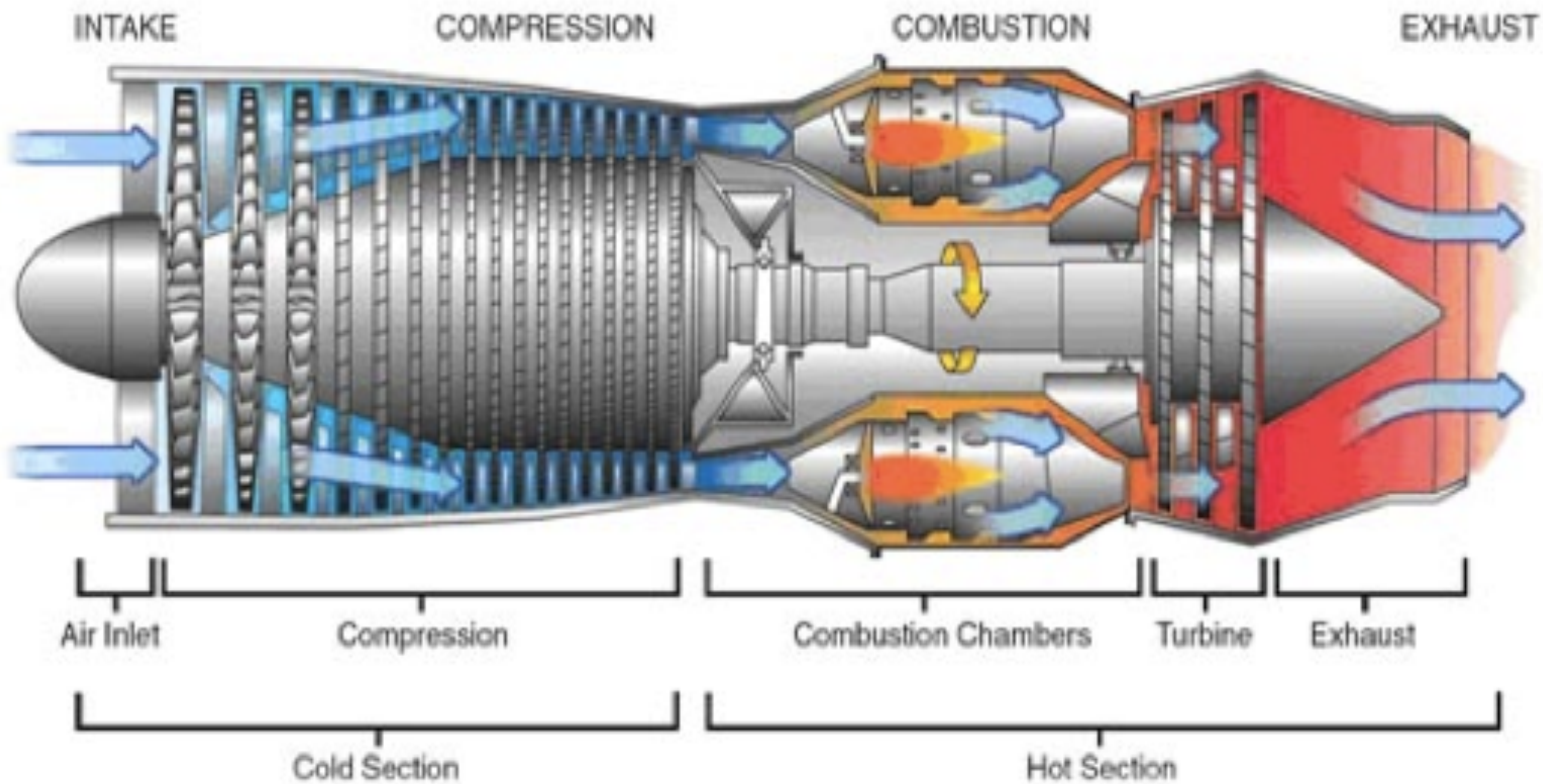
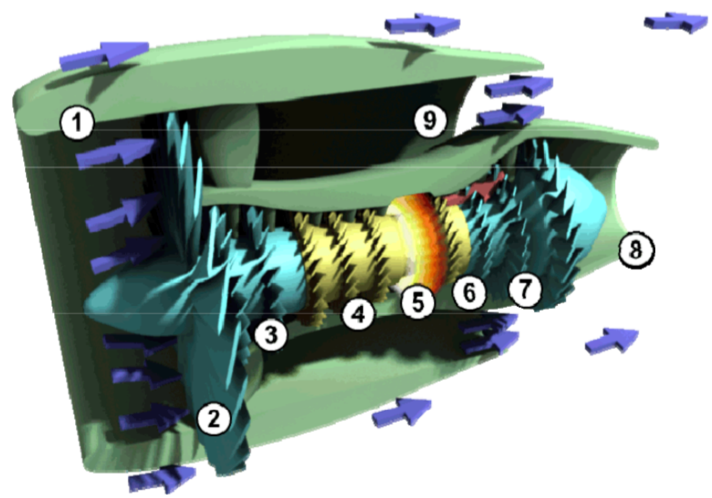
Section 5.2: The TurboJet Propulsion Cycle Massflow Matching Requirements

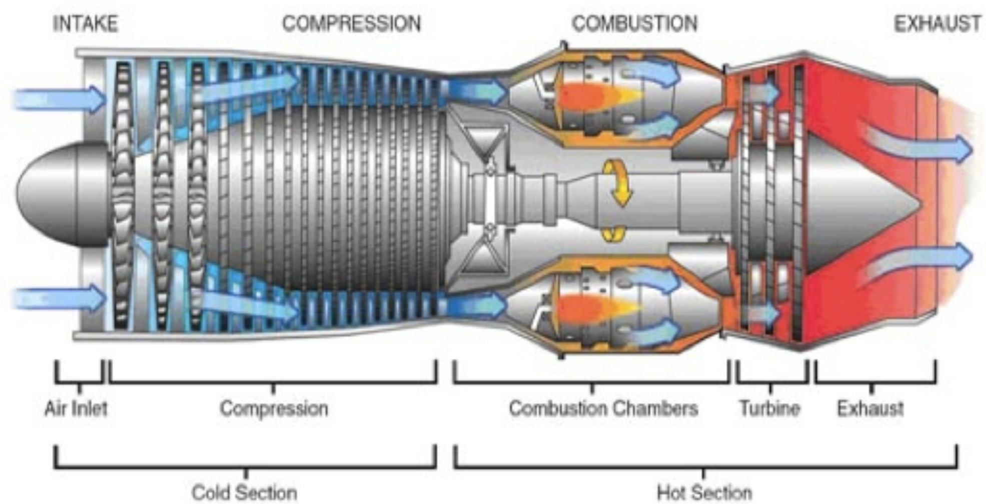


Component Matching Criteria

• The main components of a gas turbine engine are: *inlet diffuser, compressor, combustion chamber, turbine, and exhaust nozzle*.

• The individual components are designed based on established procedures and their performances are obtained from actual tests.





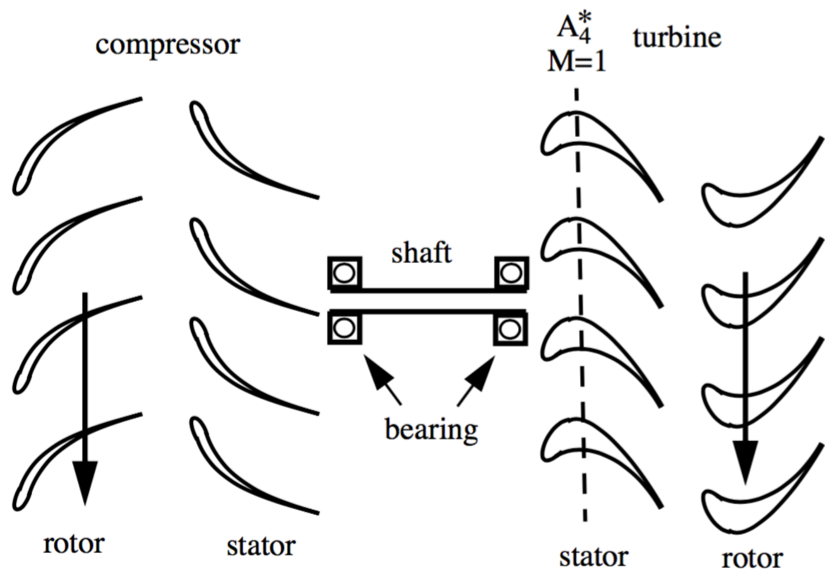
- When these components are integrated in an engine, the range of possible operating conditions is considerably reduced.
- There is considerable interdependency between components .. Referred to as "flow matching"

Turbine/Nozzle Flow Matching

- Mass balance between turbine inlet and nozzle exit is ….

$$\dot{m}_4 = \dot{m}_{exit} \rightarrow \frac{P_{0_4} \cdot A_4}{\sqrt{R_g \cdot T_{0_4}}} \frac{\sqrt{\gamma} \cdot M_4}{\left(1 + \frac{\gamma - 1}{2} M_4^{\ 2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{P_{0_{eit}} \cdot A_{exit}}{\sqrt{R_g \cdot T_{0_{eit}}}} \frac{\sqrt{\gamma} \cdot M_{exit}}{\left(1 + \frac{\gamma - 1}{2} M_{eit}^{\ 2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

- Turbine designed to provide a large pressure drop per stage.
- Drop possible because of favorable pressure gradient that stabilizes the boundary layers on turbine air- foils.
- Large pressure drop across each stage implies that at some point near the entrance to the first stage turbine stator (at the combustor exit), Flow is choked.



- Assuming near isentropic turbine flow, and choked nozzle throat, then,

$$\frac{P_{0_4} \cdot A_4^*}{\sqrt{T_{0_4}}} = \frac{P_{0_8} \cdot A_8}{\sqrt{T_{0_8}}}$$

Also ..

$$\frac{P_{0_5}}{P_{0_8}} = \left(\frac{T_{0_5}}{T_{0_8}}\right)^{\frac{\gamma}{\gamma-1}}$$

Turbine/Nozzle Flow Matching(2)

Mass Balance

$$\frac{P_{0_4}\cdot A_4^*}{\sqrt{T_{0_4}}}=\frac{P_{0_8}\cdot A_8}{\sqrt{T_{0_8}}}$$

Isentropic Flow

$$\frac{P_{0_5}}{P_{0_8}}=\left(\frac{T_{0_5}}{T_{0_8}}\right)^{\frac{\gamma}{\gamma-1}}$$

Substituting and Rearranging

$$\frac{P_{0_4}\cdot A_4^*}{\sqrt{T_{0_4}}}=\frac{P_{0_8}\cdot A_8}{\sqrt{T_{0_8}}}$$

$$\frac{P_{0_5}}{P_{0_4}}=\frac{P_{0_8}}{P_{0_4}}=\left(\frac{T_{0_8}}{T_{0_4}}\right)^{\frac{\gamma}{\gamma-1}}=\left(\frac{T_{0_5}}{T_{0_4}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\rightarrow$$

$$\frac{P_{0_8}}{P_{0_4}}=\frac{A_4^*}{A_8}\cdot\sqrt{\frac{T_{0_8}}{T_{0_4}}}=\left(\frac{T_{0_8}}{T_{0_4}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{A_4^*}{A_8}=\left(\frac{T_{0_8}}{T_{0_4}}\right)^{\frac{\gamma}{\gamma-1}-\frac{1}{2}}=\left(\frac{T_{0_8}}{T_{0_4}}\right)^{\frac{2\gamma-(\gamma-1)}{\gamma-1}}=\left(\frac{T_{0_8}}{T_{0_4}}\right)^{\frac{2\gamma-(\gamma-1)}{2(\gamma-1)}}=\left(\frac{T_{0_8}}{T_{0_4}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A_4^*}{A_8}=\left(\frac{T_{0_8}}{T_{0_4}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}=\left(\frac{T_{0_5}}{T_{0_4}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}=\left(\tau_t\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

Turbine/Nozzle Flow Matching (2)

Turbine Pressure Ratio

$$\pi_t^{\frac{\gamma-1}{\gamma}} = \tau_t \rightarrow \frac{A_4^*}{A_8} = \left(\pi_t^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \left(\pi_t\right)^{\frac{\gamma+1}{2\gamma}}$$

$$\frac{A_4^*}{A_8} = \left(\tau_t\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A_4^*}{A_8} = \left(\pi_t\right)^{\frac{\gamma+1}{2\gamma}}$$

- Temperature and pressure ratio across turbine is determined entirely by the area ratio from the turbine inlet to nozzle throat.
- As fuel flow is increased or decreased with areas fixed the temperature drop across the turbine may increase or decrease changing the amount of work done, but the temperature ratio remains constant.
- Turbine inlet and nozzle throat are choked over almost entire practical range of engine operating conditions except during brief transients at start-up and shut-down.

Free Stream- Compressor Inlet Flow Matching

- Mass balance between the free stream and the compressor face is

$$\dot{m}_\infty = \dot{m}_2 \rightarrow \frac{P_{0_\infty} \cdot A_\infty}{\sqrt{R_g \cdot T_{0_\infty}}} \frac{\sqrt{\gamma} \cdot M_\infty}{\left(1+\frac{\gamma-1}{2} M_\infty{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{P_{0_2} \cdot A_2}{\sqrt{R_g \cdot T_{0_2}}} \frac{\sqrt{\gamma} \cdot M_2}{\left(1+\frac{\gamma-1}{2} M_2{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

- Flow from free-stream to compressor face is adiabatic .. Thus … $T_{0_2} = T_{0_\infty}$ …

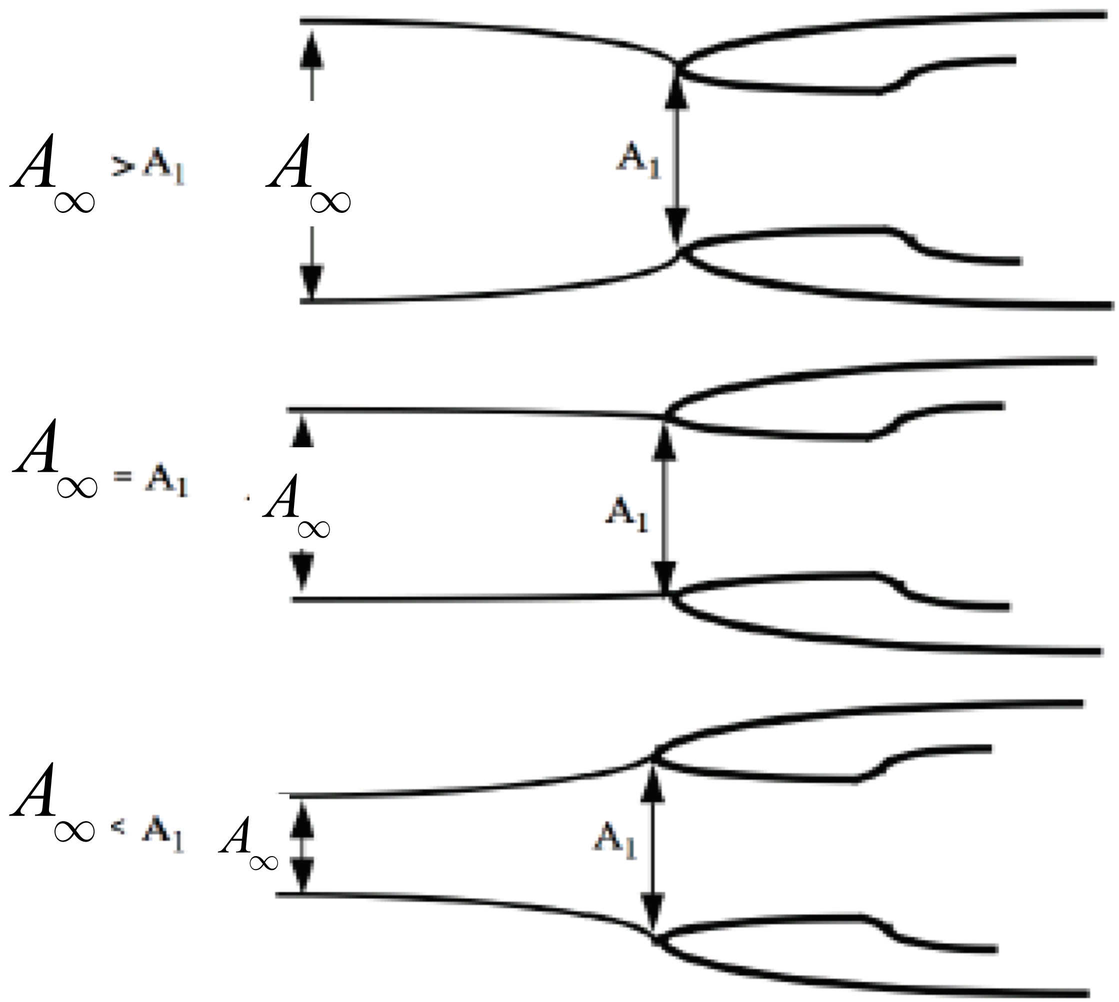
… and …

$$\pi_d = \frac{P_{0_2}}{P_{0_\infty}} \rightarrow \frac{1}{\pi_d} \frac{A_\infty}{A_2} \frac{\sqrt{\gamma} \cdot M_\infty}{\left(1+\frac{\gamma-1}{2} M_\infty{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{\sqrt{\gamma} \cdot M_2}{\left(1+\frac{\gamma-1}{2} M_2{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

- Will show that …. the fuel setting and nozzle throat area determine (M_2) independently of what is happening in free stream and inlet.

- Engine demands a certain value of M_2) and the gas dynamics of the inlet adjust A_∞

Free Stream- Compressor Inlet Flow Matching (2)



Variation of inlet capture area with engine operating point.

Compressor-Turbine Massflow Matching

• Assuming choked combustor outlet (and turbine inlet) …
Mass balance between compressor face and the turbine inlet is

$$\left(\frac{f+1}{f}\right)\dot{m}_2 = \dot{m}_4 \rightarrow \left(\frac{f+1}{f}\right)\frac{P_{0_2}\cdot A_2}{\sqrt{R_g\cdot T_{0_2}}}\frac{\sqrt{\gamma}\cdot M_2}{\left(1+\frac{\gamma-1}{2}M_2{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{P_{0_4}\cdot A_4{}^*}{\sqrt{T_{0_4}}}\sqrt{\frac{\gamma}{R_g}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$\textit{Dividing by } \left(\frac{f+1}{f}\right)\frac{P_{0_2}\cdot A_2}{\sqrt{R_g\cdot T_{0_2}}} \rightarrow \frac{\sqrt{\gamma}\cdot M_2}{\left(1+\frac{\gamma-1}{2}M_2{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{\frac{P_{0_4}\cdot A_4{}^*}{\sqrt{T_{0_4}}}\sqrt{\frac{\gamma}{R_g}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}{\left(\frac{f+1}{f}\right)\frac{P_{0_2}\cdot A_2}{\sqrt{R_g\cdot T_{0_2}}}} =$$

$$\left(\frac{f}{f+1}\right)\frac{P_{0_4}}{P_{0_2}}\cdot\sqrt{\frac{T_{0_2}}{T_{0_4}}}\cdot\frac{A_4{}^*}{A_2}\sqrt{\frac{\gamma}{R_g}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\cdot R_g} = \left(\frac{f}{f+1}\right)\frac{P_{0_4}}{P_{0_2}}\cdot\sqrt{\frac{T_{0_2}}{T_{0_4}}}\cdot\frac{A_4{}^*}{A_2}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Compressor-Turbine Massflow Matching (2)

also since diffusser is adiabatic

$\rightarrow T_{0_\infty} = T_{0_2}$

constant burner pressure

$\rightarrow \pi_b = 1$

$$\rightarrow \tau_b \cdot \tau_c = \frac{T_{0_4}}{T_\infty} \cdot \frac{T_\infty}{T_{0_3}} \cdot \frac{T_{0_3}}{T_{0_2}} = \frac{T_{0_4}}{T_\infty} \cdot \frac{T_\infty}{T_{0_3}} \cdot \frac{T_{0_3}}{T_{0_\infty}} = \frac{T_{0_4}}{T_\infty} \cdot \frac{T_\infty}{T_{0_\infty}} = \frac{\tau_\lambda}{\tau_r}$$

$$\textit{substituting} \rightarrow \frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2{}^2\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right) \cdot \frac{(\pi_c)}{\sqrt{\tau_\lambda / \tau_r}} \cdot \frac{A_4{}^*}{A_2}$$

Collected Engine-Matching Conditions

- Component matching conditions needed to understand the operation of the turbojet in order from the nozzle to the inlet are as follows.

Nozzle:

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}} \qquad \pi_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2\cdot\gamma}{\gamma+1}}$$

Turbine

$$\tau_t = 1 - \left(\frac{f}{f+1}\right)\frac{\tau_r\cdot(\tau_c - 1)}{\tau_\lambda} \rightarrow (\tau_c - 1) = \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot(1-\tau_t)$$

Compressor / Turbine Matching:

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^2\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^*}{A_2}$$

Inlet:

$$\frac{1}{\pi_d}\frac{A_\infty}{A_2}\frac{M_\infty}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{M_2}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

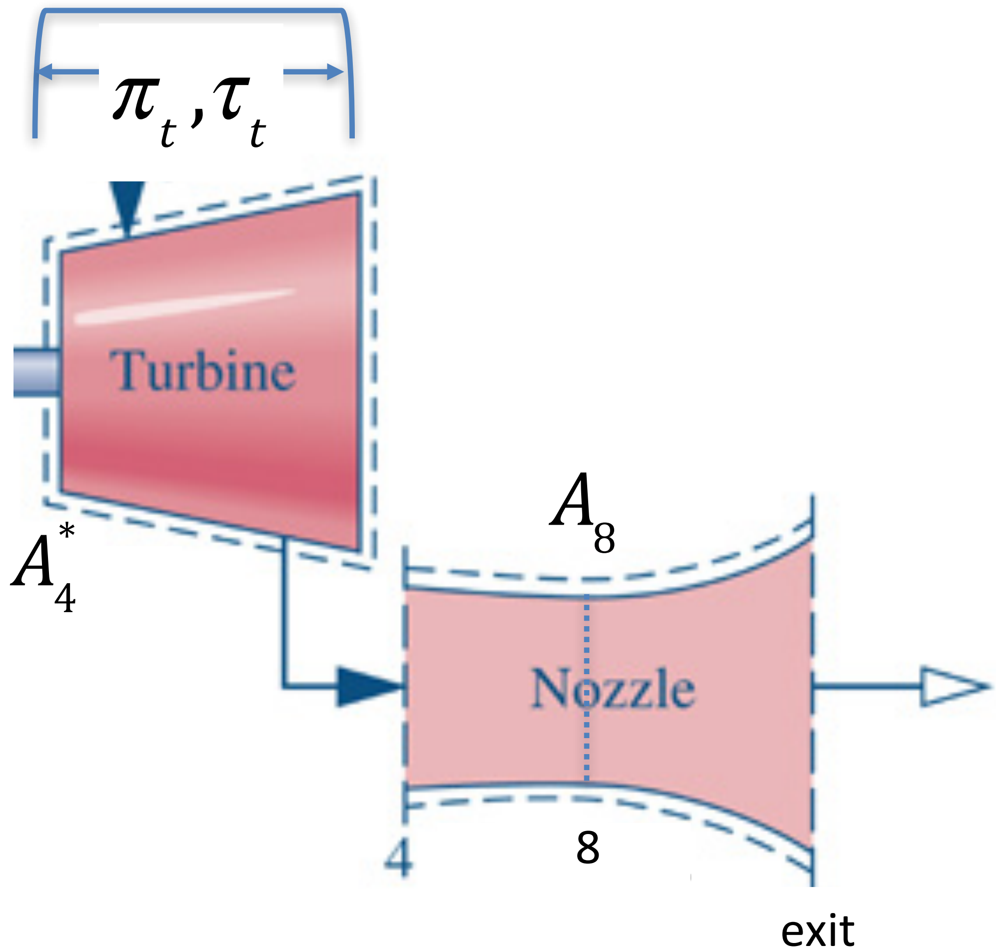
Fuel Matching/Maximum Temperature Constraint:

$$f = \frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r\cdot\tau_c}$$

Collected Engine-Matching Conditions (2)

Nozzle :

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}} \qquad \pi_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2\cdot\gamma}{\gamma+1}}$$

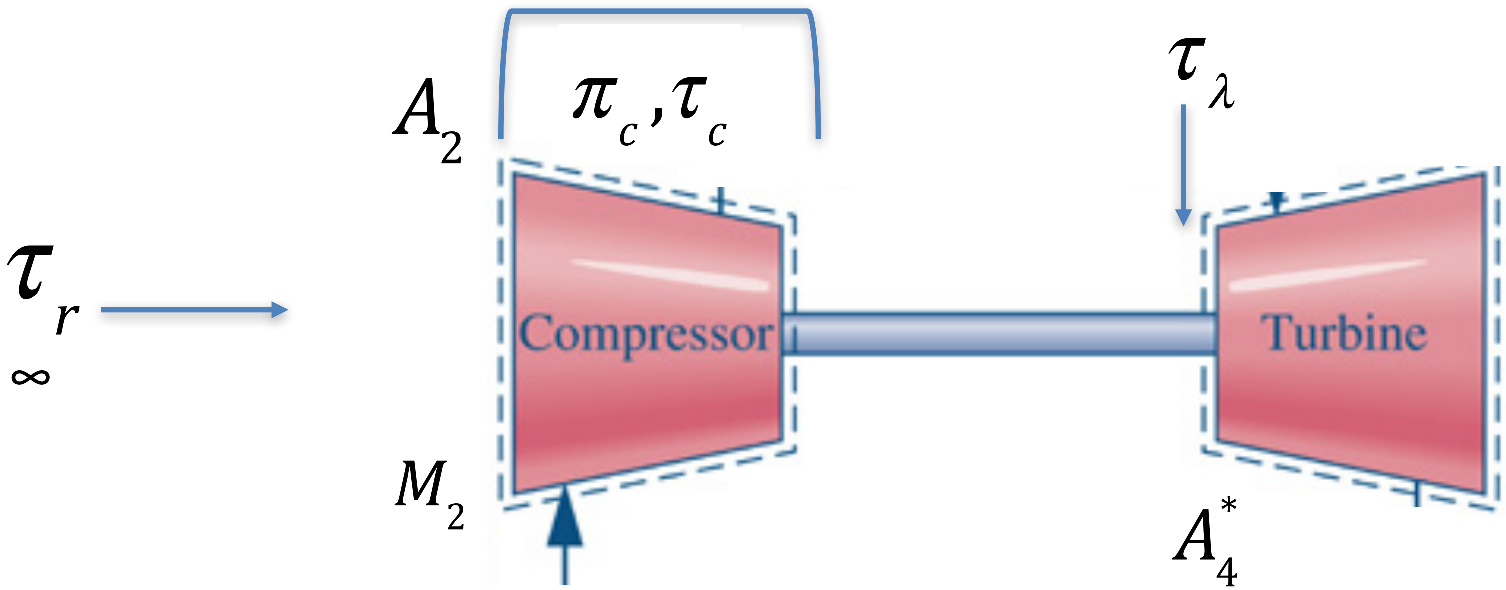


Massflow Matching across Turbine Inlet and Nozzle Throat

Collected Engine-Matching Conditions (3)

Compressor :

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}=\left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^{\,*}}{A_2}=\left(\frac{\dot{m}_{air}}{A_2 P_{0_2}}\right)\sqrt{\frac{R_g T_{0_2}}{\gamma}}$$

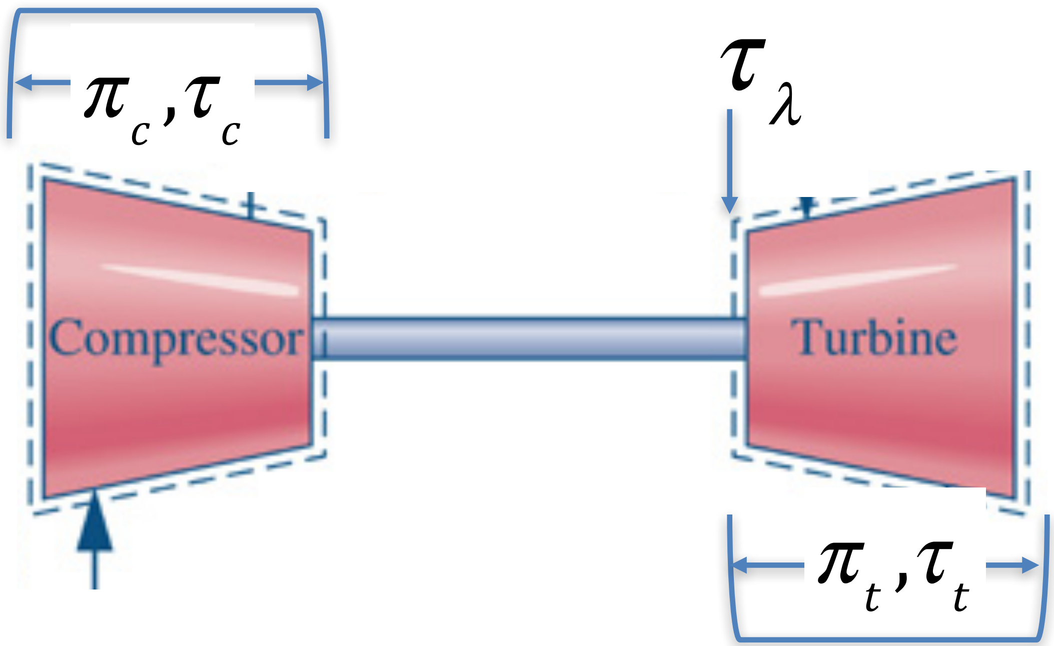


Massflow Matching across Compressor Face and Turbine Inlet

Collected Engine-Matching Conditions (4)

Turbine

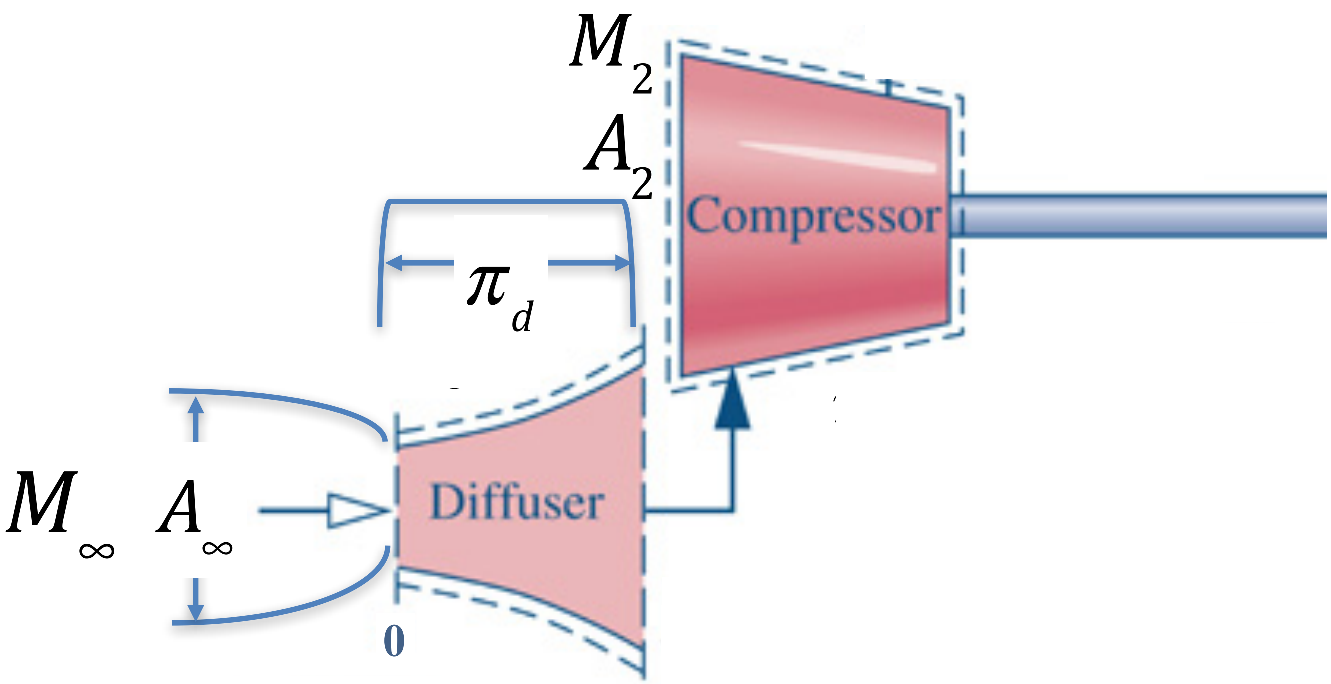
$$\tau_t = 1 - \left(\frac{f}{f+1}\right)\frac{\tau_r \cdot (\tau_c - 1)}{\tau_\lambda} \rightarrow (\tau_c - 1) = \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot(1-\tau_t)$$



Power Matching across Compressor Face and Turbine Inlet

Collected Engine-Matching Conditions (5)

$$\frac{1}{\pi_d}\frac{A_\infty}{A_2}\frac{M_\infty}{\left(1+\frac{\gamma-1}{2}M_\infty^{\ 2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}=\frac{M_2}{\left(1+\frac{\gamma-1}{2}M_2^{\ 2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$



Massflow Matching from Inlet to Compressor Face

Collected Engine-Matching Conditions (6)

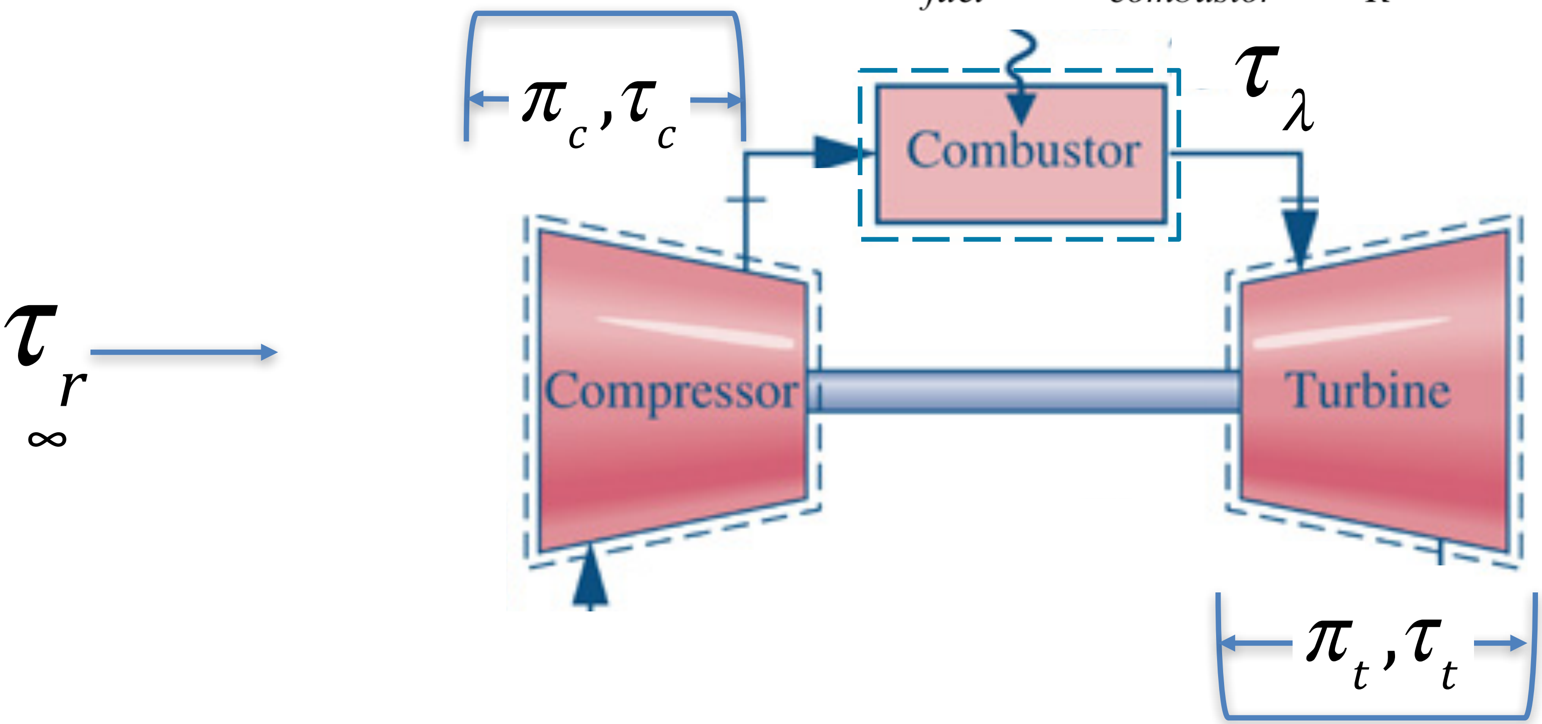
Fuel Matching/Maximum Temperature Constraint:

$$f = \frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r \cdot \tau_c}$$

Fuel Flow Allows Power Demand But is Constrained by Burner Exit Temperature

$$h_{fuel} = \eta_{combustor} \cdot Q_R$$

$$\tau_f = \frac{h_{fuel}}{C_p T_\infty}$$



Thrust of an Ideal Turbojet (15)

- Collected Parametric Turbojet Thrust Equations

$$\mathbb{T} = \frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r-1\right)\cdot\left[\left(\frac{f+1}{f}\right)\cdot\sqrt{\left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\cdot\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)}-1\right]$$

$$\left(\frac{V_{exit}}{V_\infty}\right)^2 = \left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)$$

$$\frac{1}{f} = \frac{\tau_\lambda-\tau_r\cdot\tau_c}{\tau_f-\tau_\lambda}$$

$$\tau_t = 1-\frac{\tau_r\cdot\left(\tau_c-1\right)}{\left(1+\frac{1}{f}\right)\tau_\lambda}$$

- Normalized properties depend only on $\left\{\tau_r,\tau_c,\tau_\lambda,\tau_f,\gamma\right\}$

Thrust of an Ideal Turbojet (16)

$$\{\tau_r, \tau_c, \tau_\lambda, \tau_f, \gamma\} \rightarrow$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \rightarrow$$ *Freestream Mach number reference conditions*

$$\tau_c = \frac{T_{0_3}}{T_{0_2}} \rightarrow$$ *Compressor stagnation temperature ratio measure of compressor work input*

$$\tau_\lambda = \frac{T_{0_4}}{T_\infty} \rightarrow$$ *Combustor flame temperature...Optimized up to Material limits of combustor, turbine*

$$\tau_f = \frac{h_f}{h_\infty} \rightarrow$$ *Fuel enthalpy of combustion relative to incoming air stream total enthalpy*

$$\gamma = \frac{C_p}{C_v} \rightarrow$$ *Ratio of specific heats*

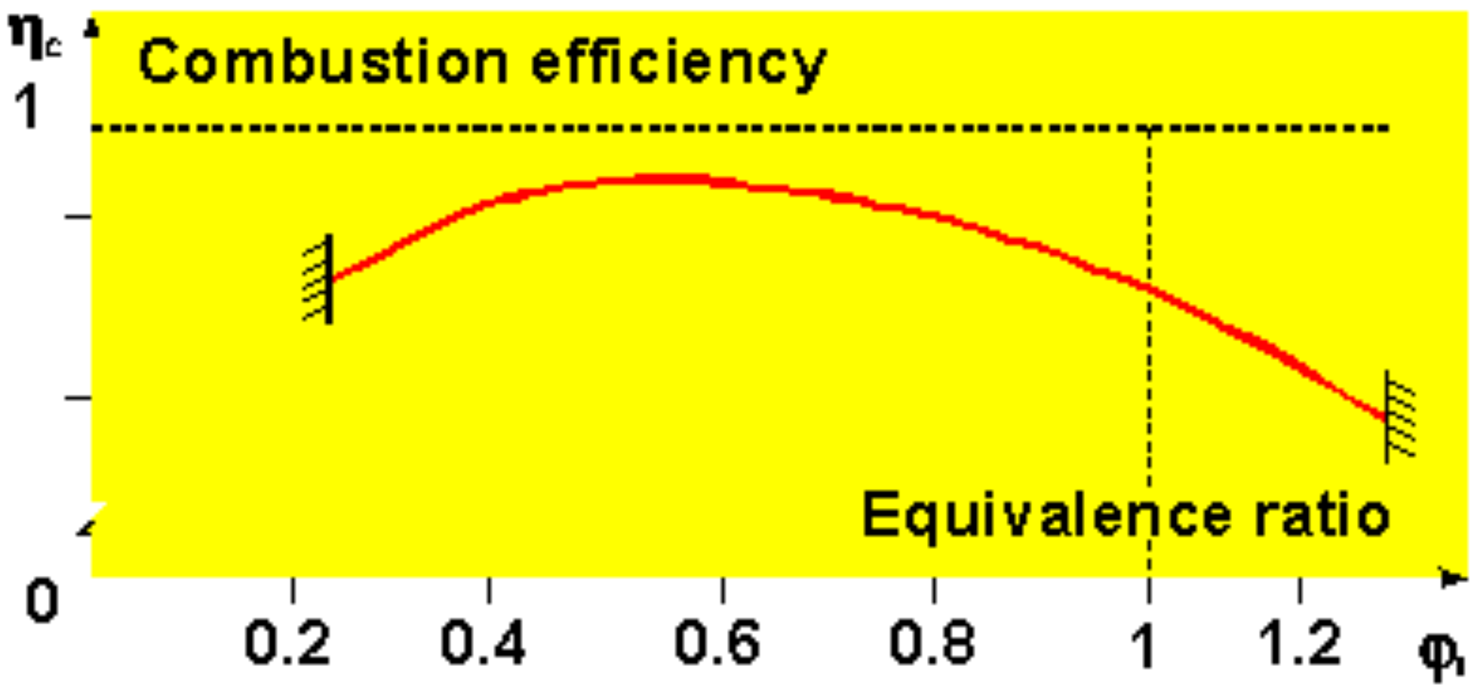
Choice of Fuel

$$\mathbb{T} = \frac{F_{thrust}}{p_\infty \cdot A_0} \rightarrow$$

- *Operating Mach Number*
- *Choice of Propellants*
- *Combustion Efficiency*
- *Compressor Work Input*

Effect of Afterburning

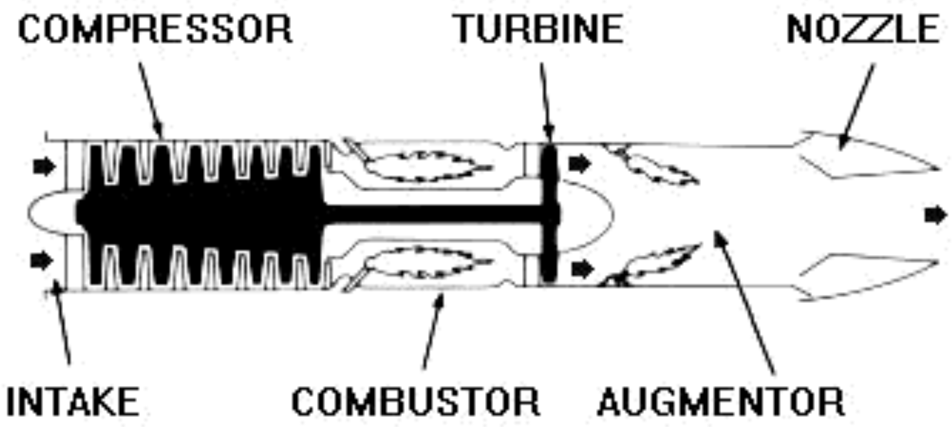
• As described previously, jet engines tend to operate lean, primarily for thermal considerations, but also for reasons of combustion efficiency.



• After- burner is a relatively simple device that includes a spray bar where fuel is injected, and a flameholder designed to provide a low speed wake where combustion takes place.

• Since flow exiting turbine is already hot and extra oxygen is present, no ignition system is needed.

Effect of Afterburning (2)

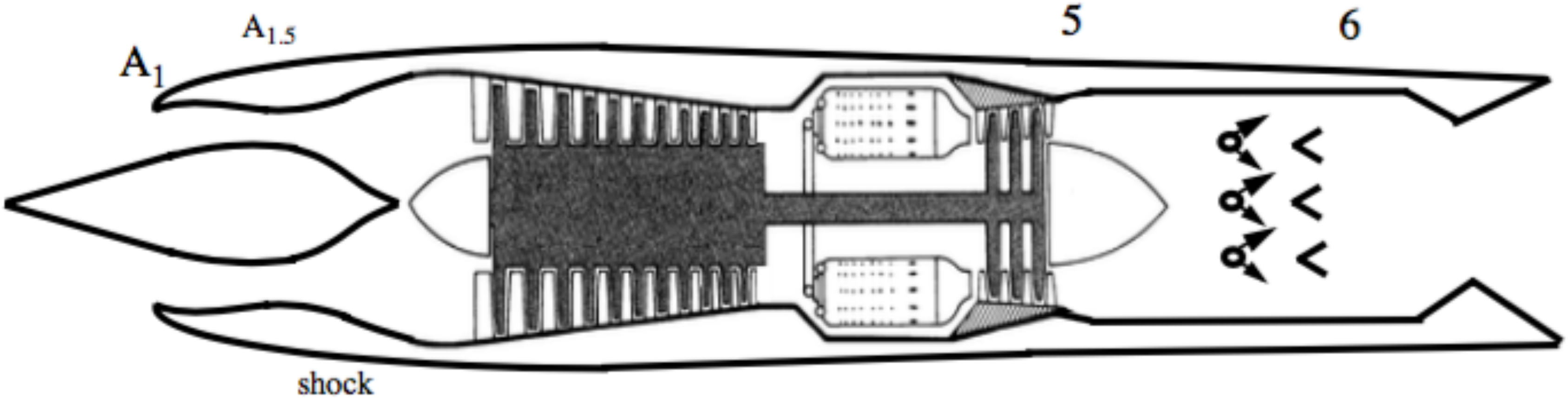


• Additional fuel is introduced into the hot exhaust and burned using excess O_2 from main combustion

• The afterburner increases the temperature of the gas ahead of the nozzle Increases exit velocity

• The result of this increase in temperature is an increase of about 40 percent in thrust at takeoff and a much larger percentage at high speeds

Effect of Afterburning (3)

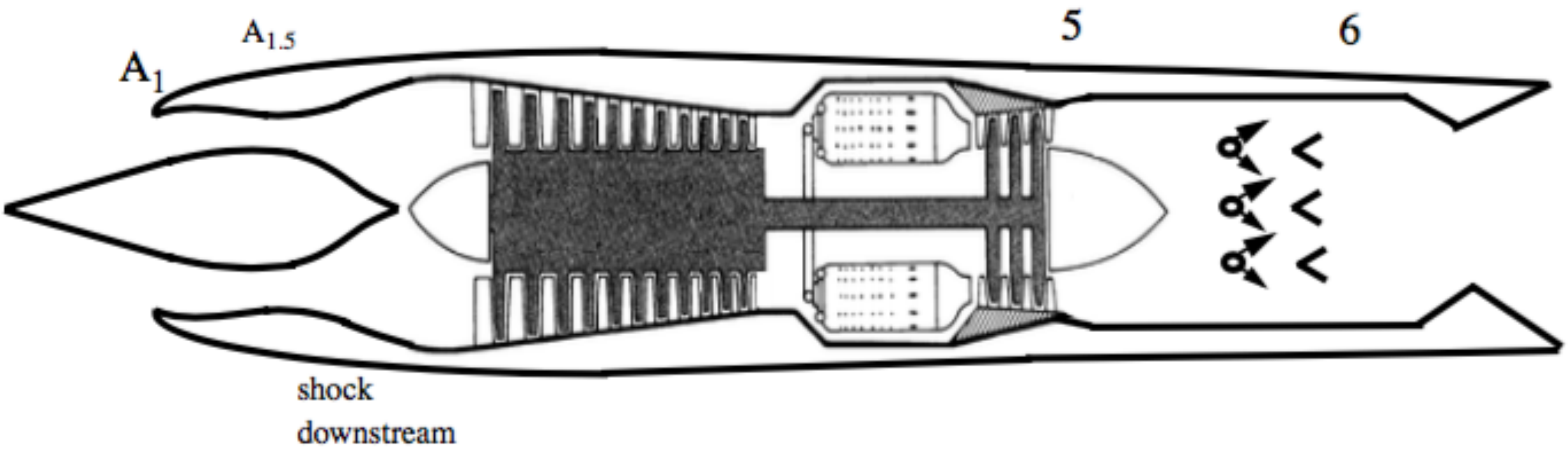


• Afterburner provides a rapid increase in thrust on demand allowing the aircraft to respond quickly to changing mission circumstances;

• Main effect of afterburner is to add heat to turbine exhaust gases while producing relatively little stagnation loss since heat addition is at relatively low Mach number.

• Exhaust Mach number is determined by nozzle area ratio and for same exit Mach number exit velocity is increased in proportion to the increase in square root of exhaust temperature.

Effect of Afterburning (4)



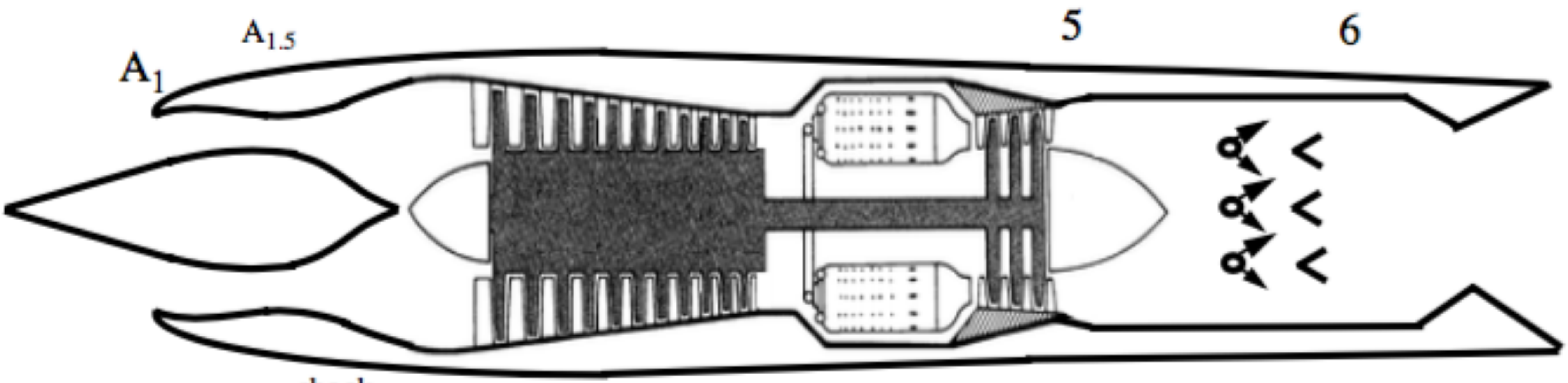
• Writing in terms of the engine parameters, this combustion occurs at relatively constant stagnation and static pressures.

$$\pi_{ab} = \frac{P_{0_6}}{P_{0_5}} \approx 1$$

• For fixed Nozzle area ratio the stagnation pressure is the same implies that the exit Mach number remains unchanged and the pressure contribution to the thrust does not change during afterburning.

$$\frac{A_{exit}}{A_\infty} \cdot \left(\frac{p_{exit}}{p_\infty} - 1 \right) \simeq \textbf{\textit{constant}}$$

Effect of Afterburning (5)



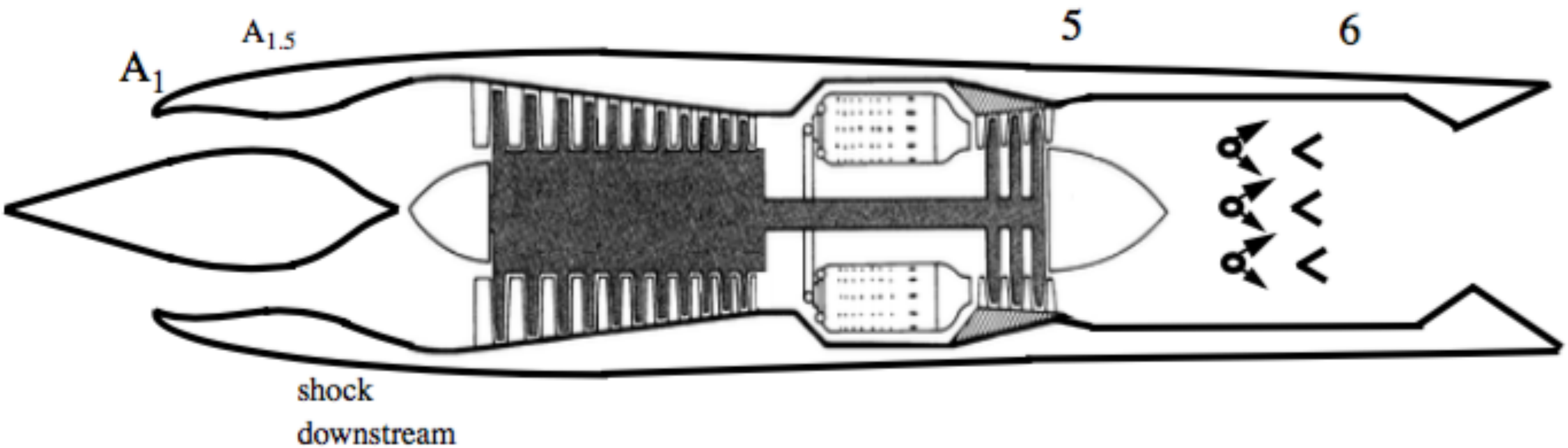
• Writing in terms of the engine parameters, this combustion occurs at relatively constant stagnation and static pressures.

$$\pi_{ab} = \frac{P_{0_6}}{P_{0_5}} \approx 1$$

• The resulting engine velocity ratio is

$$\frac{V_{exit}}{V_\infty} = \frac{M_{exit}}{M_\infty}\sqrt{\frac{T_{exit}}{T_\infty}} = \frac{M_{exit}}{M_\infty}\sqrt{\frac{T_{05}}{T_\infty}\cdot\frac{T_{06}}{T_{05}}\cdot\frac{T_{exit}}{T_{06}}} = \frac{M_{exit}}{M_\infty}\sqrt{\frac{\frac{T_{05}}{T_\infty}\cdot\tau_{ab}}{\left(1+\frac{\gamma-1}{2}M_{exit}^{\ 2}\right)}}$$

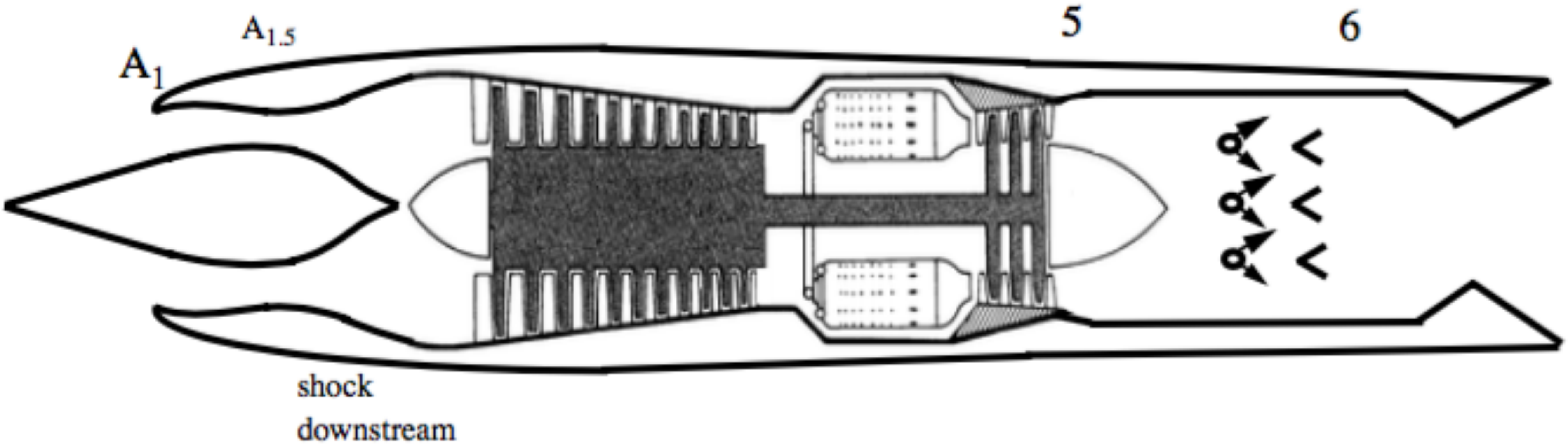
Effect of Afterburning (6)



- Looking at the resulting thrust increment

$$\Delta\left(\frac{F_{thrust}}{p_\infty \cdot A_\infty}\right) = \gamma \cdot M_\infty^2 \cdot \left(\frac{M_{exit}}{M_\infty}\sqrt{\frac{\frac{T_{05}}{T_\infty}\cdot \tau_{ab}}{\left(1+\frac{\gamma-1}{2}M_{exit}{}^2\right)}}-1\right) + \frac{A_{exit}}{A_\infty}\cdot\left(\frac{p_{exit}}{p_\infty}-1\right) - \left[\gamma \cdot M_\infty^2 \cdot \left(\frac{M_{exit}}{M_\infty}\sqrt{\frac{\frac{T_{05}}{T_\infty}}{\left(1+\frac{\gamma-1}{2}M_{exit}{}^2\right)}}-1\right) + \frac{A_{exit}}{A_\infty}\cdot\left(\frac{p_{exit}}{p_\infty}-1\right)\right] = \gamma \cdot M_\infty^2 \cdot \frac{M_{exit}}{M_\infty}\left(\sqrt{\frac{\frac{T_{05}}{T_\infty}}{\left(1+\frac{\gamma-1}{2}M_{exit}{}^2\right)}}\right)\left(\sqrt{\tau_{ab}}-1\right)$$

Effect of Afterburning (7)



- Collecting terms and writing in terms of afterburner heat addition

$$1+\frac{\Delta\left(\frac{F_{thrust}}{p_\infty \cdot A_\infty}\right)}{\gamma \cdot M_\infty^2 \cdot \frac{M_{exit}}{M_\infty}\left(\sqrt{\frac{\frac{T_{05}}{T_\infty}}{\left(1+\frac{\gamma-1}{2}M_{exit}{}^2\right)}}\right)} = 1+\frac{\Delta\left(\frac{F_{thrust}}{p_\infty \cdot A_\infty}\right)}{\gamma \cdot M_\infty^2 \cdot \left(\frac{U_{exit}}{U_\infty}\right)_{nom}} \approx \sqrt{\frac{T_{06}}{T_{05}}} = \sqrt{\tau_{ab}} = \sqrt{1+\frac{\Delta q_{ab}}{C_p T_{05}}}$$

Effect of Afterburning (8)

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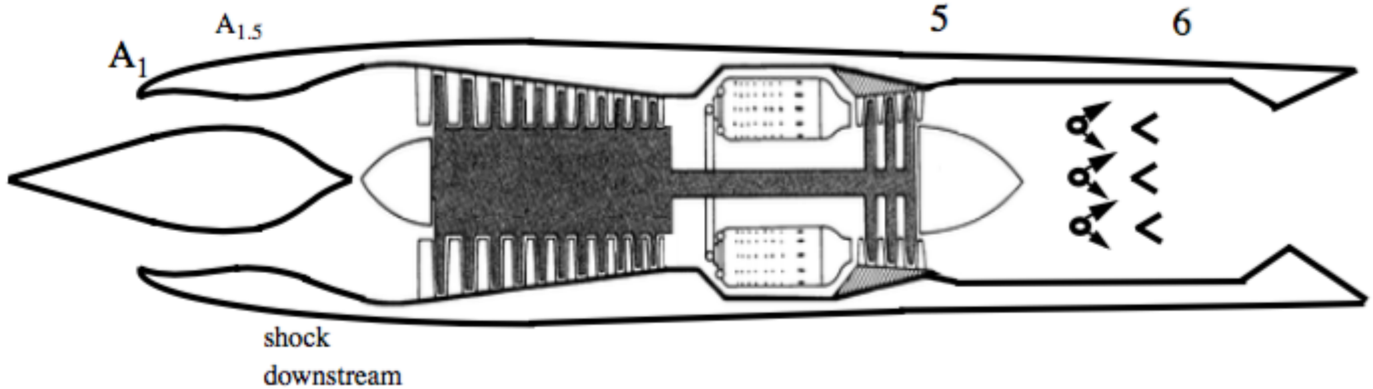

- Bottom line is that $V_{exit} \sim (\tau_{ab})^{1/2}$, Resulting Thrust Increment is proportional to square root of heat added during after-burning.
- Exit Mach remains same, but exit velocity is substantially increased
- Price is a substantial increase in fuel burn rate.
- Most military engines only spend a few hundred hours in afterburning mode over a typical engine lifetime (4000 hours) before a major overhaul.

$$1+\frac{\Delta\left(\frac{F_{thrust}}{p_\infty \cdot A_\infty}\right)_{ab}}{\gamma \cdot M_\infty^2 \cdot \left(\frac{U_{exit}}{U_\infty}\right)_{nom}} = \sqrt{1+\frac{\Delta q_{ab}}{C_p T_{05}}}$$

Effect of Afterburning (9)

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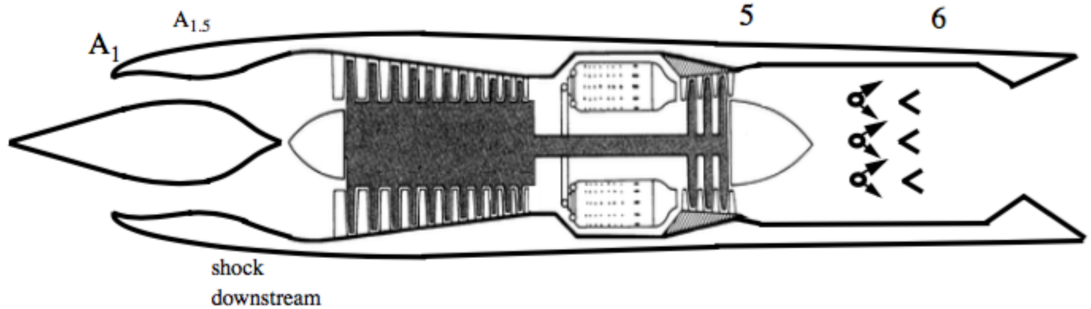

- Most military-class turbine engines employ some sort of variable area nozzle.
- Afterburning engines especially require a variable area nozzle.
- When afterburner is turned on and exit gas temperature increases according to

$$T_{0_{exit}} = T_{0_5} \cdot \tau_{ab} = 1 + \frac{\Delta q_{ab}}{C_p T_{05}}$$

- Nozzle throat area must be increased in a coordinated way to preserve engine mass flow without putting an unmanageable load on the turbine.
- Since the Nozzle is choked, with afterburner on turbine temperature ratio is

$$\tau_t = \left(\sqrt{\tau_{ab}} \cdot \frac{A^*_4}{A^*_8} \right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}$$

Effect of Afterburning (10)

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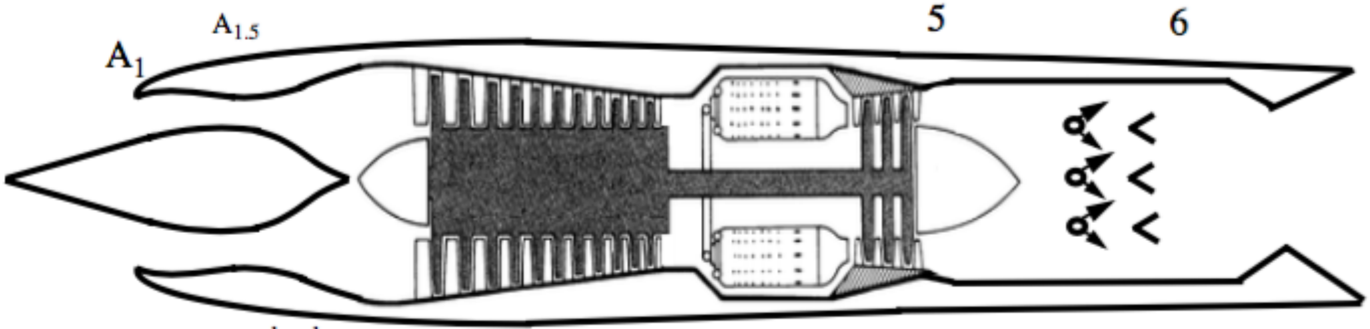


• To keep turbine temperature ratio unchanged and allow the compressor to remain at the same operating point when afterburner is turned on,

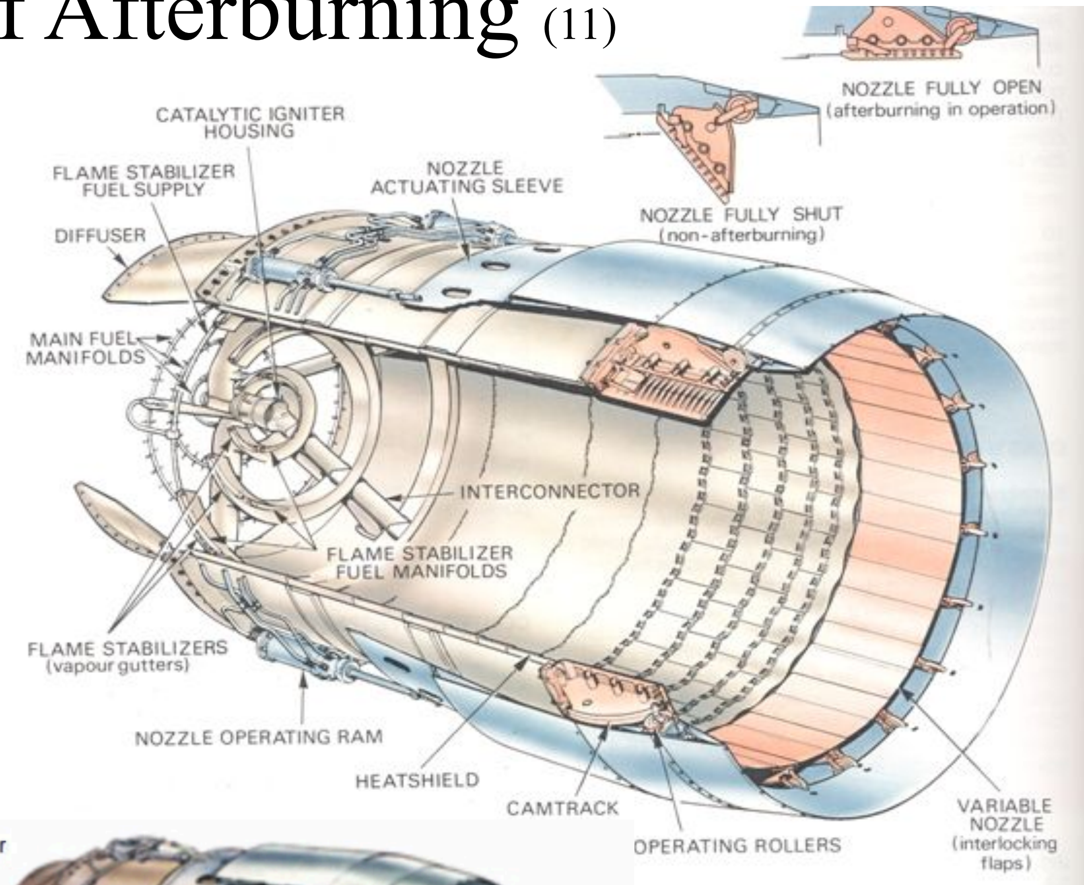
-- necessary to program nozzle area so that $\dfrac{\sqrt{\tau_{ab}}}{A^*_8}$ remains constant.

• If this ratio is not held constant turbine mass flow will decrease and desired thrust increment will not occur.

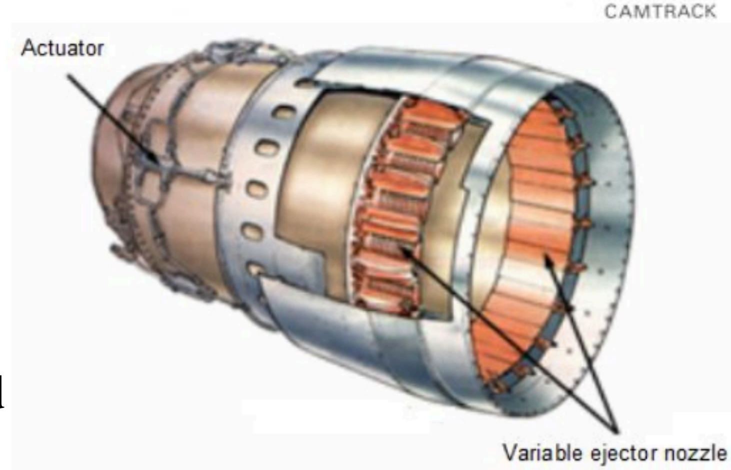
• IN an extreme case the compressor will cross over the surge line and

Effect of Afterburning (11)

- Speed of sound in the nozzle exit => variable area nozzle required!
- Aim is to maintain gas generator at same condition => variable area necessary to pass the same mass flow at a much lower density.



Sukhoi Su-35S Thrust Vectored Variable Area Nozzle (Russian)



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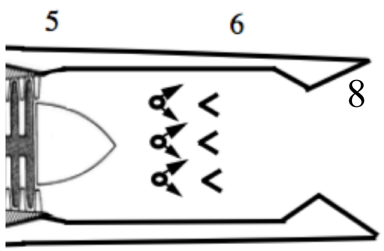
Variable ejector nozzle

Collected Engine-Matching Conditions (Updated)

• Component matching conditions needed to understand the operation of the turbojet in order from the nozzle to the inlet are as follows.

$$\tau_t = \left(\sqrt{\tau_{ab}} \cdot \frac{A^*_4}{A^*_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)} \qquad \sqrt{\frac{T_{06}}{T_{05}}} = \sqrt{\tau_{ab}} = \sqrt{1+\frac{\Delta q_{ab}}{C_p T_{05}}}$$

With Afterburner



Nozzle:

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}} \qquad \pi_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2\cdot\gamma}{\gamma+1}}$$

Turbine

$$\tau_t = 1-\left(\frac{f}{f+1}\right)\frac{\tau_r\cdot(\tau_c-1)}{\tau_\lambda} \rightarrow (\tau_c-1) = \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot(1-\tau_t)$$

Compressor / Turbine Matching:

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^2\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^*}{A_2}$$

Inlet:

$$\frac{1}{\pi_d}\frac{A_\infty}{A_2}\frac{M_\infty}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{M_2}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

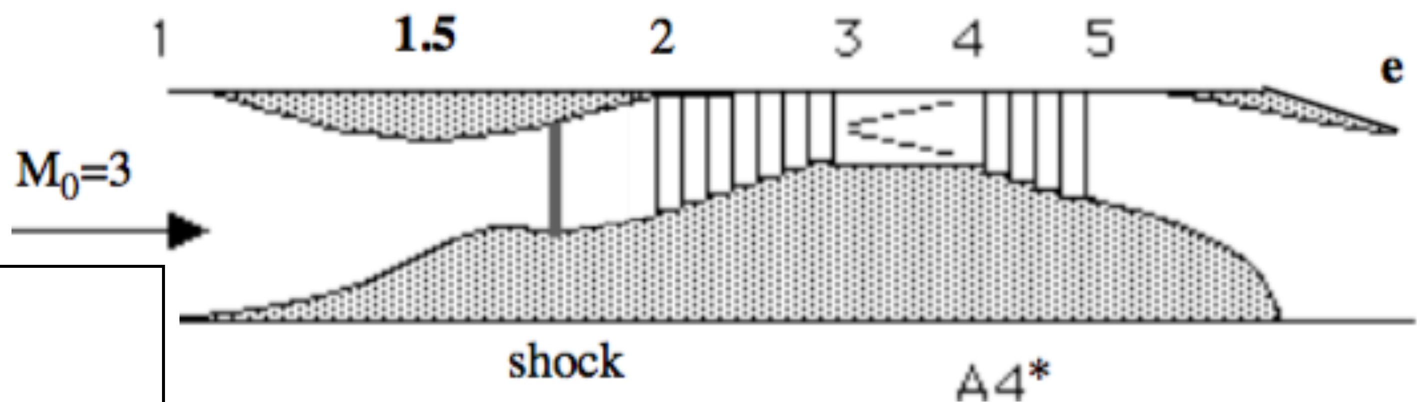
Fuel Matching/Maximum Temperature Constraint:

$$f = \frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r\cdot\tau_c}$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave

A turbojet operates supersonically at $M_0 = 3$ and $T_{t4} = 1944K$. The compressor and turbine polytropic efficiencies are $\eta_{pc} = \eta_{pt} = 1$. At the condition shown, the engine operates semi-ideally with $\pi_b = \pi_n = 1$ but $\pi_d \neq 1$ and with a simple convergent nozzle.

No afterburner



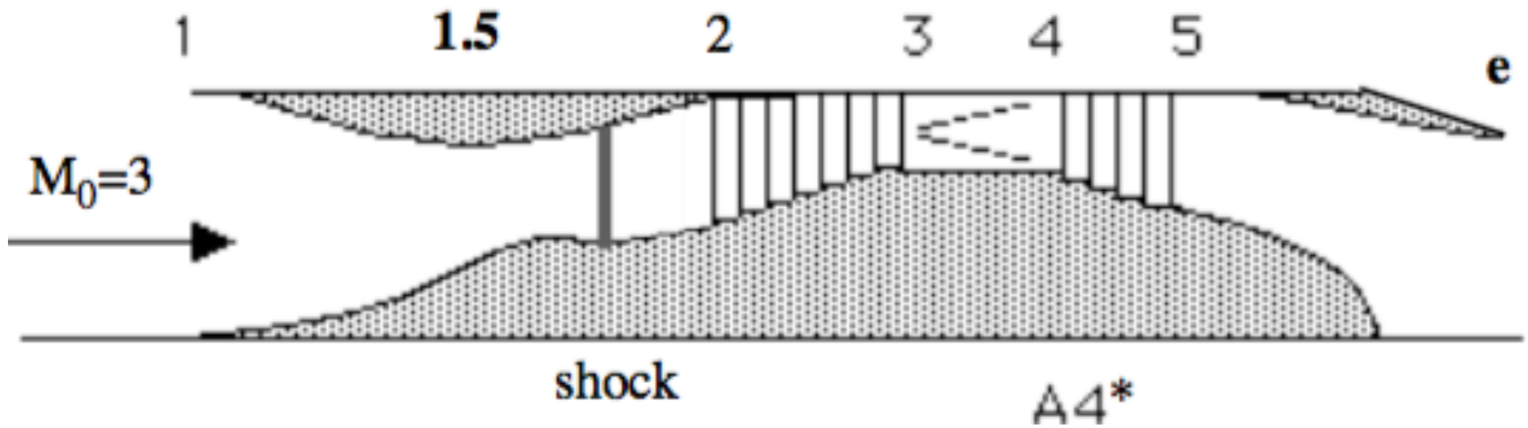
$$M_\infty = 3.0$$
$$T_{0_4} = 1944\ K$$
$$T_\infty = 216.1\ K\ \ (Stratosphere)$$
$$\eta_c = \eta_t \approx 1$$
$$\pi_b = \pi_n \approx 1$$
$$M_{b_{exit}} = M_{t_{inlet}} = M_{nozzle_{exit}} = 1$$

• Supersonic flow is established at the entrance to the inlet with a normal shock downstream of the inlet throat. "Supercritical" Operation

$$\frac{f+1}{f} = \frac{\dot{m}_{air} + \dot{m}_f}{\dot{m}_{air}} \approx 1$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave

$$M_\infty = 3.0$$
$$T_{0_4} = 1944\ K$$
$$\eta_c = \eta_t = 1$$
$$\pi_b = \pi_n = 1$$
$$M_{exit} = 1$$



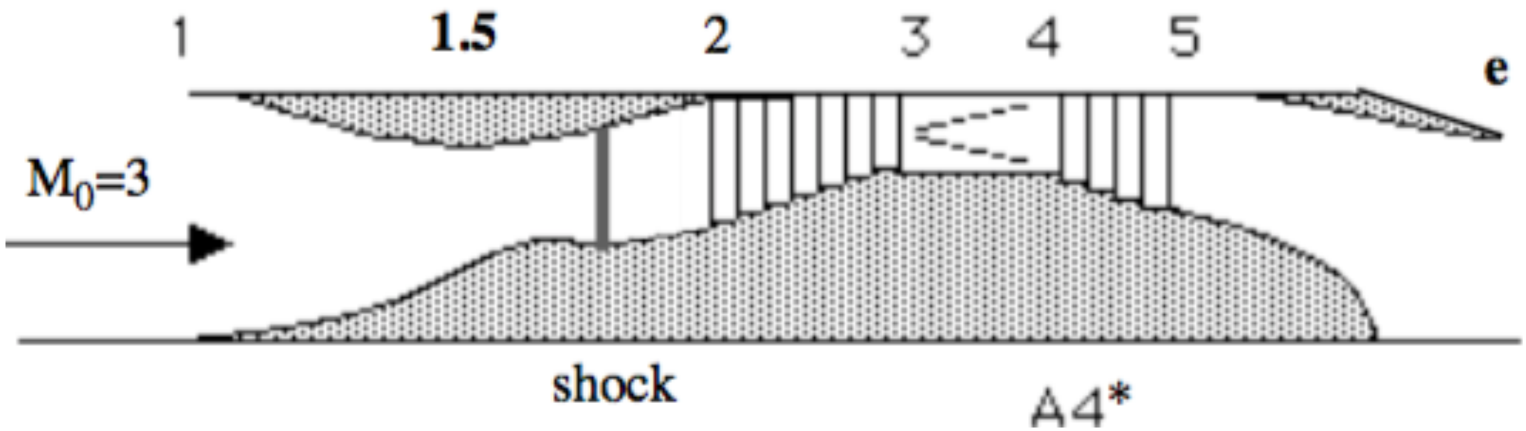
- First Calculate Reference Conditions, τ_λ

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = \left(1 + \frac{\gamma - 1}{2} M_2{}^2\right) = 1 + \frac{1.4 - 1}{\ } 3^2 = 2.8$$

$$\pi_r = \frac{P_{0_\infty}}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_2{}^2\right)^{\frac{\gamma}{\gamma - 1}} = \left(1 + \frac{1.4 - 1}{2} 3^2\right)^{\frac{1.4}{1.4 - 1}} = 36.7327$$

$$\tau_\lambda = \frac{T_{0_b}}{T_\infty} = \frac{1944}{216.1} = 8.996$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (2)



- Sketch the distribution of stagnation pressure P_0/P_{0_∞}, and stagnation temperature, T_0/T_{0_∞} through the engine.

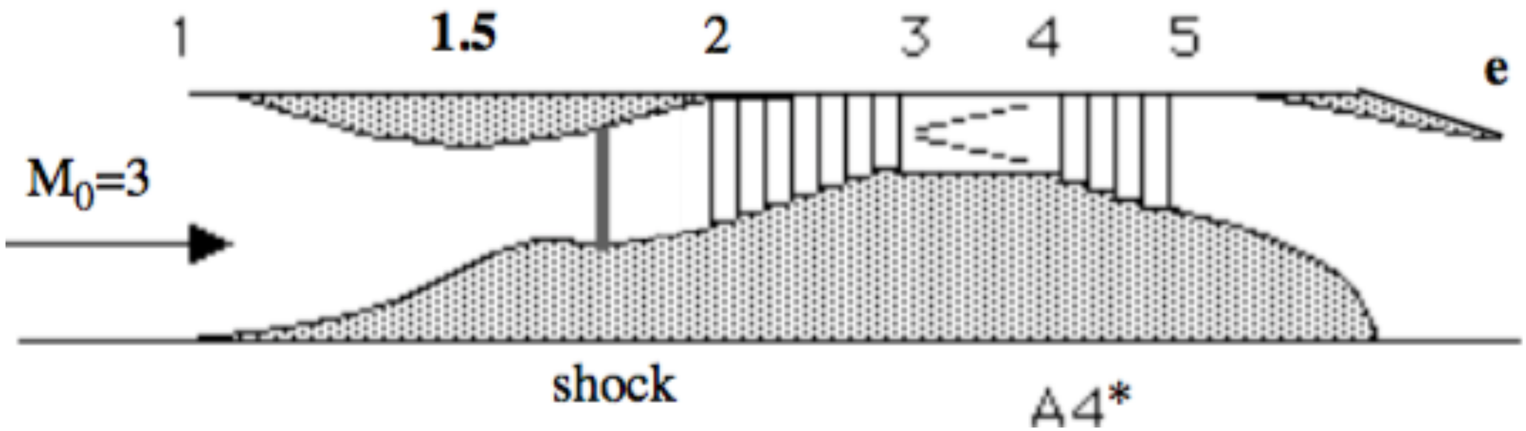
Given:

$$A_1/A_2 = 2\ , \quad A_2/A_4^* = 14,\ and\ A_e/A_4^* = 4.$$

Also...

$$\frac{A_e}{A_1} = \left(\frac{A_e}{A_4^*}\right)\cdot\left(\frac{A_4^*}{A_2}\right)\cdot\left(\frac{A_2}{A_1}\right) = (4)\cdot\left(\frac{1}{14}\right)\cdot\left(\frac{1}{2}\right) = 0.143$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (3)

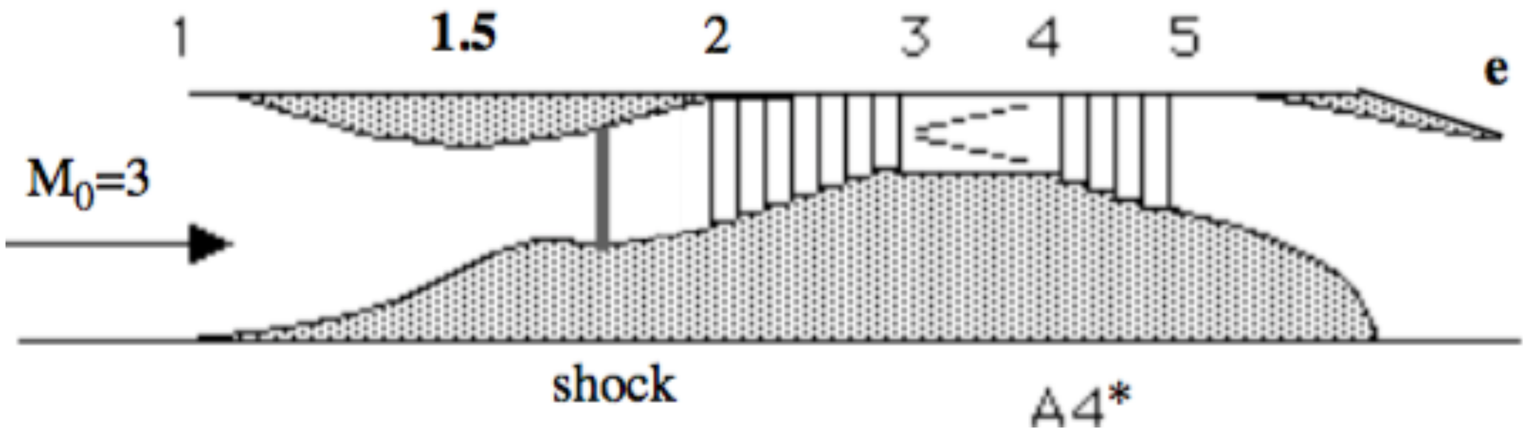


- Begin Analysis at Nozzle and work forward to determine interdependencies.
- Nozzle, Combustor Exit (Turbine Inlet) are assumed choked, Use *γ=1.4*.

$$\tau_t = \left(\frac{A_4^*}{A_e}\right)^{\frac{2(\gamma-1)}{\gamma+1}} = \left(\frac{1}{4}\right)^{\frac{2\,(1.4-1)}{1.4+1}} = 0.629961$$

$$\pi_t = \left(\frac{A_4^*}{A_e}\right)^{\frac{2\cdot\gamma}{\gamma+1}} = \left(\frac{1}{4}\right)^{\frac{2\,(1.4)}{1.4+1}} = 0.198425$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (4)

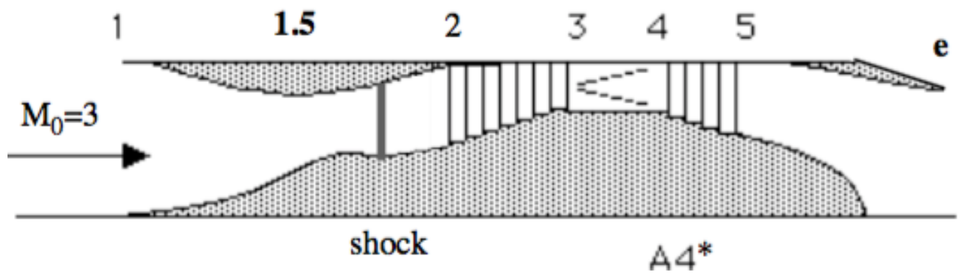


• Matching turbine and compressor work gives compressor temperature and pressure ratio.

$$\tau_c = 1 + \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot\left(1-\tau_t\right) = 1 + 1\left|\frac{8.996}{2.8}\left(1 - 0.629961\right)\right. = 2.18888$$

$$\pi_c = \left(\tau_c\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + 1\frac{8.996}{2.8}\left(1 - 0.629961\right)\right)^{\frac{1.4}{1.4-1}} = 15.516$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (5)



- Now Calculate Mach Number at Compressor Face

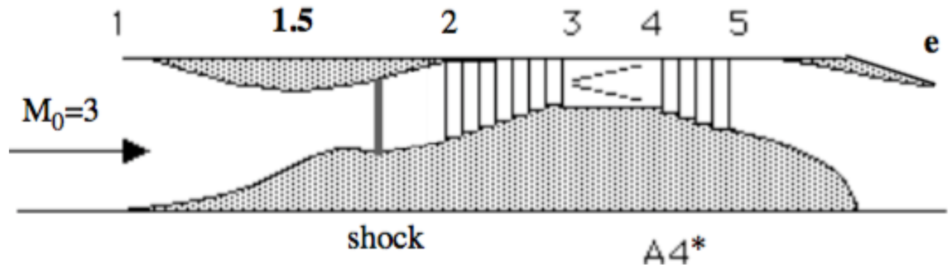
$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^*}{A_2}$$

$$\rightarrow \frac{M_2}{\left(\left(1+0.2M_2^{\,2}\right)(0.83333)\right)^3} = \left(\frac{15.52}{\sqrt{\frac{8.996}{2.8}}}\right)\cdot\left(\frac{1}{14}\right) = 0.6185$$

$$\frac{0.3919}{((1+0.2\cdot 0.3918^2)\,0.8333)^3} = 0.618547$$

$$\rightarrow M_2 = 0.392$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (6)

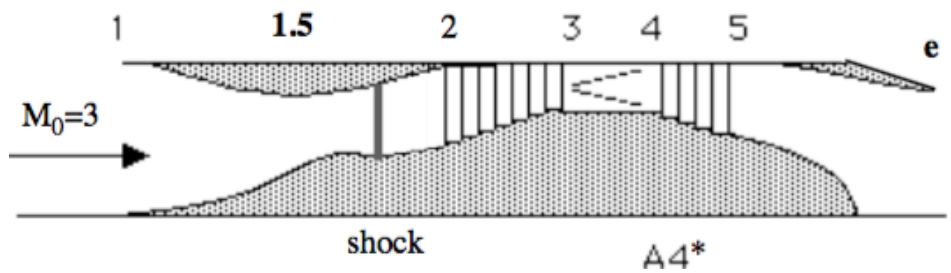


- Use free-stream-compressor-mass-flow matching to determine the stagnation pressure loss across the inlet.

$$\pi_d = \frac{A_\infty \dfrac{M_\infty}{\left(1+\dfrac{\gamma-1}{2}M_\infty{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}{A_2 \dfrac{M_2}{\left(1+\dfrac{\gamma-1}{2}M_2{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}} = \frac{\dfrac{M_\infty}{\left(1+\dfrac{\gamma-1}{2}M_\infty{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}{\dfrac{M_2}{\left(1+\dfrac{\gamma-1}{2}M_2{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}} \cdot \frac{A_\infty}{A_2} \rightarrow Assume\ A_\infty = A_1$$

$$\frac{\dfrac{3}{\left(\left(1+0.2\cdot 3^2\right)0.8333\right)^3}}{\dfrac{0.3919}{\left(\left(1+0.2\cdot 0.3918^2\right)0.8333\right)^3}} 2 = 0.763661$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (7)



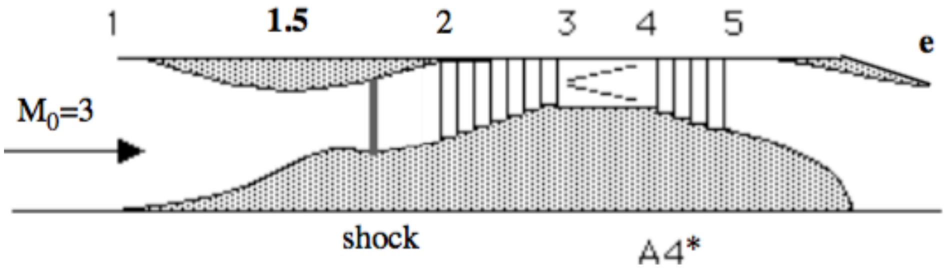
- Calculate the Stagnation Pressure Ratio Across Engine

$$\frac{P_{0_{exit}}}{P_{0_\infty}} = \pi_d \cdot \pi_c \cdot \pi_t = (0.764)\cdot(15.52)\cdot(0.1984) = 2.32$$

- Calculate Static Pressure Ratio Across Engine

$$\frac{p_{exit}}{p_\infty} = \frac{P_{0_{exit}} \Big/ \left(1+\frac{\gamma-1}{2}M_{exit}^2\right)^{\frac{\gamma}{(\gamma-1)}}}{P_{0_\infty} \Big/ \left(1+\frac{\gamma-1}{2}M_\infty^2\right)^{\frac{\gamma}{(\gamma-1)}}} = 2.34\left(\frac{1+\frac{1.4-1}{2}3^2}{1+\frac{1.4-1}{2}1^2}\right)^{\frac{1.4}{(1.4-1)}} = 45.4082$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (8)



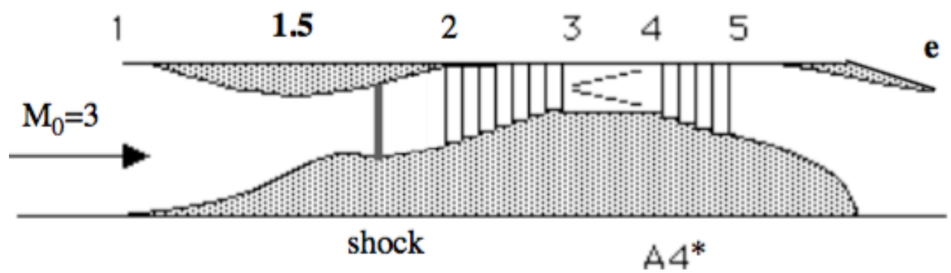
- Calculate the Stagnation Temperature Ratio Across Engine

$$\frac{T_{0_{exit}}}{T_{0_\infty}} = \frac{\tau_\lambda}{\tau_r} \cdot \tau_t = \frac{8.996}{2.8} \cdot 0.62996 = 2.024$$

- Calculate Static Temperature Ratio Across Engine

$$\frac{T_{exit}}{T_\infty} = \frac{T_{0_{exit}} \Big/ \left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)}{T_{0_\infty} \Big/ \left(1 + \frac{\gamma - 1}{2} M_\infty^{\ 2}\right)} = 2.024 \left(\frac{1 + \frac{1.4 - 1}{2} 3^2}{1 + \frac{1.4 - 1}{2} 1^2} \right) = 4.72267$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (9)



- The velocity ratio across the engine is

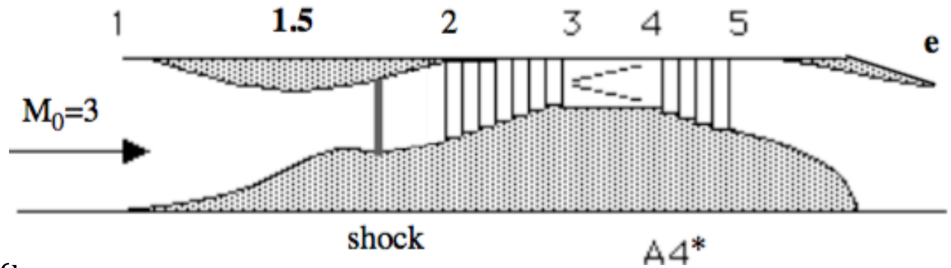
$$\frac{V_{exit}}{V_\infty} = \frac{M_{exit}}{M_\infty}\sqrt{\frac{T_{exit}}{T_\infty}} = \frac{1}{3}\sqrt{4.723} = 0.724$$

- And Normalized Thrust

$$\frac{F_{thrust}}{p_\infty \cdot A_\infty} = \gamma \cdot M_\infty^2 \cdot \left(\frac{V_{exit}}{V_\infty} - 1\right) + \frac{A_{exit}}{A_\infty} \cdot \left(\frac{p_{exit}}{p_\infty} - 1\right) =$$

$$1.4 \cdot 3^2\,(0.724 - 1)\; + 0.143\,(45.41 - 1) \qquad = 2.87303$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (10)



- Summary of results

Pressure Ratios:

$$\frac{P_{0_2}}{P_{0_\infty}} = \pi_d = 0.76; \qquad \frac{P_{0_3}}{P_{0_\infty}} = \pi_d \cdot \pi_c = 11.8$$

$$\frac{P_{0_4}}{P_{0_\infty}} = \pi_d \cdot \pi_c \cdot \pi_c = 11.8; \qquad \frac{P_{0_5}}{P_{0_\infty}} = \pi_d \cdot \pi_c \cdot \pi_c \cdot \pi_t = 2.34$$

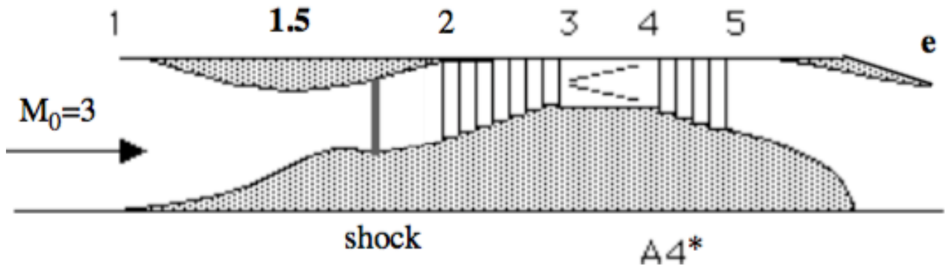
Temperature Ratios:

$$\frac{T_{0_2}}{T_{0_\infty}} = \tau_d = 1; \qquad \frac{T_{0_3}}{T_{0_\infty}} = \tau_d \cdot \tau_c = 2.19$$

$$\frac{T_{0_4}}{T_{0_\infty}} = \tau_d \cdot \tau_c \cdot \tau_c = 3.21; \qquad \frac{T_{0_5}}{T_{0_\infty}} = \tau_d \cdot \tau_c \cdot \tau_c \cdot \tau_t = 2.02$$

Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (11)

- Also



$$\frac{P_{0_{exit}}}{p_\infty} = \frac{P_{0_5}}{P_{0_\infty}} \cdot \frac{P_{0_\infty}}{p_\infty} = \frac{P_{0_5}}{P_{0_\infty}} \cdot \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right) =$$

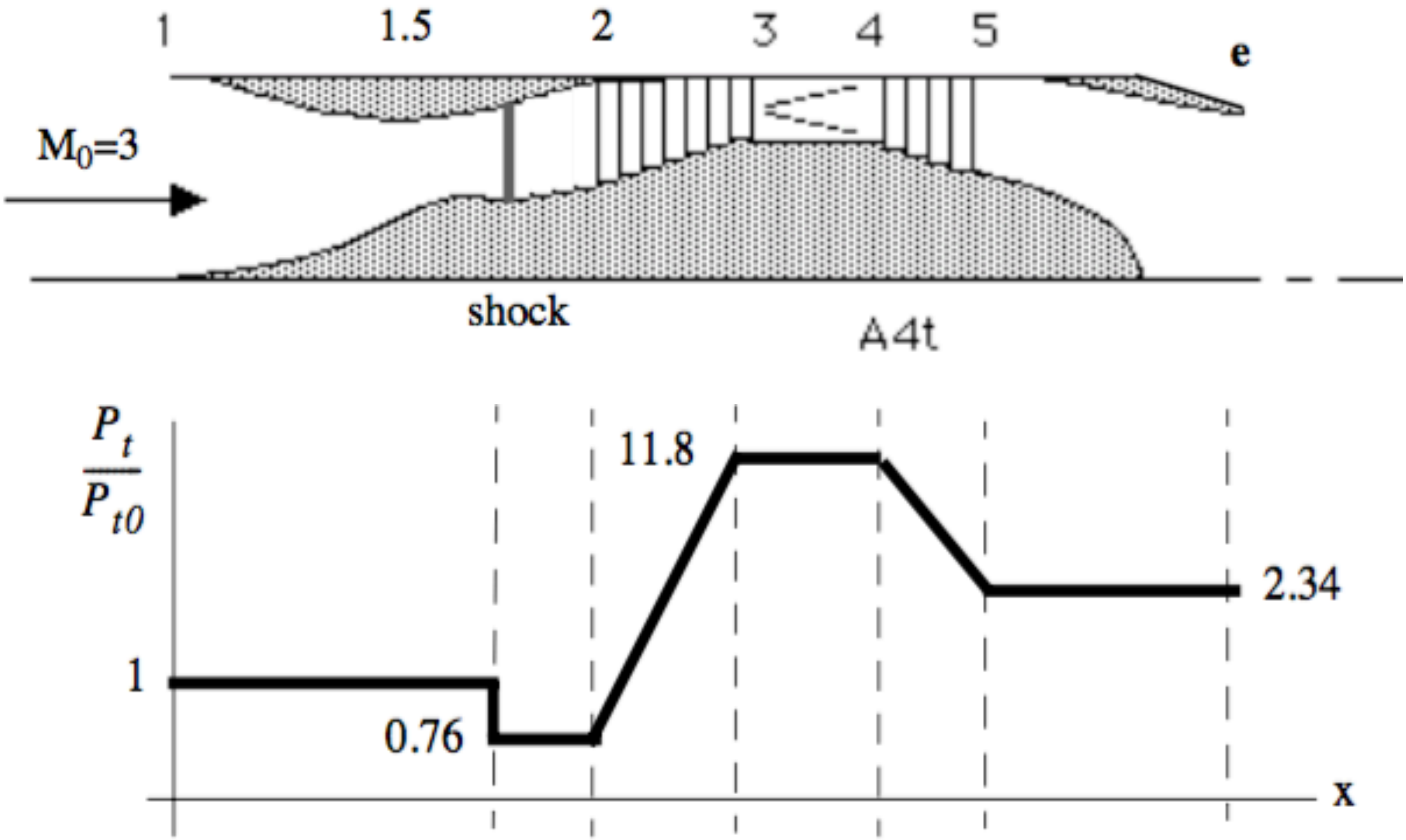
$$2.34\left(1 + \frac{1.4-1}{2}3^2\right)^{\frac{1.4}{(1.4-1)}} = 85.9546$$

$$\frac{T_{0_{exit}}}{T_\infty} = \frac{T_{0_5}}{T_{0_\infty}} \cdot \frac{T_{0_\infty}}{T_\infty} = \frac{T_{0_5}}{T_{0_\infty}} \cdot \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right) =$$

$$2.02\left(1 + \frac{1.4-1}{2}3^2\right) = 5.656$$

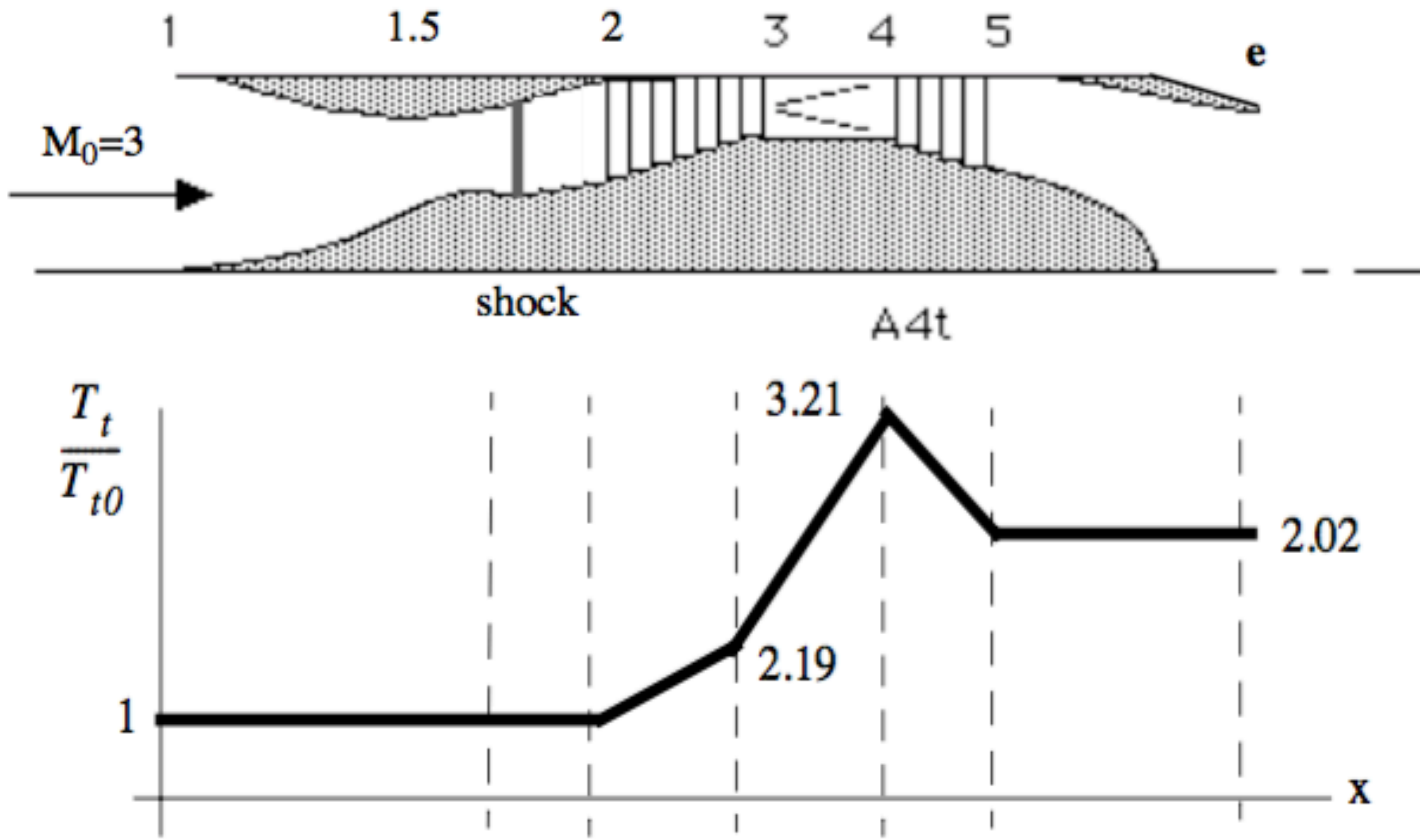
Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (11)

Pressure Ratio Profile:

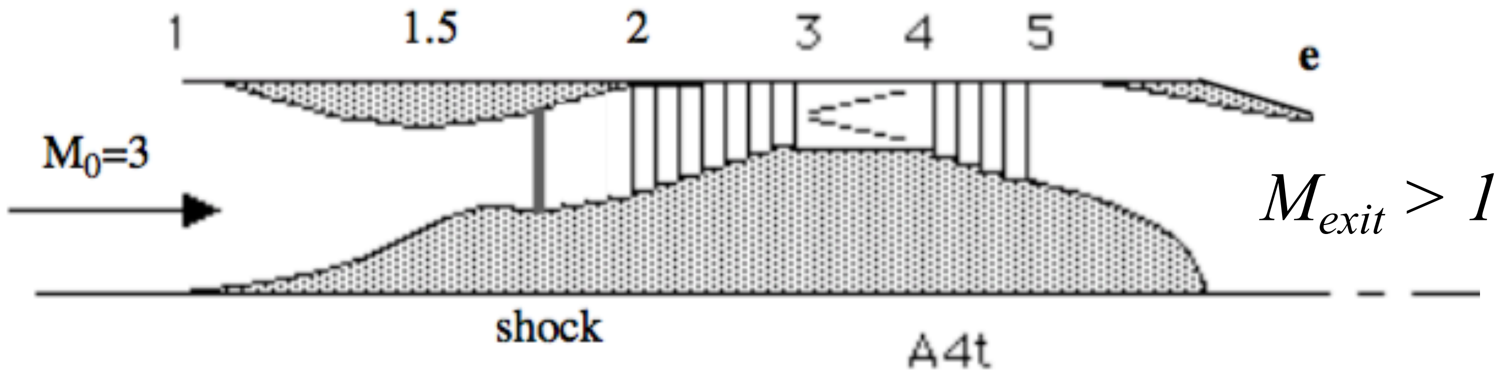


Example Calculation, Turbojet in Supersonic Flow with Inlet Shock Wave (12)

Temperature Ratio Profile:



Homework 5.2



$M_{exit} > 1$

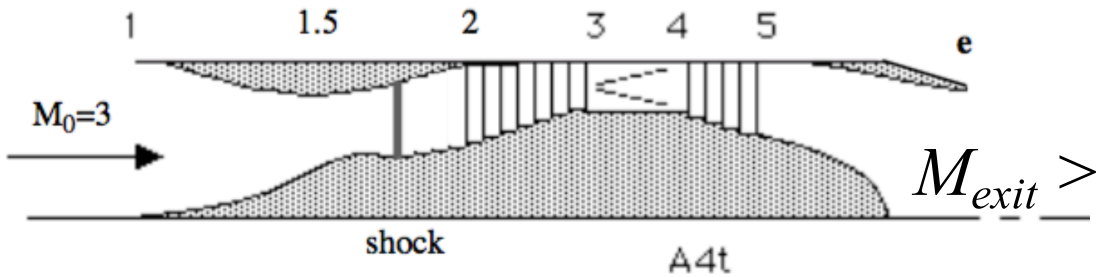
- Recall that this analysis assumes a sonic nozzle

- How would an Expanded (Supersonic) Nozzle Buy in Terms of Performance

- Find the Optimal Expansion Ratio and Exit Mach Number

- By What ratio does this Optimal expansion ratio Increase the thrust and specific Impulse of the Engine

Homework 5.2 (2)



$M_{exit} > 1$

- Hints:

$$\frac{F_{thrust}}{p_\infty \cdot A_\infty} = \frac{F_{thrust}}{p_\infty \cdot A_\infty} = \gamma \cdot M_\infty^2 \cdot \left(\frac{V_{exit}}{V_\infty} - 1\right) + \frac{A_{exit}}{A_\infty} \cdot \left(\frac{p_{exit}}{p_\infty} - 1\right) = \gamma \cdot M_\infty^2 \cdot \left(\frac{M_{exit}}{M_\infty}\sqrt{\frac{T_{exit}}{T_\infty}} - 1\right) + \frac{A_{exit}}{A_\infty} \cdot \left(\frac{p_{exit}}{p_\infty} - 1\right)$$

$$\frac{T_{exit}}{T_\infty} = \frac{T_{0_{exit}}}{T_\infty}\frac{T_{exit}}{T_{0_{exit}}} \qquad \frac{p_{exit}}{p_\infty} = \frac{P_{0_{exit}}}{p_\infty}\frac{p_{exit}}{P_{0_{exit}}}$$

$$\frac{T_{exit}}{T_{0_{exit}}} = \frac{1}{\left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)}$$

$$\frac{p_{exit}}{P_{0_{exit}}} = \left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{A_{exit}}{A_{throat}^*} = \frac{1}{M_{exit}} \cdot \left[\left(\frac{2}{\gamma - 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)\right]^{\frac{\gamma+1}{2\cdot(\gamma-1)}}$$

- Making these substitutions the normalized thrust can be written in terms of exit Mach number
- Graph Normalized Thrust and Exit expansion ratio as a function of exit Mach Number
- Verify that $p_{exit}/p_\infty = 1$ at the optimal performance condition?


Questions??