreases, $p_{o_2} < p_{o_1}$
- *f.* Velocity increases, $u_2 > u_1$

- ***Heat addition always Drives mach number towards 1***



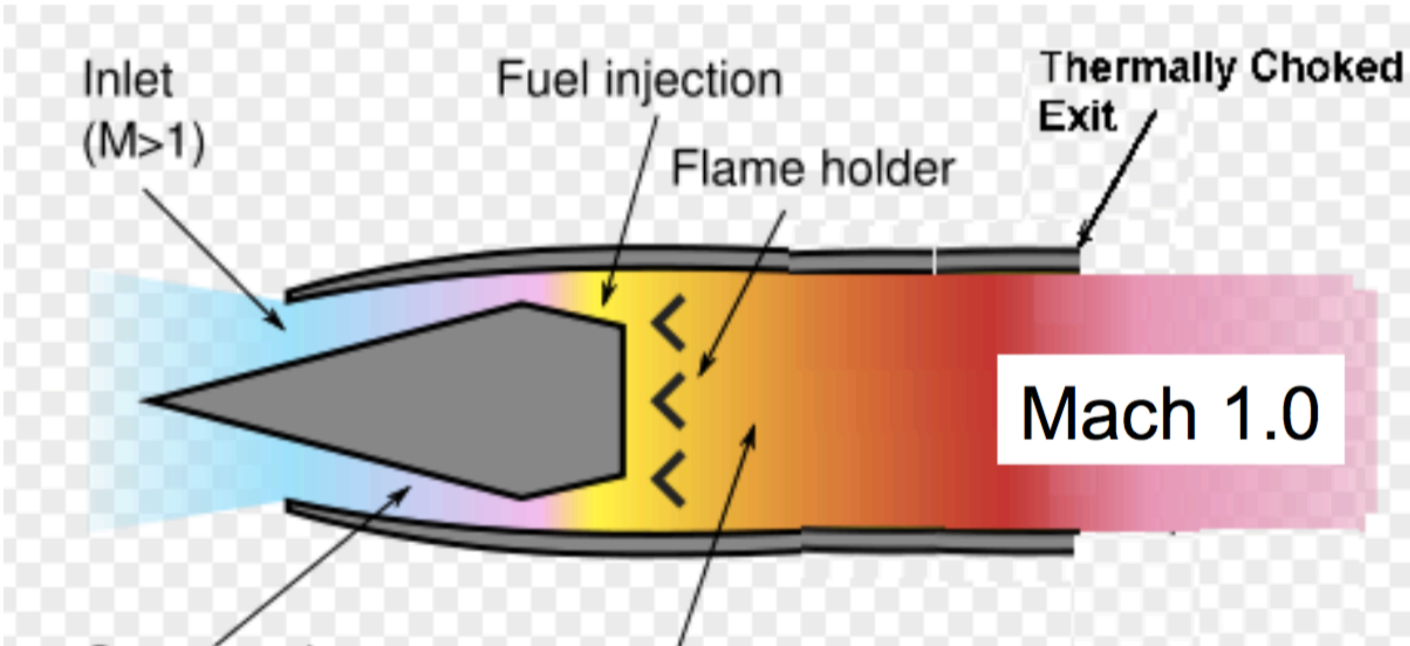
$$M_{inlet} < 1 \Leftrightarrow +\Delta q \ \textit{increases Mach} \rightarrow M_{exit} > M_{inlet}$$

$$M_{inlet} > 1 \Leftrightarrow +\Delta q \ \textit{decreases Mach} \rightarrow M_{exit} < M_{inlet}$$

$$M_{inlet} < 1 \Leftrightarrow -\Delta q \ \textit{decreases Mach} \rightarrow M_{exit} < M_{inlet}$$

$$M_{inlet} > 1 \Leftrightarrow -\Delta q \ \textit{increases Mach} \rightarrow M_{exit} > M_{inlet}$$

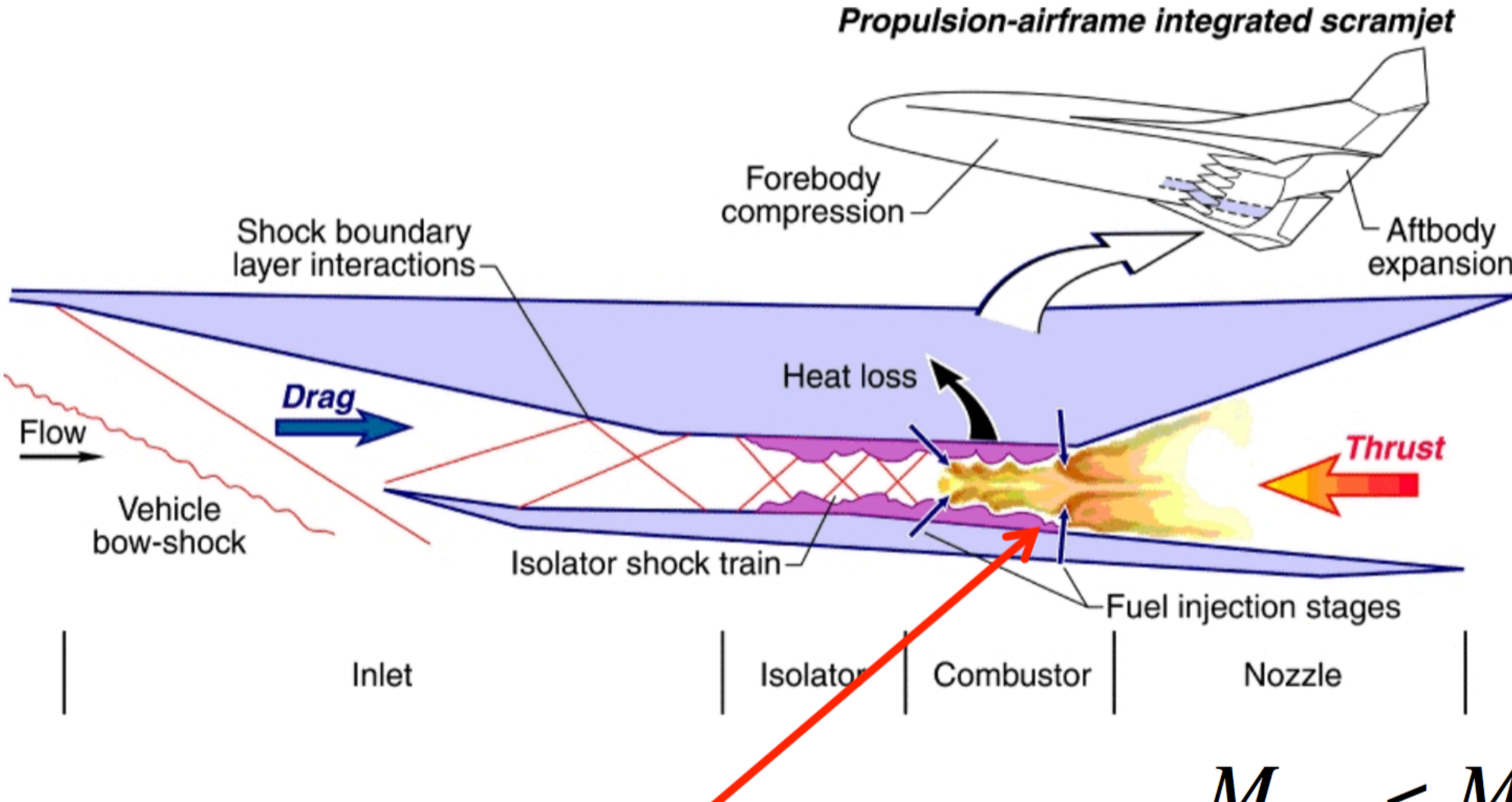
Thermal Choking in Ramjet



$$M_{exit} > M_{inlet}$$

Non-adiabatic flow is accelerated to mach 1 without divergent nozzle by adding heating

Thermal Choking in SCRAMjet



$$M_{exit} < M_{inlet}$$

Sufficient Heat Must be to Drive Combustor Exit Flow to Mach 1

Thermal Choking Comparisons

• Looking at slide 30 … There are several features shown in these plots that have important implications for the ramjet flow.

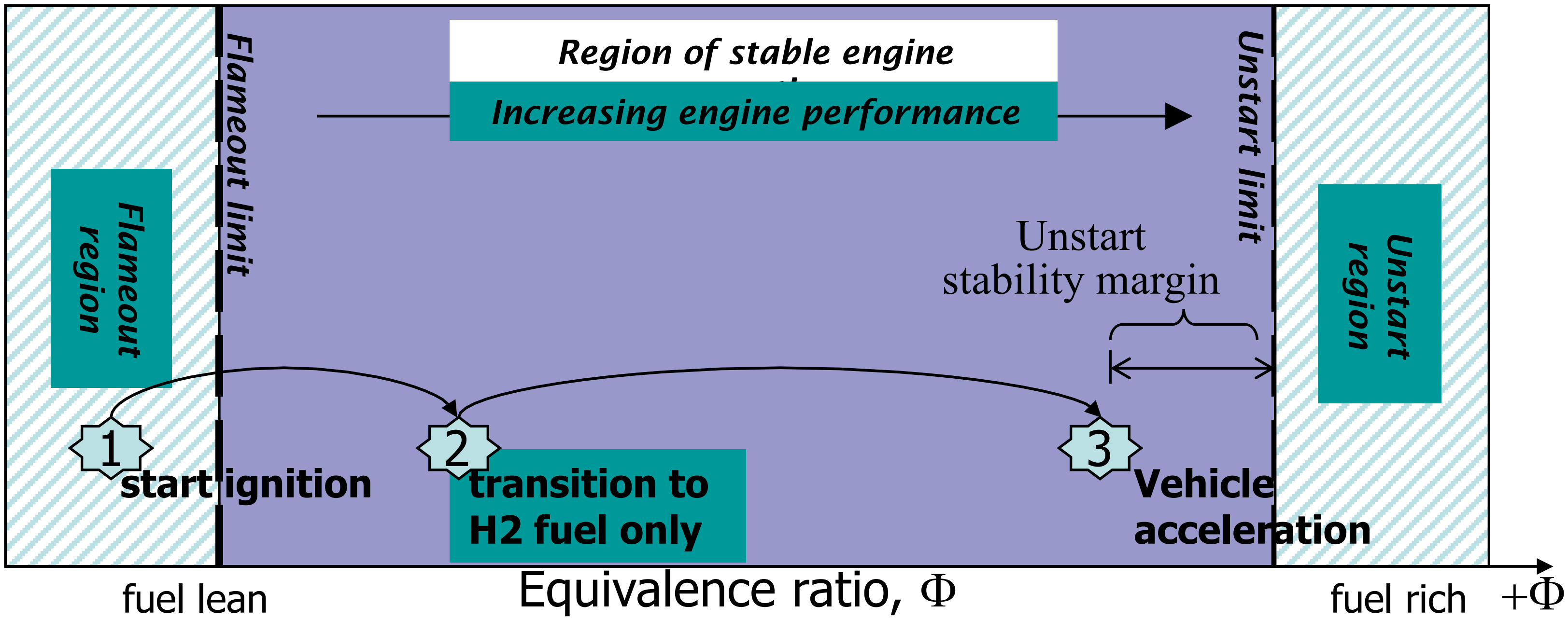
• First is that much more heat can be added to a subsonic flow than to a supersonic flow before thermal choking occurs.

• Second stagnation pressure losses due to heat addition in subsonic flow are relatively small and cannot exceed about 20% of the stagnation pressure of the flow

• ScramJets are inherently Thermally inefficient!

ScrRamjet design issues, II (cont'd)

• Supersonic flow places stringent requirements on the pressure and temperature of the flow, and requires that the fuel injection and mixing be extremely efficient.



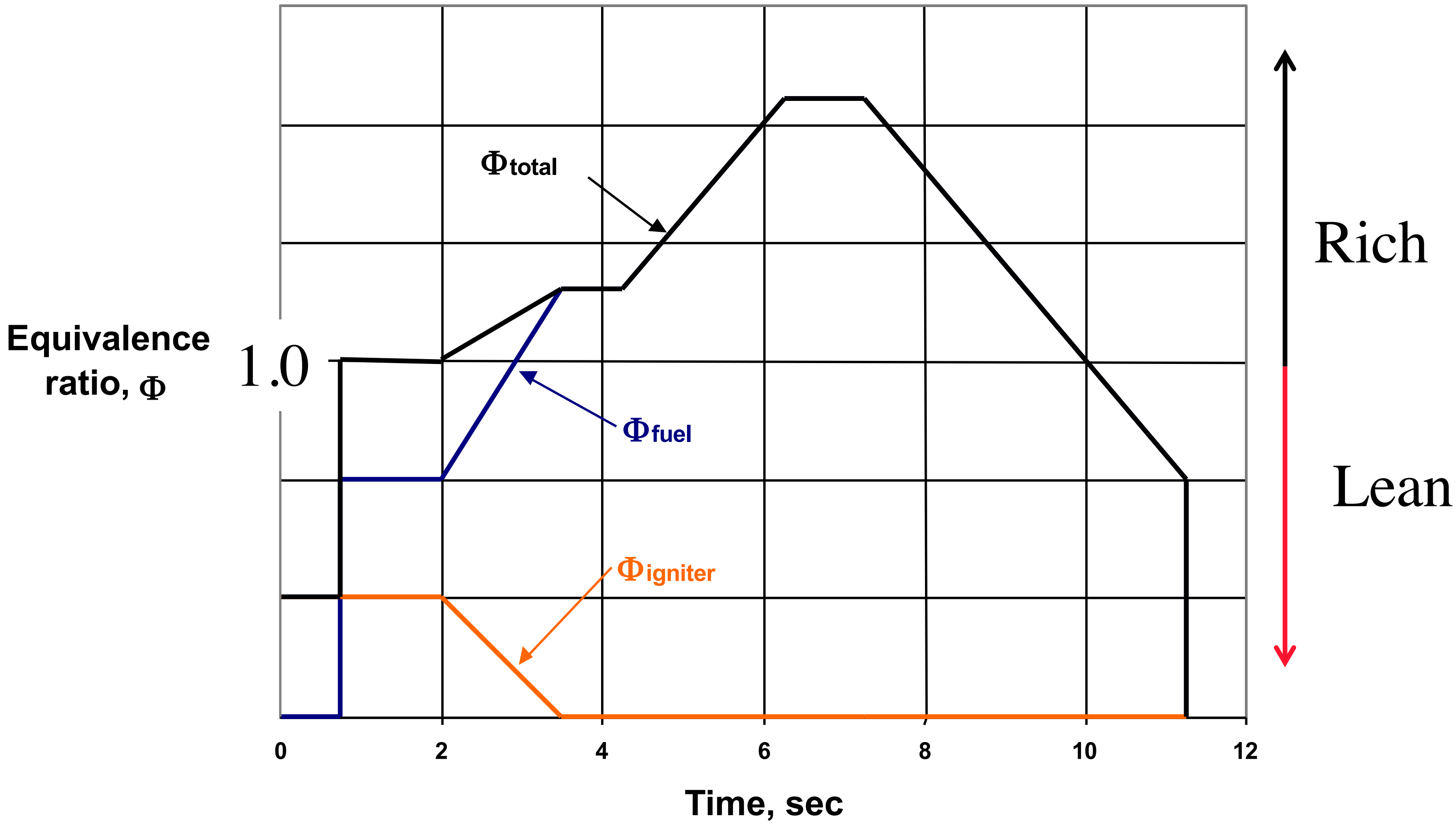
•Propulsion controller designed to maintain stable operation while achieving necessary performance

-- Flameout - low fuel flow condition where hydrogen-only combustion is not sustained

-- Unstart - high fuel flow condition where shock train moves through isolator causing causing normal shock to Occur .. Choked Nozzle result … with flow spill out … likely that shockwave forced back up to inlet .. *Very bad*

ScrRamjet design issues, II (cont'd)

- Typical Equivalence Ratio Schedule for Scramjet Burn



ScrRamjet design issues, II (cont'd)

• In typical ramjet inflow is decelerated to subsonic speeds and then reaccelerated via nozzle to supersonic speeds to produce thrust. This deceleration, which is produced by a normal shock, creates a total enthalpy loss which limits the upper operating point of a ramjet engine.

• In supersonic combustion, enthalpy of freestream air entering the scramjet engine is large compared to the heat energy released by the combustion reaction

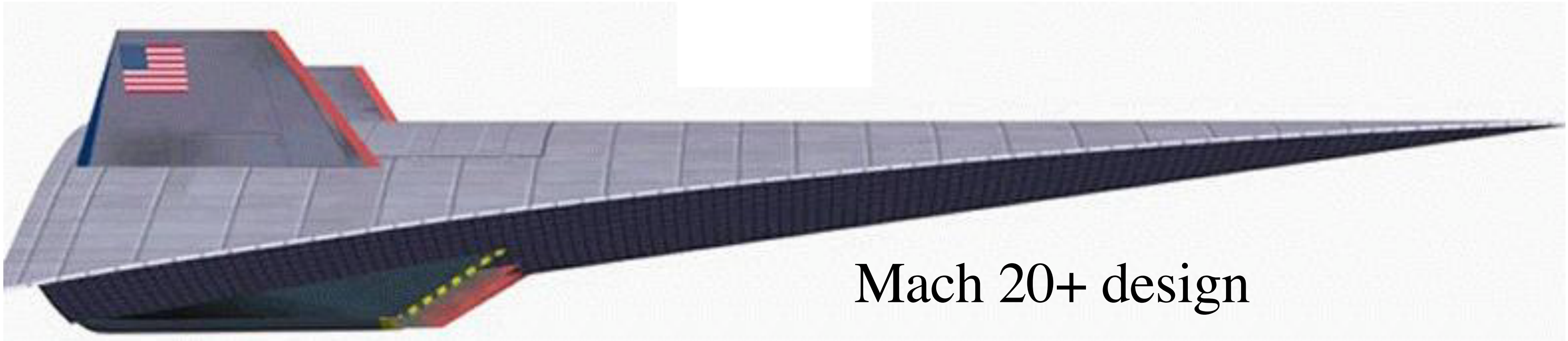
• Depending on fuel, combustion equivalence ratio, and freestream altitude,potential combustion heat release is equal to freestream flow enthalpy between Mach 8 and Mach 10.

• Heat released from combustion at Mach 25 is only 10% of total enthalpy of working fluid.

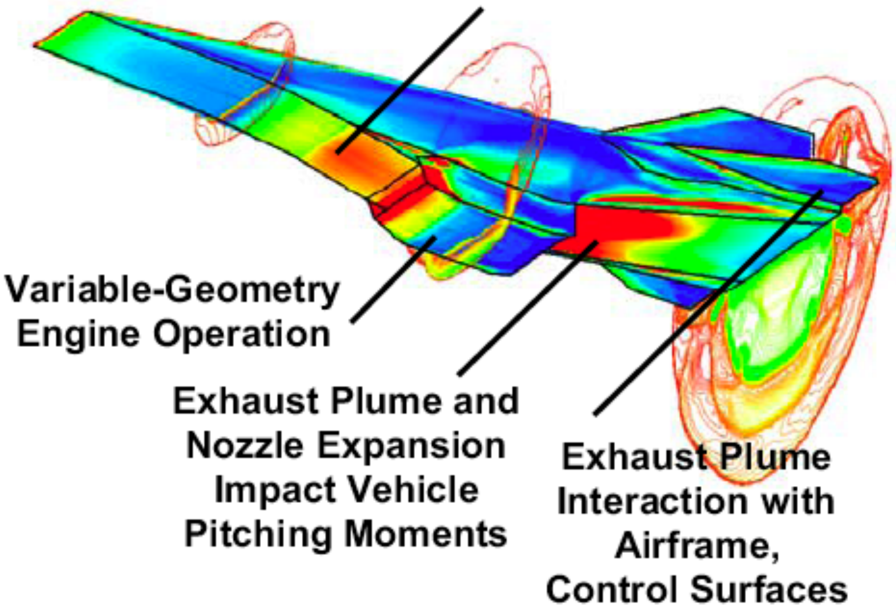
•Design of a scramjet engine is as much about minimizing drag as maximizing thrust.

• Integration of engine into airframe is Key

ScrRamjet design issues, II (cont'd)



Mach 20+ design



Mach 7-10 design

- **Integration of engine into airframe is Key**

Example Enthalpy Calculation

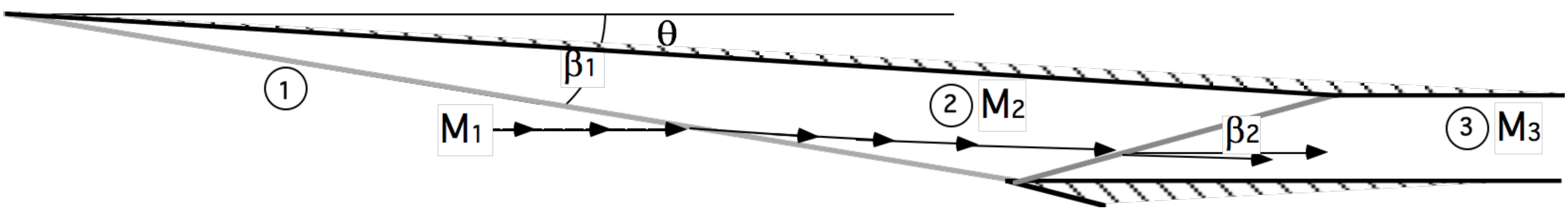
- Air enters Mach 10 SCRAM inlet at 30 km altitude

$p_\infty = 1.17180$ *kPa*, *226.65 °K*, $\theta_{ramp}=4°$

Conditions at 2:

$$\tan(\theta) = \frac{2\left\{M_1^{\,2}\sin^2(\beta)-1\right\}}{\tan(\beta)\left[2+M_1^{\,2}\left[\gamma+\cos(2\beta)\right]\right]} \rightarrow$$

$\beta_1 = 8.6531°$
M_2 ● 8.622746
$p_{\square}/p_\infty$ ● 2.474152
$T_{\square}/T_\infty$ ● 1.323222
$P0_2/P0_\infty = 0.928350$



Example Enthalpy Calculation (cont'd)

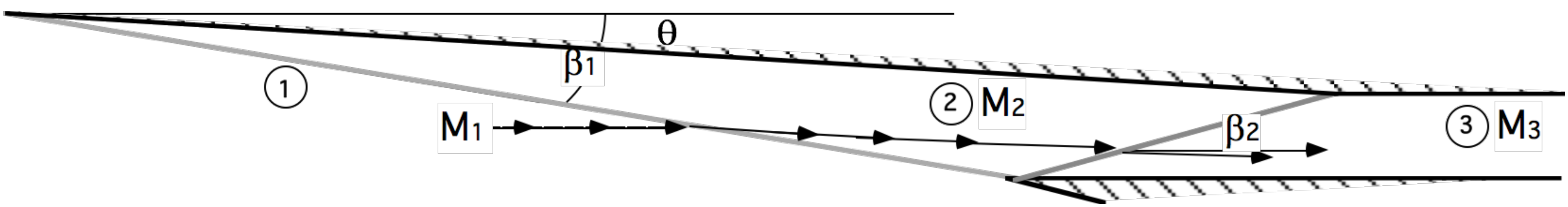
$\beta_1 = 8.6531°$
M_2 ● 8.622746
$p_{□}/p_\infty$ ● 2.474152
$T_{□}/T_\infty$ ● 1.323222
$P0_2/P0_\infty = 0.928350$

$\beta_2 = 9.521°$
M_3 ● 7.576
p_3/p_2 ● 2.20665
T_3/T_2 ● 1.271723
$P0_3/P0_2 = 0.951393$

p_3/p_∞ ● 5.4960
T_3/T_∞ ● 1.6831
$P0_3/P0_\infty = 0.8832$

$$P_3 = 5.496 \cdot 1.1718 = 6.550 \text{ kPa}$$

$$T_3 = 1.6831 \cdot 226.65 = 381.47 \text{ °K}$$



Example Enthalpy Calculation (cont'd)

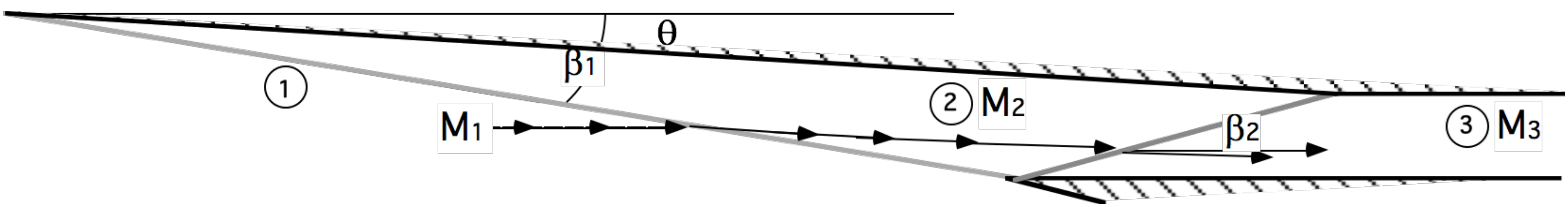
- Specific Enthalpy of flow entering combustor:

$$h_3 = C_p T_3 + \frac{\left[\sqrt{\gamma R_g T_3} M_3\right]^2}{2} = \frac{\left(\left(1.4 \cdot 287.056 \cdot 381.47\right)^{0.5} 7.576\right)^2}{2} + 1004.7 \cdot 381.47$$

$$= 4.7828 \text{ MJ/kg}$$

$$p_3 = 5.496 \cdot 1.1718 \quad = 6.550 \text{ kPa}$$

$$T_3 = 1.6831 \cdot 226.65 \quad = 381.47 \text{ °K}$$



Example Enthalpy Calculation (cont'd)

- CEA Calculation: GH_2 fuel, Φ=1

$p_3 = 6.550$ kPa

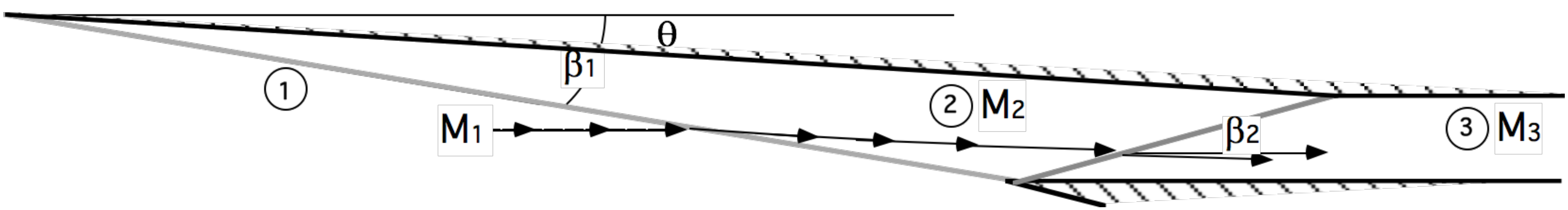
$T_3 = 381.47$ °K

T, K	**2238.92**
MW, (1/n)	**24.519**
GAMMA	**1.1565**

Combustion Enthalpy is Less than Freestream Enthalpy!

$$\Delta h_{combustion} = C_p\left[T_{combustion} - T_3\right] = \frac{R_u}{M_w}\frac{\gamma}{\gamma - 1}\left[T_{combustion} - T_3\right] =$$

$$\left(\frac{8314.4126}{24.519}\right)\left(\frac{1.1565}{1.1565 - 1}\right)(2238.92 - 381.47) = 4.655 \text{ MJ/kg}$$



Example Enthalpy Calculation (cont'd)

- Calculate thermodynamic efficiency of engine

T_B=381.47 °K

Tc= **2238.92**°K

$p_3/\ p_\infty$ ● 5.4960

$T_3\ /\ T_\infty$ ● 1.6831

$P_{0_3}/\ P_{0\ \infty}$ = 0.8832

γ ~ **(1.1565+1.4)/2=1.27825**

$$\eta = 1 - \left(\frac{p_\infty}{p_3}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_3}}{P_{0_\infty}}\right)^{\frac{\gamma-1}{\gamma}} T_3\right)}{\left(T_C - T_3\right)} =$$

$$1 - \left(\frac{1}{5.4960}\right)^{\frac{1.27825-1}{1.27825}} \frac{\left(2239.92 - \left(\left((0.8832)^{\frac{1.27825-1}{1.27825}}\right)\right)381.47\right)}{(2239.92 - 381.47)} = 0.3061$$

About 50% as efficient as out previous ramjet design (section 9.1) Operating at Mach 4 and 10km altitude ***(η=0.6)***

ScrRamjet Design Issues, III

- look at Mach 4 Ramjet problem (Section 9.1) …
  let $P_{0_B}/ P_{0_A} = 1$

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)} =$$

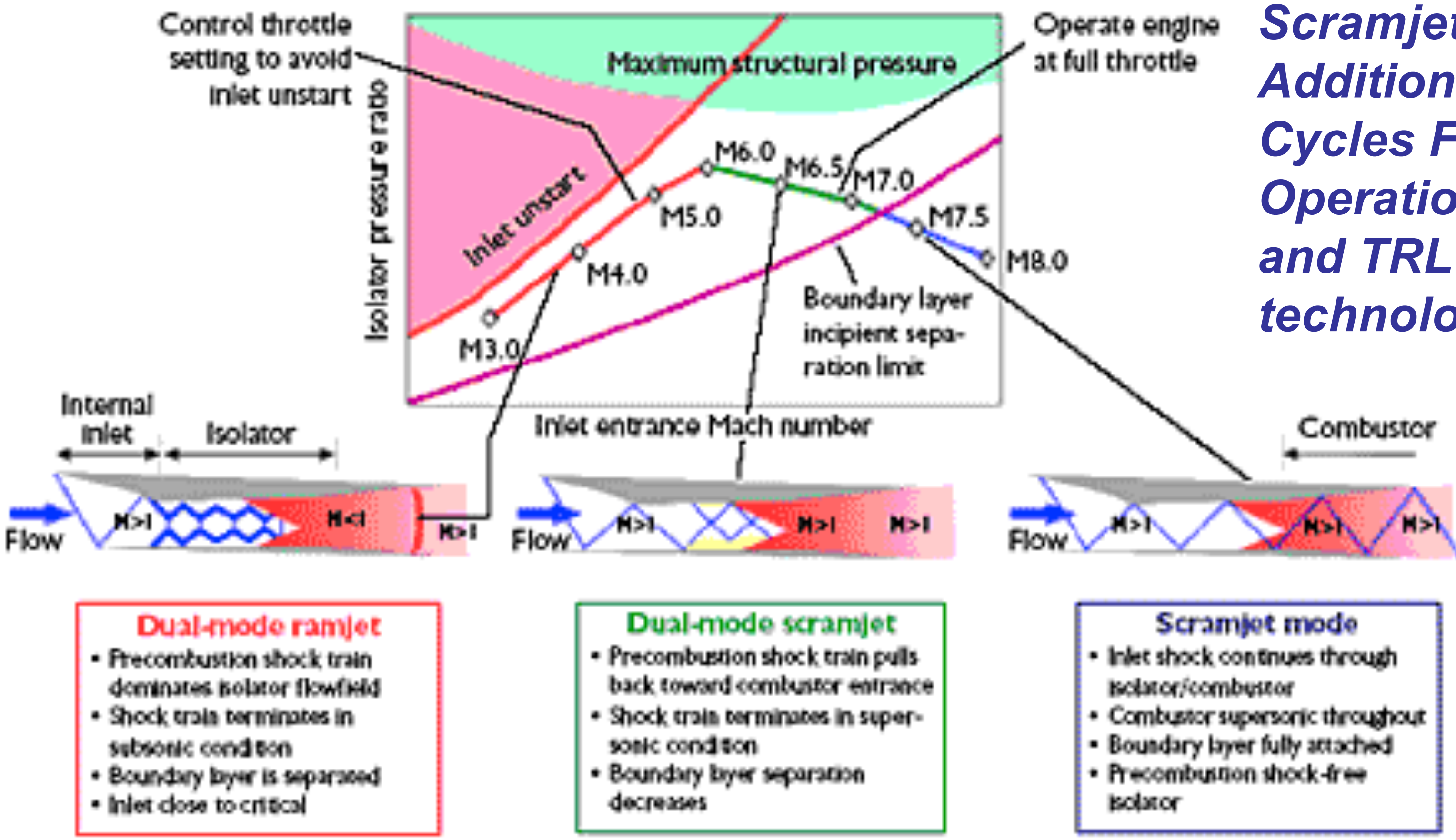
8% increase in Efficiency …
Compared to Ramjet
… but can we
Do this? … no!

$$1 - \frac{38.422^{\frac{-(1.4-1)}{1.4}} (2662.82 - 1 \cdot 856.61)}{(2662.82 - 856.61)} = 0.6475$$

ScrRamjet design issues, III (cont'd)

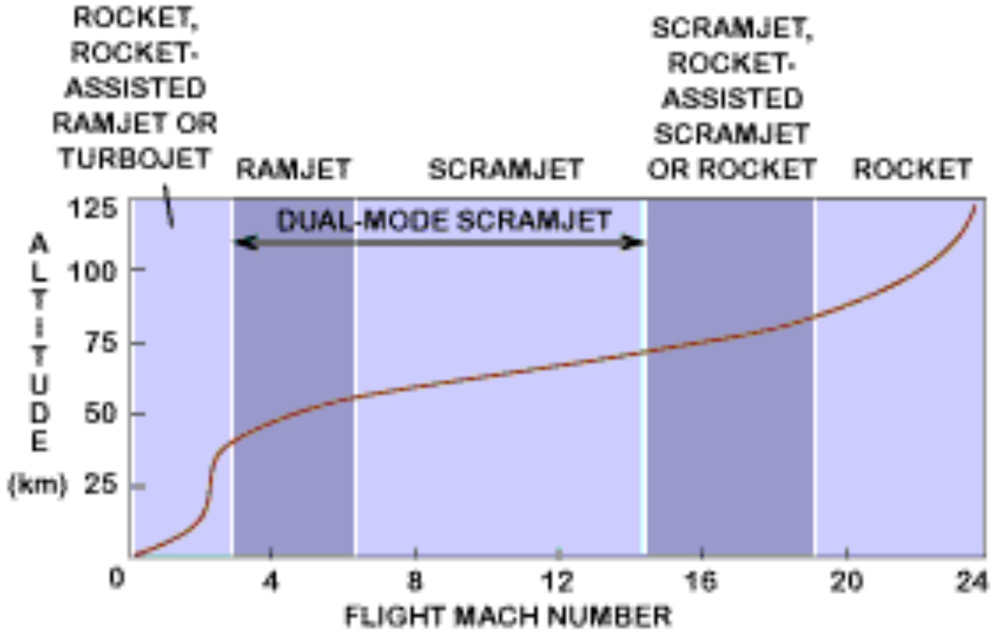
- Supersonic flow cannot maintain stable combustion below ~ Mach 6

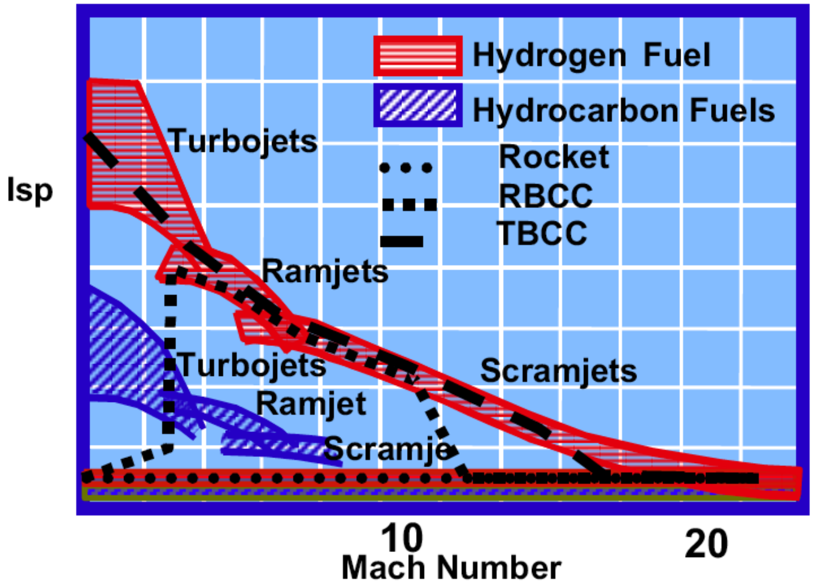
- Scramjets are feasible only for sustaining hypersonic speeds, not for achieving them from zero velocity



Scramjets Require Additional Cycles For An Operational Vehicle … and TRL for these technologies are low

ScrRamjet design issues, III (cont'd)





Combined cycle rocket engines

… Engines that are part jet engine And part rocket motor!

… or part turbojet and part SCRAMjet

Rocket + Scramjet =

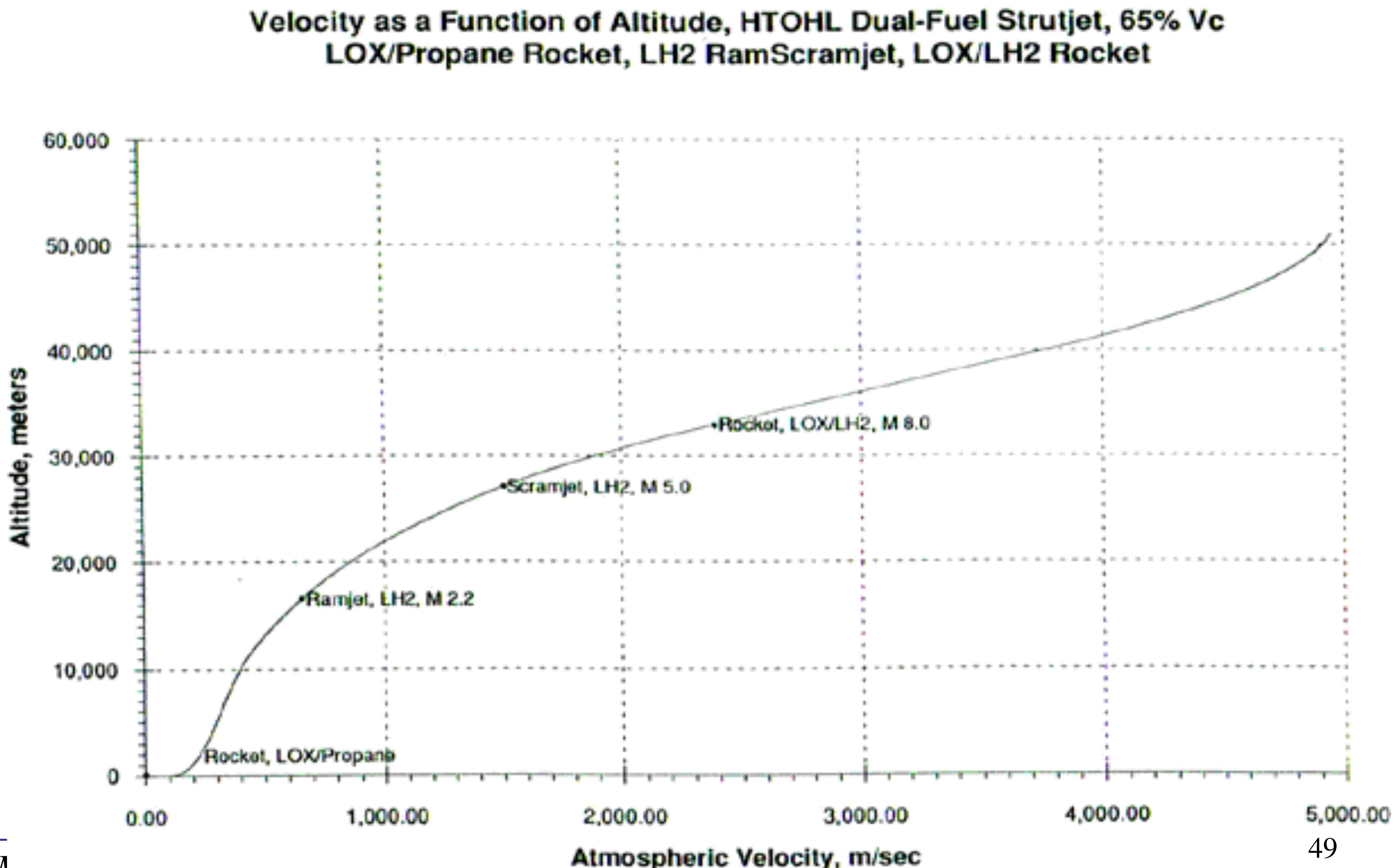
Rocket Based Combined Cycle (RBCC)

OR

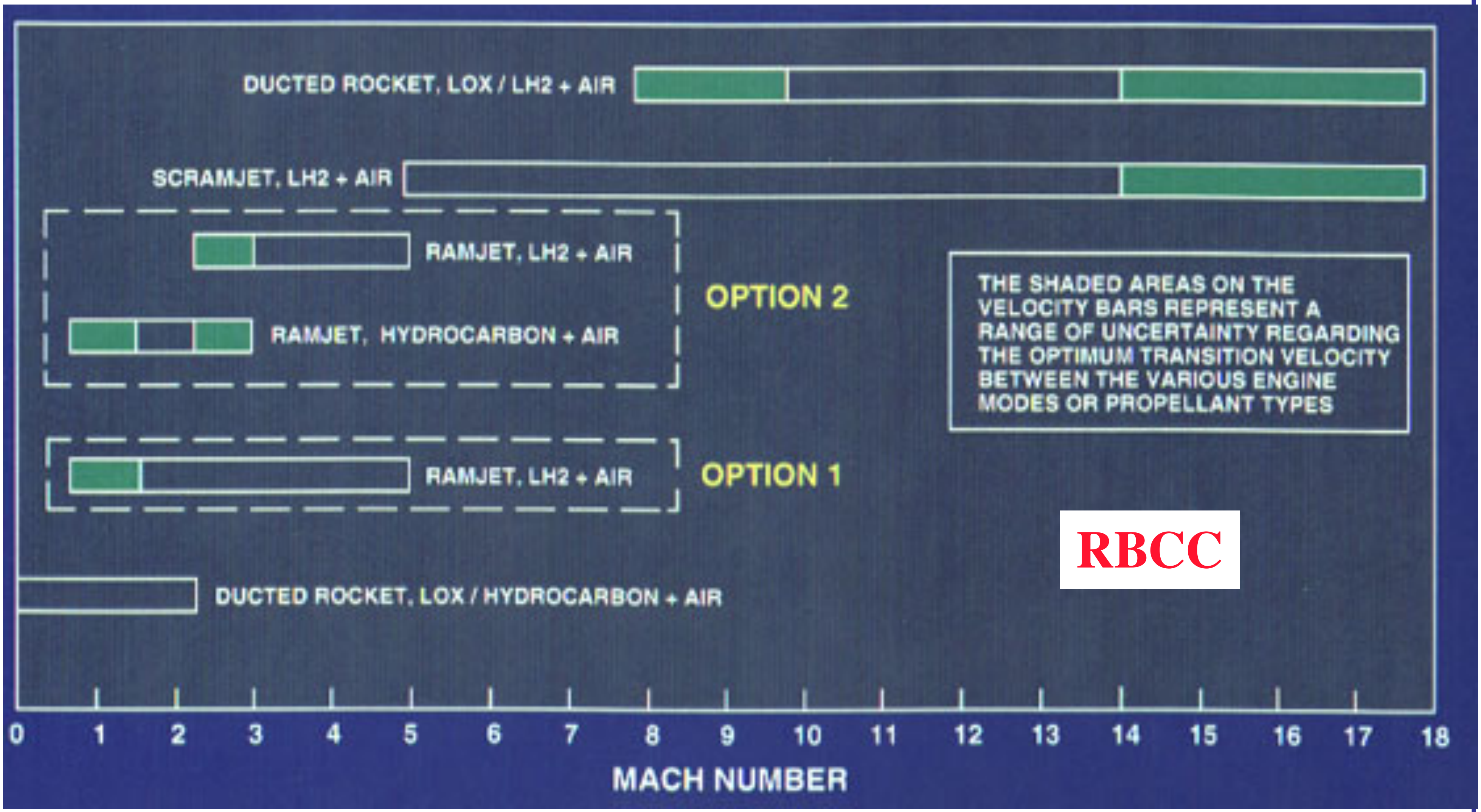
Turbine + Scramjet =

***Turbine Based Combined Cycle* (TBCC)**

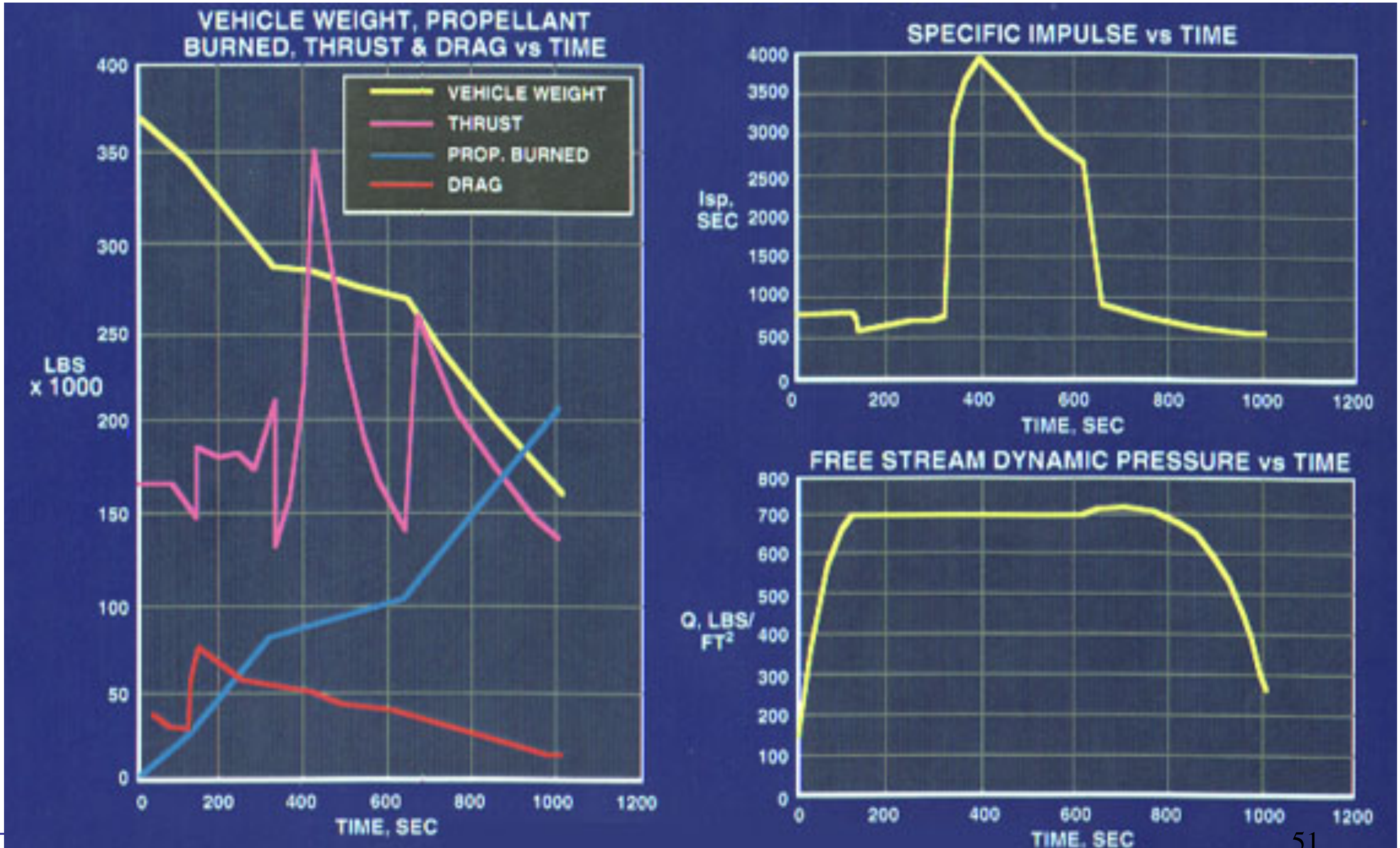
Rocket-Based Combined Cycle (RBCC) Mission Profile



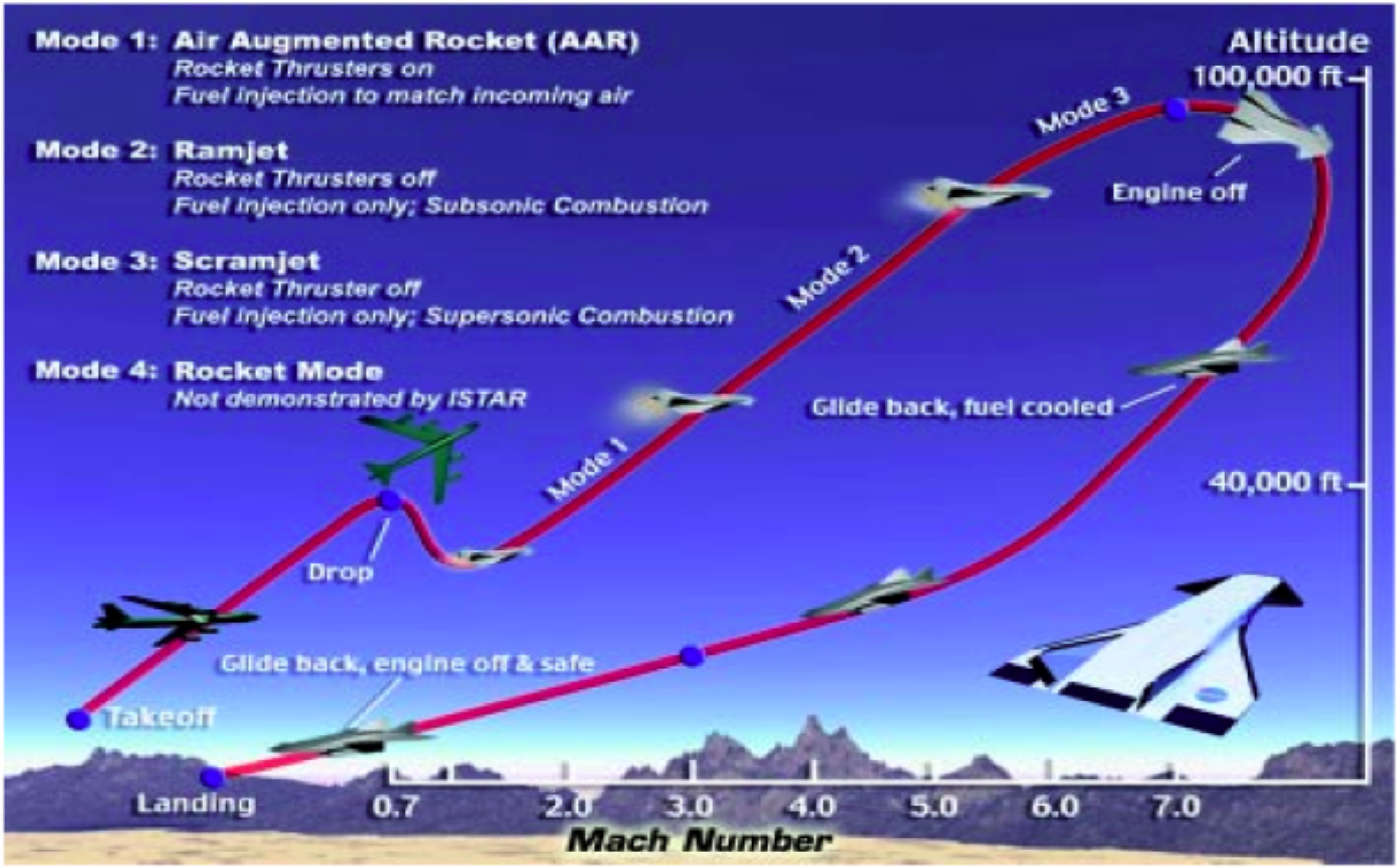
RBCC Mission Profile (2)



RBCC Mission Profile (3)



Proposed RBCC Demo Program)



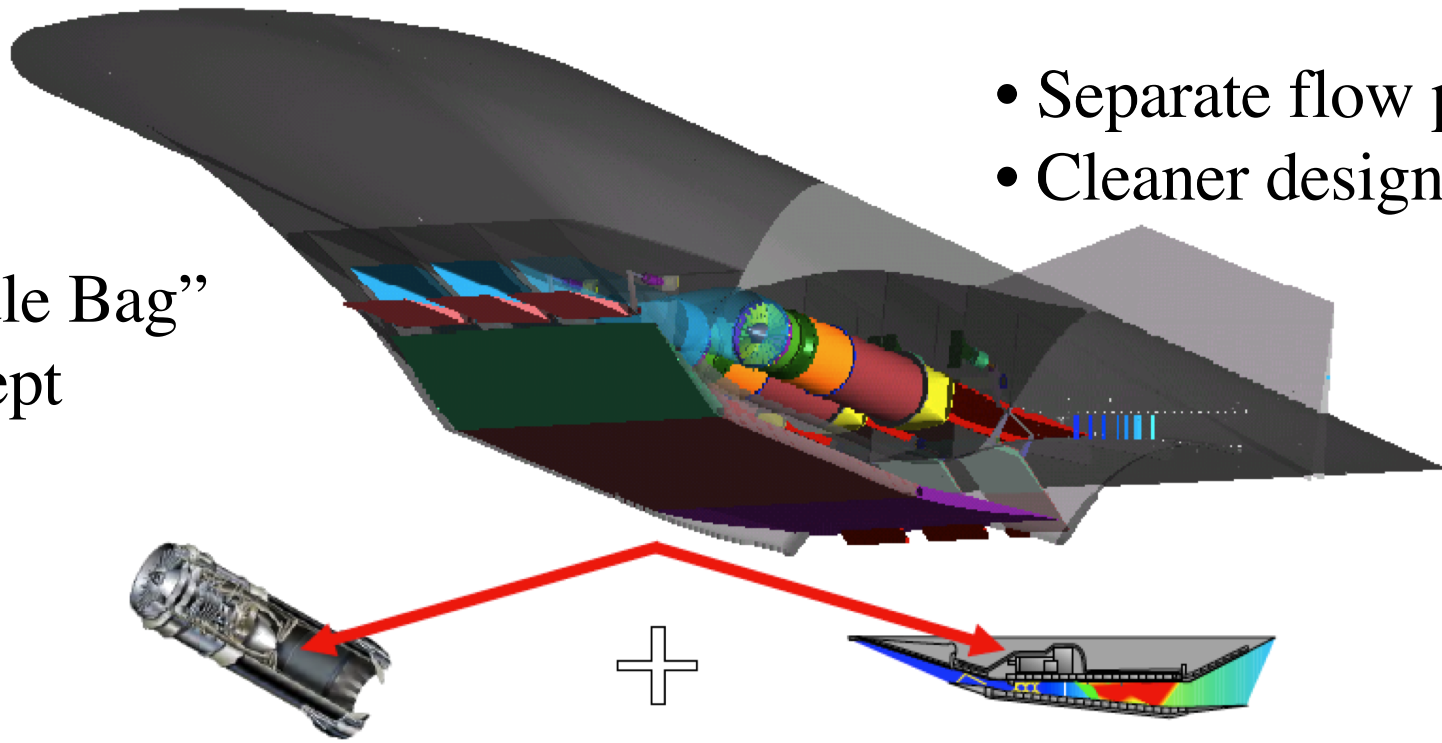
Integrated System Test of an Air-breathing Rocket (ISTAR)

Turbine Based Combined Cycle (TBCC), I

Turbine-Scramjet Combination Engine

- Separate flow paths
- Cleaner design

"Saddle Bag" Concept

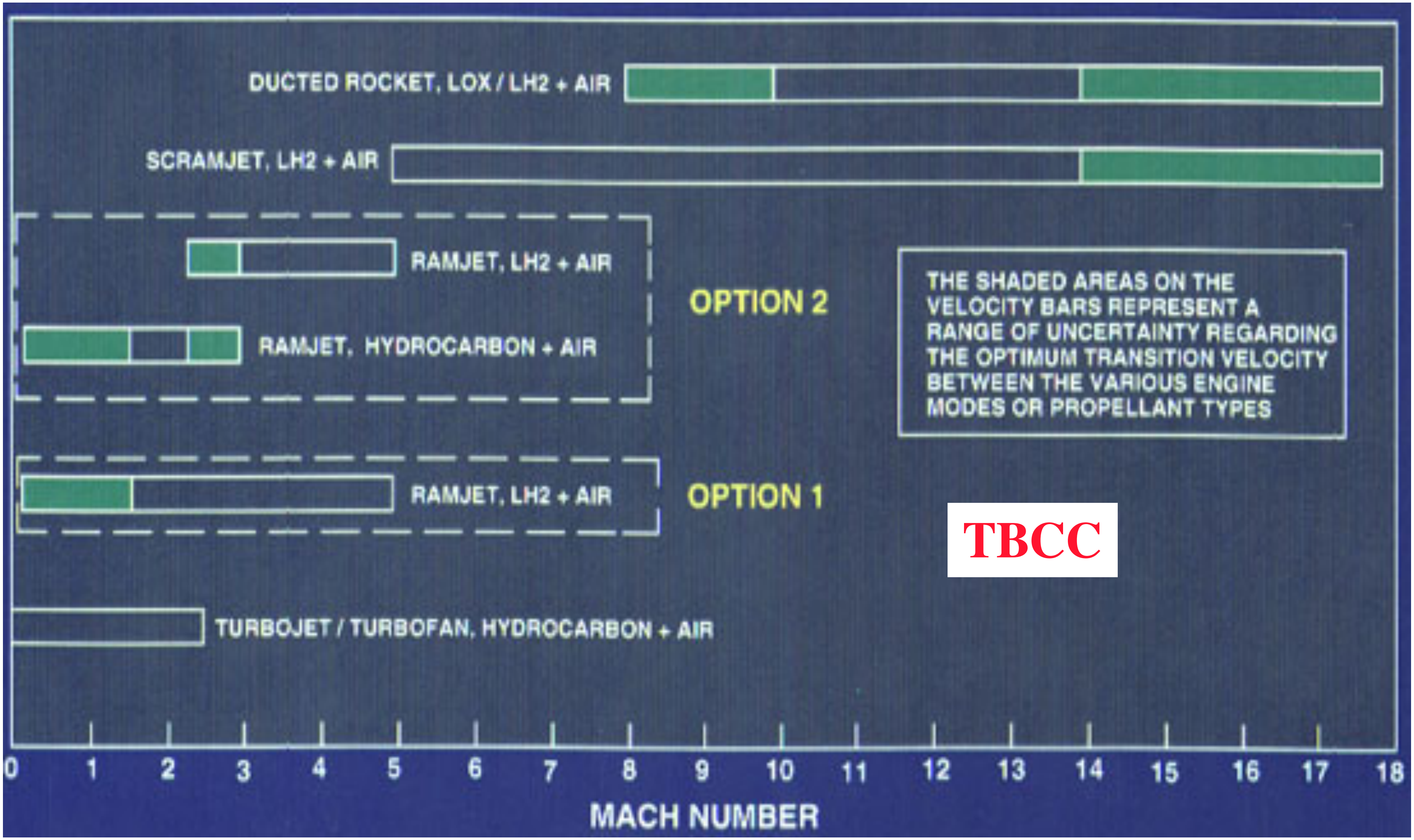


Low-Speed
Supersonic RTA (Mach 0-4+)
Revolutionary Turbine Accelerator

High-Speed
Hypersonic Scramjet (Mach 4-15)

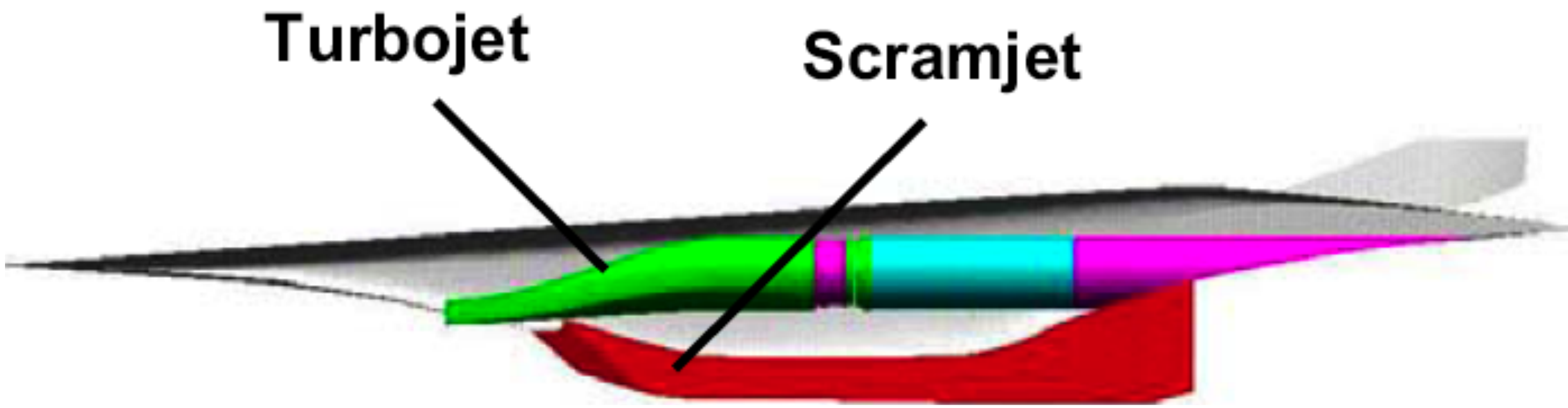
Credit: Chuck McClinton NASA

TBCC Mission Profile



Turbine Based Combined Cycle (TBCC), II

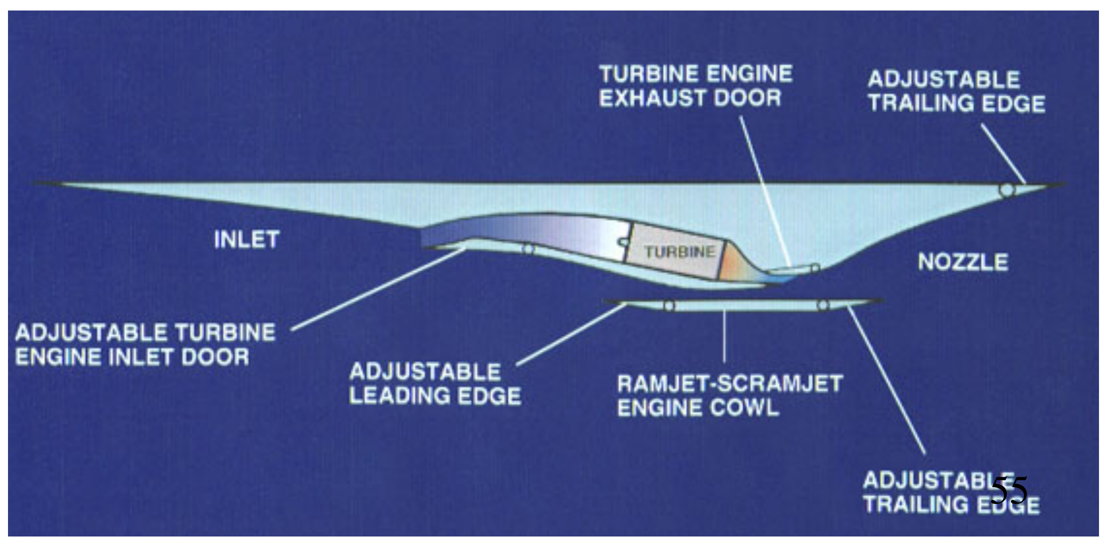
(cont'd)



Over/Under Combination Cycle Concept.

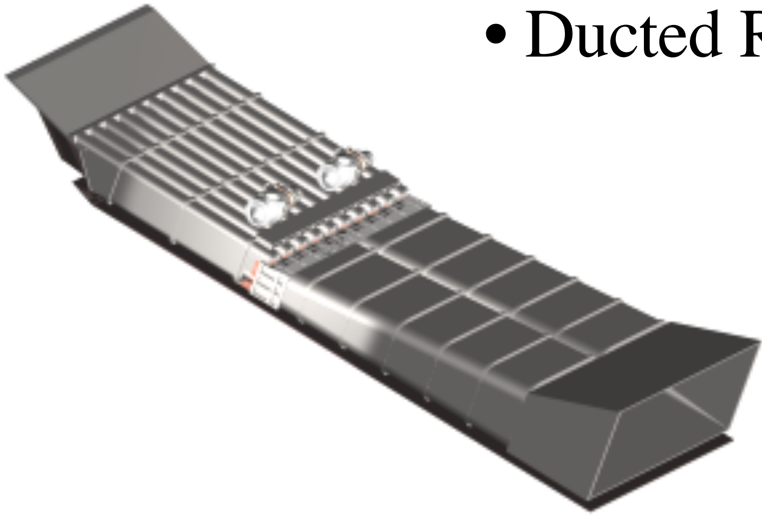
- Share parts of same flow path

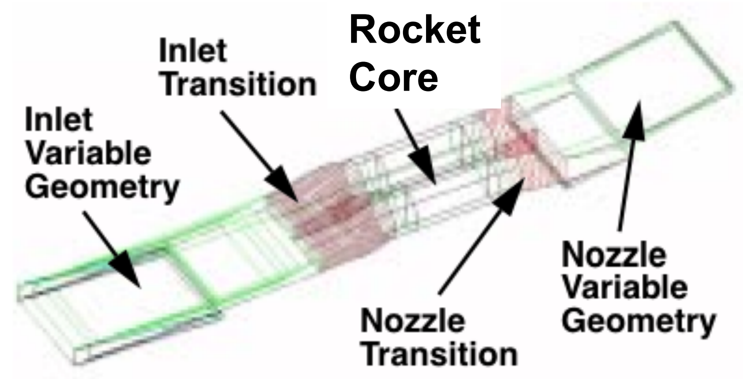
- Weight savings

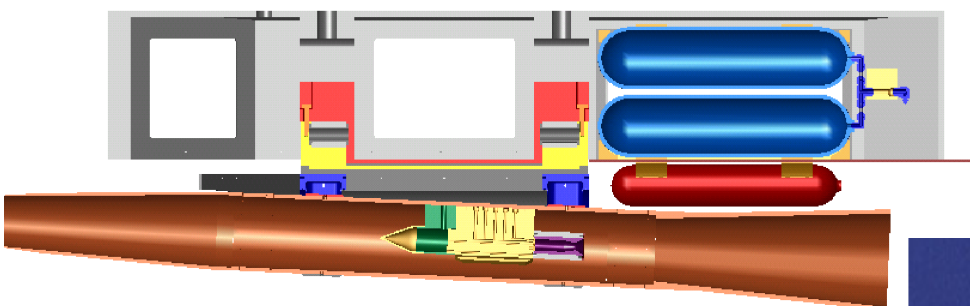


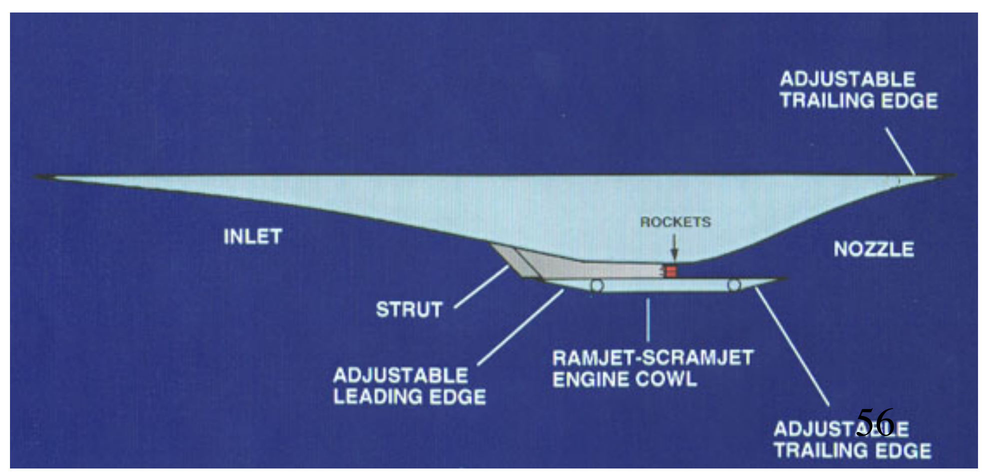
Rocket Based Combined Cycle (RBCC), I

- Ducted Rocket Approach



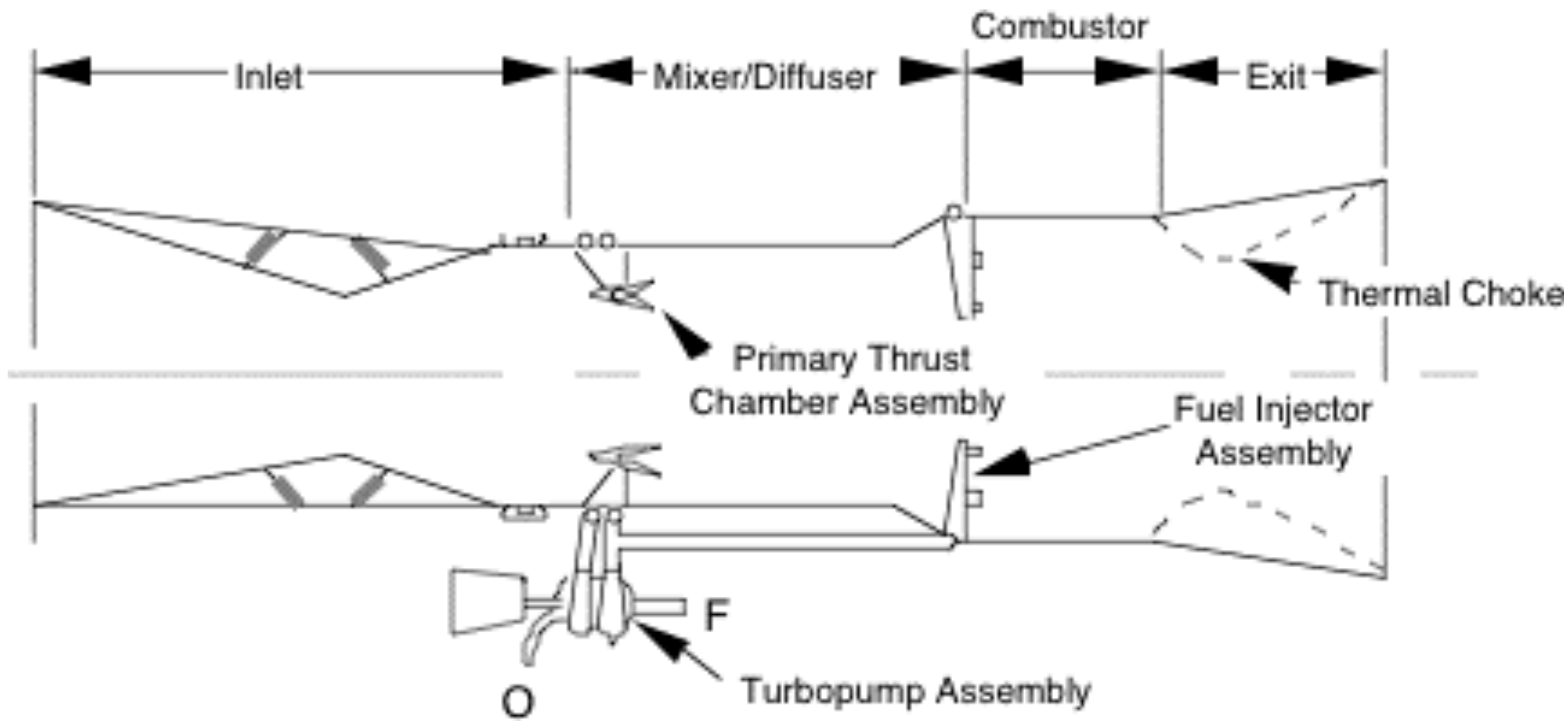






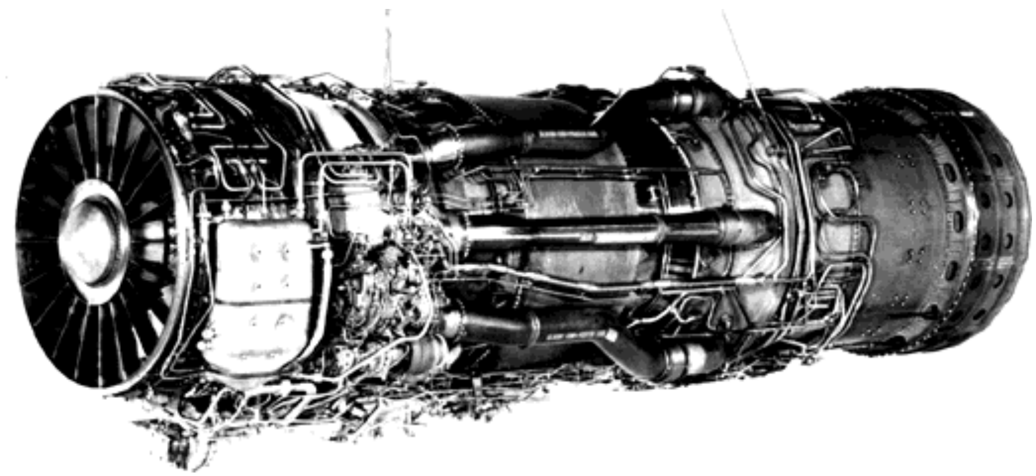
Rocket Based Combined Cycle (RBCC), II

• Dual Combustor Approach



• There is a precedent for this design

Pratt and Whitney J58 Turbojet/Ramjet Combined Cycle Engine



Part Turbo-jet

Part Ram-jet

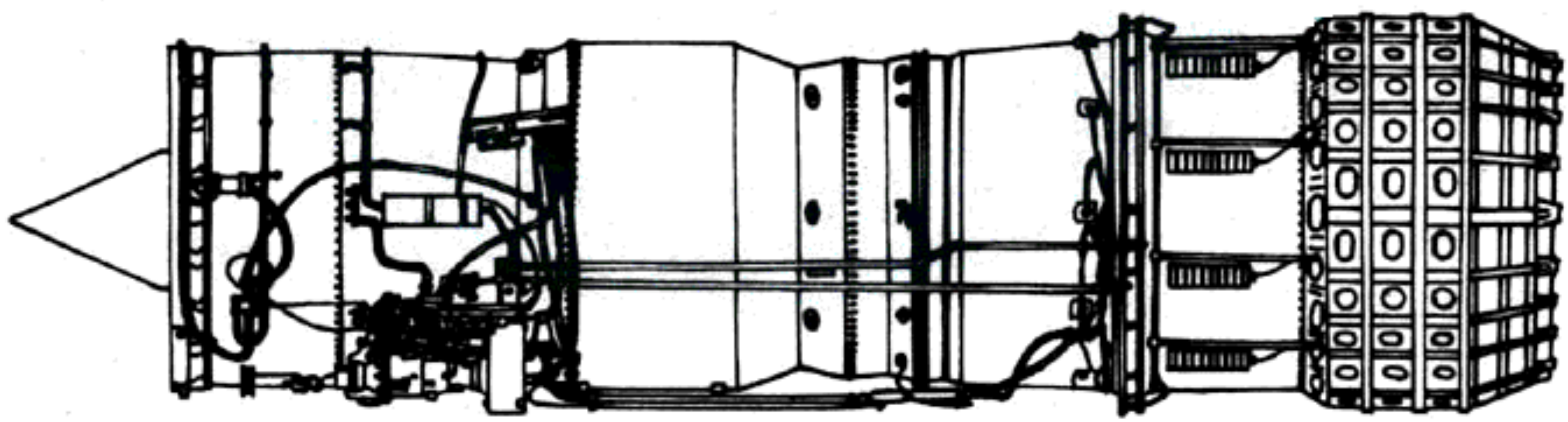
Dual Burner … Same flow path

https://www.youtube.com/watch?v=F3ao5SCedIk

Pratt and Whitney J58 Turbojet/Ramjet Combined Cycle Engine (cont'd)

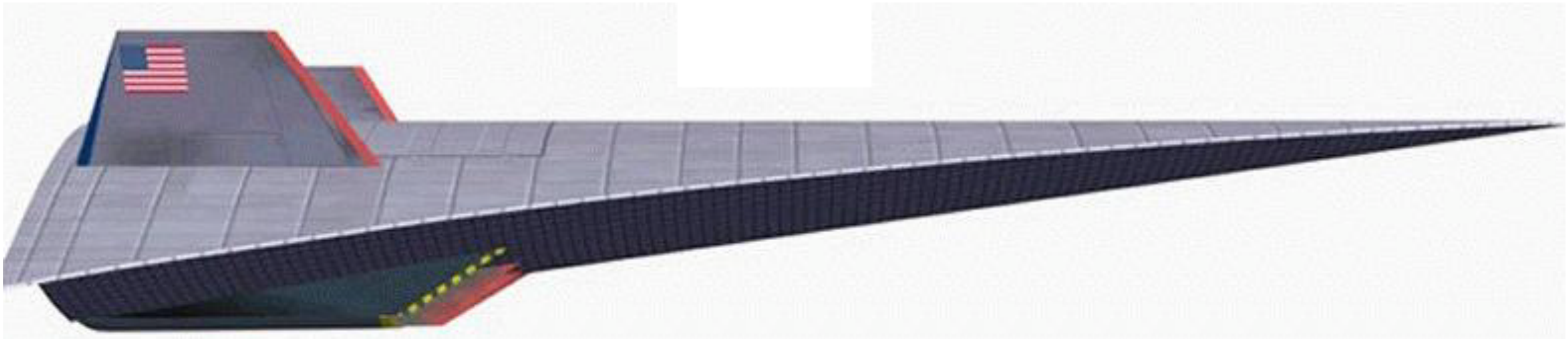
• Above mach 3 a portion of the flow bypasses the turbine and burns Directly in afterburner providing about 80% or thrust …

• At lower speeds the engine operates as a normal supersonic Turbojet … same nozzle used by both operational modes



SCRAMJET DESIGN ISSUES, IV

Thermal management

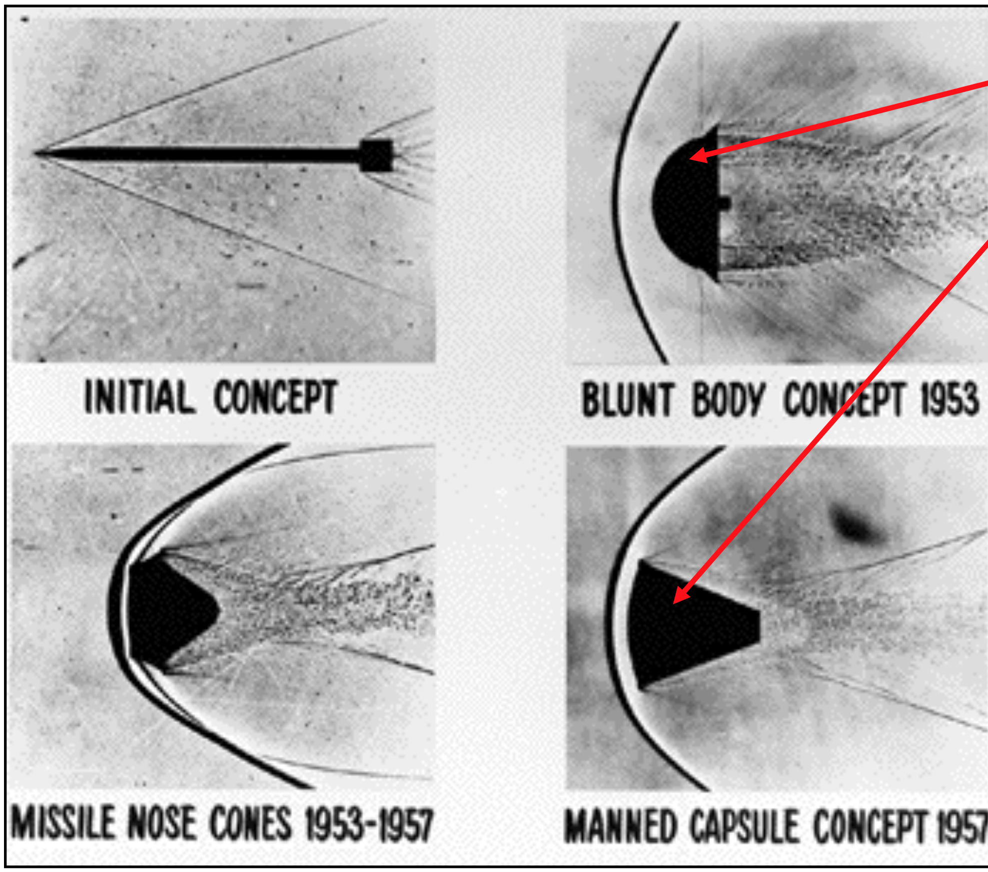


• The nature of the inlet design and need to minimize wave drag
Mandate very sharp leading edges

• Leading edges generate extreme hypersonic heating rates
In excess of 100 watts/cm^2

SCRAMJET DESIGN ISSUES, IV (cont'd)

Heating is Minimized by Blunt Body

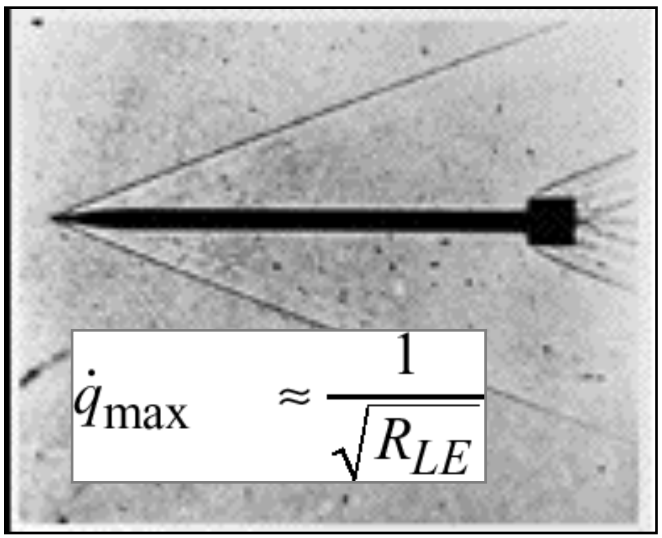


• Detached Normal Shockwave On Blunt Leading Edge Produces High level of Drag and Dissipates significant Portion of heat into flow

• **High Drag Profiles Have Lower Levels of Total Hypersonic Heating**

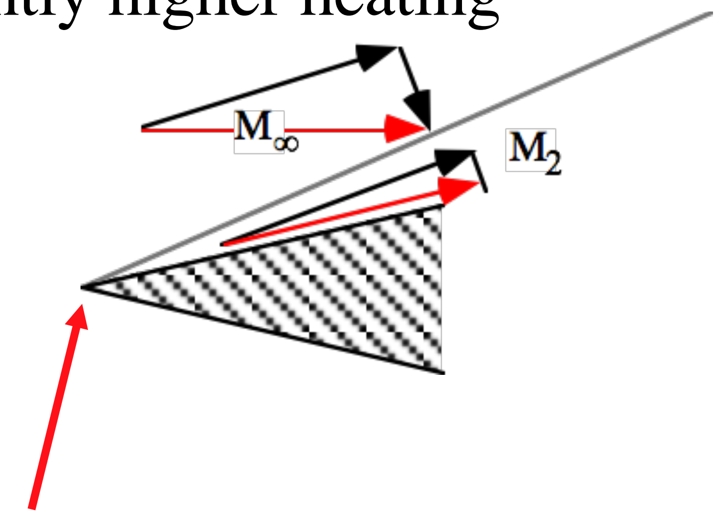
SCRAMJET DESIGN ISSUES, IV (cont'd)

- Sharp Leading Edge …Much Higher Hypersonic Lift-to-Drag, but also significantly higher heating



$$\dot{q}_{max} \approx \frac{1}{\sqrt{R_{LE}}}$$

Oblique Shockwave



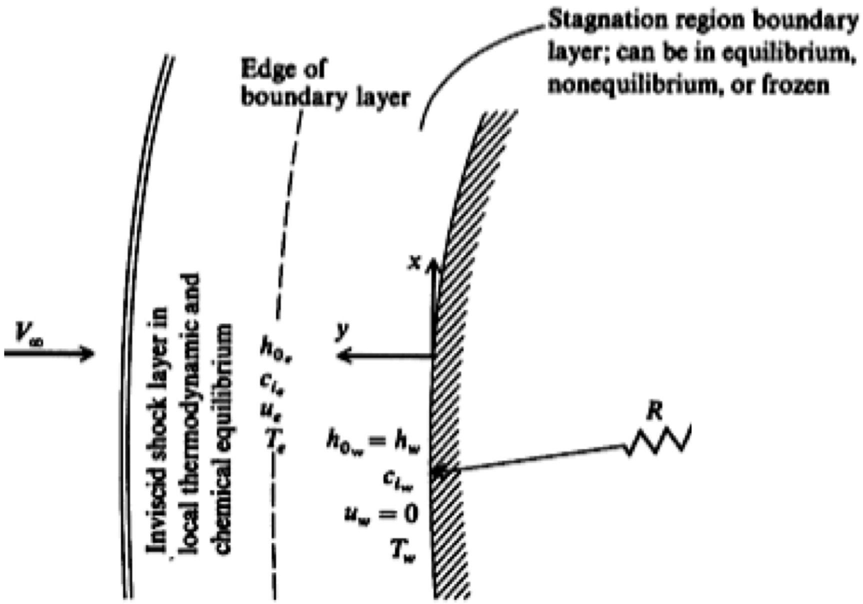
- Flow attached at leading edge Heating impinges directly

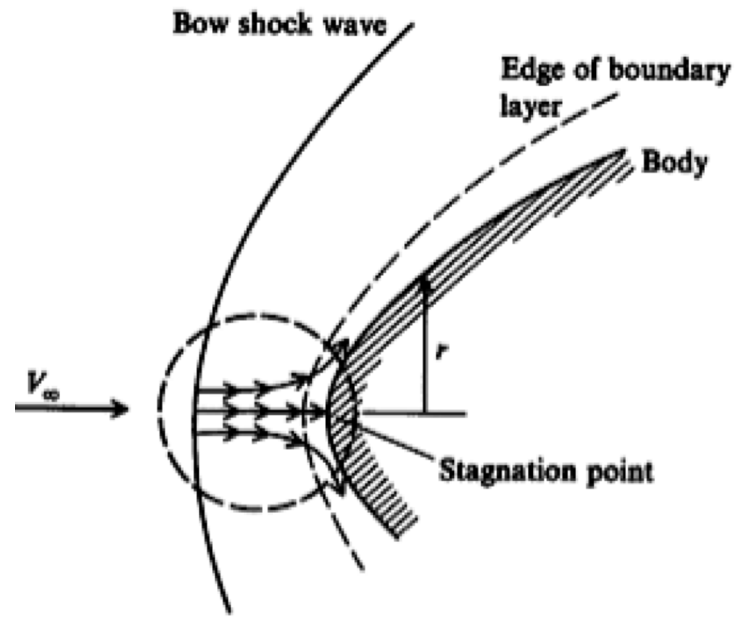
- More Exotic Thermal Protection Systems Required

SCRAMJET DESIGN ISSUES, IV (cont'd)

- Sharp leading Edge has very high heating because of small radius

- Mach number and flow density are also Key players





$$\dot{q}_{LE} = H_{tf}\left[h_0 - h_{wall}\right]\left[\frac{1}{R_{LE}}\sqrt{2\frac{\left(P_{0_2}-p_\infty\right)}{\rho_{0_2}}}\right]^{\frac{1}{2}}\left[C_{p\infty}T_\infty + \frac{V_\infty^{2}}{2} - C_{p_{wall}}T_{wall}\right]$$

Stagnation heating Rate

SCRAMJET DESIGN ISSUES, IV (cont'd)

- Small LE radius also has lower thermal capacity and the problem is compounded

Equilibrium temperature is a function of heat in
And heat out

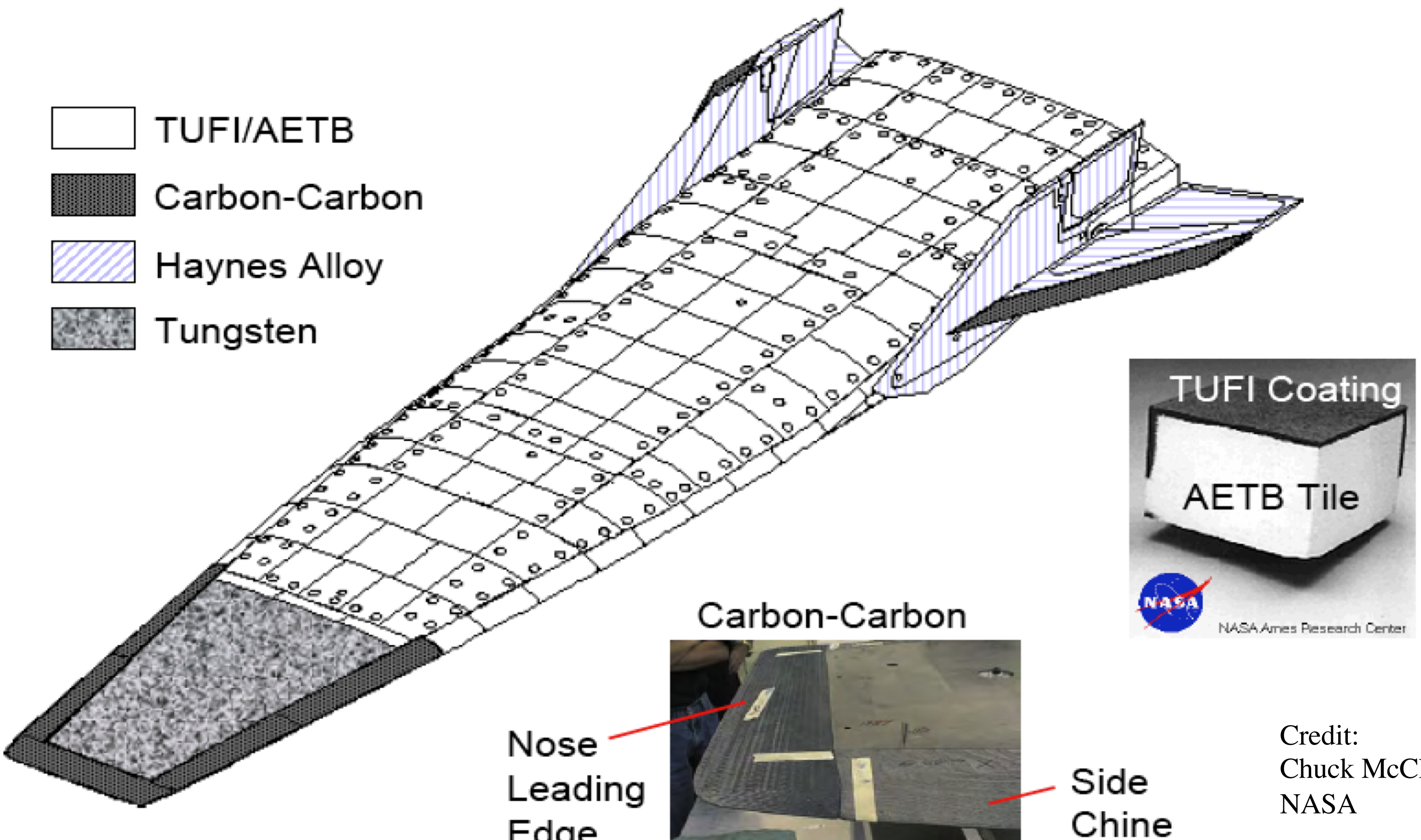
$$\dot{T}_{wall} = \frac{\left(\Phi(\theta) H_{tf}\right)\left[C_p T_\infty + \frac{V_\infty^{\ 2}}{2} - C_{p_{wall}} T_{wall}\right] + \left[\frac{\alpha}{2}\sigma T_2^{\ 4} - \varepsilon\sigma T_{wall}^{\ \ 4}\right]}{\left[\rho_{LE} C_{p\,LE} \tau_{LE}\right]}$$

- Shuttle tile manages heat by having high emissivity and very high heat capacity .. But it limited to < 2000°K

SCRAMJET DESIGN ISSUES, IV (cont'd)

X-43 Thermal Protection

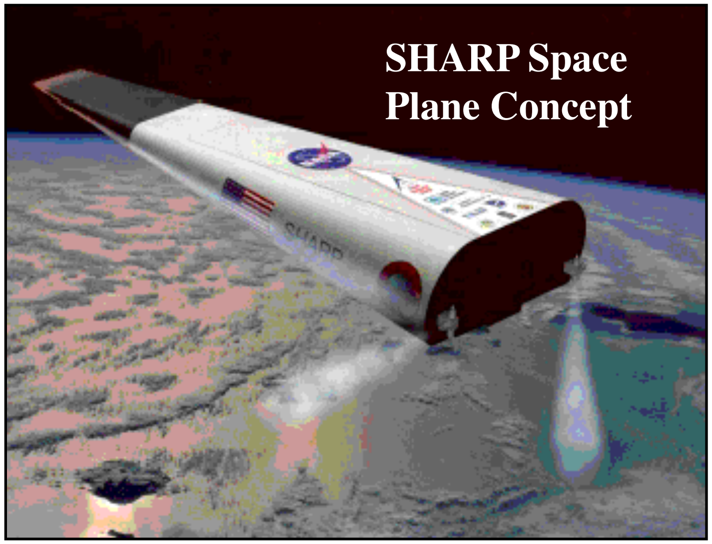
Mach 7 Vehicles (M 10 Vehicle has C-C Vertical Tail LE)



Credit:
Chuck McClinton
NASA

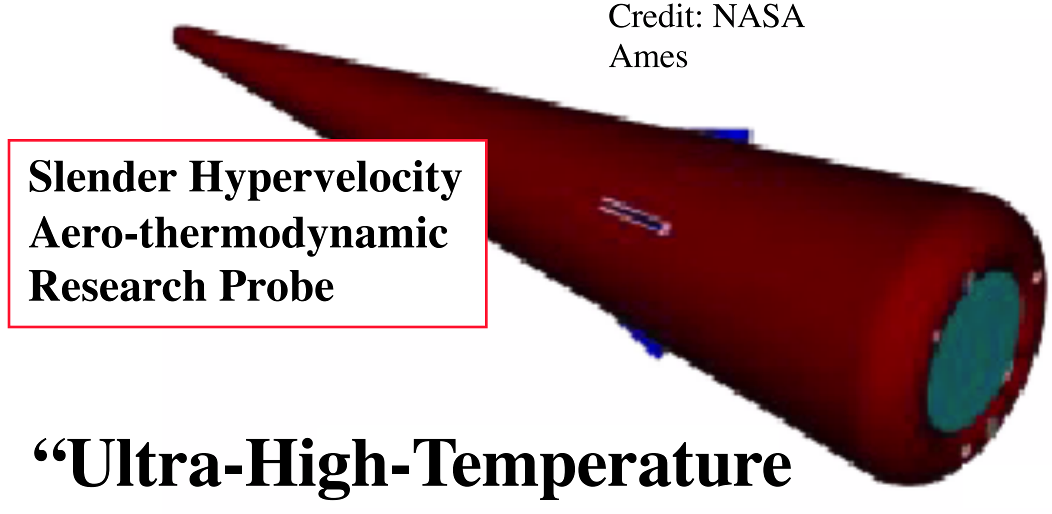
TUFI =Toughened Uni-piece Fibrous Insulation
AETB=Alumina Enhanced Thermal Barrier

SCRAMJET DESIGN ISSUES, IV (cont'd)



Mach 12+ TPS for sharp leading edge

Credit: NASA Ames

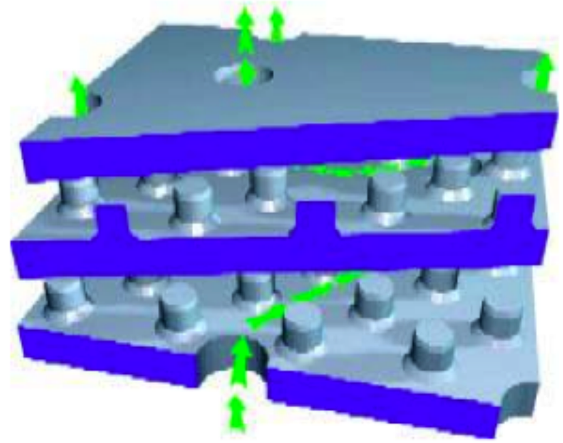


"Ultra-High-Temperature Ceramics" (UHTC)

- Technology readiness level (TRL) for UTHC TPS systems very low < 3/10

SCRAMJET DESIGN ISSUES, IV (cont'd)

• Even with matured UTHC TPS heating will have to be actively managed for long duration hypersonic flight …

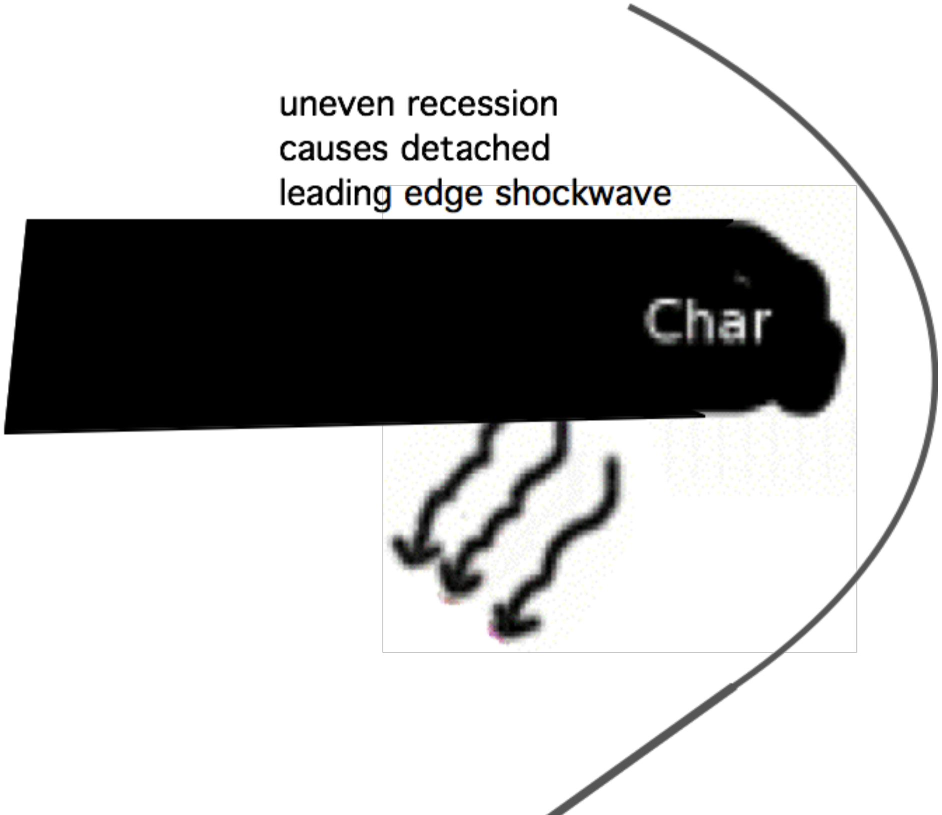


$$\dot{T}_{wall} = \frac{\left(\Phi(\theta)H_{tf}\right)\left[C_p T_\infty + \frac{V_\infty^{\;2}}{2} - C_{p_{wall}} T_{wall}\right] + \left[\frac{\alpha}{2}\sigma T_2^{\;4} - \varepsilon\sigma T_{wall}^{\;4}\right] - \left(\dot{q}\right)_{removed}}{\left[\rho_{LE} C_{p\,LE} \tau_{LE}\right]}$$

Where do you put the heat you remove?

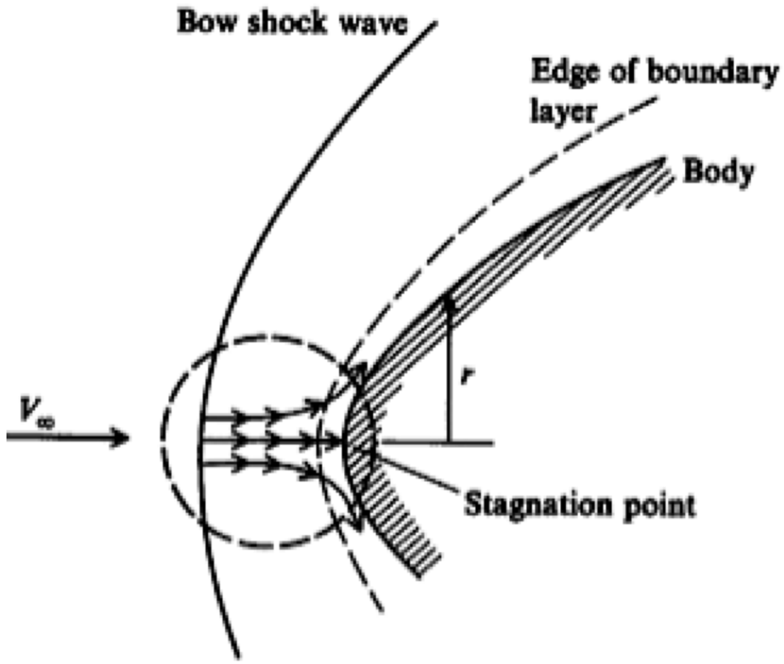
SCRAMJET DESIGN ISSUES, IV (cont'd)

- How About Ablative leading edges



- **Detached shockwave effects inlet flow path increases drag**

- **Emitted gases can effect mixture ratio of engine**



- Non-receding charring ablative (NRCA)

SCRAMjet flight tests

• The high cost of flight testing and the unavailability of full enthalpy ground facilities have hindered scramjet development.

• A large amount of the experimental work on scramjets has been undertaken in cryogenic facilities, direct-connect tests, or burners, each of which simulates one aspect of the engine operation.

• Further, vitiated facilities, storage heated facilities, arc facilities and the various types of shock tunnels each have limitations which have prevented perfect simulation of scramjet operation.

• Full Enthalpy, full dynamic pressure data is a *REAL RARITY*

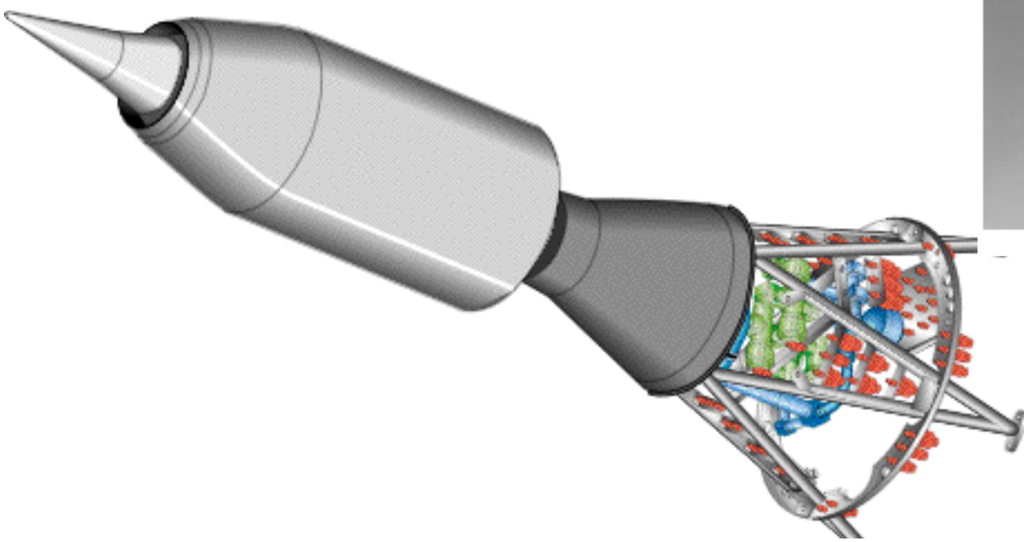
SCRAMjet flight tests (cont'd)

WHY FLIGHT TESTS?

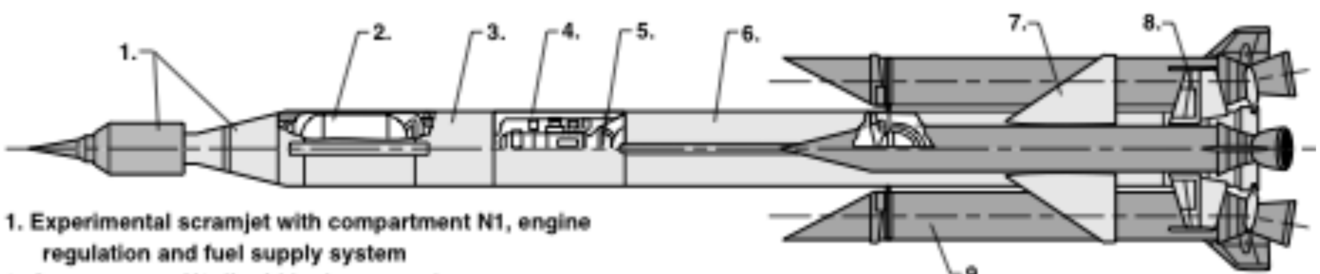
"...to separate the real from the imagined and to make known the overlooked and the unexpected problems..." Hugh L. Dryden

SCRAMjet flight tests, CIAM

- Russian CIAM … mid 1990's



- Supersonic Combustion Never verified by peer review … debate rages

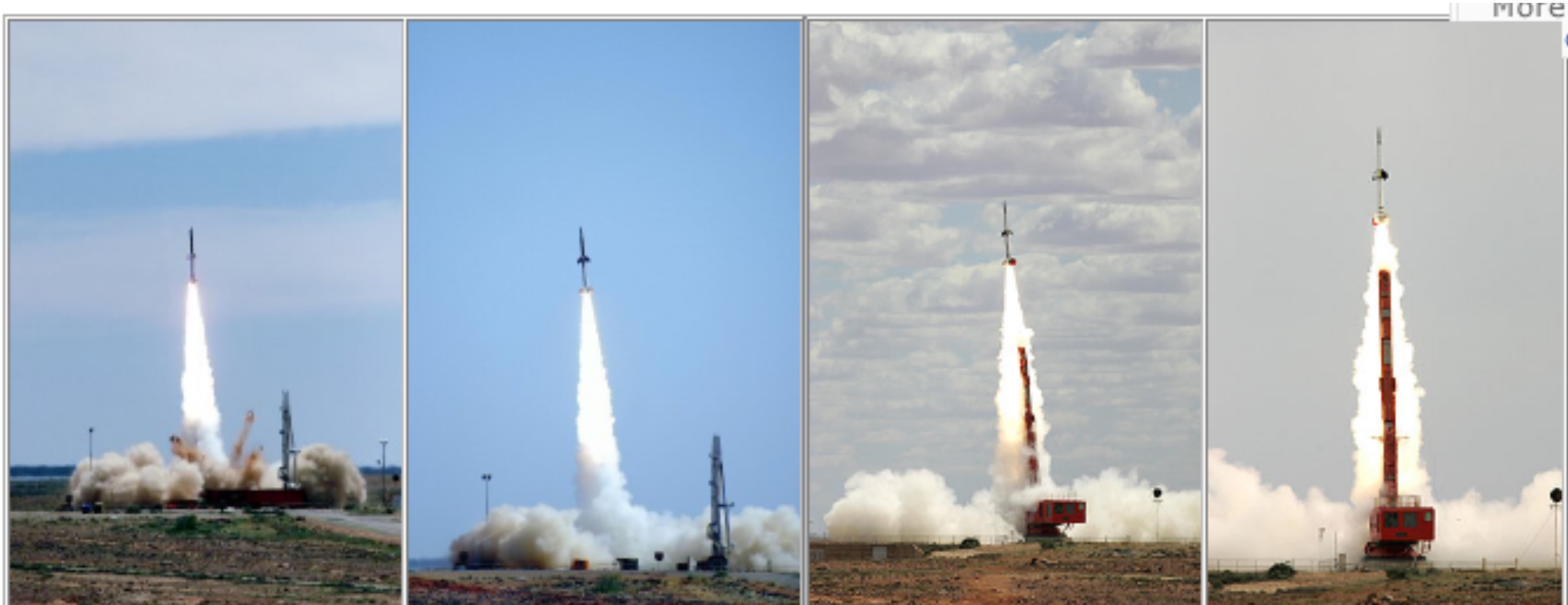


1. Experimental scramjet with compartment N1, engine regulation and fuel supply system
2. Compartment N2, liquid hydrogen tank
3. Compartment N3A, nitrogen/helium pressure supply system
4. Compartment N3B, flight control system and power supply
5. Propellant tank control system
6. SA-5 rocket motor
7. Fin
8. Roll control surface
9. Solid booster rocket

Central Institute for Aviation Motors (CIAM)

SCRAMjet flight tests, HyShot

- U. Queensland (Australian) Hyshot Flight tests



HyShot I Launch 30 October 2001 | HyShot II launch 30 July 2002 | HyShot III launch 25 March 2006 | HyShot IV launch 30 March 2006

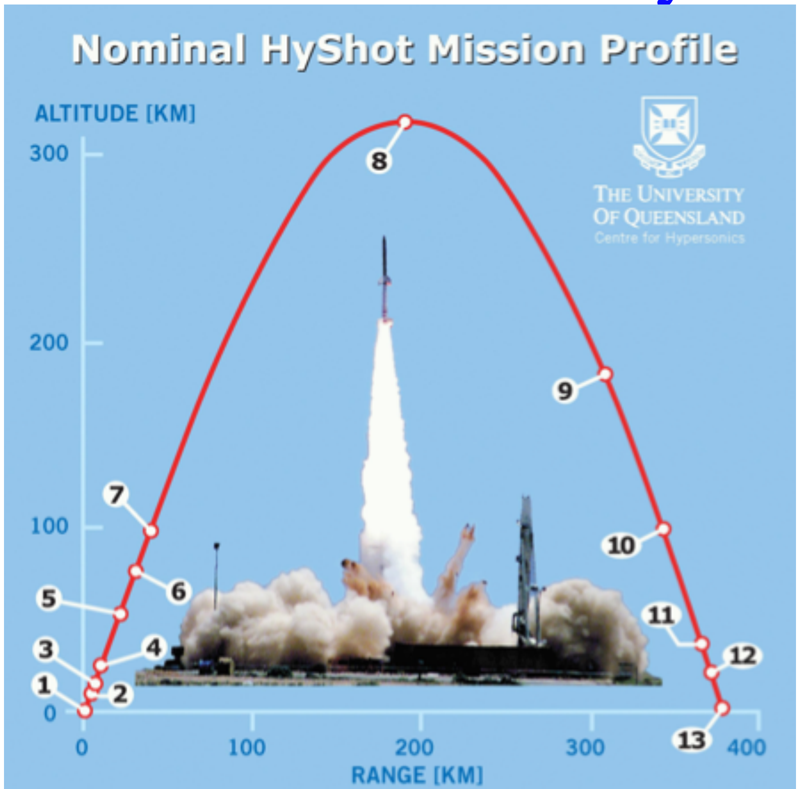
- Flight 1 failed, Flights 2-4 successful

SCRAMjet flight tests, HyShot (cont'd)

- U. Queensland (Australian) Hyshot Flight tests
- Hyshot II First verified SCRAM flight operation July 30, 2002
- Engine only tests, not an integrated vehicle .. Hyshot I, II Flowpath tests …Never intended to produce more thrust than drag

Terrier-Orion sounding rocket

HyShot II Mission Profile



		Time	Altitude	Speed – Mach
1	Terrier Ignition	– 0 SEC,	0 KM,	M0
2	Terrier Burnout	– 6.03 SEC,	3.44 KM,	M3.6
3	Stage Separation	– 6.04 SEC,	3.5 KM,	M3.6
4	Orion Ignition	– 16 SEC,	12.8 KM,	M3.3
5	Orion Burnout	– 42.4 SEC,	56.4 KM,	M7.1
6	Nosecone Eject	– 63 SEC,	100 KM,	–
7	Start Attitude Control Manoeuvre	– 73 SEC,	115 KM,	–
8	Apogee	– 281 SEC,	315 KM,	–
9	Re-enter Atmosphere	– 510 SEC,	80 KM,	M8.0
10	‡ Start Experiment	– 529 SEC,	35 KM,	M7.6
11	‡ Stop experiment	– 535 SEC,	23 KM,	M7.6
12	Impact	– 565 SEC,	0 KM,	M0.67

- Terrier-Orion Mk 70 rocket
- Max liftoff spd: Mach 8+
- Liftoff accl: 22 g (60 g for 0.5 s)
- Apogee: 330 km
  - Nose is pushed over, cone
  - ejected (Bang-Bang maneuver)
- Max descent spd: Mach 7.6
  - Scramjet stage
  - Hydrogen Fueled

SCRAMjet flight tests, HyShot (cont'd)

- HyShot III Flight, March 25, 2006
- More Sophisticated 4-chamber axi-symmetric inlet design
- Teaming with British company Qinetiq
- Positive thrust accelerated vehicle from Mach 6.8 to Mach 8.0
- Hyshot IV data still being analyzed

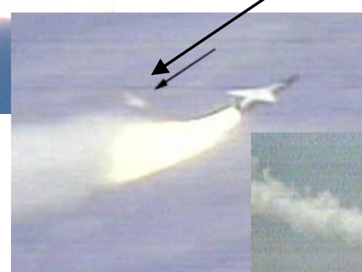
SCRAMjet flight tests, HyShot (cont'd)

SCRAMjet flight tests, X-43A

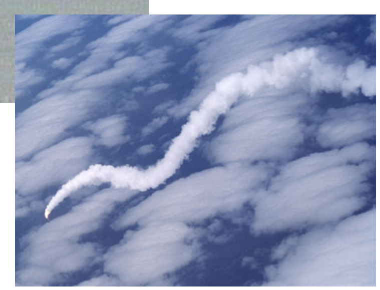
- NASA X-43A, three flights

Flight 1, June 2 2001 … booster failure, terminated flight

Not a good sign

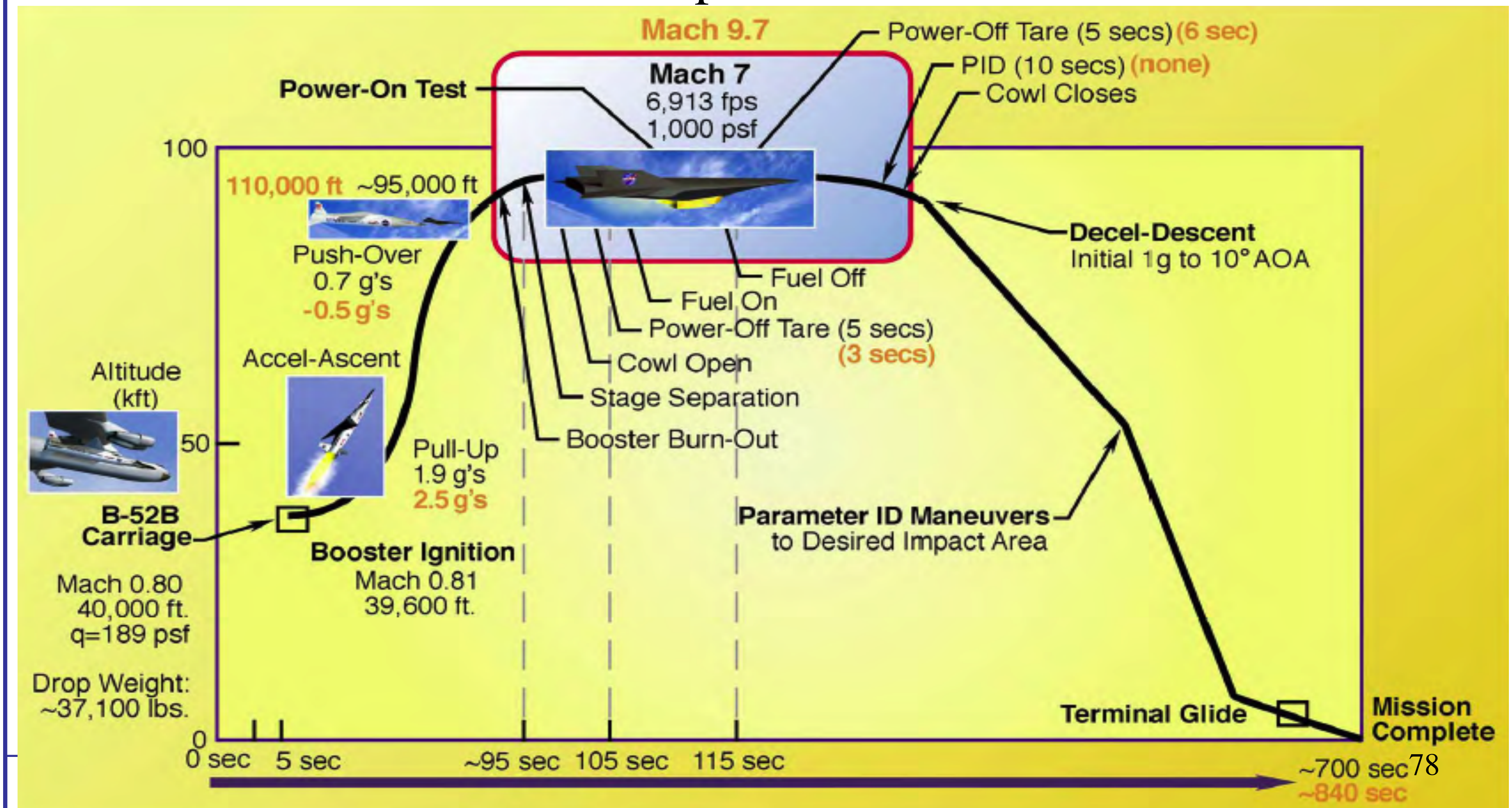


Here Comes the MIB.

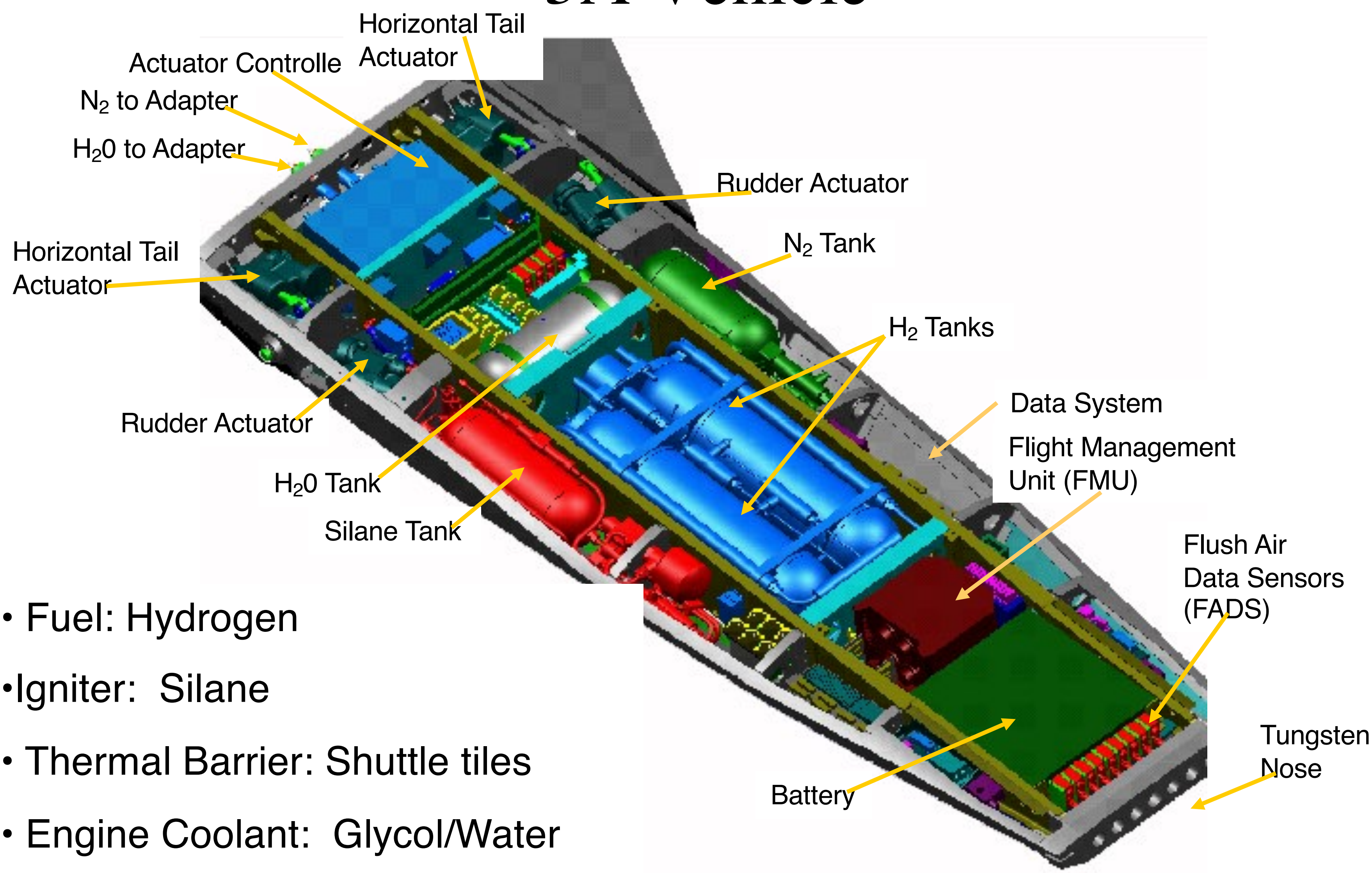


SCRAMjet flight tests, X-43A (cont'd)

- Took three years to get Problem fixed … flight 2, *March 27, 2004* successful Mach 7 max engine operation, *Flight 3, November 16, 2004*, Mach 10 successful operation



X-43A Vehicle



- Fuel: Hydrogen
- Igniter: Silane
- Thermal Barrier: Shuttle tiles
- Engine Coolant: Glycol/Water
- Nitrogen Purge
- Electric Actuators

X-43A firsts (cont'd)

- **Firsts**
  - First flight of Integrated Scramjet Vehicle
  - **Successful high dynamic pressure, high Mach, non-symmetrical stage separation (required for TSTO)**
- **Verified performance, operability and controllability**
  - **Airframe-integrated Scramjet**
  - **Integrated, powered, hypersonic airbreathing Vehicle**
- **Verified engineering application of NASA-Industry-University hypersonic vehicle design tools**

Tools	**Disciplines**	**Physics**
- Experimental	- Propulsion	
- Analysis	- Aerodynamic	
CFD - Numerical	- Structural	
Analytical	- Thermal	
Empirical	- Boundary layer transition	
- MDOE for engine/vehicle design optimization	- Flight and engine controls	
	- Vehicle synthesis	

Credit: Chuck McClinton NASA

X-43A Lessons learned

- **X-43 airframe drag (and lift) was slightly higher than nominal predicted, but within uncertainty prediction**
- **Scramjet engine performance was very close to preflight predictions (positive acceleration for M 7, Cruise for M 10)**
- **Control deflections to trim engine induced moments were very close to preflight predictions**
- **Other hypersonic vehicle technologies were as predicted**
  - **Aerodynamic stability and control**
  - **Natural and Tripped boundary layer transition**
  - **Airframe and wing structure**
  - **Thermal loads/Gap heating**
  - **TPS**
  - **Internal environment**
  - **Launch vehicle stiffness**

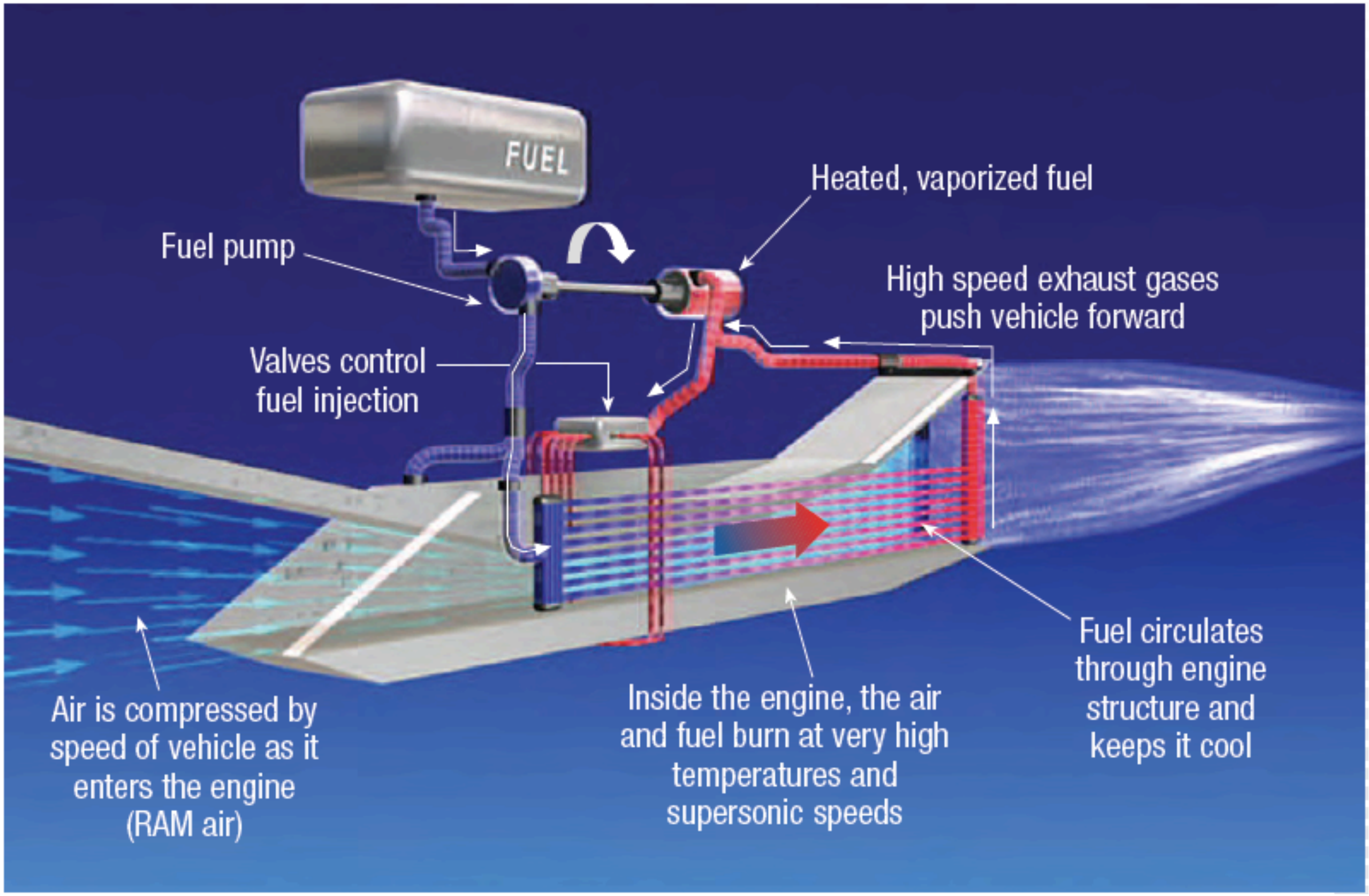
Credit: Chuck McClinton NASA

USAF X-51A

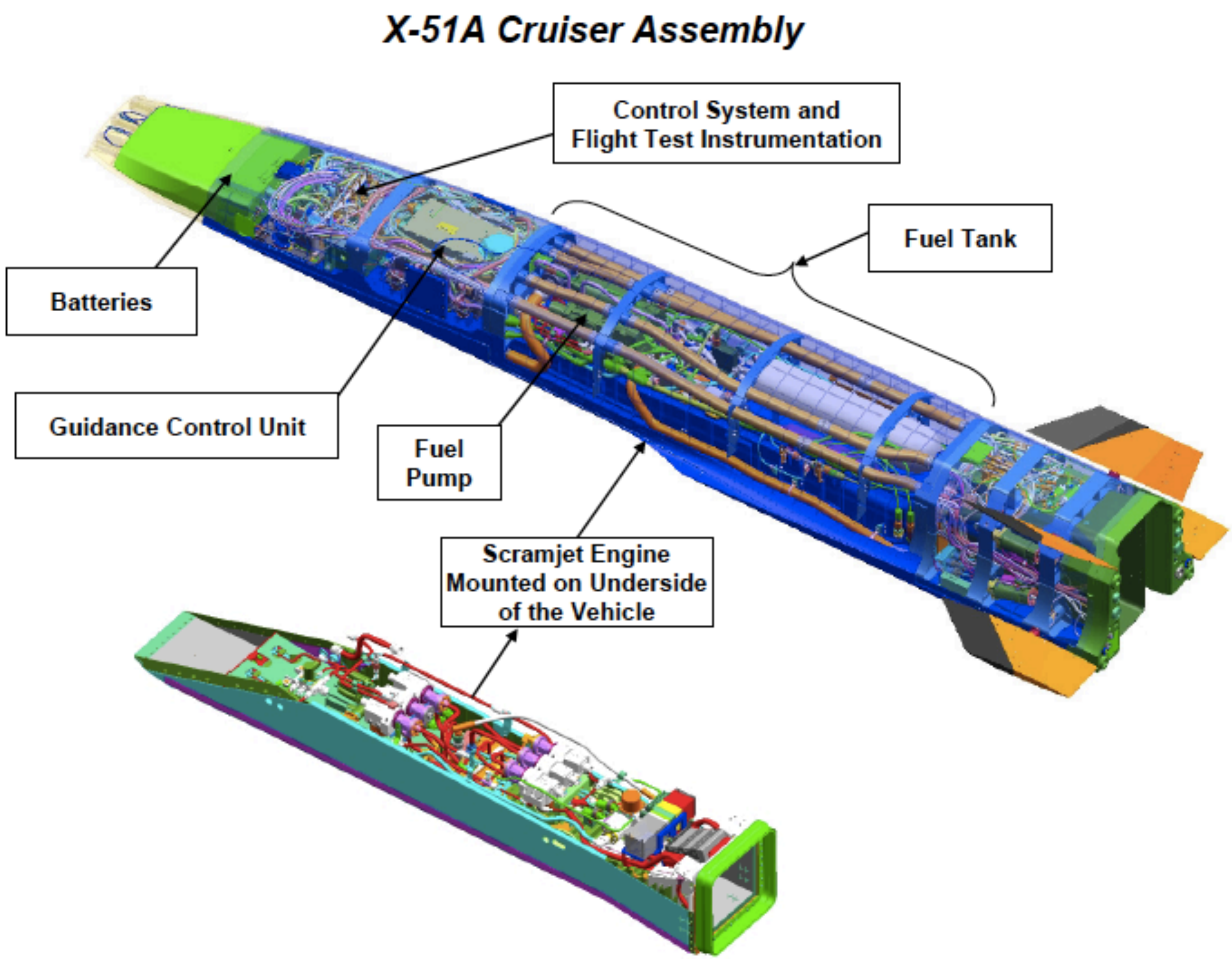

X-51A
FIRST FLIGHT
May 2010
PTE
GDE-1
GDE-2
X-1
X-2
Pratt & Whitney Rocketdyne
Pratt & Whitney
A United Technologies Company
PTE = Performance Test Engine • GDE = Ground Demonstrator Engine • X = Experimental

USAF X-51A (2)

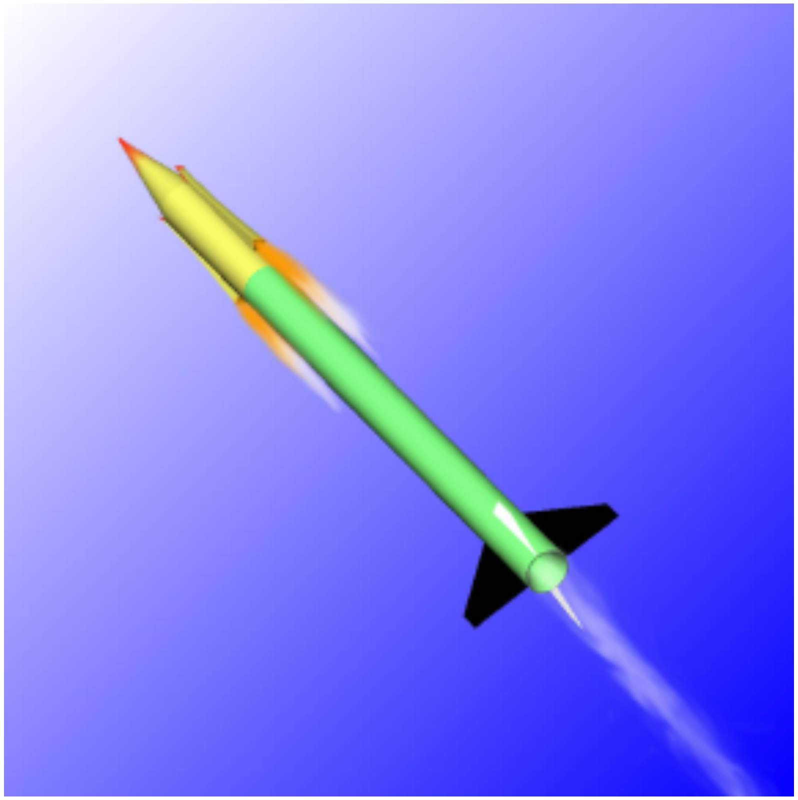
- First Hydrocarbon Fueled ScramJet



USAF X-51A (3)


X-51A Cruiser Assembly
Control System and Flight Test Instrumentation
Fuel Tank
Batteries
Guidance Control Unit
Fuel Pump
Scramjet Engine Mounted on Underside of the Vehicle

Small Scale SCRAM experiments



- Simplified Approach to Scramjet Testing (SAST)
- Propulsion & Performance Branch (RP), NASA Dryden
- Small directionally-symmetric Mach 6 Scramjet design
- Configuration is aerodynamically stable

Small Scale SCRAM experiments

(cont'd)

SAST Design / Test Team

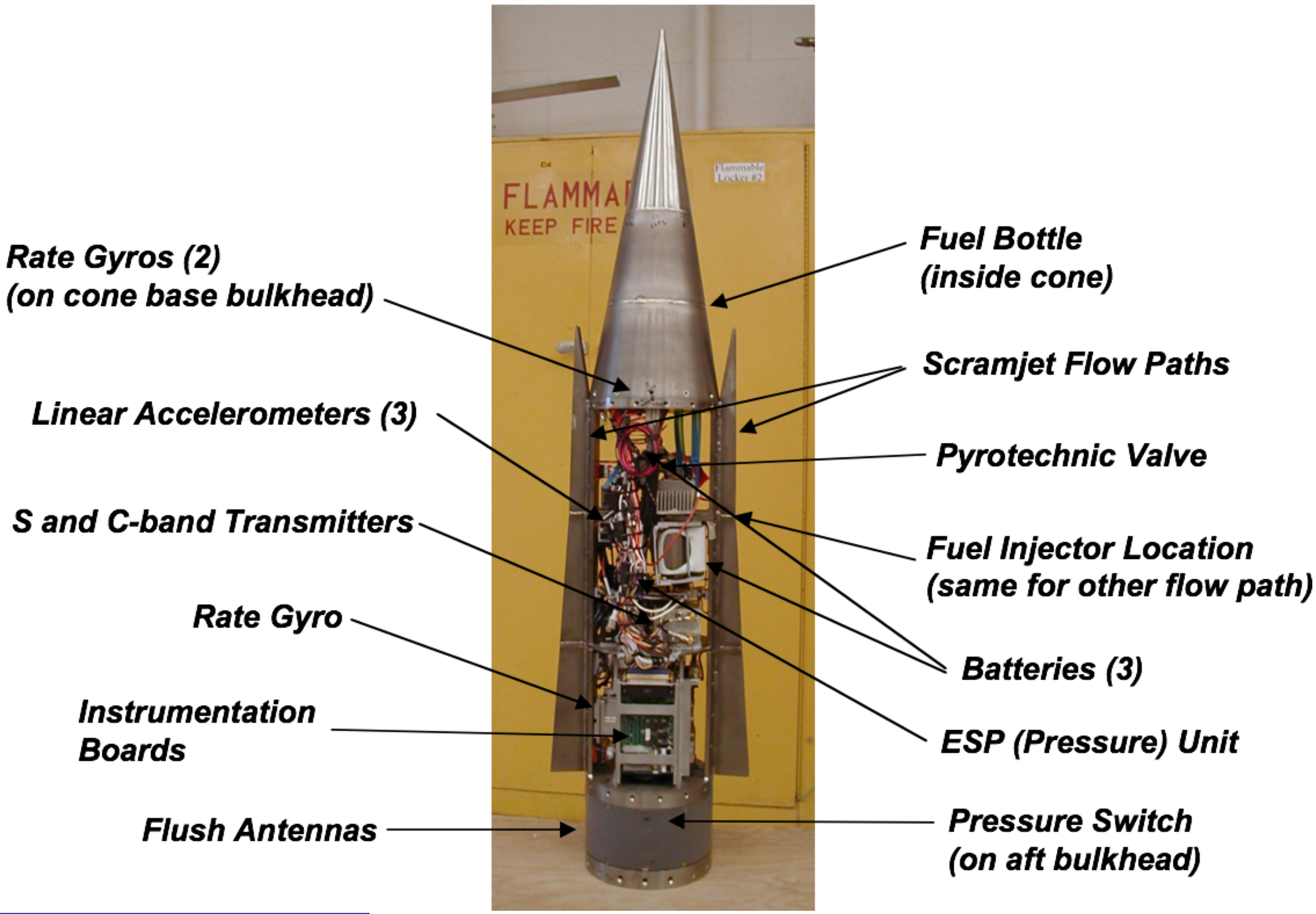
- ***NASA Dryden, Edwards, CA***
  - *Vehicle and experiment design, fabrication, and analysis.*
  - *Data acquisition / telemetry (Code FT).*
- ***NAWC-Weapons Division, Research Rockets Branch, White Sands Missile Range, NM***
  - *Ground, range, and flight operations and safety.*
- ***NAWC-Weapons Division, Pt. Mugu, CA***
  - *Flush mount antenna design, fabrication, and test.*
- ***NAWC, China Lake, CA***
  - *Payload welding.*
- ***Industrial Solid Propulsion (ISP), Las Vegas, NV***
  - *Rocket motors.*

SAST Objectives

- Build experience with hypersonic flight test techniques and instrumentation at NASA Dryden.
- Evaluate the feasibility and value of a simple, low cost hypersonic flight testbed.
- Obtain hypersonic propulsion flight data for a simplified scramjet engine.
- Get operational expertise through high flight rate

Simplified Approach to SCRAMjet Testing (SAST)

SAST Payload Internal Arrangement



Simplified Approach to SCRAMjet Testing (SAST)

Scramjet Engine Design

- ***Simple cone (9.5°) forebody***
  - ***High dynamic pressure flight delivers high combustor entrance pressure using simple conical shock-on-lip, shock on shoulder configuration.***
- ***Self-starting scoop inlet***
  - ***Swept sidewall, spilling design allows fixed geometry inlet to start at about Mach 3 and obtain full capture at design Mach of 6.***
- ***Diverging isolator (1°)***
  - ***Diverging isolator prevents combustor-inlet interactions.***
- ***Single orifice, normal fuel injection***
  - ***Fuel injection scheme chosen for simplicity but is easily changed to more optimum designs.***
- ***High pressure, gaseous hydrogen-silane fuel***
  - ***Proven hydrogen-silane pyrophoric fuel used for auto-ignition.***
- ***Diverging combustor / nozzle (4°)***
  - ***Conservative expansion ratio provides measurable engine thrust with low external aerodynamic drag.***

Simplified Approach to SCRAMjet Testing (SAST)

SAST Trajectory

- ***Launch at elevation angle, Q_E, of 79 deg.***
- ***Ballistic trajectory to impact. As directed by WSMR, no destruct system will be used.***
- ***Test point (at rocket motor burnout) of Mach 5.2, 17,000 ft. approx. 6 seconds after launch.***
- ***Maximum altitude of approx. 150,000 ft.***
- ***Impact point approx. 18 nmi. downrange.***
- ***Total flight time of approx. 220 seconds.***
- ***Dispersion footprint within WSMR range.***

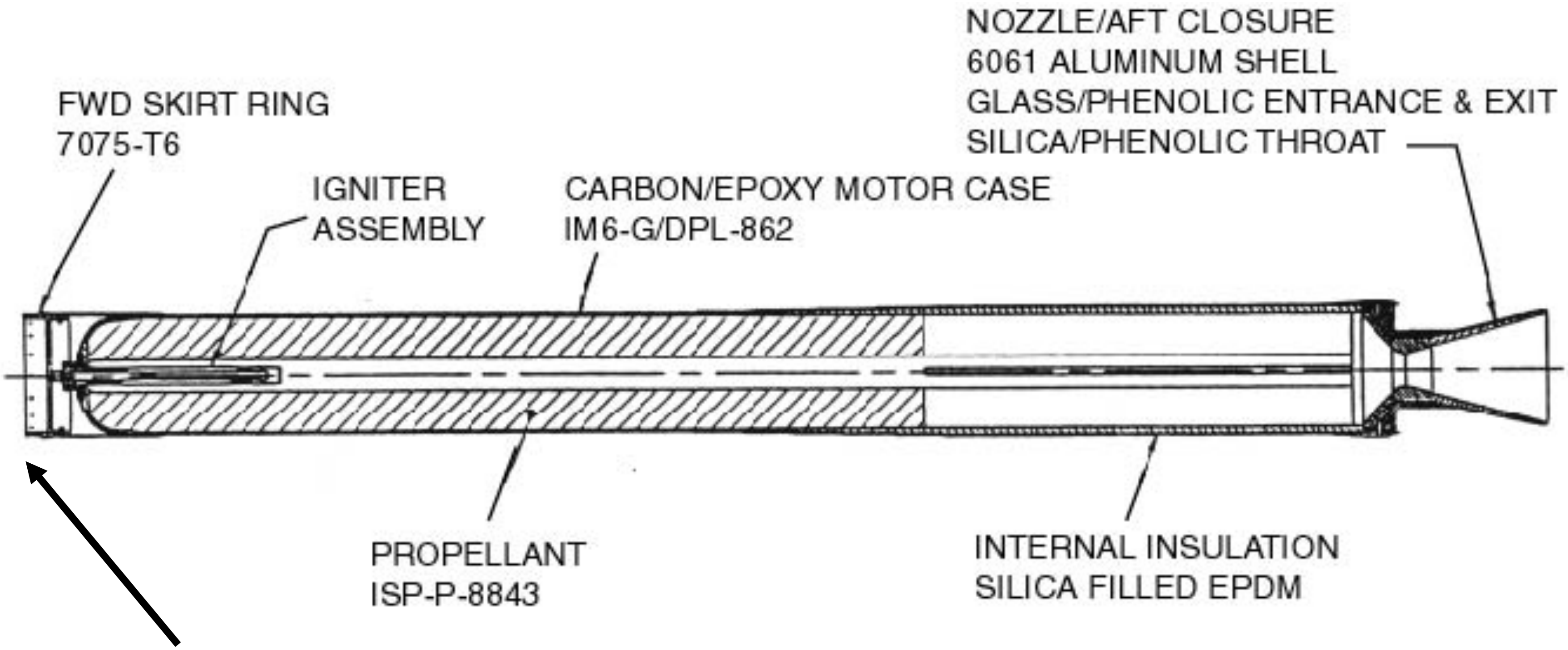
Rocket Motor Description

- Viper-V Block-II solid rocket motor
- Manufacturer -- Industrial Solid Propulsion, Inc.
- Propellant -- 87% solids HTPB/AP/AL
- Dimensions -- 131 in. length and 7 in. dia.
- Weight -- 225 lbs.
- Thrust -- approx. 6,000 lbs for 5.5 seconds
- Motor case -- carbon/epoxy composite
- Launch lugs -- fixed T-rail aft lug, ejectable T-rail forward lug

Scramjet Fuel System

- High pressure, gaseous blow down system.
- Gaseous hydrogen-silane fuel stored in fuel tank at 1800 psi.
- Pyrotechnic valve used to release fuel to fuel injectors.
- Pressure switch sensing booster burn-out opens pyrotechnic valve.
- Fuel injectors sized for initial ER=0.2.
- Scramjet burn time of about 2 sec. (with decreasing ER).
- Predicted peak combustion pressure of about 300 psi and change in force of about 150 lbs.

Viper V Block II Motor Cross Section



INDUSTRIAL SOLID PROPULSION Inc.

First Flight March 2000

- Guess What? Booster Failure

- Dust yourself off, try it again!!!

Hypersonic Physics - Propulsion

- Natural and forced boundary layer transition
- Turbulence
- Separation caused by shock-boundary layer interaction
- Shock-shock interaction heating (Type 3 and 4)
- Isolator shock trains
- Cold-wall heat transfer
- Fuel injection, penetration and mixing
- Finite rate chemical kinetics
- Turbulence-chemistry interaction
- Boundary layer relaminarization
- Recombination chemistry
- Catalytic wall effects

- **Lot of Promise but Long way to go**

\- Most of these phenomena were modeled in the design tools. Some were avoided by application of a uncertainty factors.

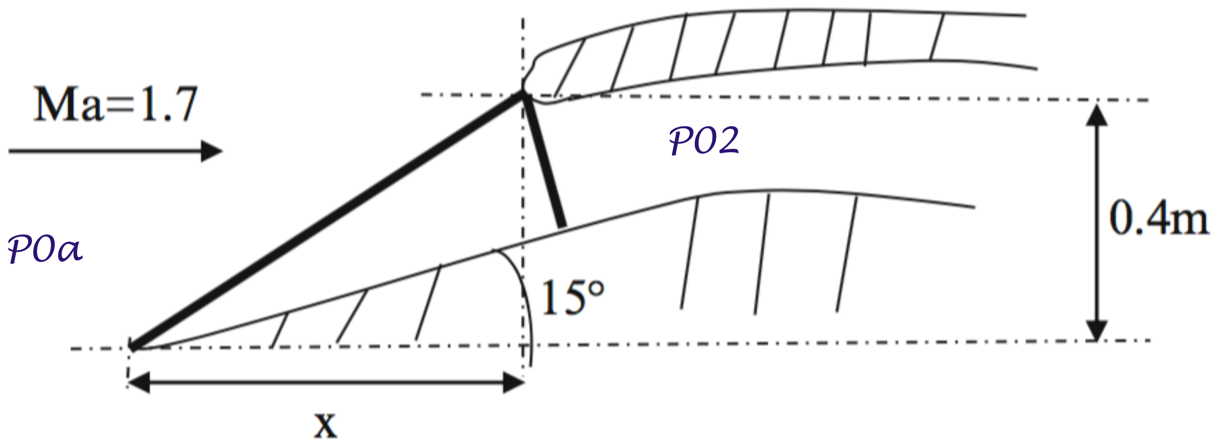
\- X-43 success demonstrates an engineering level understanding of "the physics". A better understanding of these issues will be beneficial for optimization of vehicle performance, but not "enabling"

\- All designs share the same physics

Credit: Chuck McClinton NASA

Homework 4.3, Part 1

A ramjet operates at an altitude of 10,000 m $(T_a = 223\ K,\ P_a = 0.26\ atm,\ \gamma = 1.4)$ at a Mach number of 1.7. The external diffusion is based on an oblique shock and on a normal shock, as described in the shown figure.

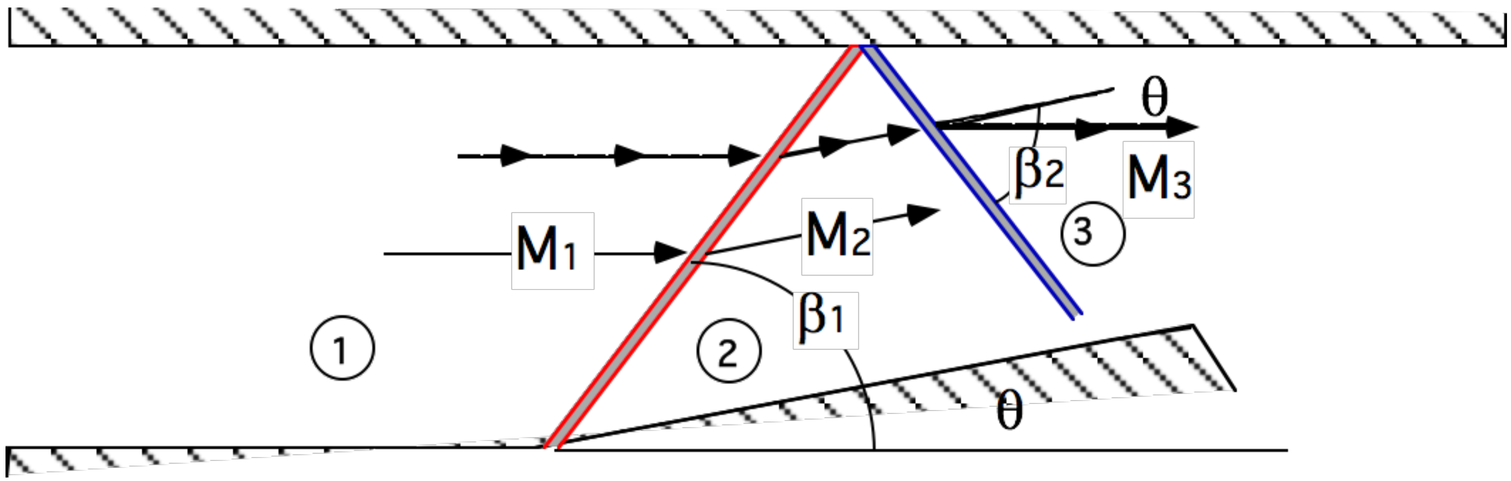


Calculate

Assume ∞ = a

- Stagnation pressure recovery, $\frac{P_{02}}{P_{0a}}$?
- At what Mach number does the oblique shock become detached?
- What is the distance x, from the cone tip to the outer inlet lip, for the condition described in the figure?
- What is the best turning angle θ in terms of highest pressure ratio, $\frac{P_{02}}{P_{0a}}$?

Homework 4.3, Part 2



- Calculate

$\beta_1 = 30°$
M_1=2.8
p_1=1 atm
T_1=300°K

β_2
M_3
p_3
T_3

- Assume γ=1.4


Questions??