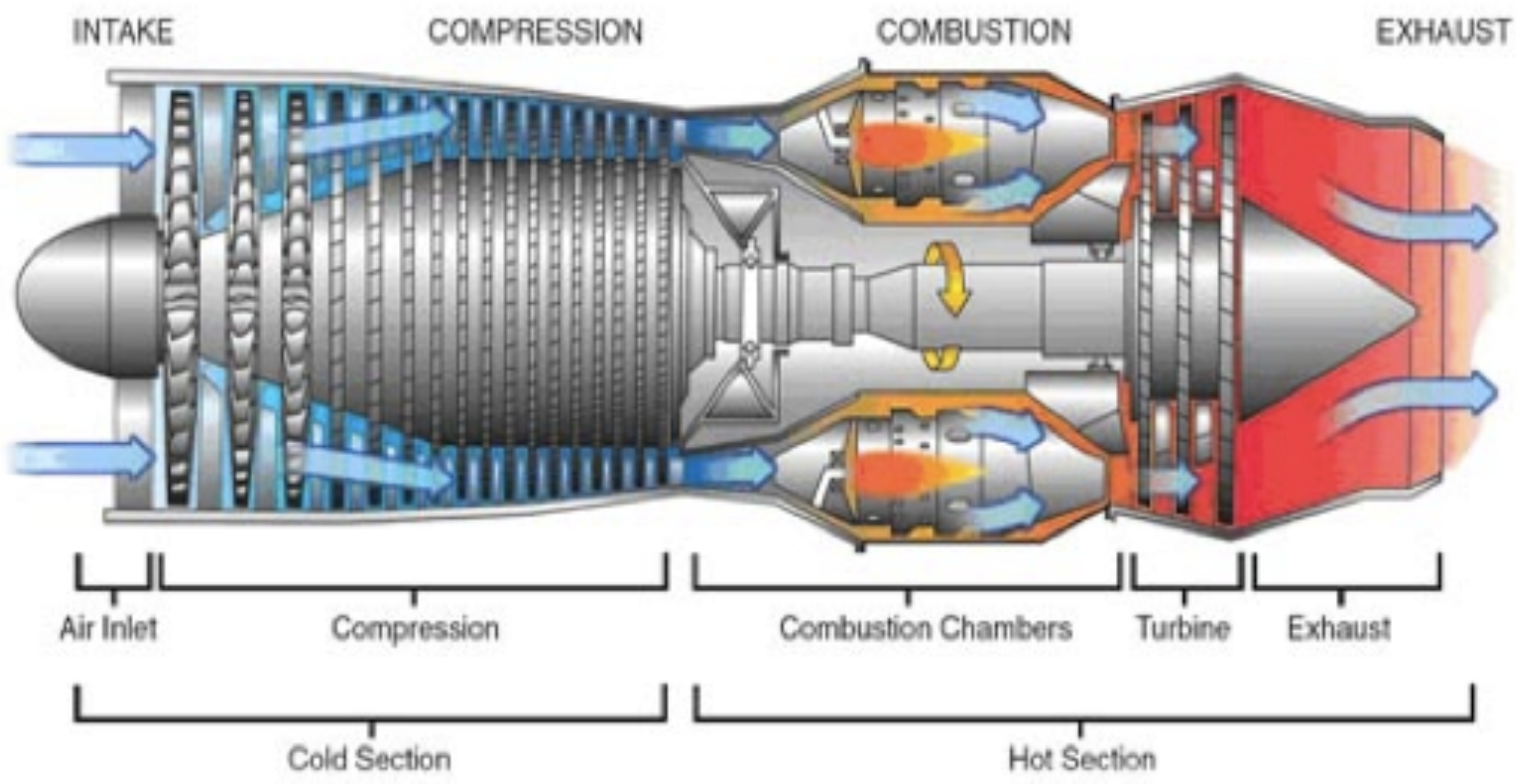
Section 5.1: The TurboJet Propulsion Cycle



Overview

- **Revisit Thermal Efficiency of a RamJet …**

$$\rightarrow \quad \eta_{thermal} = 1 - \frac{Heat\ Rejected\ During\ Cycle}{Heat\ Input\ During\ Cycle} = 1 - \frac{\left(\dot{m}_{air} + \dot{m}_{fuel}\right)\left(h_{exit}\right) - \left(\dot{m}_{air}\right)\left(h_{\infty}\right)}{\left(\dot{m}_{air} + \dot{m}_{fuel}\right)\left(h_{0_{exit}}\right) - \left(\dot{m}_{air}\right)\left(h_{0_{\infty}}\right)} =$$

$$1 - \frac{\left(1 + \frac{1}{f}\right) C_{p_{exit}} T_{exit} - C_{p_{air}} T_{\infty}}{\left(1 + \frac{1}{f}\right) C_{p_{exit}} T_{0_{exit}} - C_{p_{air}} T_{0_{\infty}}} = 1 - \frac{\left(1 + \frac{1}{f}\right) C_{p_{exit}} T_{exit} - C_{p_{air}} T_{\infty}}{\left(1 + \frac{1}{f}\right) C_{p_{exit}} T_{0_{exit}} - C_{p_{air}} T_{0_{\infty}}} = 1 - \frac{C_{p_{air}} T_{\infty}}{C_{p_{air}} T_{0_{\infty}}} \frac{\left(1 + \frac{1}{f}\right) \frac{C_{p_{exit}} T_{exit}}{C_{p_{air}} T_{0_{\infty}}} - 1}{\left(1 + \frac{1}{f}\right) \frac{C_{p_{exit}} T_{0_{exit}}}{C_{p_{air}} T_{0_{\infty}}} - 1}$$

Since

$$\frac{T_{\infty}}{T_{0_{\infty}}} = \frac{1}{1 + \frac{\gamma - 1}{2} M_{\infty}^2}$$

$$\rightarrow T_{0_{\infty}} = T_{\infty}\left(1 + \frac{\gamma - 1}{2} M_{\infty}^2\right)$$

$$T_{0_{exit}} = T_{exit}\left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)$$

Substitution into the above equation gives ….. →

$$\eta_{thermal} = 1 - \frac{1}{1 + \frac{\gamma - 1}{2} M_{\infty}^2} \frac{\left(1 + \frac{1}{f}\right) \frac{C_{p_{exit}} T_{exit}}{C_{p_{air}} T_{\infty}} - 1}{\left(1 + \frac{1}{f}\right) \frac{C_{p_{exit}} T_{0_{exit}}}{C_{p_{air}} T_{0_{\infty}}} - 1}$$

Overview (2)

- **… Thermal Efficiency of a RamJet …**

$$\rightarrow \eta_{thermal} = 1 - \frac{1}{1+\frac{\gamma-1}{2}M_\infty^2}\,\frac{\left(1+\frac{1}{f}\right)\frac{C_{p_{exit}}T_{exit}}{C_{p_{air}}T_\infty}-1}{\left(1+\frac{1}{f}\right)\frac{C_{p_{exit}}T_{0_{exit}}}{C_{p_{air}}T_{0_\infty}}-1} = \frac{\left(1+\frac{\gamma-1}{2}M_\infty^2\right) - \frac{\left(1+\frac{1}{f}\right)\frac{C_{p_{exit}}T_{exit}}{C_{p_{air}}T_\infty}-1}{\left(1+\frac{1}{f}\right)\frac{C_{p_{exit}}T_{0_{exit}}}{C_{p_{air}}T_{0_\infty}}-1}}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)} =$$

$$\frac{\left(1+\frac{\gamma-1}{2}M_\infty^2\right) - \frac{\left(1+\frac{1}{f}\right)\frac{C_{p_{exit}}T_{exit}}{C_{p_{air}}T_\infty}-1}{\left(1+\frac{1}{f}\right)\frac{C_{p_{exit}}T_{exit}\cdot\left(1+\frac{\gamma-1}{2}M_{exit}^2\right)}{C_{p_{air}}T_\infty\left(1+\frac{\gamma-1}{2}M_\infty^2\right)}-1}}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)}$$

Investigate what happens to thermal efficiency as Mach ~0

Overview (3)

Investigate what happens to thermal efficiency as Mach ~0

$$\left(\eta_{thermal}\right)\Bigg|_{M_{exit}=0}^{M_\infty=0} = (1) - \frac{\left(1+\frac{1}{f}\right)\frac{C_{p_{exit}} T_{exit}}{C_{p_{air}} T_\infty} - 1}{\left(1+\frac{1}{f}\right)\frac{C_{p_{exit}} T_{exit}}{C_{p_{air}} T_\infty} - 1} = (1) - (1) = 0$$

• The thermal efficiency of the ideal ramjet is entirely determined by the flight Mach number. As the Mach number goes to zero the thermal efficiency goes to zero and the engine produces no thrust. ..

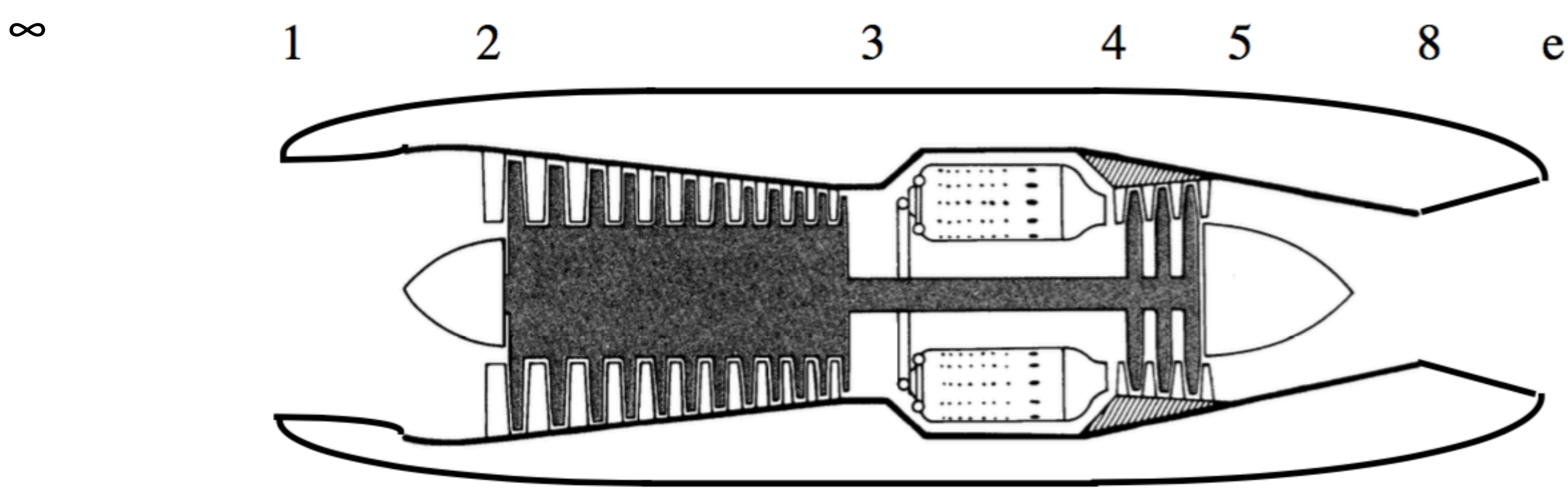
• Ram air is necessary for sufficient flow compression to hold the combustor flame

Overview: TurboJet as an Alternative Cycle

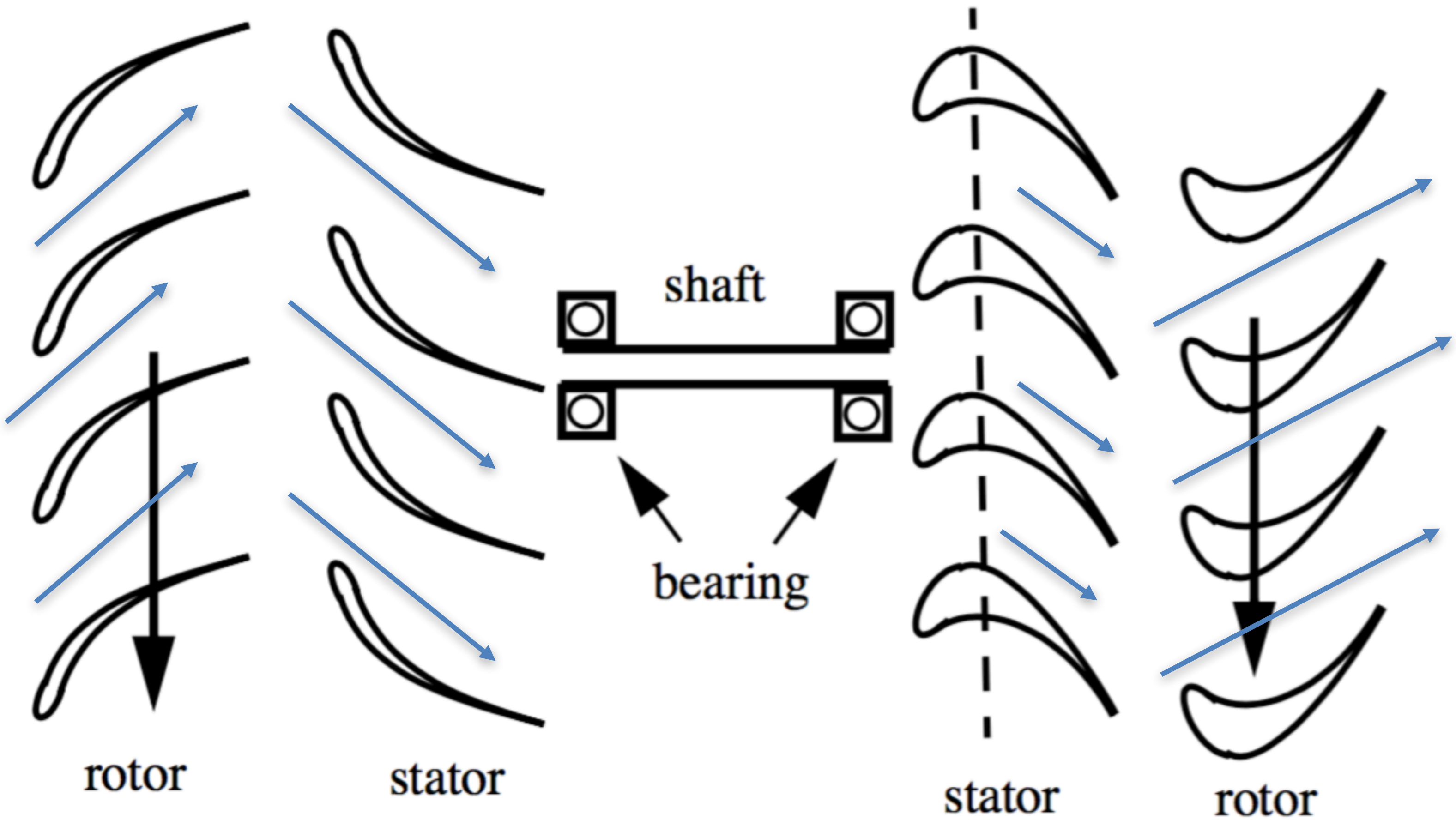
• To allow zero-velocity starts … a engine cycle that produces its own compression is necessary.

• Form of this cycle uses a compressor ahead of the diffuser, driven by a turbine that uses some of the exhaust flow power to drove the compressor

A sketch of a turbojet engine is shown below.



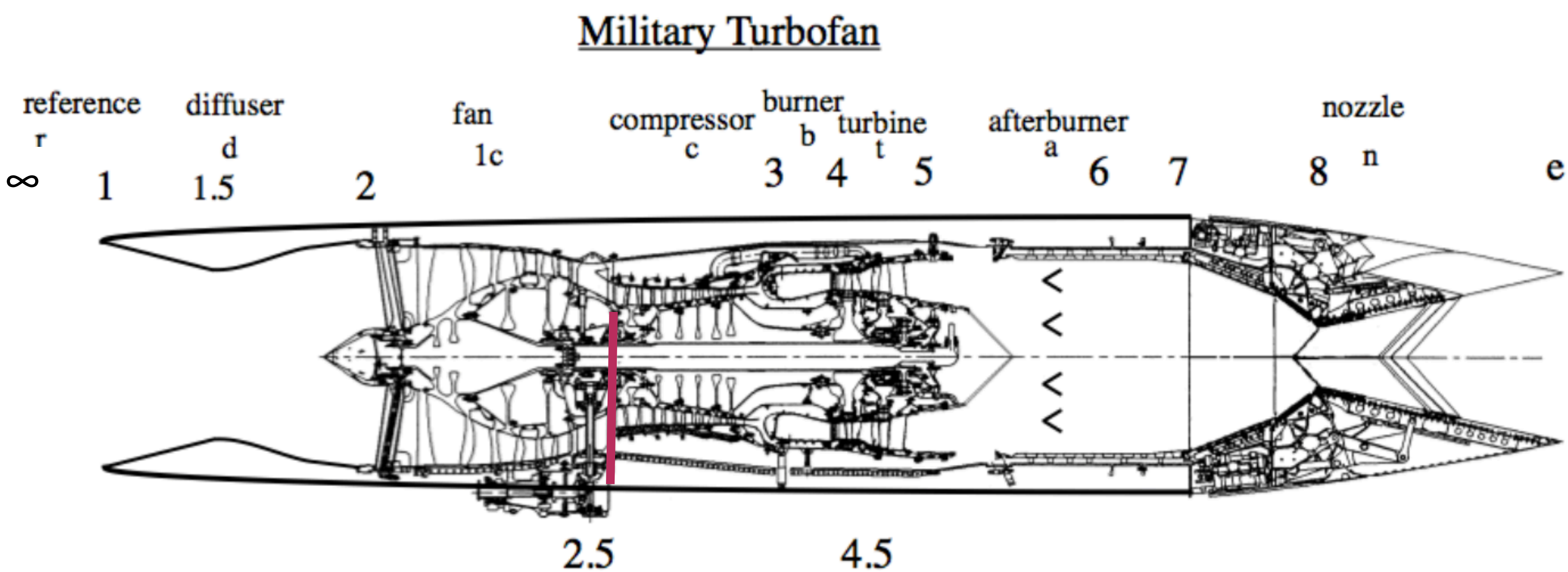
Overview: TurboJet as an Alternative Cycle (2)



Turbojet engine and compressor-turbine blade diagram.

TurboJet as an alternative Cycle (2)

- Notice that the turbojet ”map” is broken up into more individual stations (10) than was the ramjet “map.”
- This industry standard notation accounts for the more flow path associated with the turbojet combustion cycle.



Credit Brian Cantwell, Stanford AAE

Engine numbering and component notation

TurboJet Station Definitions

Station 0 - This is the *reference state* of the gas well upstream of the engine entrance. The temperature nd pressure parameters are

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \qquad \pi_r = \frac{P_{0_\infty}}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}$$

Station 1 - Entrance to the engine inlet. The purpose of the inlet is to reduce the Mach number of the incoming flow to a low subsonic value with as small a stagnation pressure loss as possible. From the entrance to the end of the inlet there is always an increase in area and so the component is appropriately called a diffuser.

Station 1.5 - The inlet throat.

TurboJet Station Definitions *(2)*

Station 2 - The fan or compressor face. The temperature/pressure parameters across the diffuser are

$$\tau_d = \frac{T_{0_2}}{T_{0_1}} \qquad \pi_d = \frac{p_{0_2}}{p_{0_1}}$$

The inlet is usually modeled as adiabatic flow so stagnation temperature is constant; however the stagnation pressure decreases due to the presence of viscous losses in diffuser and possible shock waves in the diffuser path

$$T_{0_\infty} = T_{0_1} = T_{0_2} \qquad P_{0_\infty} = P_{0_1} \geq P_{0_2}$$

TurboJet Station Definitions *(3)*

Station 2.5 - All modern turbofan engines use at least two compressor spools, a fan and a compressor.

The fan is usually accompanied by a low pressure compressor driven by a low pressure turbine through a shaft along the centerline of the engine.

A concentric shaft connects the high pressure turbine and high pressure compressor.

Station 2.5 is the interface between the low and high pressure compressor. Some (Roll Royce) turbofans commonly employ three spools with the high pressure compressor broken into two spools.

TurboJet Station Definitions *(4)*

Station 3 - High pressure compressor Exit. The temperature/pressure parameters across the compressor are

$$\tau_c = \frac{T_{0_3}}{T_{0_2}} \qquad \pi_c = \frac{p_{0_3}}{p_{0_2}}$$

The total compression includes that due to the fan. From a cycle perspective it is usually not necessary to distinguish the high and low pressure sections of the compressor. The goal of the designer is to produce a compression system that is as near to isentropic as possible.

TurboJet Station Definitions (5)

Station 4 **–** Combustor Exit. Temperature/pressure parameters across the burner are

$$\tau_b = \frac{T_{0_4}}{T_{0_3}} \qquad \pi_{\mathbf{b}} = \frac{p_{0_4}}{p_{0_3}}$$

Burner exit temperature is highest temperature in the Brayton cycle. Burner is designed to allow an influx of cooler compressor air to mix with the combustion gases bringing the temperature down to a level that the high pressure turbine structure can tolerate.

Modern engines use sophisticated cooling methods to enable operation at values of T_04 that approach *3700 ºR (2050 ºK),* well above the melting temperature of the turbine materials.

TurboJet Station Definitions (6)

Station 4.5 **–**Interface of the high and low pressure turbines.

Station 5 - Exit of the turbine. The temperature/pressure parameters across the turbine are

$$\tau_t = \frac{T_{0_5}}{T_{0_4}} \qquad \pi_t = \frac{P_{0_5}}{P_{0_4}}$$

As with the compressor the goal of the designer is to produce a turbine system that operates as isentropically as possible.

exceeds the power demand of the compressors

TurboJet Station Definitions *(7)*

Station 6 **–** The afterburner exit (for an afterburning system) The temperature/pressure parameters across the afterburner are

$$\tau_a = \frac{T_{0_6}}{T_{0_5}} \qquad \pi_a = \frac{P_{0_6}}{P_{0_5}}$$

The Mach number entering the afterburner is fairly low and so the stagnation pressure ratio of the afterburner is fairly close to, and always less than, one.

TurboJet Station Definitions (8)

Station 7 - The entrance to the nozzle.

Station 8 - The nozzle throat. Over the vast range of operating conditions of modern
engines the nozzle throat is choked or very nearly so.

Station e - The nozzle exit. The temperature/pressure component parameters across the nozzle are

$$\tau_n = \frac{T_{0_e}}{T_{0_7}} \qquad \pi_n = \frac{P_{0_e}}{P_{0_7}}$$

In absence of an afterburner, nozzle parameters are referenced to the turbine exit condition

$$\tau_n = \frac{T_{0_e}}{T_{0_5}} \qquad \pi_n = \frac{P_{0_e}}{P_{0_5}}$$

TurboJet Station Definitions (9)

• Generally the design goal is to minimize heat and stagnation pressure loss through the inlet, burner and nozzle.

• There are two more very important parameters that need to be defined. The first is one we encountered before when we compared the fuel enthalpy to the ambient air enthalpy.

$$\tau_f = \frac{h_{fuel}}{C_p T_\infty}$$

• The second parameter is, in a sense, the most important quantity needed to characterize the performance of an engine.

$$\tau_\lambda = \frac{T_{0_4}}{T_\infty}$$

Every performance measure of the engine gets better as τ_λ is increased and a tremendous investment has been made over the years to devise turbine cooling and ceramic coating schemes that permit ever higher turbine inlet temperatures, $T_0 4$.

The Turbojet as a Brayton Cycle

- Assuming no shaft losses, work performed by turbine matches the work performed by the compressor,

Turbine Compressor

$$\left(\dot{m}_a + \dot{m}_f\right)\cdot\left(h_{0_4} - h_{05}\right) \overset{>}{=} \left(\dot{m}_a\right)\cdot\left(h_{0_3} - h_{02}\right)$$

- Across the combustor the enthalpy balance is

Outlet Inlet

$$\left(\dot{m}_a + \dot{m}_f\right)\cdot\left(h_{0_4}\right) = \left(\dot{m}_a\right)\cdot\left(h_{0_3}\right) + \left(\dot{m}_f\right)\cdot\left(h_f\right)$$

The Turbojet as a Brayton Cycle (2)

- Inlet and nozzle flows are ~ adiabatic. Enthalpy Balance is

Nozzle Inlet/Diffuser + combustor

$$\left(\dot{m}_a + \dot{m}_f\right)\cdot\left(h_{0_e}\right) = \left(\dot{m}_a\right)\cdot\left(h_{0_\infty}\right) + \left(\dot{m}_f\right)\cdot\left(h_f\right)$$

- The resulting thermal efficiency is

$$\eta_{th} = \frac{\left(\dot{m}_a + \dot{m}_f\right)\cdot\left(h_{0_e} - h_e\right) - \left(\dot{m}_a\right)\cdot\left(h_{0_\infty} - h_\infty\right)}{\left(\dot{m}_a + \dot{m}_f\right)\cdot\left(h_{0_4}\right) - \left(\dot{m}_a\right)\cdot\left(h_{0_3}\right)} =$$

$$1 - \frac{\left(\dot{m}_a + \dot{m}_f\right)\cdot\left(h_e\right) - \left(\dot{m}_a\right)\cdot\left(h_\infty\right)}{\left(\dot{m}_a + \dot{m}_f\right)\cdot\left(h_{0_4}\right) - \left(\dot{m}_a\right)\cdot\left(h_{0_3}\right)} = 1 - \frac{Cycle\ Heat\ Rejected}{Cycle\ Heat\ Input}$$

The Turbojet as a Brayton Cycle (3)

- Neglecting the change in properties across the combustor and substituting in for temperatures, thermal efficiency reduces to

$$\eta_{th} = 1 - \frac{\mathit{Cycle\ Heat\ Rejected}}{\mathit{Cycle\ Heat\ Input}} = 1 - \frac{T_\infty}{T_{0_3}} \frac{\left(1+\frac{1}{f}\right)\cdot\frac{T_e}{T_\infty} - 1}{\left(1+\frac{1}{f}\right)\cdot\frac{T_{0_4}}{T_{0_3}} - 1}$$

- Since for the ideal Brayton cycle compression process from free stream to station 3 is assumed isentropic.

- Similarly expansion from station 4 to exit is also assumed to be isentropic. Thus

$$\frac{T_{0_3}}{T_\infty} = \left(\frac{P_{0_3}}{p_\infty}\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{T_{0_4}}{T_e} = \left(\frac{P_{0_4}}{p_e}\right)^{\frac{\gamma}{\gamma-1}}$$

The Turbojet as a Brayton Cycle (4)

- Finally, for constant pressure combustor and optimal nozzle exit

$$P_{0_4} = P_{0_3} \qquad p_\infty = p_e$$

- Subbing into the thermal efficiency and reducing terms

$$\left(\eta_{th}\right)_{\substack{ideal\\turbojet}} = 1 - \frac{T_\infty}{T_{0_3}} = 1 - \frac{1}{\tau_r \cdot \tau_c}$$

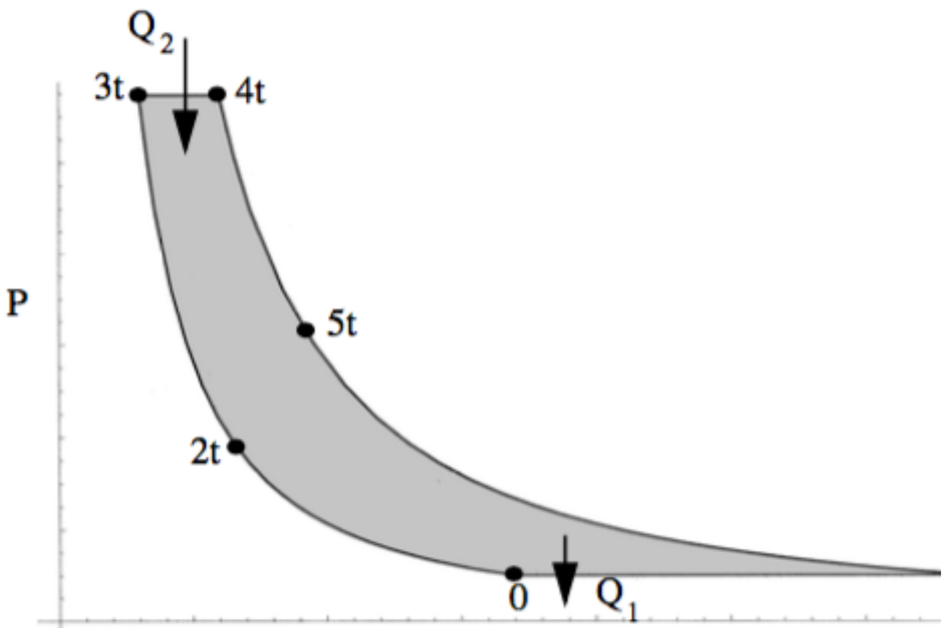
- When freestream Mach number approaches zero

$$M_\infty \rightarrow 0 \quad .... \quad \tau_r = 1 \rightarrow \left(\eta_{th}\right)_{\substack{ideal\\turbojet}} = 1 - \frac{1}{\tau_3} = 1 - \frac{1}{T_{0_3} / T_{0_2}} = \frac{T_{0_3} - T_{0_2}}{T_{0_3}}$$

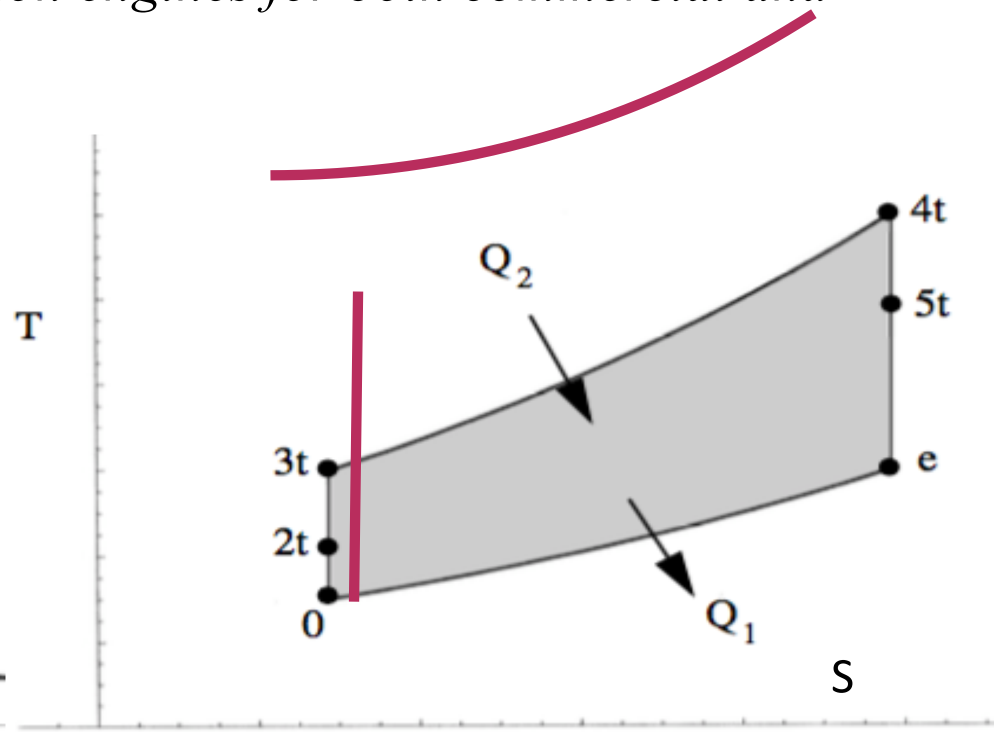
- Efficiency is positive and proportional to the total temperature increase across the compressor.

The Turbojet as a Brayton Cycle (5)

Impact of compression process on thermal efficiency is major factor behind historical trend toward higher compression engines for both commercial and military applications.



P-V diagram of ideal turbo jet cycle

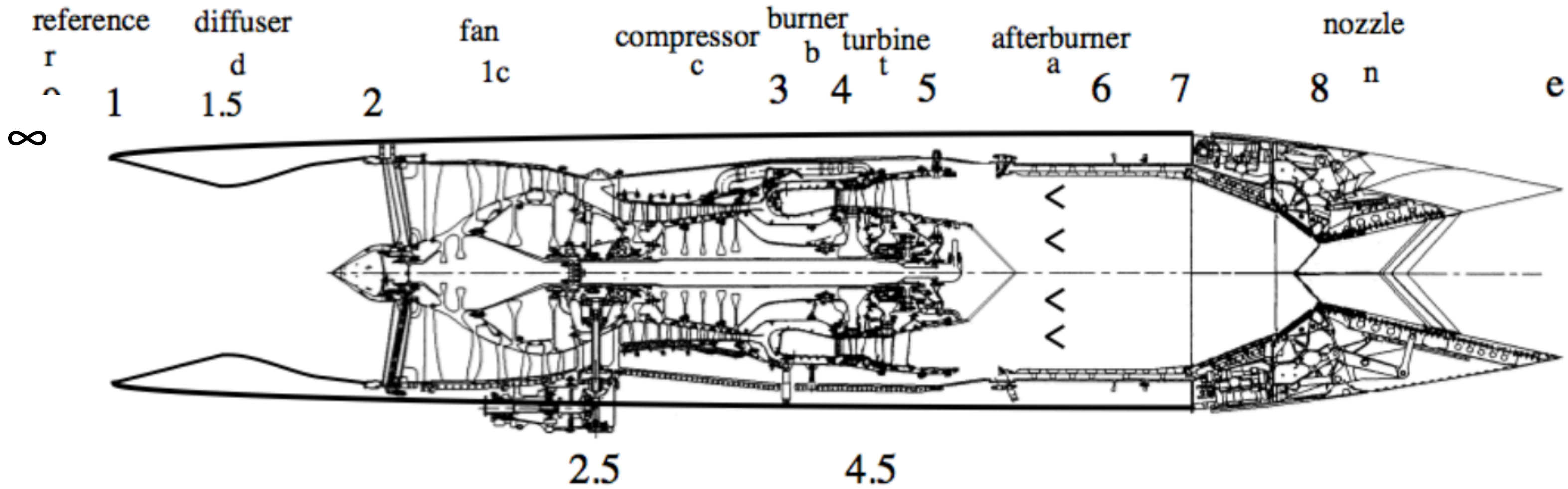


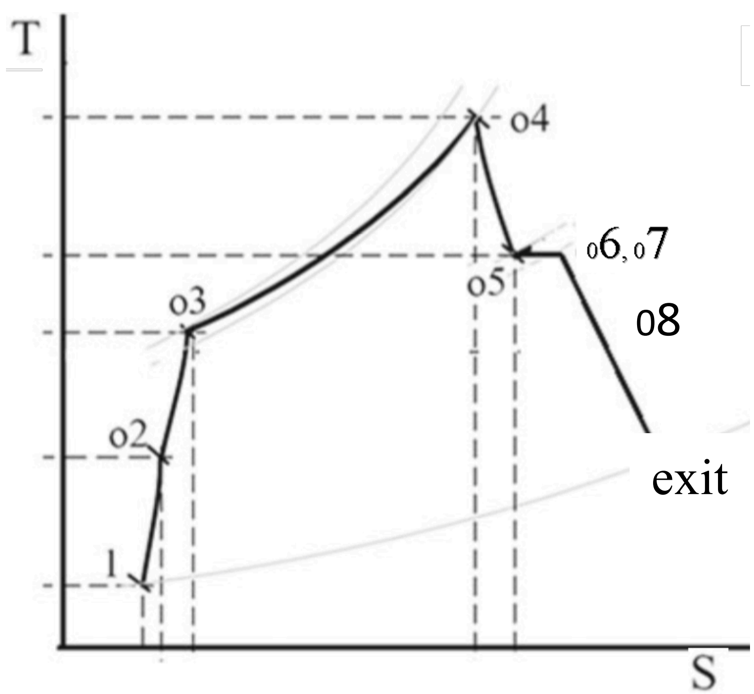
T-S diagram of the ideal turbojet cycle.

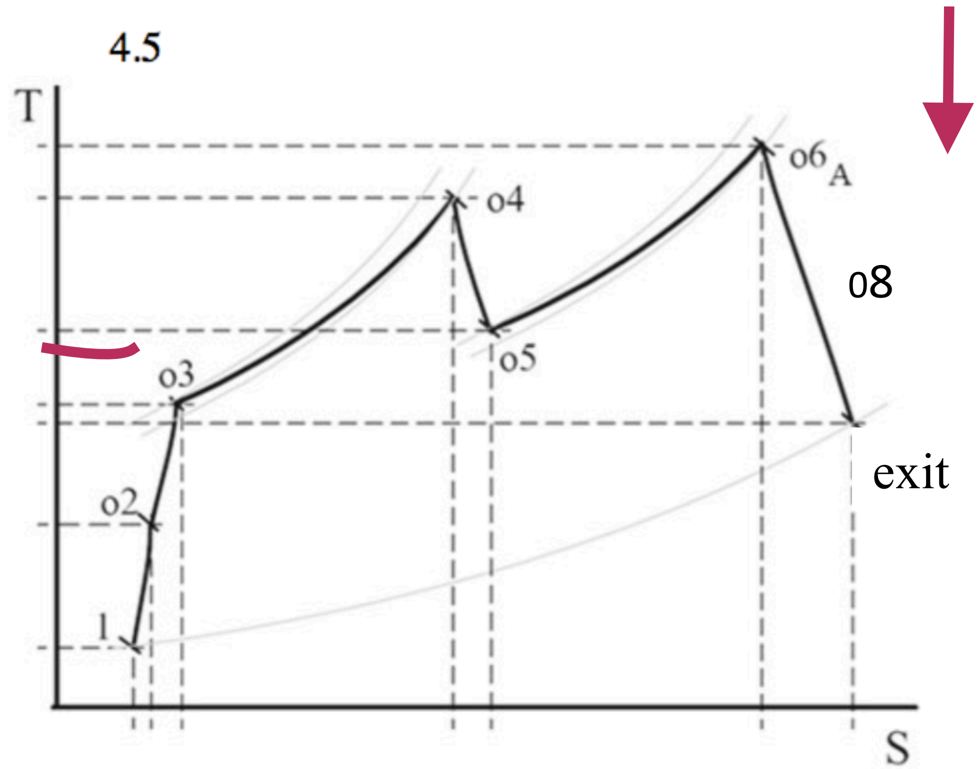
Station numbers with "t" refer to the stagnation state of the gas at that point.

The Turbojet as a Brayton Cycle (6)

True Brayton Cycle (with loses)







Afterburner Not Operating

Thrust of an Ideal Turbojet

- Normalized Thrust Equation

$$\mathbb{T} \equiv \frac{F_{thrust}}{p_\infty \cdot A_0} = \frac{\left(\dot{m}_{air} + \dot{m}_{fuel}\right)\cdot V_{exit} - \left(\dot{m}_{air}\right)\cdot V_\infty + A_{exit}\cdot\left(p_{exit} - p_\infty\right)}{p_\infty \cdot A_0} =$$

$$\mathbb{T} = \frac{F_{thrust}}{p_\infty \cdot A_0} = \frac{\dot{m}_{air}\cdot V_\infty}{p_\infty \cdot A_0}\left[\left(\frac{f+1}{f}\right)\cdot\frac{V_{exit}}{V_\infty} - 1\right] + \frac{A_{exit}}{A_0}\cdot\left(\frac{p_{exit}}{p_\infty} - 1\right)$$

$$\rightarrow \frac{\dot{m}_{air}\cdot V_\infty}{p_\infty \cdot A_0} = \frac{\left(\rho_\infty \cdot A_0 \cdot V_\infty\right)\cdot V_\infty}{p_\infty \cdot A_0} = \frac{\rho_\infty \cdot {V_\infty}^2}{p_\infty} = \frac{{V_\infty}^2}{R_g \cdot T_\infty} = \frac{\gamma \cdot {V_\infty}^2}{\gamma \cdot R_g \cdot T_\infty} = \gamma \cdot M_\infty^2$$

$$\rightarrow \boxed{\mathbb{T} = \gamma \cdot M_\infty^2 \cdot \left[\left(\frac{f+1}{f}\right)\cdot\frac{V_{exit}}{V_\infty} - 1\right] + \frac{A_{exit}}{A_0}\cdot\left(\frac{p_{exit}}{p_\infty} - 1\right)} \qquad \boxed{f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}}$$

Thrust of an Ideal Turbojet (2)

- For Fully-Expanded (Optimal) Nozzle

$$\mathbb{T} = \gamma \cdot M_\infty^2 \cdot \left[\left(\frac{f+1}{f} \right) \cdot \frac{V_{exit}}{V_\infty} - 1 \right]$$

- Calculate Thrust level that gives velocity ratio

$$\frac{V_{exit}}{V_\infty} = \frac{M_{exit}}{M_\infty} \sqrt{\frac{T_{exit}}{T_\infty}}$$

- Start with stagnation pressure loss across engine

$$P_{0_{exit}} = p_{\infty} \cdot \left(\frac{P_{0_\infty}}{p_\infty} \right) \cdot \left(\frac{P_{0_2}}{P_{0_\infty}} \right) \cdot \left(\frac{P_{0_3}}{P_{0_2}} \right) \cdot \left(\frac{P_{0_4}}{P_{0_3}} \right) \cdot \left(\frac{P_{0_5}}{P_{0_4}} \right) \cdot \left(\frac{P_{0_{exit}}}{P_{0_5}} \right) =$$

$$p_\infty \cdot \pi_r \cdot \pi_d \cdot \pi_c \cdot \pi_b \cdot \pi_t \cdot \pi_n = p_{exit} \left(1 + \frac{\gamma - 1}{2} M_{exit}^2 \right)^{\frac{\gamma}{\gamma - 1}}$$

Thrust of an Ideal Turbojet (3)

- For Fully Expanded Nozzle

$$p_\infty = p_{exit} \rightarrow \pi_r \cdot \pi_d \cdot \pi_c \cdot \pi_b \cdot \pi_t \cdot \pi_n = \left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)^{\frac{\gamma}{\gamma-1}}$$

- Using isentropic assumption for diffuser and nozzle, stagnation Pressure losses are ignored

$$\pi_d = \pi_n = 1 \rightarrow \pi_r \cdot \pi_c \cdot \pi_b \cdot \pi_t = \left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)^{\frac{\gamma}{\gamma-1}}$$

- Assuming that compressor and turbine work is reversible, Stagnation pressure written in terms of stagnation temperature ratio

$$\pi_c = (\tau_c)^{\frac{\gamma}{\gamma-1}} \qquad \pi_t = (\tau_t)^{\frac{\gamma}{\gamma-1}} \text{ ... also ... } \pi_t = (\tau_t)^{\frac{\gamma}{\gamma-1}}$$

$$\rightarrow (\tau_t)^{\frac{\gamma}{\gamma-1}} \cdot (\tau_c)^{\frac{\gamma}{\gamma-1}} \cdot \pi_b \cdot (\tau_t)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)^{\frac{\gamma}{\gamma-1}}$$

Thrust of an Ideal Turbojet (4)

- Solve for Mach number

$$\left(\pi_b\right)^{\frac{\gamma-1}{\gamma}} \cdot \left(\tau_r \cdot \tau_c \cdot \tau_t\right) = \left(1 + \frac{\gamma-1}{2} M_{exit}^2\right) \rightarrow \boxed{M_{exit}^2 = \frac{2}{\gamma-1}\left[\left(\pi_b\right)^{\frac{\gamma-1}{\gamma}} \cdot \left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1\right]}$$

- Write Freestream Mach number in terms of τ_r

$$\left(1 + \frac{\gamma-1}{2} M_\infty^2\right) = \frac{T_{0_\infty}}{T_\infty} = \tau_r \rightarrow M_\infty^2 = \frac{2}{\gamma-1}\left(\tau_r - 1\right)$$

- Resulting Mach number ratio is

$$\boxed{\frac{M_{exit}^2}{M_\infty^2} = \frac{\frac{2}{\gamma-1}\left[\left(\pi_b\right)^{\frac{\gamma-1}{\gamma}} \cdot \left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1\right]}{\frac{2}{\gamma-1}\left(\tau_r - 1\right)} = \frac{\left(\pi_b\right)^{\frac{\gamma-1}{\gamma}} \cdot \left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1}{\left(\tau_r - 1\right)}}$$

Thrust of an Ideal Turbojet (5)

- For a constant pressure combustor

$$\left(\pi_b\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_{04}}{P_{03}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_4 \cdot \left(1+\frac{\gamma-1}{2}M_4^2\right)^{\frac{\gamma}{\gamma-1}}}{p_3 \cdot \left(1+\frac{\gamma-1}{2}M_3^2\right)^{\frac{\gamma}{\gamma-1}}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\left(1+\frac{\gamma-1}{2}M_4^2\right)}{\left(1+\frac{\gamma-1}{2}M_3^2\right)}$$

≈ 1 for low burner mach number

- Thus for constant pressure burner with low internal Mach numbers

$$M_{exit}^2 \approx \frac{2}{\gamma-1}\left[\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1\right]$$

$$\boxed{\frac{M_{exit}}{M_\infty} \approx \sqrt{\frac{\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1}{\left(\tau_r - 1\right)}}}$$

Thrust of an Ideal Turbojet (6)

- Similar Approach Finds Temperature Ratio Across Engine

$$T_{0_{exit}} = T_{\infty} \cdot \left(\frac{T_{0_\infty}}{T_{\infty}}\right) \cdot \left(\frac{T_{0_2}}{T_{0_\infty}}\right) \cdot \left(\frac{T_{0_3}}{T_{0_2}}\right) \cdot \left(\frac{T_{0_4}}{T_{0_3}}\right) \cdot \left(\frac{T_{0_5}}{T_{0_4}}\right) \cdot \left(\frac{T_{0_{exit}}}{T'_{0_5}}\right) =$$

$$T_{\infty} \cdot \tau_r \cdot \tau_d \cdot \tau_c \cdot \tau_b \cdot \tau_t \cdot \tau_n = T_{exit}\left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)$$

- Diffuser and Nozzle are Adiabatic …

$$\tau_d = 1 \quad \tau_n = 1 \quad \rightarrow T_{\infty} \cdot \tau_r \cdot \tau_c \cdot \tau_b \cdot \tau_t = T_{exit}\left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)$$

- And from previous slide ...

$$M_{exit}^2 \approx \frac{2}{\gamma - 1}\left[\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1\right]$$

Thrust of an Ideal Turbojet (7)

- Substituting and Simplifying

$$\tau_d = 1 \quad \tau_n = 1 \quad \rightarrow T_\infty \cdot \tau_r \cdot \tau_c \cdot \tau_b \cdot \tau_t = T_{exit}\left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)$$

$$\frac{2}{\gamma - 1}\left[\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1\right] = M_{exit}^2$$

- $$T_\infty \cdot \tau_r \cdot \tau_c \cdot \tau_b \cdot \tau_t = T_{exit}\left(1 + \frac{\gamma - 1}{2} \frac{2}{\gamma - 1}\left[\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1\right]\right) =$$

$$T_\infty \cdot \tau_r \cdot \tau_c \cdot \tau_b \cdot \tau_t = T_{exit} \cdot \left(\tau_r \cdot \tau_c \cdot \tau_t\right)$$

Thrust of an Ideal Turbojet (8)

- Solving for Burner Temperature Ratio

$$\frac{T_{exit}}{T_\infty} = \tau_b \equiv \frac{T_{0_4}}{T_{0_3}}$$

- Defining

$$\tau_\lambda = \frac{T_{0_4}}{T_\infty}$$

$$\tau_\lambda = \frac{T_{0_4}}{T_\infty} \rightarrow \tau_\lambda = \frac{T_{0_4}}{T_{0_3}} \cdot \frac{T_{0_3}}{T_\infty} \cdot \frac{T_{0_2}}{T_{0_2}} = \frac{T_{0_4}}{T_{0_3}} \cdot \frac{T_{0_3}}{T_{0_2}} \cdot \frac{T_{0_1}}{T_\infty} = \tau_b \cdot \tau_c \cdot \tau_r \rightarrow \boxed{\tau_b = \frac{\tau_\lambda}{\tau_c \cdot \tau_r}}$$

τ_λ → Parameter that is optimized under the constraints of the the highest temperature that can be tolerated by the turbine materials

$$\frac{T_{exit}}{T_\infty} = \frac{\tau_\lambda}{\tau_c \cdot \tau_r}$$

Thrust of an Ideal Turbojet (9)

- Substituting into the Velocity Ratio

$$\frac{V_{exit}}{V_\infty} = \frac{M_{exit}}{M_\infty}\sqrt{\frac{T_{exit}}{T_\infty}} \rightarrow \left| \begin{array}{l} \frac{M_{exit}}{M_\infty} = \sqrt{\frac{\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1}{\left(\tau_r - 1\right)}} \\ \frac{T_{exit}}{T_\infty} = \frac{\tau_\lambda}{\tau_c \tau_r} \end{array} \right. \rightarrow \frac{V_{exit}}{V_\infty} = \sqrt{\frac{\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1}{\left(\tau_r - 1\right)}}\sqrt{\frac{\tau_\lambda}{\tau_c \tau_r}}$$

- and Normalized Thrust Equation

$$\mathbb{T} = \gamma \cdot M_\infty^2 \cdot \left[\left(\frac{f+1}{f}\right) \cdot \frac{V_{exit}}{V_\infty} - 1\right] \rightarrow \frac{V_{exit}}{V_\infty} = \sqrt{\frac{\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1}{\left(\tau_r - 1\right)}}\sqrt{\frac{\tau_\lambda}{\tau_c \tau_r}}$$

$$\rightarrow \mathbb{T} = \gamma \cdot M_\infty^2 \cdot \left[\left(\frac{f+1}{f}\right) \cdot \sqrt{\left(\frac{\left(\tau_r \cdot \tau_c \cdot \tau_t\right) - 1}{\left(\tau_r - 1\right)}\right) \cdot \left(\frac{\tau_\lambda}{\tau_c \tau_r}\right)} - 1\right]$$

Thrust of an Ideal Turbojet (10)

- Substituting the resulting velocity ratio into the Normalized Thrust Equation

$$\mathbb{T} = \gamma \cdot M_\infty^2 \cdot \left[\left(\frac{f+1}{f} \right) \cdot \frac{V_{exit}}{V_\infty} - 1 \right] \rightarrow \frac{V_{exit}}{V_\infty} = \sqrt{\frac{(\tau_r \cdot \tau_c \cdot \tau_t) - 1}{(\tau_r - 1)}} \sqrt{\frac{\tau_\lambda}{\tau_c \tau_r}}$$

$$\rightarrow \mathbb{T} = \gamma \cdot M_\infty^2 \cdot \left[\left(\frac{f+1}{f} \right) \cdot \sqrt{\left(\frac{(\tau_r \cdot \tau_c \cdot \tau_t) - 1}{(\tau_r - 1)} \right) \cdot \left(\frac{\tau_\lambda}{\tau_c \tau_r} \right)} - 1 \right]$$

$$\rightarrow M_\infty^2 = \frac{2}{\gamma - 1} \cdot (\tau_r - 1)$$

$$\rightarrow \boxed{\mathbb{T} = \frac{2 \cdot \gamma}{\gamma - 1} \cdot (\tau_r - 1) \cdot \left[\left(\frac{f+1}{f} \right) \cdot \sqrt{\left(\frac{(\tau_r \cdot \tau_c \cdot \tau_t) - 1}{(\tau_r - 1)} \right) \cdot \left(\frac{\tau_\lambda}{\tau_c \tau_r} \right)} - 1 \right]}$$

Thrust of an Ideal Turbojet (11)

- Re-Write air-fuel ratio in terms of τ parameters by considering combustor enthalpy balance

$$\left(\dot{m}_{air} + \dot{m}_f\right) h_{0_4} = \dot{m}_{air} h_{0_3} + \dot{m}_f \cdot h_f$$

$$\rightarrow \left(\dot{m}_{air} + \dot{m}_f\right) h_{0_4} = \dot{m}_{air} h_{0_3} + \dot{m}_f \cdot h_f \rightarrow \left(1 + \frac{1}{f}\right) h_{0_4} = h_{0_3} + \frac{1}{f} \cdot h_f$$

$$\rightarrow \frac{1}{f} = \frac{h_{0_4} - h_{0_3}}{h_f - h_{0_4}} = \frac{h_{0_4} - h_{0_2} \cdot \frac{h_{0_3}}{h_{0_2}}}{h_f - h_{0_4}} = \frac{h_{0_4} - h_{0_\infty} \cdot \frac{h_{0_3}}{h_{0_2}}}{h_f - h_{0_4}} = \frac{\frac{h_{0_4}}{h_\infty} - \frac{h_{0_\infty}}{h_\infty} \frac{h_{0_3}}{h_{0_2}}}{\frac{h_f}{h_\infty} - \frac{h_{0_4}}{h_\infty}} = \frac{\tau_\lambda - \tau_r \cdot \tau_c}{\tau_f - \tau_\lambda}$$

Thrust of an Ideal Turbojet (12)

- Solve for *1/f*

$$\frac{1}{f}=\frac{\tau_\lambda-\tau_r\cdot\tau_c}{\tau_f-\tau_\lambda}$$

- Insert into Normalized thrust equation

$$\mathbb{T}=\frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r-1\right)\cdot\left[\left(1+\frac{\tau_\lambda-\tau_r\cdot\tau_c}{\tau_f-\tau_\lambda}\right)\cdot\sqrt{\left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r\tau_e}\right)}-1\right]$$

$$\mathbb{T}=\frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r-1\right)\cdot\left[\left(\frac{\tau_f-\tau_r\cdot\tau_c}{\tau_f-\tau_\lambda}\right)\cdot\sqrt{\left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r\tau_e}\right)}-1\right]$$

Thrust of an Ideal Turbojet (13)

- Since the Turbine and Compressor Work Inputs are Equal

$$\left(\dot{m}_{air} + \dot{m}_{fuel}\right)\cdot\left(h_{0_4} - h_{0_5}\right) = \dot{m}_{air}\left(h_{0_3} - h_{0_2}\right)$$

$$\left[\begin{array}{l} f = \dfrac{\dot{m}_{air}}{\dot{m}_{fuel}} \\ \tau_c = \dfrac{h_{0_3}}{h_{0_2}} \\ \tau_t = \dfrac{h_{0_5}}{h_{0_4}} \end{array}\right] \rightarrow \left(1 + \frac{1}{f}\right)\cdot\left(1 - \tau_t\right)h_{0_4} = \left(\tau_c - 1\right)h_{0_2}$$

Thrust of an Ideal Turbojet (14)

- Since the Turbine and Compressor Work Inputs are Equal

$$f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

$$\tau_c = \frac{h_{0_3}}{h_{0_2}}$$

$$\tau_t = \frac{h_{0_5}}{h_{0_4}}$$

$$\tau_\gamma = \frac{h_{0_4}}{h_\infty}$$

lambda

$$\tau_r = \frac{h_{0_\infty}}{h_\infty}$$

$$h_{0_2} = h_{0_\infty}$$

$$\left(1+\frac{1}{f}\right)\cdot\left(1-\tau_t\right)h_{0_4} = \left(\tau_c - 1\right)h_{0_2} = \left(1+\frac{1}{f}\right)\cdot\left(1-\tau_t\right)\frac{h_{0_4}}{h_\infty} = \left(\tau_c - 1\right)\frac{h_{0_2}}{h_\infty} =$$

$$\rightarrow \left(1+\frac{1}{f}\right)\cdot\left(1-\tau_t\right)\tau_\lambda = \left(\tau_c - 1\right)\tau_r \rightarrow \boxed{\tau_t = 1 - \frac{\tau_r\cdot\left(\tau_c - 1\right)}{\left(1+\frac{1}{f}\right)\cdot\tau_\lambda}}$$

Thrust of an Ideal Turbojet (15)

- Collected Parametric Turbojet Thrust Equations

$$\mathbb{T} = \frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r - 1\right)\cdot\left[\left(\frac{f+1}{f}\right)\cdot\sqrt{\left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\cdot\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)}-1\right]$$

$$\left(\frac{V_{exit}}{V_\infty}\right)^2 = \left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)$$

$$\frac{1}{f} = \frac{\tau_\lambda - \tau_r\cdot\tau_c}{\tau_f - \tau_\lambda}$$

$$\tau_t = 1 - \frac{\tau_r\cdot\left(\tau_c - 1\right)}{\left(1+\frac{1}{f}\right)\tau_\lambda}$$

- Normalized properties depend only on $\left\{\tau_r, \tau_c, \tau_\lambda, \tau_f, \gamma\right\}$

Thrust of an Ideal Turbojet (16)

$$\{\tau_r, \tau_c, \tau_\lambda, \tau_f, \gamma\} \rightarrow$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \rightarrow$$ *Freestream Mach number reference conditions*

$$\tau_c = \frac{T_{0_3}}{T_{0_2}} \rightarrow$$ *Compressor stagnation temperature ratio measure of compressor work input*

$$\tau_\lambda = \frac{T_{0_4}}{T_\infty} \rightarrow$$ *Combustor flame temperature...Optimized up to Material limits of combustor, turbine*

$$\tau_f = \frac{h_f}{h_\infty} \rightarrow$$ *Fuel enthalpy of combustion relative to incoming air stream total enthalpy*

$$\gamma = \frac{C_p}{C_v} \rightarrow$$ *Ratio of specific heats*

Choice of Fuel

$$\mathbb{T} = \frac{F_{thrust}}{p_\infty \cdot A_0} \rightarrow$$

- *Operating Mach Number*
- *Choice of Propellants*
- *Combustion Efficiency*
- *Compressor Work Input*

Finally, Calculate Normalized Isp

$$I_{sp} = \left(\frac{F_{thrust}}{g_0 \cdot \dot{m}_f}\right) \rightarrow \mathbb{I} \equiv \frac{I_{sp} \cdot g_0}{c_\infty} = \left(\frac{F_{thrust}}{g_0 \cdot \dot{m}_f} \cdot \frac{g_0}{c_\infty}\right) = \left(\frac{F_{thrust}}{\dot{m}_f \cdot c_\infty}\right) = \frac{\dot{m}_{air}}{\dot{m}_f}\left(\frac{F_{thrust}}{\dot{m}_{air} \cdot c_\infty}\right) =$$

$$f \cdot \left(\frac{F_{thrust}}{\dot{m}_{air} \cdot c_\infty}\right) = \frac{f \cdot F_{thrust}}{\left(\rho_\infty \cdot A_0 \cdot V_\infty\right) \cdot c_\infty} = \frac{f \cdot F_{thrust}}{\left(\rho_\infty \cdot A_0 \cdot V_\infty\right) \cdot \sqrt{\gamma \cdot R_g \cdot T_\infty}} =$$

$$\frac{\gamma \cdot R_g \cdot T_\infty}{\gamma \cdot p_\infty} \frac{f \cdot F_{thrust}}{\left(A_0 \cdot V_\infty\right) \cdot \sqrt{\gamma \cdot R_g \cdot T_\infty}} = \frac{f \cdot F_{thrust}}{p_\infty \cdot A_0} \frac{1}{\gamma \cdot \left(\frac{V_\infty}{\sqrt{\gamma \cdot R_g \cdot T_\infty}}\right)} = \frac{f \cdot F_{thrust}}{p_\infty \cdot A_0} \frac{1}{\gamma \cdot M_\infty} = \mathbb{T} \cdot \frac{f}{\gamma \cdot M_\infty}$$

$$\boxed{\mathbb{I} = \mathbb{T} \cdot \frac{f}{\gamma \cdot M_\infty} = f \cdot \frac{F_{thrust}}{\dot{m}_{air} \cdot c_\infty}} \rightarrow f = \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

Example Calculation

- **Stiochiometric Combustion of Gasoline (Octane) with air**

Substance	Heat of Combustion (kJ mol^{-1})	Combustion Reaction	$\Delta H_{reaction}$ (kJ mol^{-1})
octane	5460	$C_8H_{18(g)} + {}^{25}/_2 (O_{2(g)} + {}^{0.79}/_{0.21} N_{2(g)}) \rightarrow 8CO_{2(g)} + 9H_2O_{(l)} + {}^{25}/_2 ({}^{0.79}/_{0.21} N_{2(g)})$	$\Delta H = -5460$

$$C_8H_{18} \rightarrow M_w = 8 \cdot 12 + 18 = 114_{kg/kg-mol}$$

$$h_f = \frac{5460_{\frac{KJ}{mol}}}{114_{kg/kg-mol}} \cdot 1000_{\frac{mol}{kg-mol}} = 49.47_{\frac{MJ}{kg}}$$

$$f = \frac{m_{air}}{m_{air}} = \frac{(25/2)_{kg-mol} \cdot \left(32_{kg/kg-mol} + \frac{0.79}{0.21} \cdot 28_{kg/kg-mol} \right)}{114_{kg/kg-mol}} = 15.06$$

At Stoichiometric Point

Example Calculation (2)

- **non-Stiochiometric Combustion of Gasoline (Octane) with air**

Assume M=0.80, h= 11 km (36,090 ft), T_{burner} = 1600 °C (1873.17 K)

$$\rightarrow \begin{array}{l} M_\infty = 0.8 \\ h = 11_{km}(36{,}090_{ft}) \end{array}$$

$$\rightarrow \begin{array}{l} p_\infty = 22.632_{kPa} \\ T_\infty = 216.65_K \end{array}$$

$$\rightarrow P_{0_\infty} = p_\infty \cdot \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}} = 22.632\left(1 + \frac{1.4-1}{2}0.8^2\right)^{\frac{1.4}{(1.4-1)}} = 89.3511 \text{ kPa}$$

$$T_{0_\infty} = T_\infty \cdot \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right) = 216.65\left(1 + \frac{1.4-1}{2}0.8^2\right) = 244.381 \text{ Deg. K}$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1.128$$

$$\tau_f = \frac{h_f}{h_\infty} = \frac{49.47 \cdot 10^6}{1004.96 \cdot 216.65} = 227.214$$

$$\tau_\lambda = \frac{T_{0_4}}{\tau_\infty} = \frac{1873.17}{216.65} = 8.6461$$

Calculate non – stoichiometric f in terms of $\{t_c, t_r, t_f\}$

$$f = \frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r \cdot \tau_c} = \frac{218.568}{8.6461 - 1.128 \cdot \tau_c}$$

Example Calculation (3)

- Plot Parametric Equations as function of τ_c

$$\mathbb{T} = \frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r-1\right)\cdot\left[\left(\frac{f+1}{f}\right)\cdot\sqrt{\left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\cdot\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)}-1\right]$$

$$\left(\frac{V_{exit}}{V_\infty}\right)^2 = \left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)$$

$$f = \frac{\tau_f-\tau_\lambda}{\tau_\lambda-\tau_r\cdot\tau_c} = \frac{218.568}{8.6461-1.128\cdot\tau_c}$$

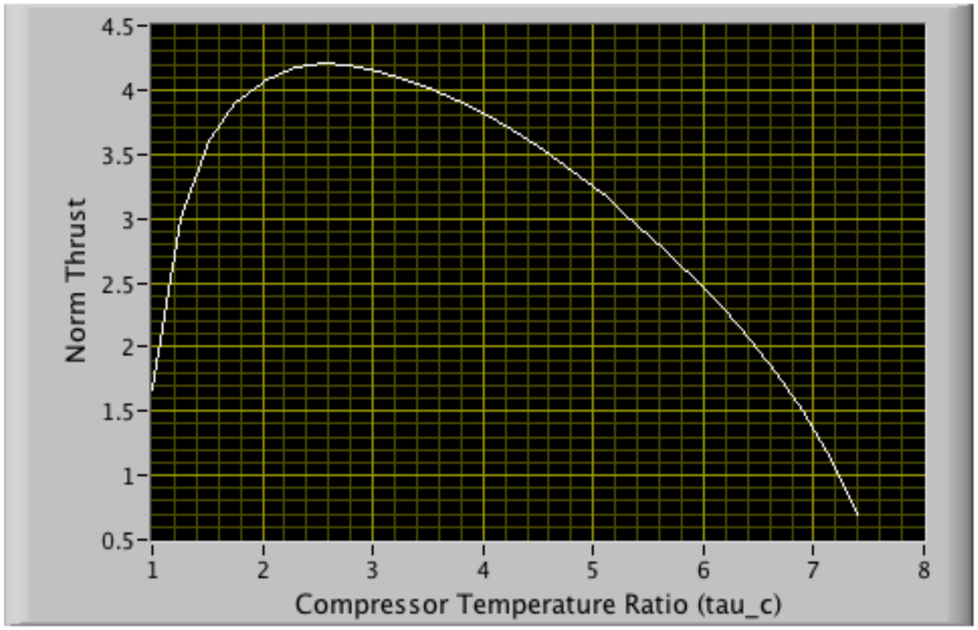
$$\tau_t = 1-\frac{\tau_r\cdot\left(\tau_c-1\right)}{\left(1+\frac{1}{f}\right)\tau_\lambda} \qquad \mathbb{I} = \mathbb{T}\cdot\frac{f}{\gamma\cdot M_\infty}$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1.128 \qquad \tau_\lambda = \frac{T_{0_4}}{\tau_\infty} = \frac{1873.17}{216.65} = 8.6461$$

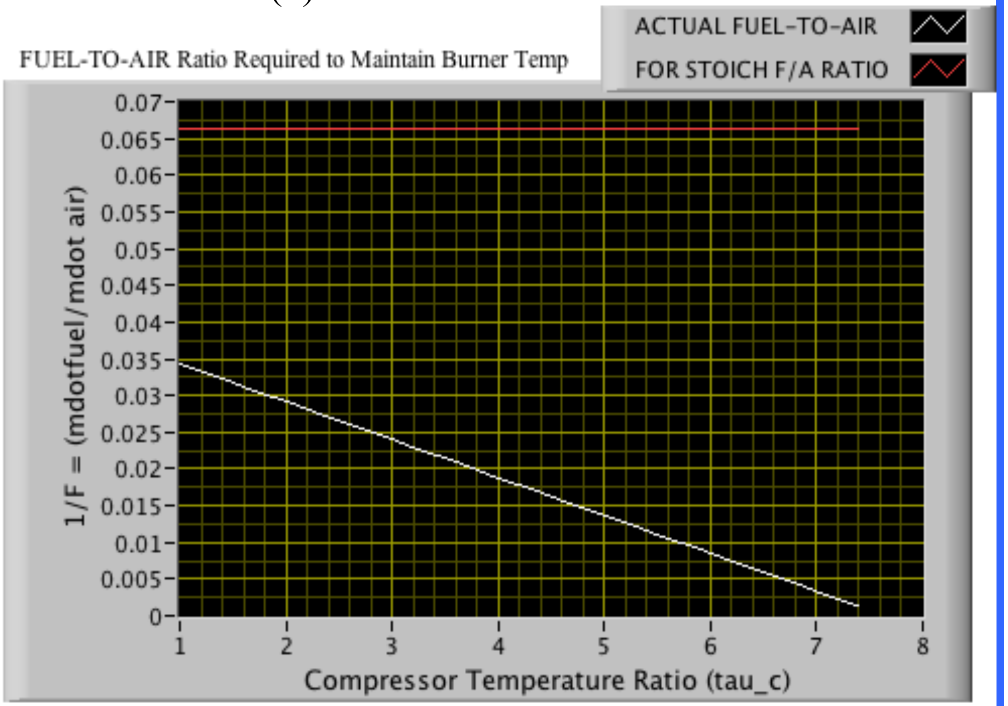
$$\tau_f = \frac{h_f}{h_\infty} = 227.214 \qquad \gamma = 1.4$$

Example Calculation (4)

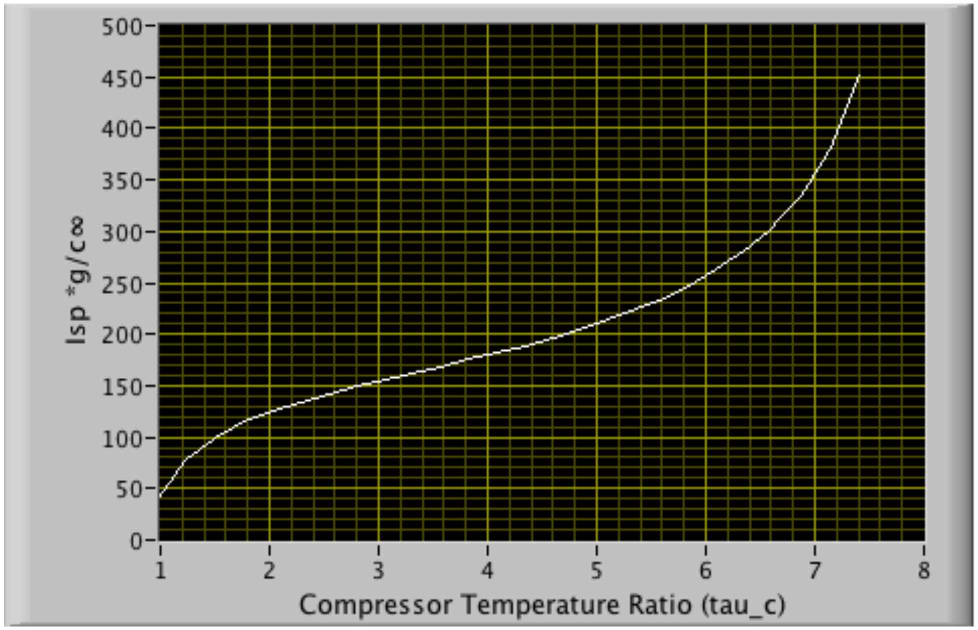
Normalized Thrust Plot



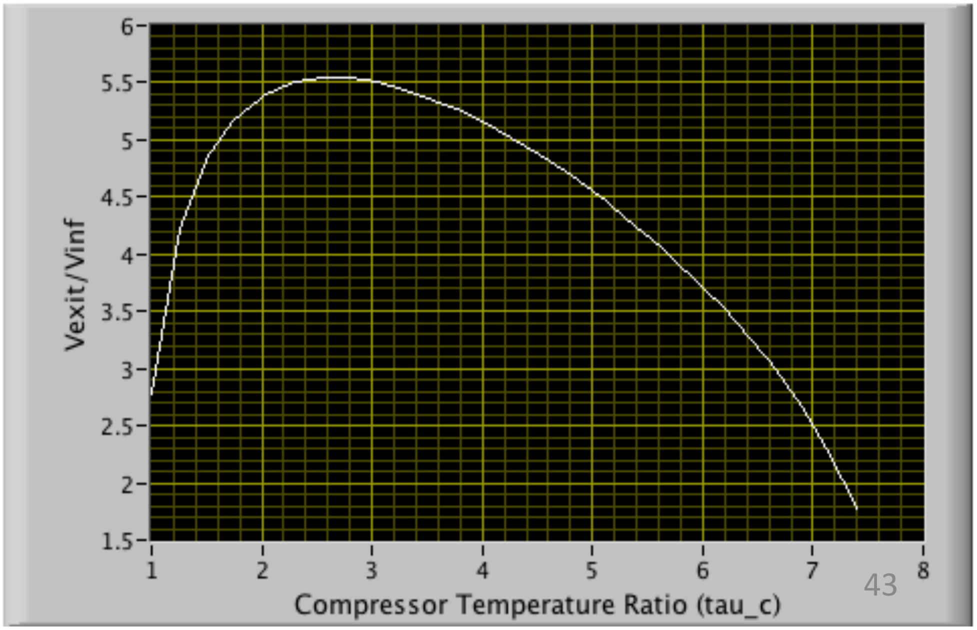
FUEL-TO-AIR Ratio Required to Maintain Burner Temp



Normalized Isp



Velocity Ratio



Example Calculation (5)

- Based on the plots of the previous slide, there exists an optimal choice for τ_c that gives maximum thrust at a given burner temperature and operating condition

$$0 = \frac{\partial}{\partial \tau_c}\left(\frac{V_{exit}}{V_\infty}\right)^2 = \frac{\partial}{\partial \tau_c}\left(\left(\frac{(\tau_r \cdot \tau_c \cdot \tau_t) - 1}{(\tau_r - 1)}\right)\left(\frac{\tau_\lambda}{\tau_c \tau_r}\right)\right) = \left(\frac{\tau_r \cdot \tau_t + \tau_r \cdot \tau_c \cdot \frac{\partial \tau_t}{\partial \tau_c}}{\tau_r - 1}\right)\left(\frac{\tau_\lambda}{\tau_c \tau_r}\right) - \left(\frac{(\tau_r \cdot \tau_c \cdot \tau_t) - 1}{(\tau_r - 1)}\right)\left(\frac{\tau_\lambda}{\tau_r}\right)\frac{1}{\tau_c^{\,2}}$$

$$\rightarrow \left(\frac{\tau_r \cdot \tau_t + \tau_r \cdot \tau_c \cdot \frac{\partial \tau_t}{\partial \tau_c}}{\tau_r - 1}\right)\left(\frac{\tau_\lambda}{\tau_c \tau_r}\right) - \left(\frac{(\tau_r \cdot \tau_c \cdot \tau_t) - 1}{(\tau_r - 1)}\right)\left(\frac{\tau_\lambda}{\tau_r}\right)\frac{1}{\tau_c^{\,2}} = 0 \rightarrow \textit{simplify} \rightarrow \frac{1}{\tau_c} + \tau_r \cdot \tau_c \cdot \frac{\partial \tau_t}{\partial \tau_c} = 0$$

$$\rightarrow \tau_t = 1 - \frac{\tau_r \cdot (\tau_c - 1)}{\left(1 + \frac{1}{f}\right)\tau_\lambda} \rightarrow \frac{\partial \tau_t}{\partial \tau_c} = -\frac{\tau_r}{\left(1 + \frac{1}{f}\right)\tau_\lambda} \rightarrow \textit{substitute} \rightarrow \frac{1}{\tau_c} - \tau_r \cdot \tau_c \cdot \frac{\tau_r}{\left(1 + \frac{1}{f}\right)\tau_\lambda} = 0$$

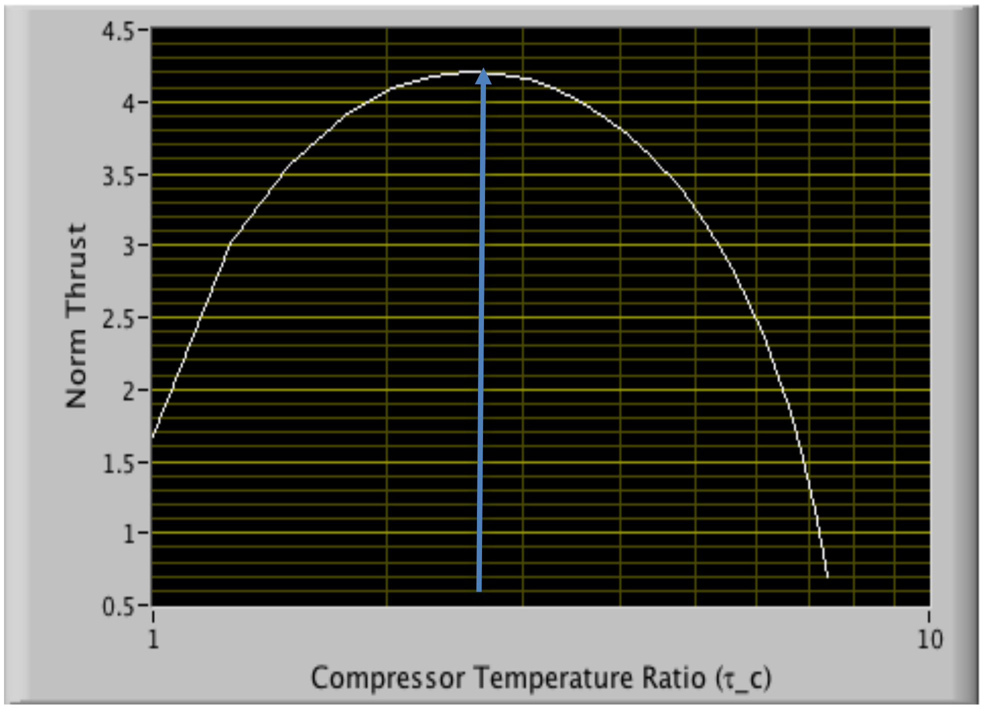
$$\rightarrow \textit{solve} \;....\; \tau^2{}_c = \left(1 + \frac{1}{f}\right)\frac{\tau_\lambda}{\tau_r^{\,2}} \rightarrow \boxed{\tau_c = \frac{1}{\tau_r} \cdot \sqrt{\left(1 + \frac{1}{f}\right)\tau_\lambda}}$$

Example Calculation (6)

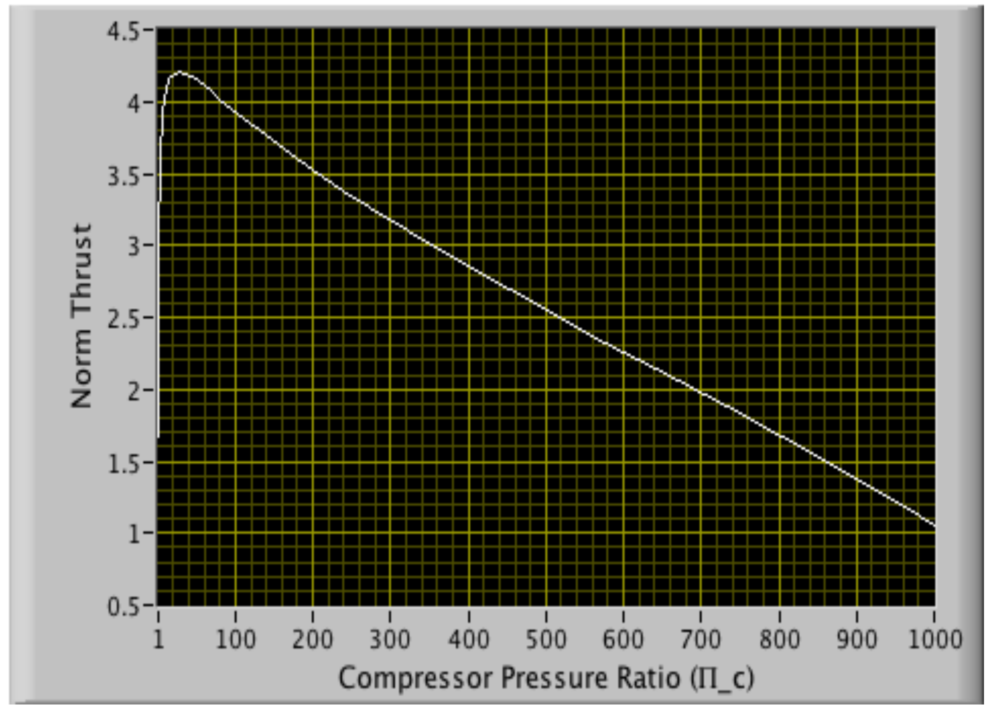
- From previous page

$$\left(\tau_c\right)_{opt} = \frac{1}{\tau_r} \cdot \sqrt{\left(1 + \frac{1}{f}\right)\tau_\lambda} = \frac{1}{1.128}\left(\left(1 + \frac{1}{40}\right)8.6461\right)^{0.5} = 2.63914$$

Normalized Thrust Plot, vs Compressor Temperaure Ratio

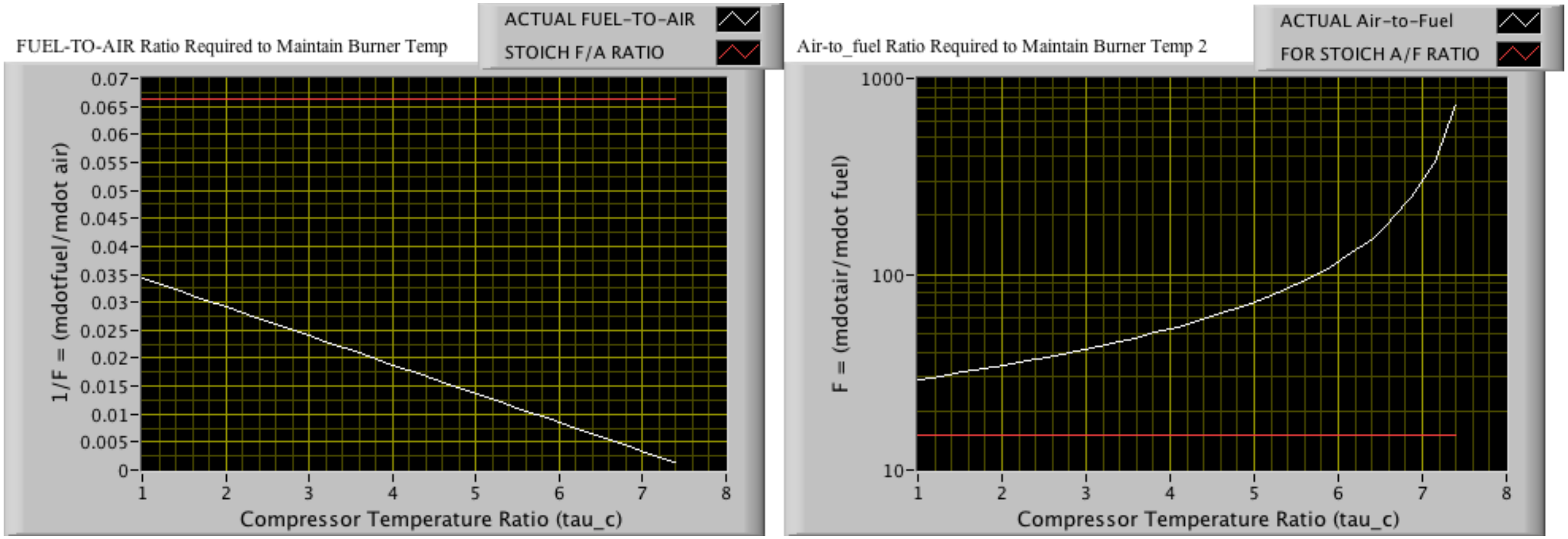


Normalized Thrust Plot, vs Compressor Pressure Ratio



$$\left(\Pi_c\right)_{opt} = \left(\tau_c^{\frac{\gamma}{\gamma-1}}\right)_{opt} = 2.63914^{\left(\frac{1.4}{1.4-1}\right)} = 29.862$$

Discussion of Example



• In order to maintain Burner temperature at 1600 º C as compression temperature ratio Increases, fuel mixture must become increasingly lean. In fact at the given temperature, the engine can never operate near the stoichiometric point.

• Thus, it become clear that the combustor material design temperature is a limiting factor for the engine.

• Similarly, since fuel to air ratio is $\frac{1}{f} = \frac{\tau_\lambda - \tau_r \cdot \tau_c}{\tau_f - \tau_\lambda}$ fuel cutoff occurs when $\tau_c = \frac{\tau_\lambda}{\tau_r}$

Discussion of Example (2)

• As a result an engine designed to cruise at low Mach number (a low value of τ_r) will be designed with a relatively large compressor generating a relatively high value of τ_c as indicated by (4.42).

• But as the flight Mach number increases the compression goes down until at

$$1 = \frac{1}{\tau_r} \cdot \sqrt{\left(1 + \frac{1}{f}\right)\tau_\lambda} \rightarrow \tau_\lambda = \frac{f}{f+1}\tau_r^{\,2}$$

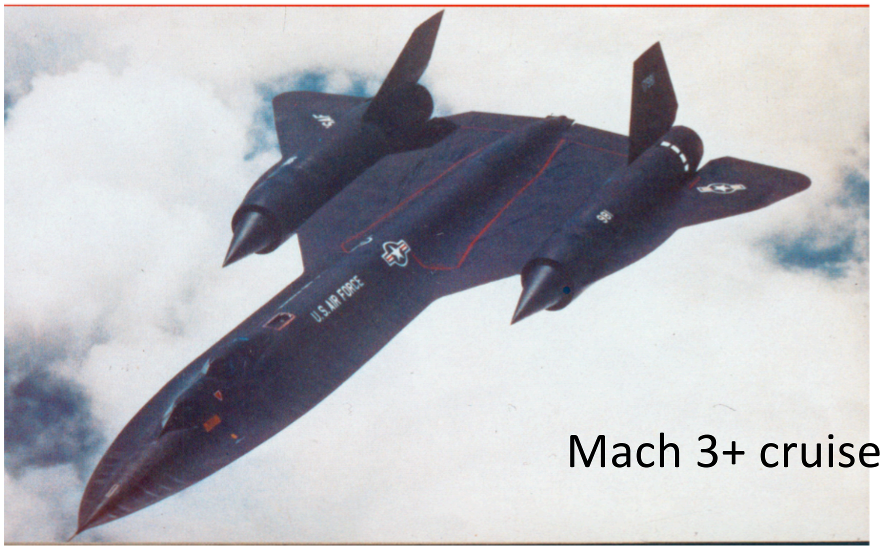
The optimal solution gets rid of the compressor all together, and the best engine becomes a ramjet!

• This result also shows something about the general trend of engine design history.

• As higher temperature turbine materials and better cooling schemes have been developed, allowing higher τ_1, newer engines have been designed with correspondingly higher compression ratios leading to higher specific impulse and better fuel efficiency.

Discussion of Example (3)

• In light of the discussion of the Previous page …. Revisit the SR-71 J-58 Engine



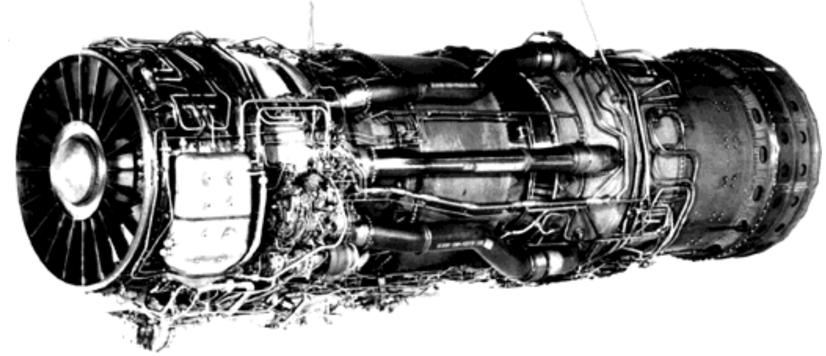
Mach 3+ cruise

$$\tau_{c_{max}|\, opt} = \frac{\sqrt{\tau_\lambda}}{\tau_r}. \qquad f \sim 1$$

• Thus as Mach number grows large, so does τ_r, and it becomes more and more difficult to achieve the optimal temperature cross the combustor with over temperature of the materials

• Above Mach 3 a portion of the flow bypasses the turbine and burns Directly in afterburner providing about 80% of th thrust …

• At lower speeds the engine operates as a normal supersonic Turbojet … same nozzle used by both operational modes

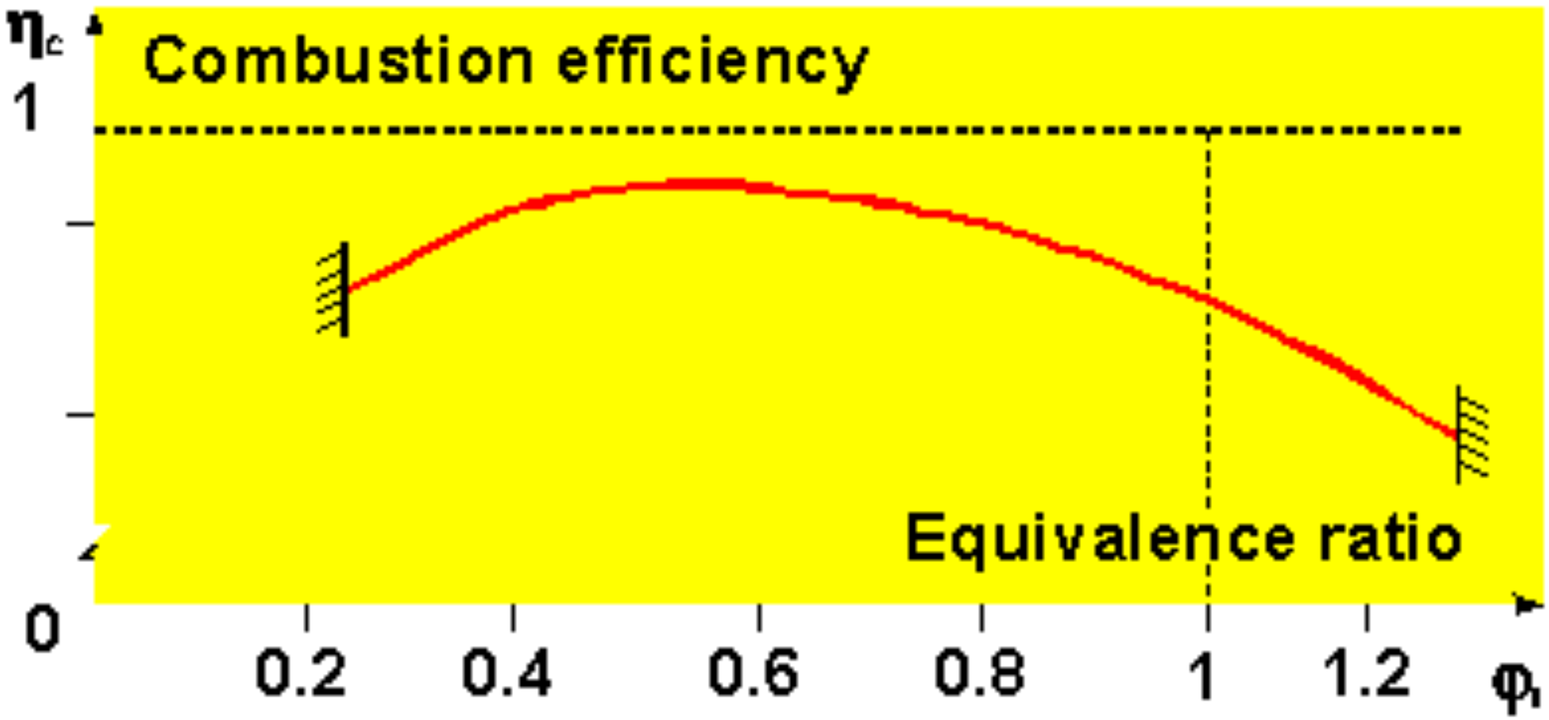


Discussion of Example (4)

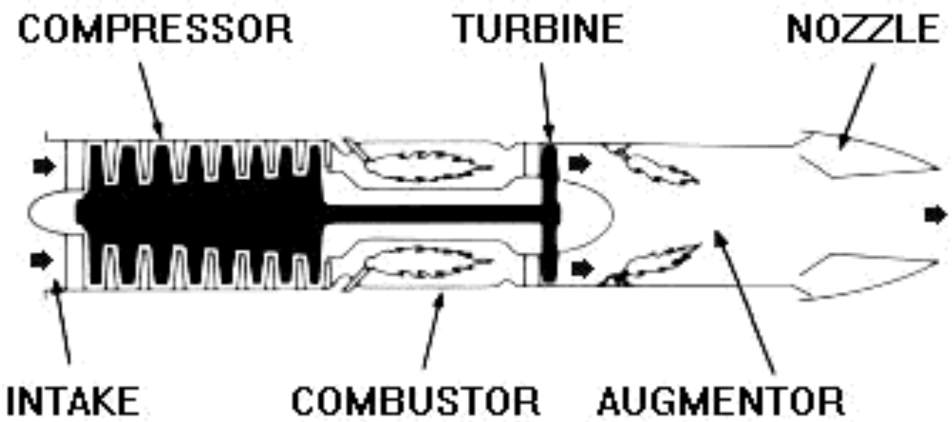
• This result also shows why turbine-driven jet engines tend to operate fuel-lean – temperature limits in the combustor and the downstream turbine materials.

• This limit is why afterburners are used for thrust augmentation, where fuel is burned downstream of the turbine .. instead of just dumping more fuel into the combustor …

THERMAL LIMITS OF THE MATERIALS!



Discussion of Example (5)



- … that is why afterburners work … left over O_2 after combustion

Additional fuel is introduced into the hot exhaust and burned using excess O_2 from main combustion

- The afterburner increases the temperature of the gas ahead of the nozzle Increases exit velocity

- The result of this increase in temperature is an increase of about 40 percent in thrust at takeoff and a much larger percentage at high speeds


Questions??