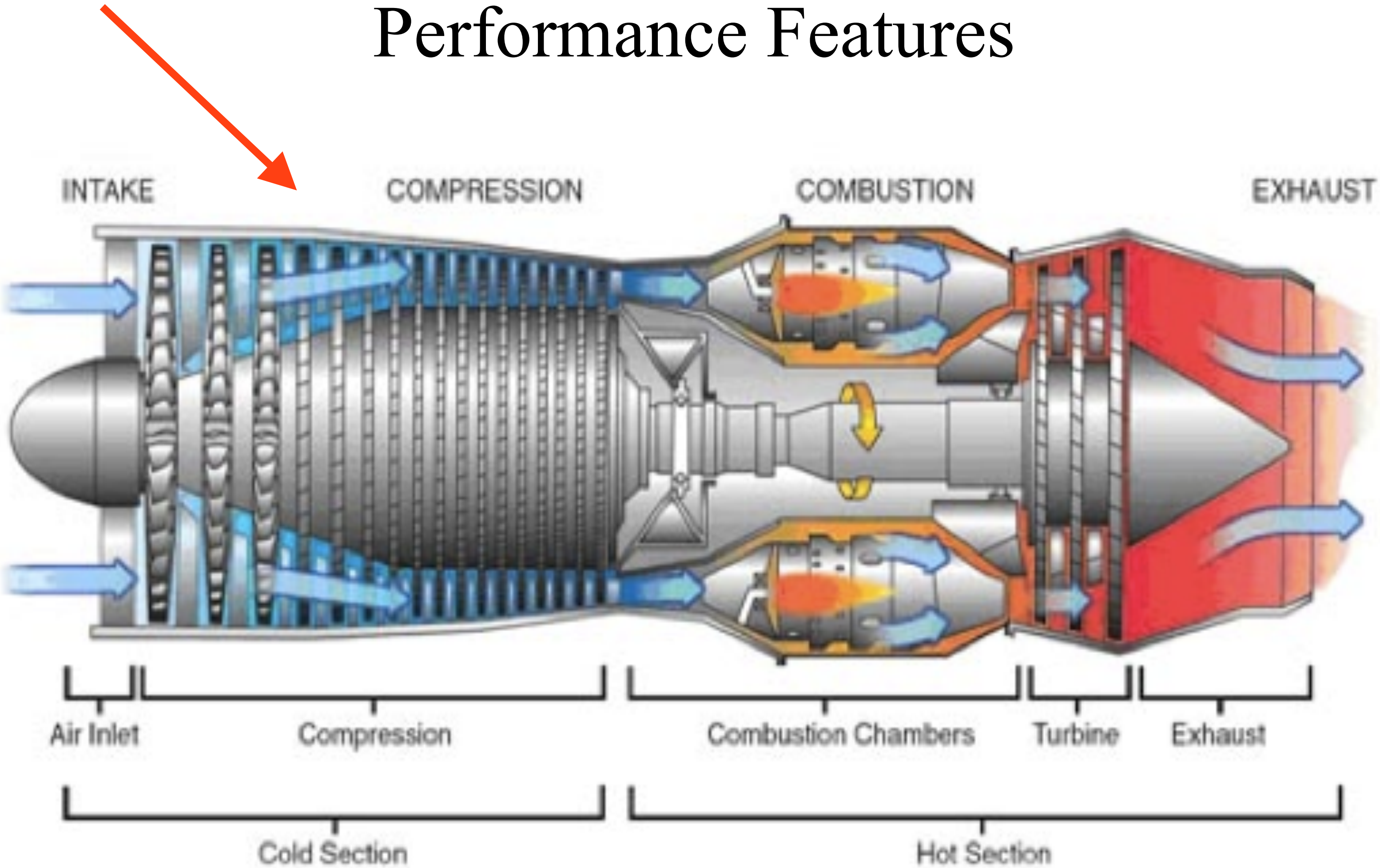
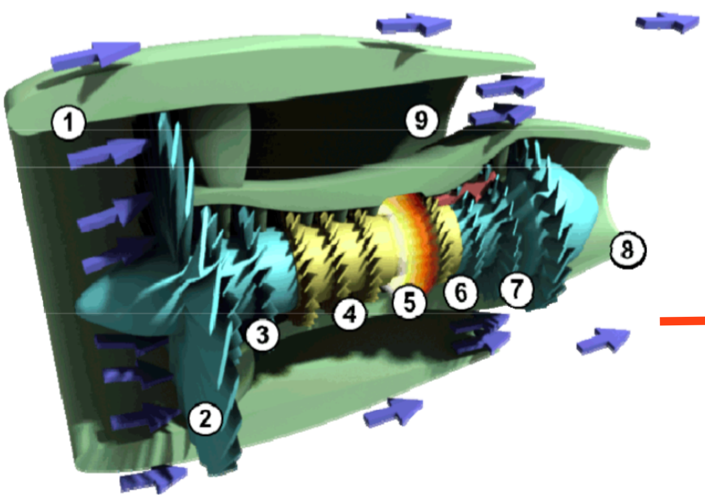
Section 5.3: TurboJet Compressor Design and Performance Features

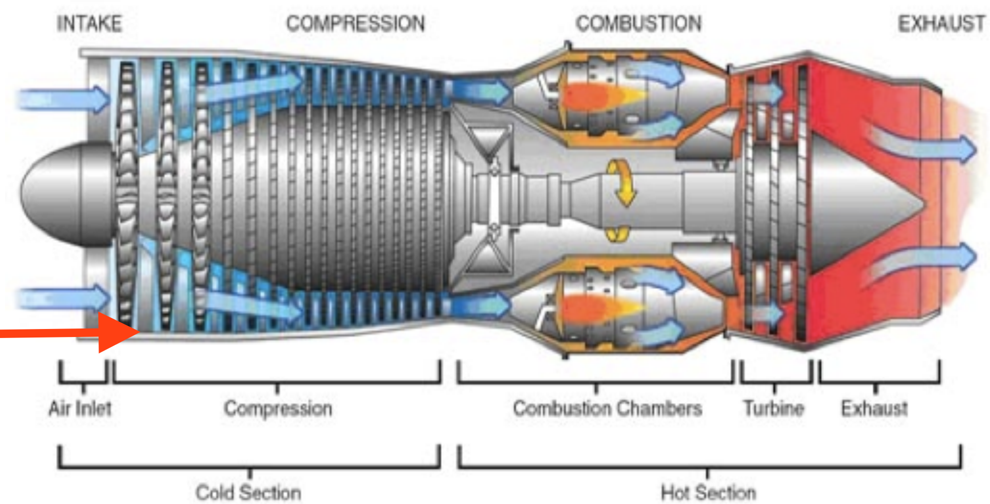


Component Matching Conditions

• The main components of a gas turbine engine are: *inlet diffuser, compressor, combustion chamber, turbine, and exhaust nozzle*.

• The individual components are designed based on established procedures and their performances are obtained from actual tests.





- When these components are integrated in an engine, the range of possible operating conditions is considerably reduced.
- There is considerable interdependency between components .. Referred to as "flow matching"

Collected Engine-Matching Conditions

- Component matching conditions needed to understand the operation of the turbojet in order from the nozzle to the inlet are as follows.

Nozzle:

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}} \qquad \pi_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2\cdot\gamma}{\gamma+1}}$$

Turbine

$$\tau_t = 1 - \left(\frac{f}{f+1}\right)\frac{\tau_r\cdot(\tau_c - 1)}{\tau_\lambda} \rightarrow (\tau_c - 1) = \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot(1-\tau_t)$$

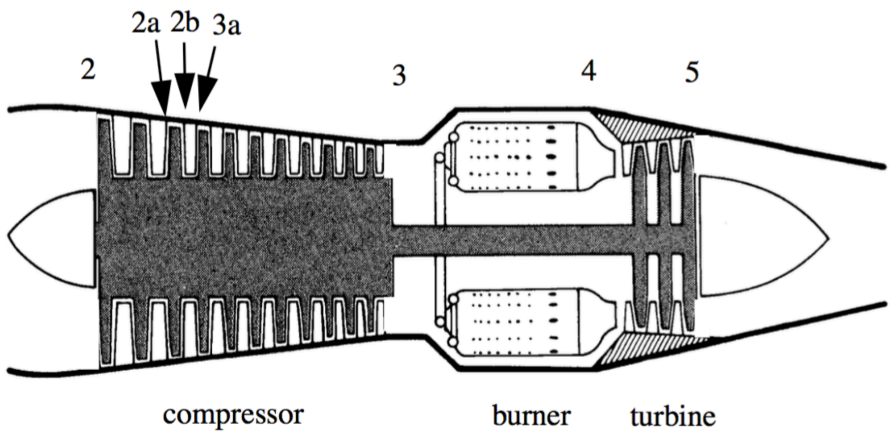
Compressor / Turbine Matching:

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^*}{A_2}$$

Inlet:

$$\frac{1}{\pi_d}\frac{A_\infty}{A_2}\frac{M_\infty}{\left(1+\frac{\gamma-1}{2}M_\infty^{\,2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{M_2}{\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Gas Generator Compressor Description



- Combination of compressor, burner and turbine shown below is called the *gas generator*.

- Compressor is the first and most-important component of the gas generator.

- Compressor provides combustor with the required massflow at high static pressures.

- Compression of incoming volume of air accomplished by a rotating disk containing many airfoils, set at an angle of attack to compressor face.

- As disk with blades is forced to rotate by the turbine, each blade axially and circumferentially accelerates the air, pumping it downstream.

- Circumferential acceleration does not contribute to thrust, is partially compensated for corrected by putting another row of airfoils behind rotating disk section.

- This second row is stationary and its airfoils are at opposing angle to the rotating airfoils.

Gas Generator Compressor Description (2)

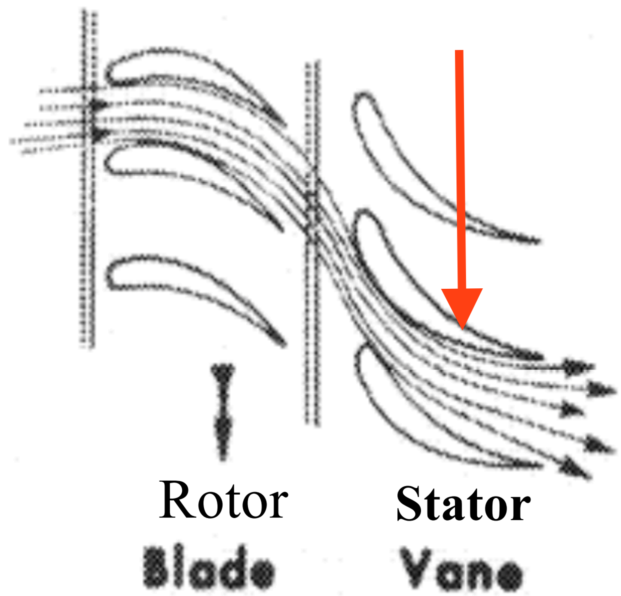
- Compression is typically achieved by multiple stages, with each stage consisting of a rotating disk, followed by another row of non-rotating airfoils.

- The for each compression stage the rotating disk section is referred to as the compressor stage *rotor*, the fixed disk station is the compressor stage *stator*.

- A single stage of compression consists of a set of *rotor blades* attached to a rotating disk, followed by *stator vanes* attached to a stationary ring.

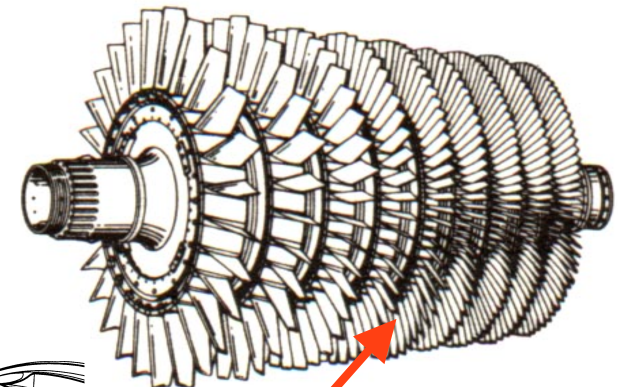
- Compressor *rotor blades* convert mechanical energy into gaseous energy. This energy conversion greatly increases total pressure P_0 by increasing the axial velocity.

- Stator *vanes* slow the air by means of their divergent duct shape, converting dynamic pressure to higher static pressure.

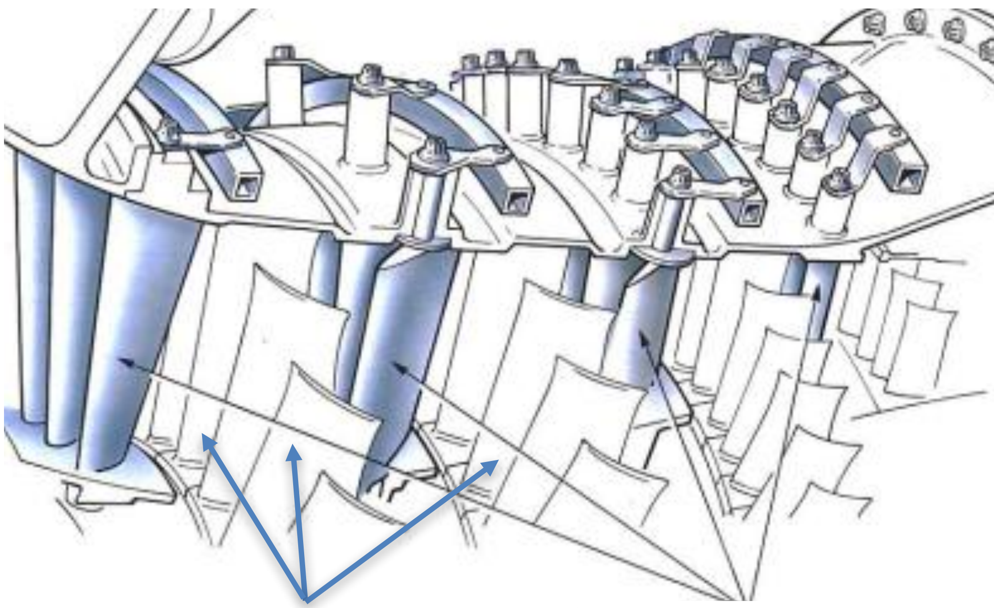


Gas Generator Compressor Description (3)

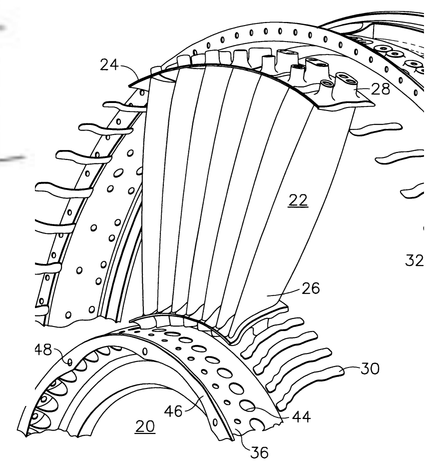
- In addition to multiple stages of blades and vanes, compressor also incorporates inlet / outlet guide vanes located at compressor inlet and outlet.
- Guide vanes are neither divergent nor convergent, but direct air to first stage compressor blades at the "best" angle.
- The outlet guide vanes "straighten" the air to provide the combustor with the proper airflow direction.



9-stage compressor rotor assembly.



Rotor Blades **Stator Vanes**



Inlet Guide Vanes

Compressor Operating Line

- Recall that across the compressor …

$$\tau_c = 1 + \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot\left(1-\tau_t\right)$$

and

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{\left(\pi_c\right)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^{\,*}}{A_2}$$

- Solve for τ_λ/τ_r in Both eqs. →

$$\tau_c = 1 + \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot\left(1-\tau_t\right) \rightarrow \boxed{\tau_\lambda/\tau_r = \left(\frac{f}{f+1}\right)\left(\frac{\tau_c-1}{1-\tau_t}\right)}$$

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{\left(\pi_c\right)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^{\,*}}{A_2} \rightarrow \boxed{\tau_\lambda/\tau_r = \left(\left(\frac{f}{f+1}\right)\cdot\pi_c\cdot\frac{A_4^{\,*}}{A_2}\cdot\frac{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_2}\right)^2}$$

Compressor Operating Line (2)

- Eliminate τ_λ/τ_r. →

$$\rightarrow\left(\frac{\tau_c - 1}{1-\tau_t}\right) = \pi_c \cdot \frac{A_4^*}{A_2} \cdot \frac{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_2}$$

- Assume quasi-isentropic comprssor ... Let → $\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}$ and substitute

$$\frac{\pi_c^{\,2}}{\pi_c^{\frac{\gamma-1}{\gamma}} - 1} = \frac{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\tau_t}\right)}{\left(\frac{A_4^*}{A_2} \cdot \frac{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_2}\right)^2}$$

Compressor Operating Line (2)

- Since, Choking at the turbine inlet and nozzle throat determine the turbine temperature and pressure ratio …. As derived previously →

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}$$

- Substituting ... And taking square root of each side

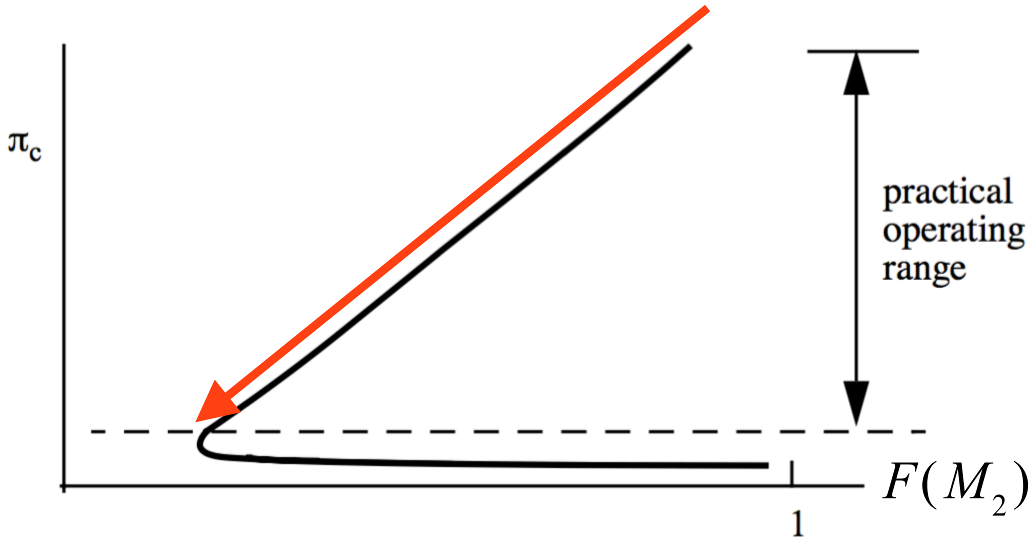
$$\frac{\pi_c}{\sqrt{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}} = \sqrt{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right)} \cdot \left(\frac{A_2}{A_4^*} \cdot \frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^2\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\right)$$

- Ths equation defines the operating line of the compressor.

  - Letting $F(M_2) = \frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^2\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$ and plotting

Compressor Operating Line (3)

• The qualitative representation of the operating line is a Monotonic, ”straight-line,” relationship except for values of the compressor ratio <1, which is not a practical operating condition.



• Compressor Mach number is not truly an independent variable, as its value is partially dependent on the rotational velocity N of the compressor, and the isentropic efficiency of the compressor η.

• Currently missing is the relationship between τ_c, π_c and the actual compressor rotational velocity (N) and massflow, $\dot{m}_{air}$

Corrected Massflow

- From Previous page

$$\frac{\pi_c}{\sqrt{\pi_c^{\frac{\gamma-1}{\gamma}}-1}}=\sqrt{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right)}\cdot\left(\frac{A_2}{A_4^*}\cdot\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\right)$$

- Recall that for quasi-1D flow

$$\dot{m}=A\cdot\frac{P_0}{\sqrt{T_0}}\cdot\sqrt{\frac{\gamma}{R_g}}\cdot\frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

$$\text{multiply by}\,\frac{P_0}{\sqrt{T_0}}\rightarrow\dot{m}\cdot\frac{\sqrt{T_0}}{P_0}=A\cdot\sqrt{\frac{\gamma}{R_g}}\cdot\frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

$$\text{again multiply by}\,\frac{p_{sl}}{\sqrt{T_{sl}}}\rightarrow\rightarrow\dot{m}\cdot\frac{\sqrt{\frac{T_0}{T_{sl}}}}{\frac{P_0}{p_{sl}}}=A\cdot p_{sl}\sqrt{\frac{\gamma}{R_g\cdot T_{sl}}}\cdot\frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

Corrected Massflow (2)

Define $\Theta \equiv \frac{T_{0_\infty}}{T_{S.L.}}$

$\Delta \equiv \frac{P_{0_\infty}}{p_{S.L.}}$

$$\dot{m} \cdot \frac{\sqrt{\frac{T_0}{T_{sl}}}}{\frac{P_0}{p_{sl}}} = A \cdot p_{sl} \sqrt{\frac{\gamma}{R_g \cdot T_{sl}}} \cdot \frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}} \rightarrow \boxed{\dot{m}_c = \dot{m} \cdot \left(\sqrt{\frac{T_0}{T_{sl}}} \Big/ \frac{P_0}{p_{sl}}\right) \rightarrow \text{"corrected massflow"}}$$

$$Substitute \rightarrow \dot{m}_c = A \cdot p_{sl} \sqrt{\frac{\gamma}{R_g \cdot T_{sl}}} \cdot \frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}} \rightarrow$$

$T_{S.L.} = 288.15 \ _K$

$p_{S.L.} = 101{,}325 \ _{pa}$

$\gamma = 1.4$

$R_g = 287.056 \ _{\text{j/kg-K}}$

$$Evaluate\ constant \rightarrow \dot{m}_c = A \cdot p_{sl} \sqrt{\frac{\gamma}{R_g \cdot T_{sl}}} \cdot \frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}} =$$

$$416.858_{kg/m^2-\sec} \cdot A \cdot \frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

Corrected Massflow (3)

- From Previous Page $$\frac{\dot{m}_c}{A\cdot p_{sl}\sqrt{\dfrac{\gamma}{R_g\cdot T_{sl}}}}=\frac{M}{\left(1+\dfrac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

- Thus for an isentropic inlet/diffuser

$$F(M_2)=\frac{M_2}{\left(\left(1+\dfrac{\gamma-1}{2}M_2{}^2\right)\left(\dfrac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}=\frac{M}{\left(1+\dfrac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}\cdot\frac{1}{\left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}=\frac{\dot{m}_c}{A_2\cdot p_{sl}\sqrt{\dfrac{\gamma}{R_g\cdot T_{sl}}\left(\dfrac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}}$$

- And compressor pressure ratio operating line is written as a function of corrected massflow

$$\frac{\pi_c}{\sqrt{\pi_c{}^{\frac{\gamma-1}{\gamma}}-1}}=\sqrt{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\left(\dfrac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right)}\cdot\left(\frac{A_2}{A_4^*}\frac{M_2}{\left(\left(1+\dfrac{\gamma-1}{2}M_2{}^2\right)\left(\dfrac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\right)=$$

$$\sqrt{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\left(\dfrac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right)}\cdot\left(\frac{A_2}{A_4^*}\frac{\dot{m}_c}{A_2\cdot p_{sl}\sqrt{\dfrac{\gamma}{R_g\cdot T_{sl}}\left(\dfrac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}}\right)=\frac{1}{A_4^*}\sqrt{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\left(\dfrac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right)}\cdot\left(\frac{\dot{m}_c}{p_{sl}\sqrt{\dfrac{\gamma}{R_g\cdot T_{sl}}\left(\dfrac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}}\right)$$

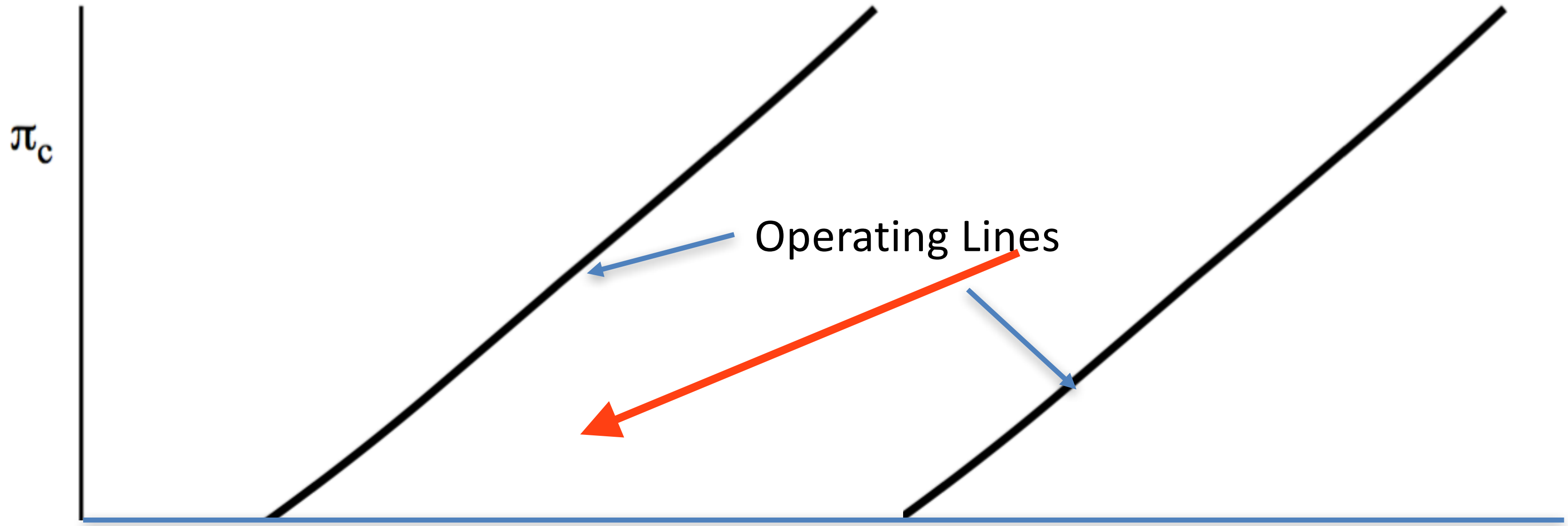
Corrected Massflow (4)

- Corrected Massflow

$$\boxed{\dot{m}_c = \dot{m} \cdot \left(\sqrt{\frac{T_0}{T_{sl}}} \Big/ \frac{P_0}{p_{sl}} \right) \rightarrow \text{"}\textit{corrected massflow}\text{"}}$$

Corrected Mass Flow is the **mass flow** that would pass through a device (e.g. compressor, bypass duct, etc.) if the inlet pressure and temperature corresponded to ambient conditions at Sea Level, on a Standard Day, *e.g. 101,325 pa, 288.15 K (14.696 lbf/in², 518.7 R).*

Compressor Operating Line (4)



$$\dot{m}_c = \dot{m}_{air} \cdot \left(\sqrt{\Theta}/\Delta\right)$$

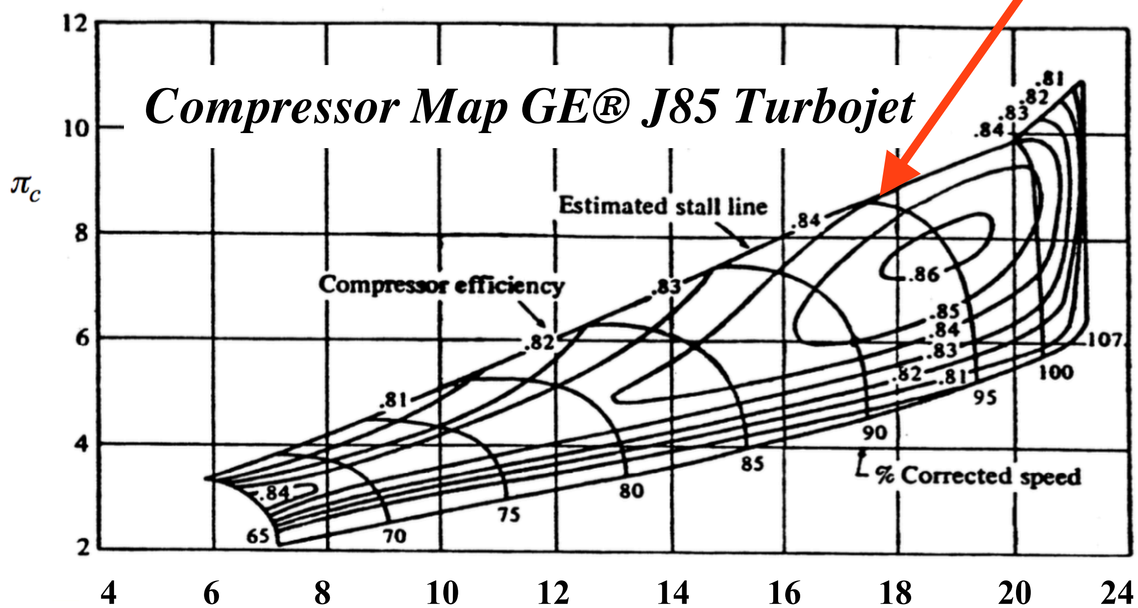
$$\frac{\pi_c}{\sqrt{\pi_c^{\frac{\gamma-1}{\gamma}}-1}} = \frac{1}{A_4^*}\sqrt{\left(\frac{f+1}{f}\right)\left[\frac{1}{1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right]} \cdot \left(\frac{\dot{m}_c}{p_{sl}\sqrt{\frac{\gamma}{R_g \cdot T_{sl}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}}\right) = \frac{1}{A_4^*}\sqrt{\left(\frac{f+1}{f}\right)\left[\frac{1}{1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right]} \cdot \left(\frac{\dot{m}_c}{241.237_{kg/m^2-\sec}}\right)$$

Compressor Performance Maps

- The corrected massflow is not truly an independent variable, as its value is partially dependent on the rotational velocity *N* of the compressor, and the isentropic efficiency of the compressor η … that is the relationship between the turbine pressure and temperature.

- This relationship depends on the internal workings of the compressor blades and is extremely complex, and is typically characterized by a compressor "performance map."

- *The compressor map* which describes the functional relationship between compressor pressure ratio, mass flow, efficiency, and compressor rotor speed.



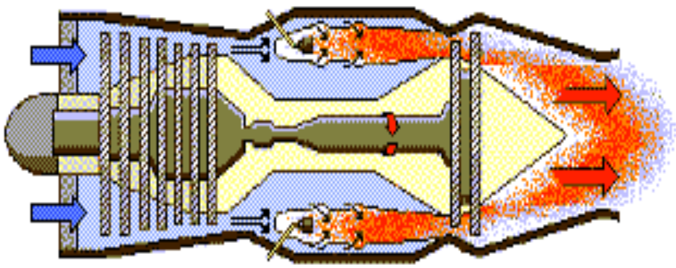
$$\dot{m}_c = \dot{m}_{air} \cdot \left(\sqrt{\Theta}/\Delta\right), kg/\sec$$

- Like massflow, the effect of Rotor speed N on the flow rate is also a function of the Freestream conditions

- *N* also is typically corrected to Sea Level Conditions for Compressor map

Corrected Engine RPM

- Look at turbine power output (or work performed by compressor) per unit mass



$$\left(\frac{\dot{W}}{\dot{m}}\right)_{turbine} = \left(\frac{\delta W}{\delta m}\right)_{turbine} = \frac{\mathfrak{T}_{turbine}\cdot\omega_{turbine}}{\dot{m}_e} \rightarrow \mathfrak{T}_{turbine}\ldots\text{"Torque"}$$

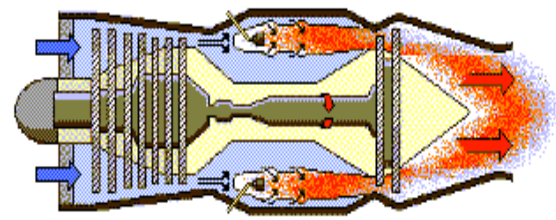
k is an arbitrary constant

$$\mathfrak{T}_{turbine}\ \textit{proportional to fluid density} \rightarrow \left(\frac{\delta W}{\delta m}\right)_{turbine} = \frac{k\cdot\rho_e\cdot\omega_{turbine}}{\dot{m}_e}$$

$$\left(\frac{\delta W}{\delta m}\right)_{turbine} = \frac{k\cdot\rho_e\cdot\omega_{turbine}}{\dot{m}_e} = \frac{k\cdot\dfrac{p_e}{R_g T_e}\cdot\omega_{turbine}}{A_e\cdot\sqrt{\dfrac{\gamma}{R_g}}\cdot\dfrac{P_0}{\sqrt{T_0}}\cdot\dfrac{M_e}{\left(1+\dfrac{\gamma-1}{2}M_e^{\,2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}} = \frac{\dfrac{k}{R_g}\cdot\dfrac{P_0}{\left(1+\dfrac{\gamma-1}{2}M_e^{\,2}\right)^{\frac{\gamma}{(\gamma-1)}}}\dfrac{\left(1+\dfrac{\gamma-1}{2}M_e^{\,2}\right)}{T_0}\omega_{turbine}}{A_e\cdot\sqrt{\dfrac{\gamma}{R_g}}\cdot\dfrac{P_0}{\sqrt{T_0}}\cdot\dfrac{M_e}{\left(1+\dfrac{\gamma-1}{2}M_e^{\,2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}} =$$

$$\left(\frac{k}{A_e\sqrt{\gamma R_g T_0}}\cdot\right)\frac{\sqrt{1+\dfrac{\gamma-1}{2}M_e^{\,2}}}{M_e}\omega_{turbine}$$

Corrected Engine RPM (2)



- From Previous …

$$\left(\frac{\delta W}{\delta m}\right)_{turbine} = \left(\frac{k}{A_e\sqrt{\gamma R_g T_0}}\cdot\right)\frac{\sqrt{1+\frac{\gamma-1}{2}M_e^{\,2}}}{M_e}\omega_{turbine}$$

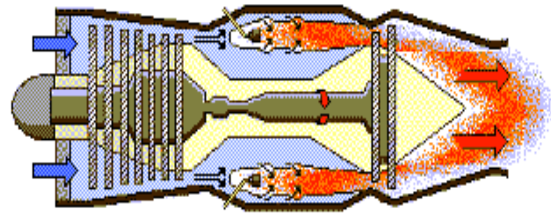
- Define "corrected" conditions as …being based on sea level conditions

$$\left(\left(\frac{\delta W}{\delta m}\right)_{turbine}\right)_c = \left(\frac{k}{A_e\sqrt{\gamma R_g T_{sl}}}\cdot\right)\frac{\sqrt{1+\frac{\gamma-1}{2}M_e^{\,2}}}{M_e}\left(\omega_{turbine}\right)_c$$

- Then, the ratio of corrected-to-actual specific work is

$$\frac{\left(\left(\frac{\delta W}{\delta m}\right)_{turbine}\right)_c}{\left(\frac{\delta W}{\delta m}\right)_{turbine}} = \frac{\left(\frac{k}{A_e\sqrt{\gamma R_g T_{sl}}}\cdot\right)\frac{\sqrt{1+\frac{\gamma-1}{2}M_e^{\,2}}}{M_e}\left(\omega_{turbine}\right)_c}{\left(\frac{k}{A_e\sqrt{\gamma R_g T_0}}\cdot\right)\frac{\sqrt{1+\frac{\gamma-1}{2}M_e^{\,2}}}{M_e}} = \frac{\left(\omega_{turbine}\right)_c}{\omega_{turbine}}\cdot\sqrt{\frac{T_0}{T_{sl}}}$$

Corrected Engine RPM (3)



Set … $\left(\left(\frac{\delta W}{\delta m}\right)_{turbine}\right)_c = \left(\frac{\delta W}{\delta m}\right)_{turbine}$

Then at flight condition the turbine rotating at speed $\left(\omega_{turbine}\right)_c$ produces the same work output as the turbine at sea level rotating at speed $\omega_{turbine}$

In Terms of RPM …

$$\left(\omega_{turbine}\right)_c = \frac{\omega_{turbine}}{\sqrt{\frac{T_0}{T_{sl}}}} \equiv \frac{\omega_{turbine}}{\sqrt{\theta_c}} \rightarrow \theta_c = \frac{T_0}{T_{sl}} \rightarrow N = \frac{\omega_{turbine}}{2\pi}\cdot 60_{\frac{sec}{min}} \Rightarrow \boxed{N_c = \frac{N}{\sqrt{\frac{T_0}{T_{sl}}}} \equiv \frac{N}{\sqrt{\theta}}}$$

- Compressor efficiency is primarily determined by smoothness of the airflow.
- Any deviation from maximum rated rotor speed changes compressor airflow characteristics
- Blades and vanes are no longer positioned at their optimum angles.
- Compressor efficiency is a function of ratio operating rotor speed to design speed

$$\%\ Corrected\ Speed = \frac{N}{\sqrt{\theta_c}} \Big/ \left.\frac{N}{\sqrt{\theta_c}}\right._{design}$$

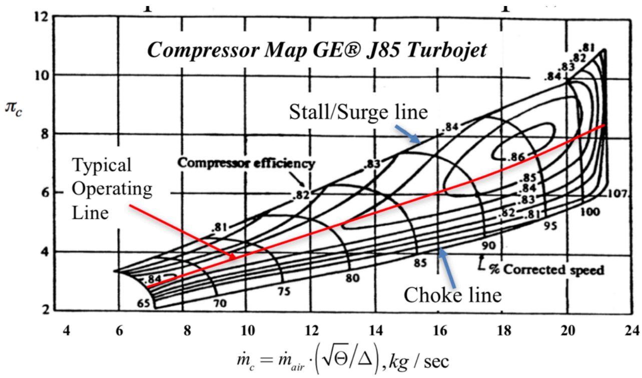
Corrected Massflow and Rotor Speed

$$\boxed{\dot{m}_c = \dot{m} \cdot \left(\sqrt{\frac{T_0}{T_{sl}}} \Big/ \frac{P_0}{p_{sl}} \right)}$$

- **Corrected Mass Flow** is the **mass flow** that would pass through a device (e.g. compressor, bypass duct, etc.) if the inlet pressure and temperature corresponded to ambient conditions at Sea Level, on a Standard Day, *e.g. 101,325 pa, 288.15 K (14.696 lbf/in², 518.7 R)*.
- **Corrected Rotor Speed**
- At flight condition the turbine rotating at speed $\left(\omega_{turbine}\right)_c$ produces the same work output as the turbine at sea level rotating at speed $\omega_{turbine}$

$$\boxed{N_c = \frac{N}{\sqrt{\frac{T_0}{T_{sl}}}} \equiv \frac{N}{\sqrt{\theta}}}$$

$$\boxed{\%\ Corrected\ Speed = \frac{N}{\sqrt{\theta_c}} \Big/ \frac{N}{\sqrt{\theta_c}}_{design}}$$



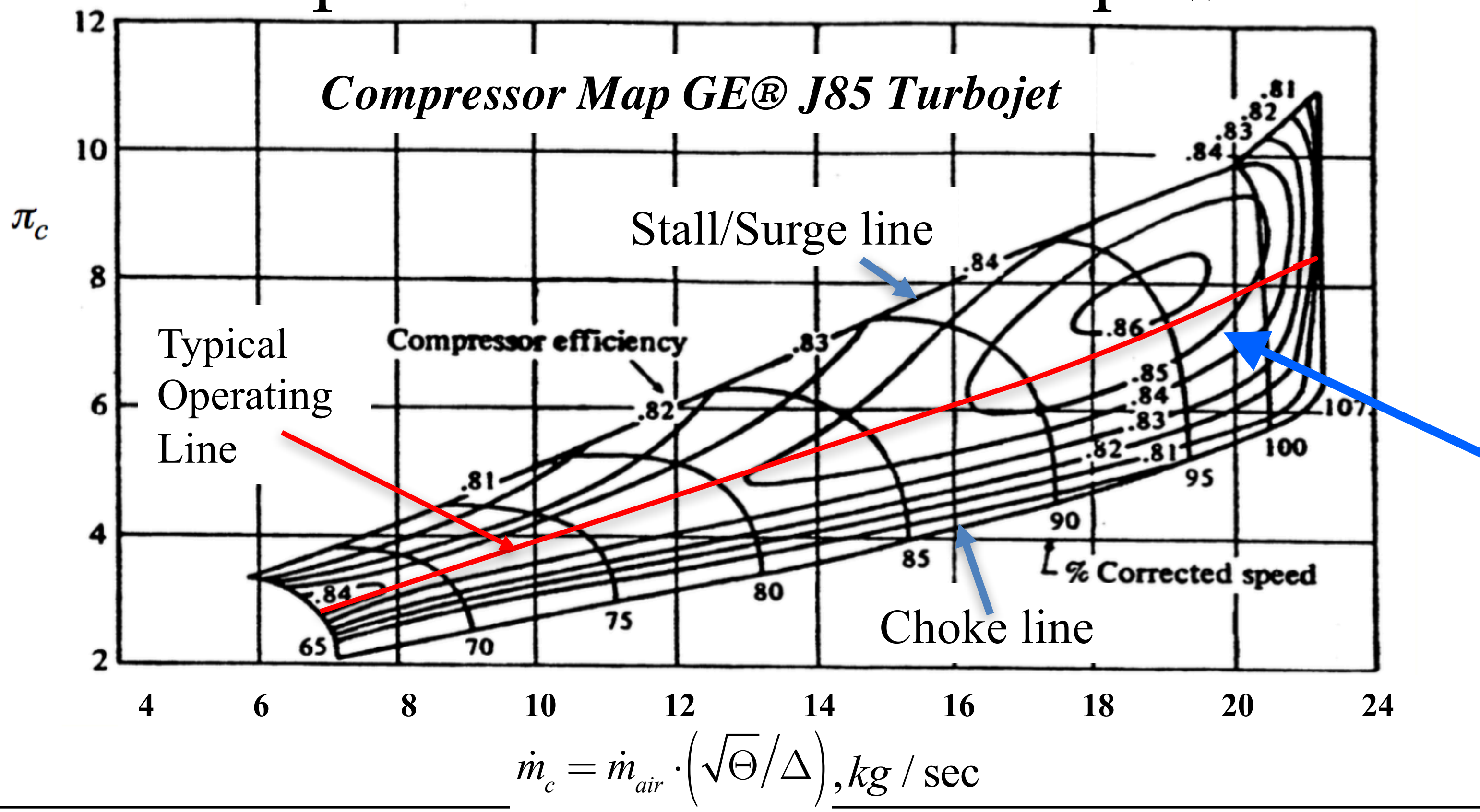
• Compressor efficiency is a function of ratio operating rotor speed to design speed

- Compressor efficiency is primarily determined by smoothness of the airflow.

- Any deviation from maximum rated rotor speed changes compressor airflow characteristics

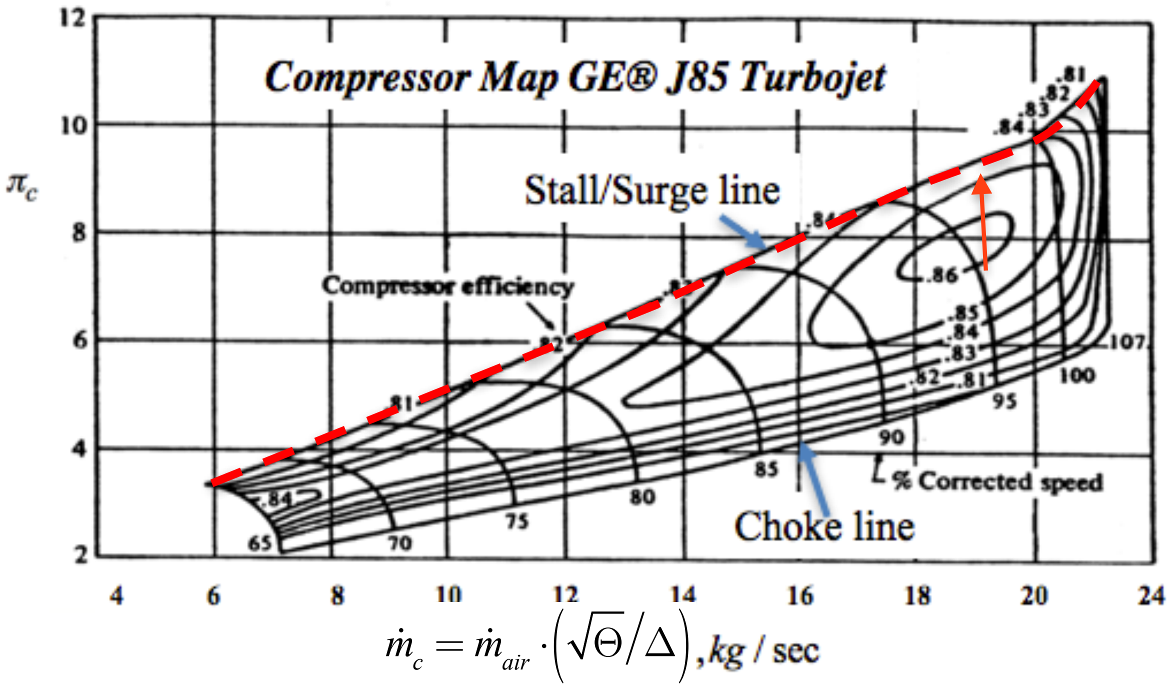
- Blades and vanes are no longer positioned at their optimum angles.

Compressor Performance Maps (2)



- Compressor map can be regarded as a cross plot of compressor pressure ratio with respect to three independent variables 1) corrected massflow, 2) compressor efficiency, and 3) rotational speed of the compressor.
- As pressure ratio across the compressor increases, then a higher rotor speed is required in order to maintain the same corrected massflow
- Two critical regions for every compressor map, 1) stall/surge line and 2) choking line.
- Area enclosed by surge line and choke lines is normal operating range for compressor.
- Best efficiency is centered between the compressor stall and choke lines

Compressor Stall/Surge Line



• Above the stall/surge line is a region of unstable flow where speed contours drop with respect to pressure ratio.

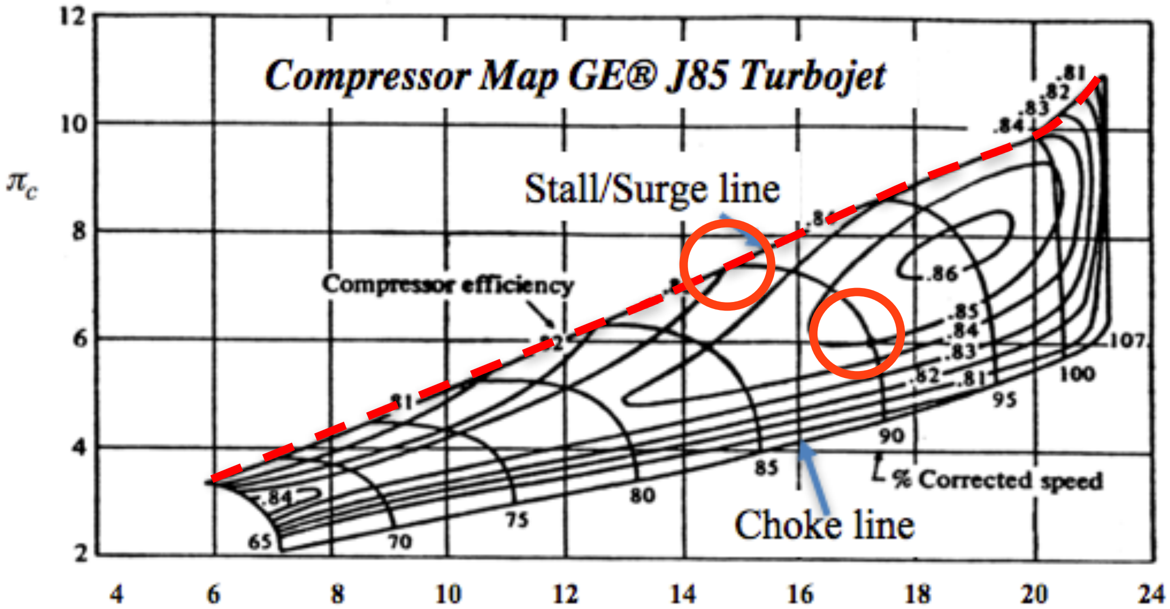
• Surge is associated with a drop in the pressure ratio, and which can lead to pulsations in the mass flow and even reverse it.

• A ***compressor surge*** typically causes an abrupt reversal of massflow, as the pumping action of the airfoils stall (akin to an aircraft wing stalling).

• Compressor surge can cause considerable damage to the compressor, and must be avoided.

• ***Surge Margin*** -- distance of the operating point on compressor map from surge line.

Compressor Stall/Surge Line (2)



$$\dot{m}_c = \dot{m}_{air} \cdot \left(\sqrt{\Theta}/\Delta\right), kg/\sec$$

- ***Surge Margin*** -- distance of the operating point on compressor map from surge line.
- Measure of "safety" of the current operating condition

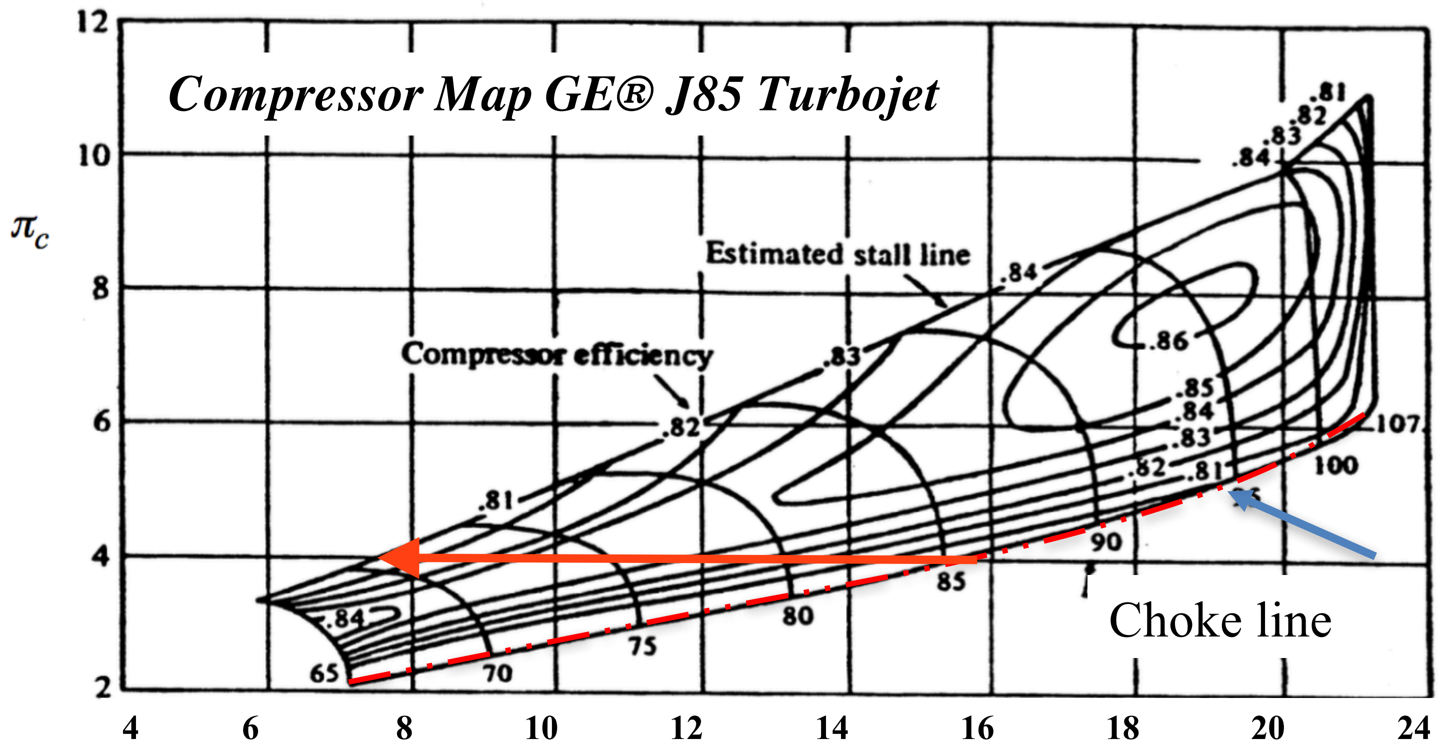
$$SM = 100\% \cdot \frac{\dot{m}_w - \dot{m}_s}{\dot{m}_w}$$

where:

$\dot{m}_w$ is the mass flow at the operating point, be it steady state or transient

$\dot{m}_s$ is the mass flow at surge, at same corrected speed as $\dot{m}_w$

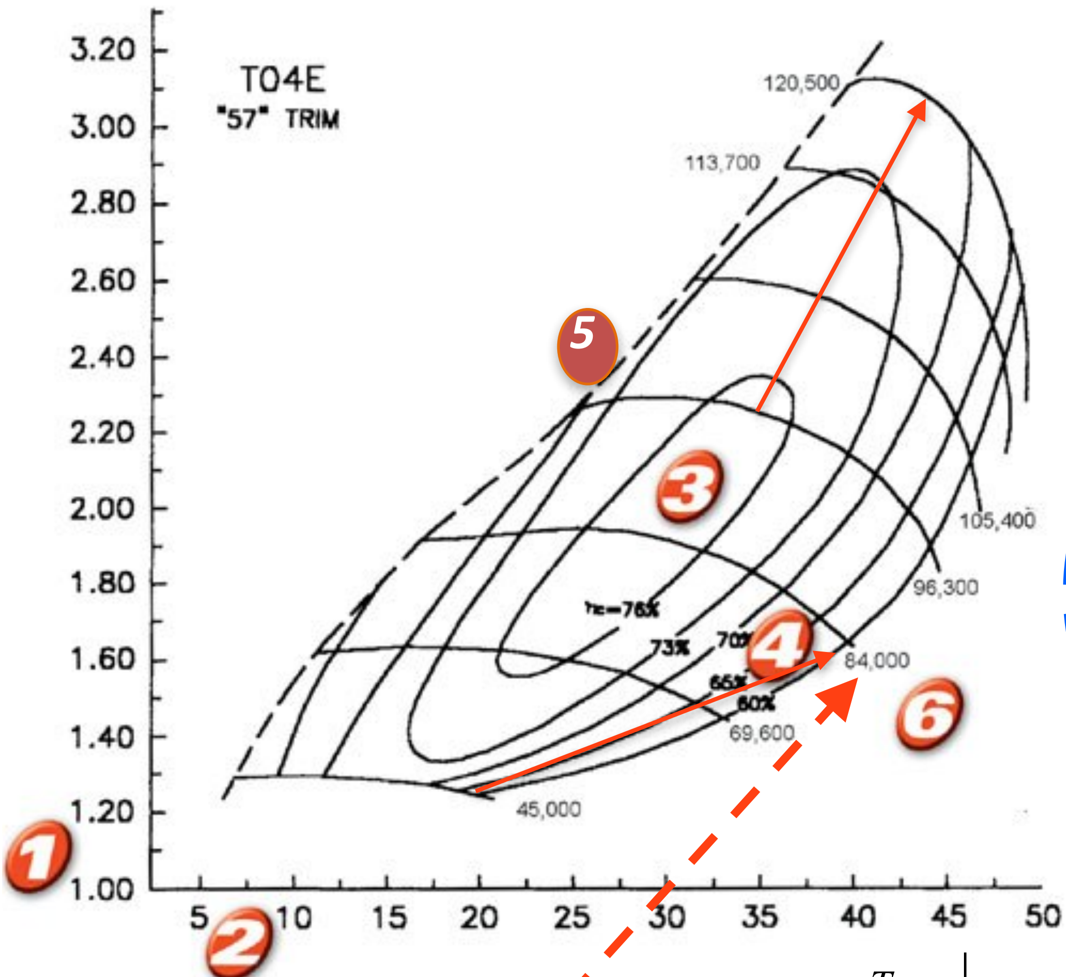
Compressor Choking Line



$$\dot{m}_c = \dot{m}_{air} \cdot \left(\sqrt{\Theta}/\Delta\right), kg/\sec$$

- The compressor choke point is when the flow in the compressor reaches Mach 1 at the blade throat, a point where no more flow can pass through the compressor.
- Compressor choking occurs when operating at low discharge pressure and high massflow rates.
- Any further decrease in outlet resistance will not lead to increase in compressor output.

Compressor Map Key



1) Compression (Pressure) Ratio
2) Corrected Massflow
3) Efficiency Islands
4) Corrected Rotor Speed
5) Surge/Stall Line
6) Choke Line

$$\frac{\pi_c}{\sqrt{\pi_c^{\frac{\gamma-1}{\gamma}}-1}} = \frac{1}{A_4^*}\sqrt{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right)}\cdot\left(\frac{\dot{m}_c}{p_{sl}\sqrt{\frac{\gamma}{R_g\cdot T_{sl}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}}\right)$$

Define

$$\Theta \equiv \frac{T_{0_\infty}}{T_{S.L.}}$$

$$\Delta \equiv \frac{P_{0_\infty}}{p_{S.L.}}$$

$$\dot{m}_c = \dot{m}\cdot\left(\sqrt{\frac{T_0}{T_{sl}}}\Big/\frac{P_0}{p_{sl}}\right) \rightarrow \text{"corrected massflow"}$$

Two-Spool Compressor Design

• A single stage of compression can achieve perhaps 1.5:1 or 2.5:1 decrease in the air's volume.

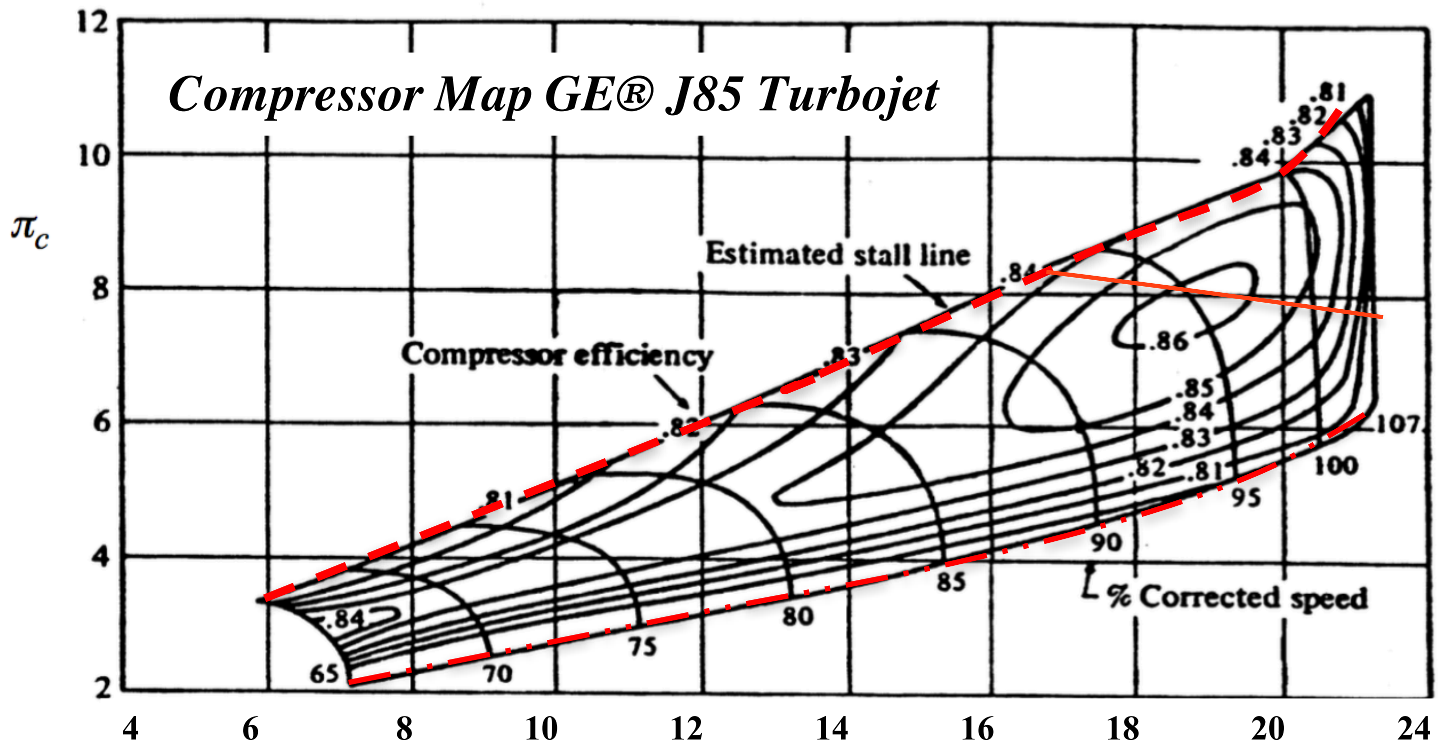
• Compression of the air increases the energy that can be extracted from the air during combustion and exhaust (which provides the thrust).

• In order to achieve the 10:1 to 15:1 total compression needed for the engine to develop adequate power, the engine is built with many stages of compressors stacked in a line.

• Multiple compression stages increases the overall compression while limiting the total stagnation pressure loss during compression.

• Depending upon the engine design, there may be as many as 10 to 15 stages in the total compressor.

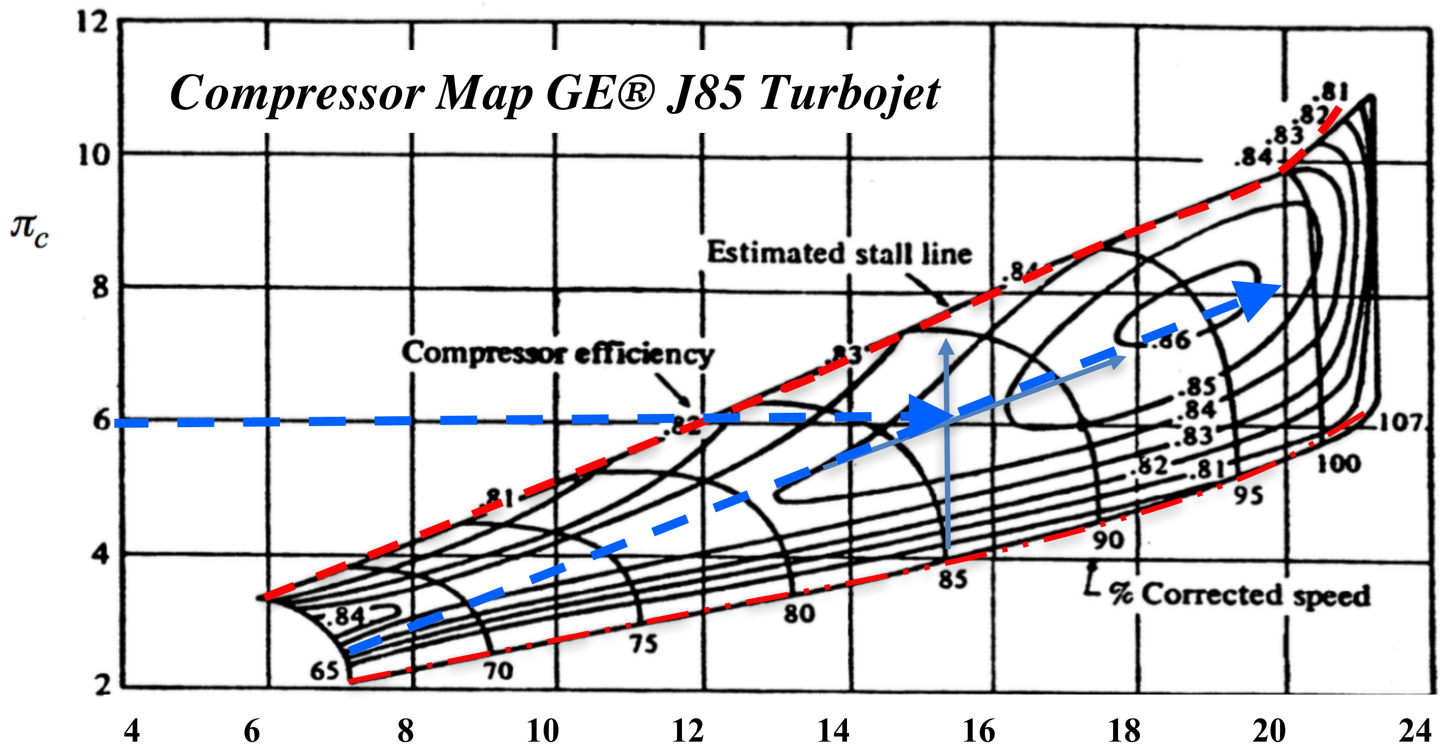
Two-Spool Compressor Design (2)



$$\dot{m}_c = \dot{m}_{air} \cdot \left(\sqrt{\Theta}/\Delta\right), kg/\sec$$

- As pressure increases across each stage, more and more work is required to impart a given pressure increment.
- Typically, this additional work is provided by increasing the blade angles of attack, and number of blades for successive stages.
- Eventually, the stage blade angle of attack is limited by the stall line and the spacing is limited by the choke line.

Two-Spool Compressor Design (4)



$$\dot{m}_c = \dot{m}_{air} \cdot \left(\sqrt{\Theta}/\Delta\right), kg/\sec$$

• As pressure ratio across the compressor increases, then a higher rotor speed is required in order to maintain the same massflow

N2 > than N1 for most designs

Two-Spool Compressor Design (3)

• Solution is to break the problem into two ”spools” with each spool operating at an independent speed, while still providing the same massflow level.

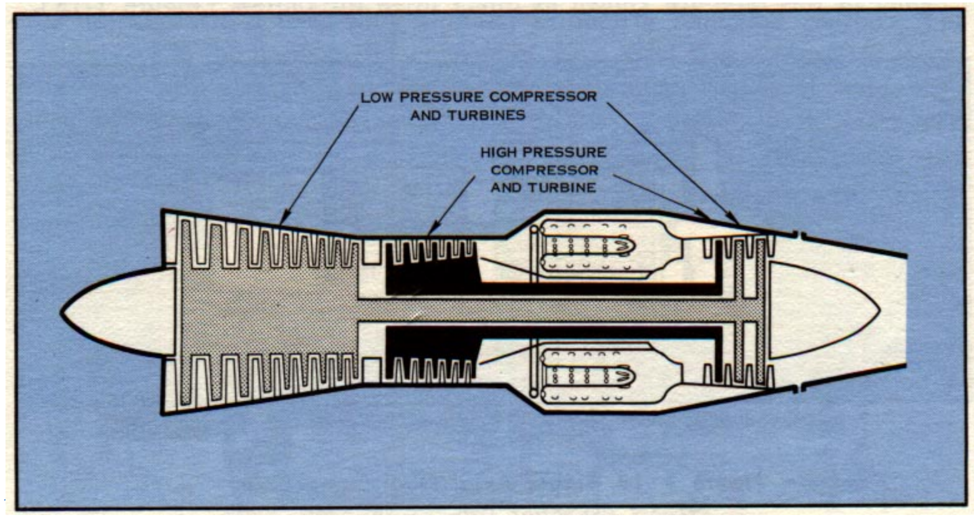
• Rear high-pressure compressor, is connected by a hollow shaft to a high-pressure turbine. The high rotor is often referred to as *N2* for short.

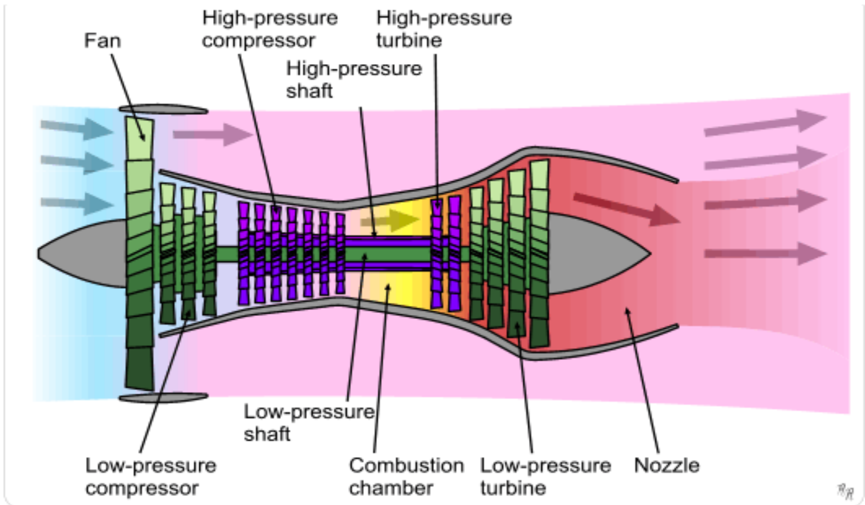
• The front low-pressure compressor is driven by a low-pressure turbine by a shaft that goes through the hollow shaft of the high rotor. The low-pressure rotor is called *N1* for short.

• *N1 and N2* rotors are not connected mechanically in any way. There is no gearing between them. As the air flows through the engine, each rotor is free to operate at its own efficient speed.

• Most modern turbojet engines are use dual-rotor compressor/turbine systems.

• Both rotors have their own individual redline limits.





Two-Spool Compressor Design (4)

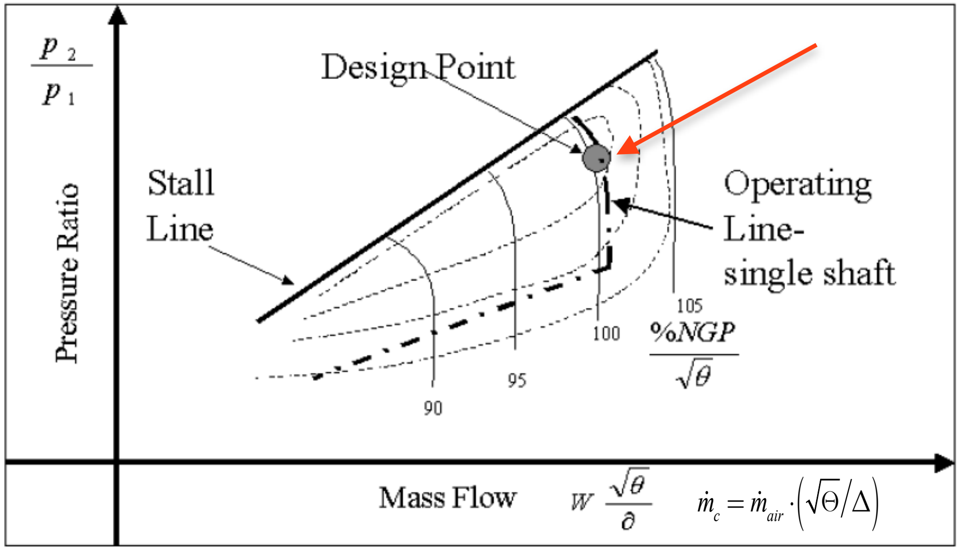
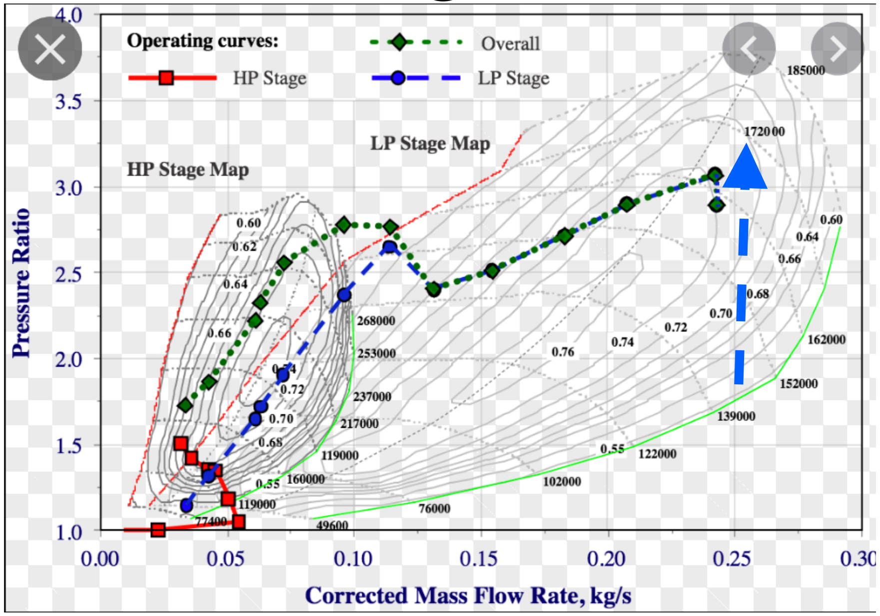
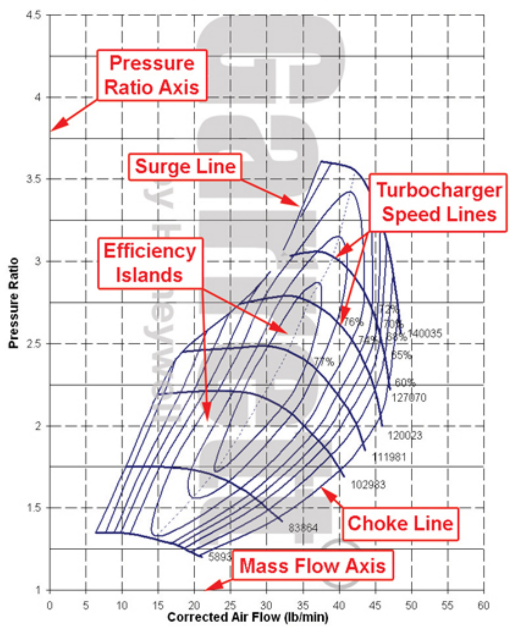
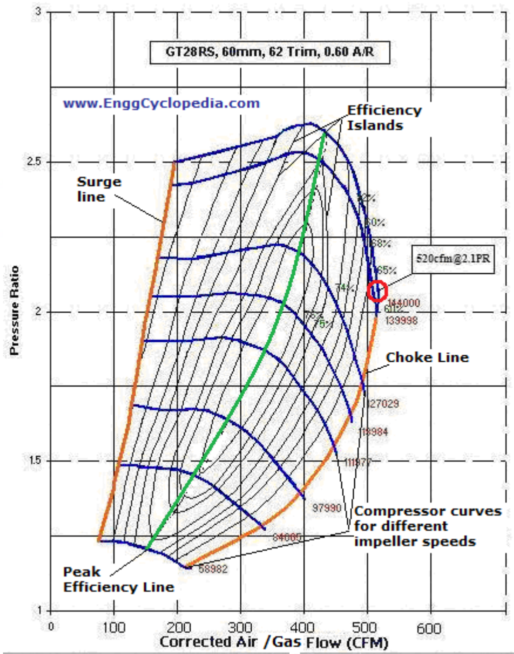


Figure 7. Typical Compressor Performance Map with Operating Lines for a Single Shaft Engine.







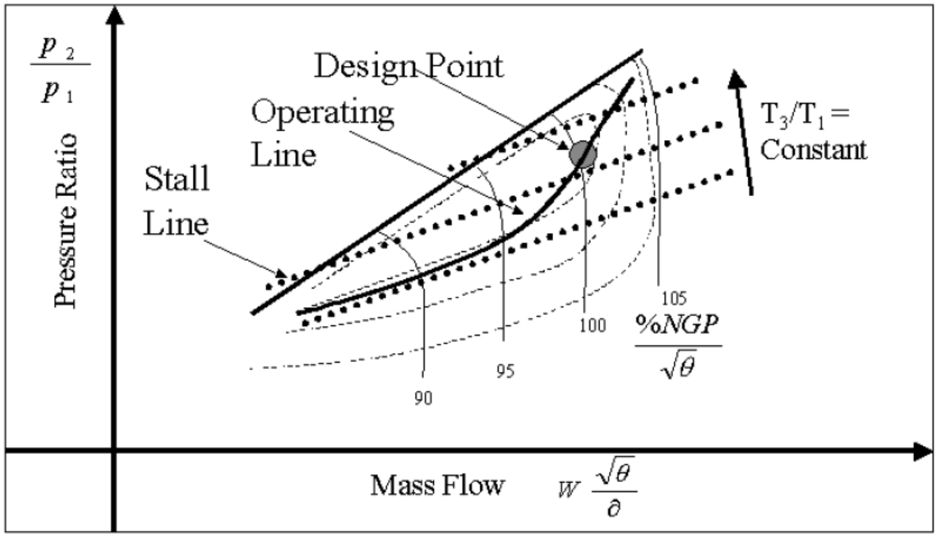
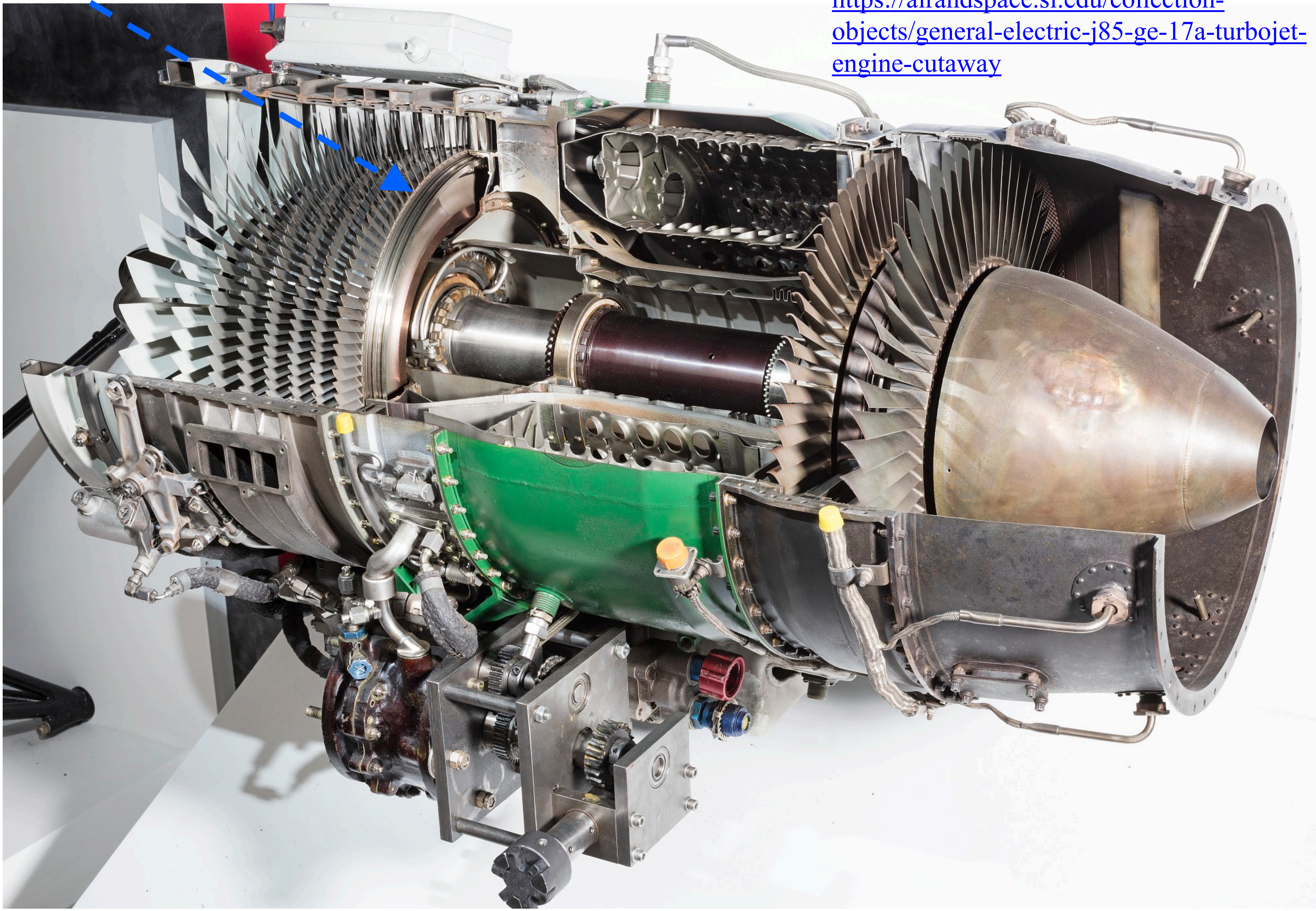


Figure 8. Typical Compressor Performance Map with Operating Lines for a Two Shaft Engine.

J-85 Engine Cutaway

https://airandspace.si.edu/collection-objects/general-electric-j85-ge-17a-turbojet-engine-cutaway



Effect of Compressor Operation on Engine Control

Two main inputs to the control of the engine are:

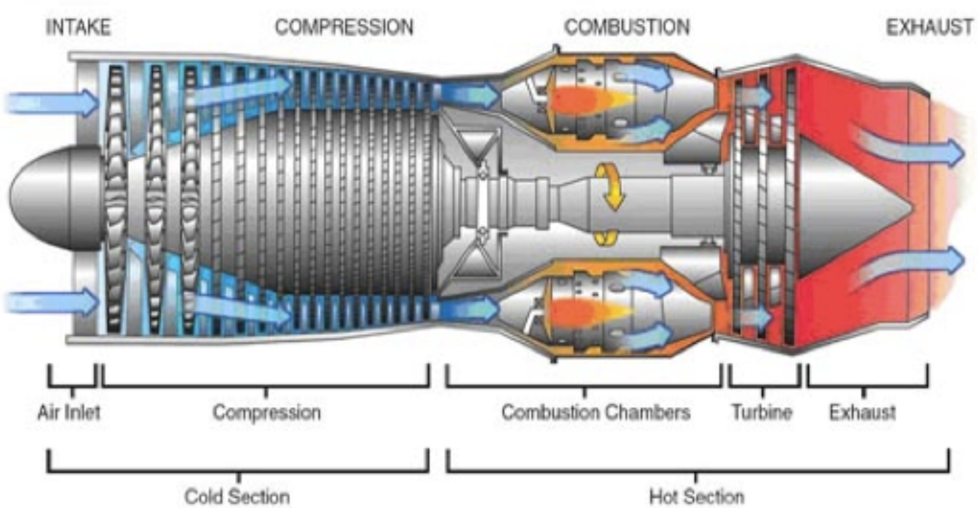
1) Throttle

-- Controls fuel flow which in turn controls T_{04} and τ_{λ}.

2) Nozzle throat area *A*8* .

-- For variable geometry configuration

… collectively referred to as Power level Angle (PLA)



Boeing 747 Throttle

Effect of Compressor Operation on Engine Control (2)

Case 1: Variable Nozzle Exit Are with constant fuel flow rate, A^*_4, τ_λ, τ_r fixed

a) Calculate Turbine Temperature Ratio Response to A_8

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}$$

b) Calculate Compressor Temperature, Pressure Ratio Response to τ_t.

$$\tau_c = 1 + \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot\left(1-\tau_t\right) \qquad \pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}$$

c) Calculate corrected massflow, and Mach number at compressor face

$$\dot{m}_c = A_4^* \cdot p_{sl} \cdot \sqrt{\frac{\gamma}{R_g \cdot T_{sl}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \cdot \pi_c \cdot \sqrt{\frac{\left(\frac{f}{f+1}\right)\left(1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}\right)}{\pi_c^{\frac{\gamma-1}{\gamma}}-1}}$$

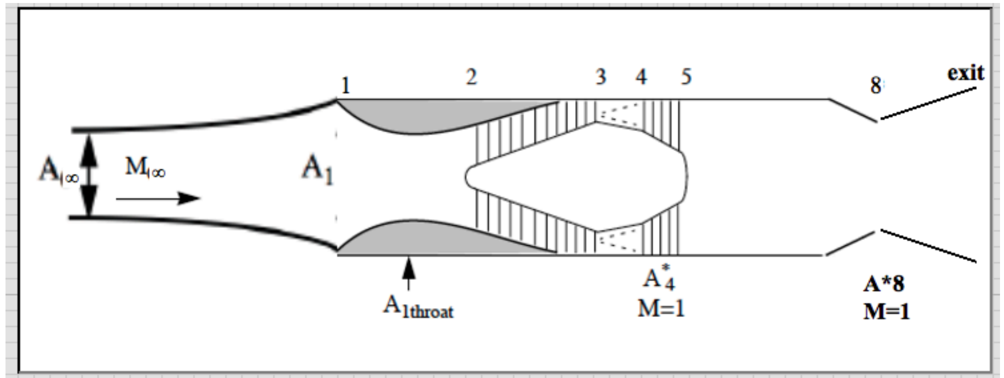
$$\frac{M}{\left[\left(1+\frac{\gamma-1}{2}M^2\right)\left(\frac{2}{\gamma+1}\right)\right]^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}} = \frac{\dot{m}_c}{A_2 \cdot p_{sl}\sqrt{\frac{\gamma}{R_g \cdot T_{sl}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}}$$

Effect of Compressor Operation on Engine Control

(3)

Case 1: Variable Nozzle Exit Are with constant fuel flow rate, A^*_4, τ_λ, τ_r fixed

d) Calculate Inlet Capture Area from Inlet massflow Matching

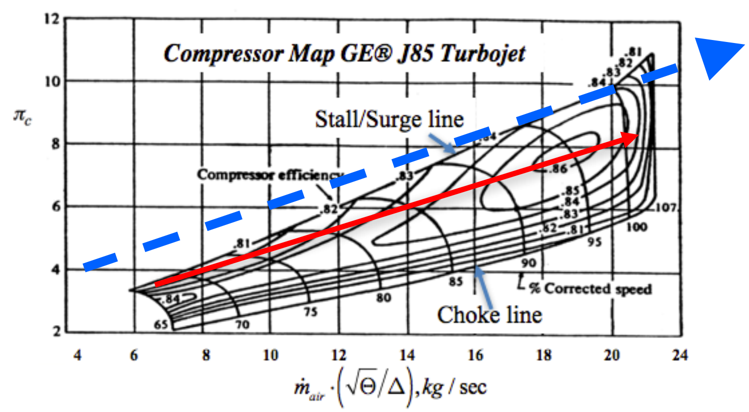


$$A_\infty = \pi_d \cdot A_2 \cdot \frac{M_2}{M_\infty} \left(\frac{1 + \frac{\gamma - 1}{2} M_\infty^{\ 2}}{1 + \frac{\gamma - 1}{2} M_2^{\ 2}} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

e) The changes of corrected massflow, $f(M_2)$ and π_c are achieved by an increase in compressor rotor speed (*N1 or N2*) ,according to the behavior indicated on the compressor map. The compressor operating point moves along a constant

$$\tau_\lambda / \tau_r = \frac{T_{combustor}}{T_{0_\infty}}$$

characteristic line.



Effect of Compressor Operation on Engine Control (4)

Case 2: Variable fuel flow (combustor temperature) with fixed nozzle throat area i.e. vary τ_λ, and keep A^*_8 constant.

a. Calculate Compressor Temperature, Pressure Ratio Response to τ_λ.

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)} \qquad \tau_c = 1 + \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot\left(1-\tau_t\right) \qquad \pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}$$

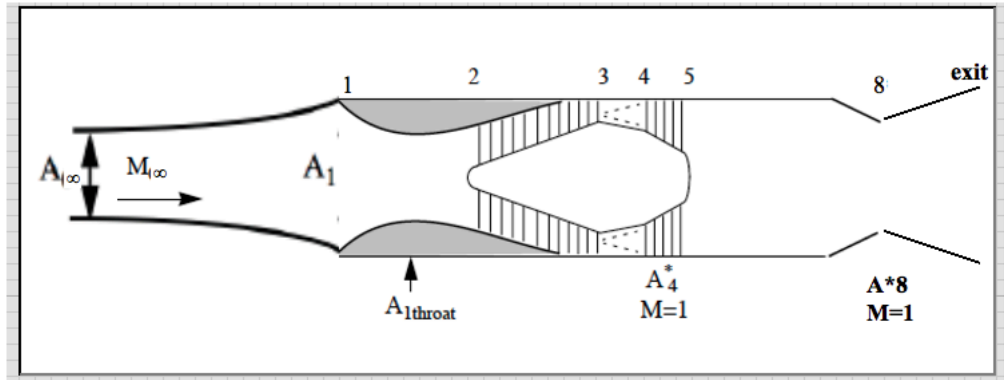
b. Calculate corrected massflow and compressor face massflow

$$\dot{m}_c = A_4^* p_{sl}\sqrt{\frac{\gamma}{R_g \cdot T_{sl}}}\cdot\pi_c\cdot\sqrt{\frac{\left(\frac{f}{f+1}\right)\left[1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}\right]}{\pi_c^{\frac{\gamma-1}{\gamma}}-1}}$$

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{\dot{m}_c}{A_2\cdot p_{sl}\sqrt{\frac{\gamma}{R_g\cdot T_{sl}}}}$$

Effect of Compressor Operation on Engine Control (5)

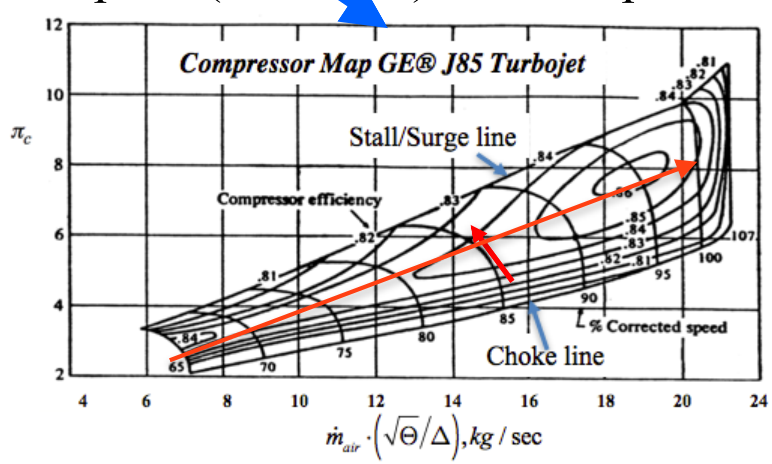
Case 2: Variable fuel flow (combustor temperature) with fixed nozzle throat area i.e. vary τ_λ, and keep A^*_8 constant.



d) Calculate Inlet Capture Area from Inlet massflow Matching

$$A_\infty = \pi_d \cdot A_2 \cdot \frac{M_2}{M_\infty}\left(\frac{1+\frac{\gamma-1}{2}M_\infty^{\ 2}}{1+\frac{\gamma-1}{2}M_2^{\ 2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

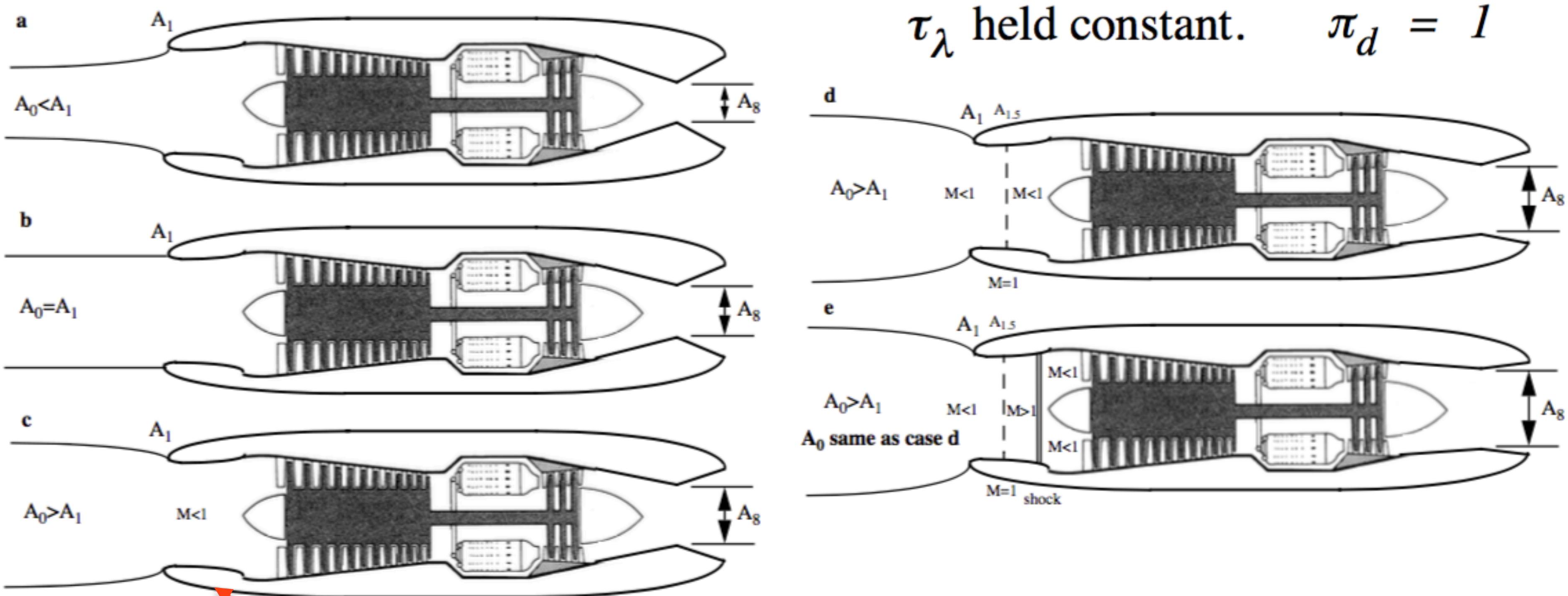
e) The changes of corrected massflow, $f(M_2)$ and π_c are achieved by an increase in flame temperature and no by compressor rotor speed (*N1 or N2*) The compressor operating point moves along a constant rotors speed And crosses constant lines of τ_λ/τ_r.



Effect of Compressor Operation on Engine Control

(6)

τ_λ held constant. $\pi_d = 1$

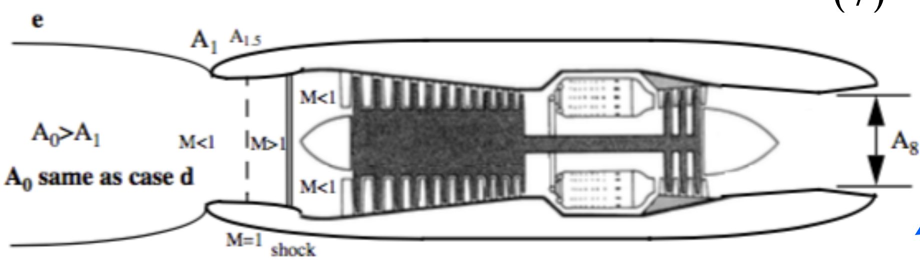


Inlet behavior with increasing nozzle throat area in subsonic flow

Mach number entering compressor (station 2) increases from top to bottom. In figures (*a, b, c, d*) there is no inlet shock and so, neglecting skin friction, only way for increase in M_2 is for capture area A_∞ to increase leading to an increase in the air mass flow through the engine. As A_8 is increased further the inlet eventually chokes *(Figure e)*.

Effect of Compressor Operation on Engine Control

(7)



Condition for inlet choking

- Mass Balance between inlet throat (station 1.5) and compressor face (Station 2)

$$\frac{P_{0_2} \cdot A_2}{\sqrt{T_{0_2}}} f\left(M_2\right) = \frac{P_{0_{1.5}} \cdot A_{1throat}}{\sqrt{T_{0_{1.5}}}} f\left(M_{1.5}\right)$$

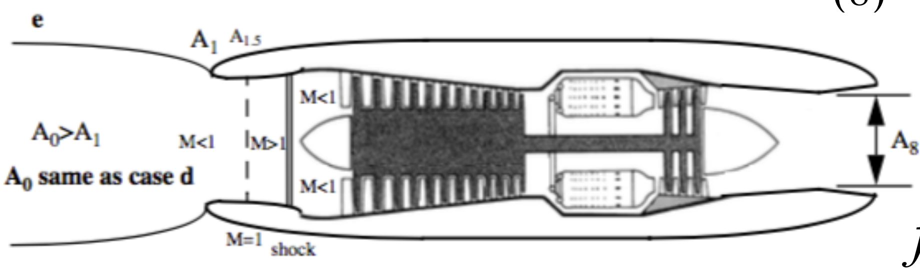
$\rightarrow$ *assume approximately isentropic in diffuer* $\rightarrow$

$$\frac{P_0}{\sqrt{T_0}} \rightarrow \text{constant} \rightarrow f\left(M_2\right) = f\left(M_{1.5}\right)\frac{A_{1throat}}{A_2}$$

- For choked inlet $\rightarrow f\left(M_{1.5}\right)_{\substack{choked \\ inlet}} = \left(\left(1+\frac{\gamma-1}{2}1_2^{\ 2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 1$

$$f\left(M_2\right)_{\substack{choked \\ inlet}} = \frac{A_{1throat}}{A_2}$$

Effect of Compressor Operation on Engine Control (8)



Condition for inlet choking

$$f\left(M_2\right)_{\substack{choked\\inlet}} = \frac{A_{1throat}}{A_2}$$

• If the nozzle area is increased beyond this point there is no change in A_∞ , the air mass flow remains fixed and a shock wave forms downstream of the inlet throat (case e).

• Matching condition satisfied by stagnation pressure loss across the shock ($\pi_d < 1$).

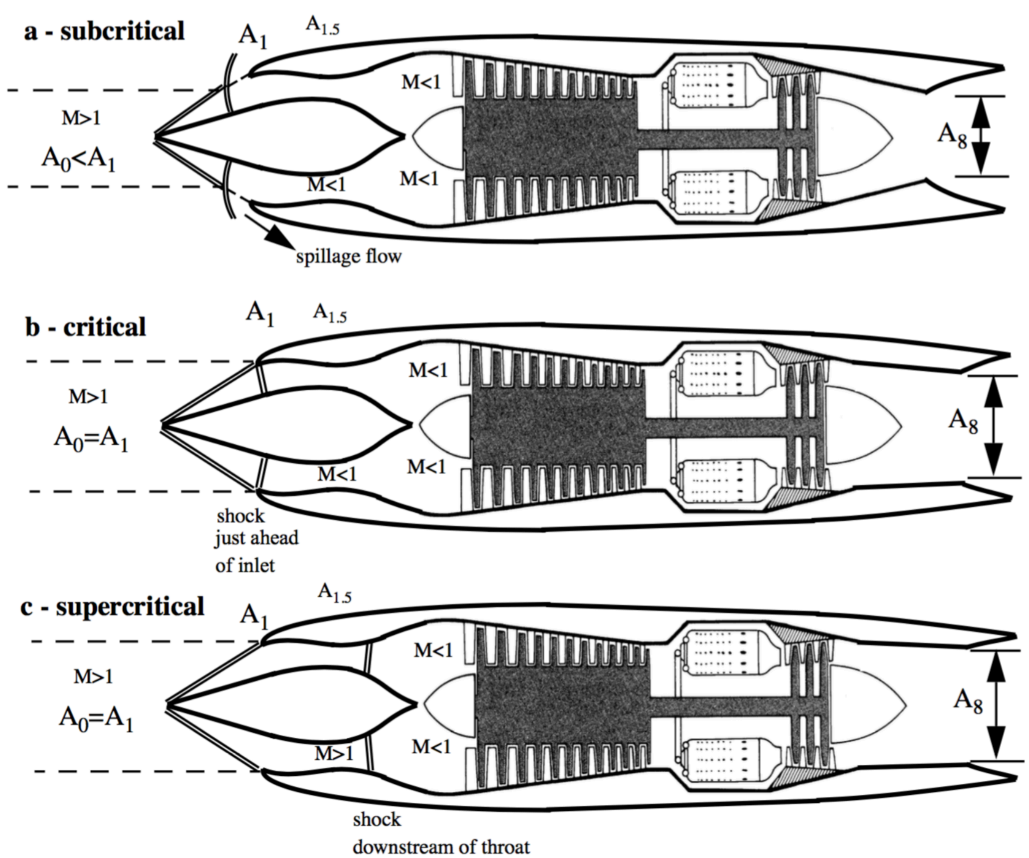
• Shock becomes stronger as M_2 is further increased.

• Whole mechanism is referred to as the pumping characteristic of the engine.

• Once an inlet shock begins to form engine performance (thrust and efficiency) drop off rather rapidly. i.e. and a well designed sub-sonic system is designed to avoid internal shock wave formation.

Supersonic Inlet Flow

- In supersonic flow inlet is routinely designed to accommodate an inlet shock and/or a system of external shocks that may be needed to decelerate a high Mach number flow to subsonic value at the compressor face
- Basic operation of inlet throat in supersonic flow is similar to that of a subsonic inlet. Key is to minimize stagnation pressure **loss** due to external shock system as well as variable losses, due to the movement of inlet shock.
- Figure below depicts the effect of increasing A^*_8 on inlet flow for an engine operating in a supersonic stream.



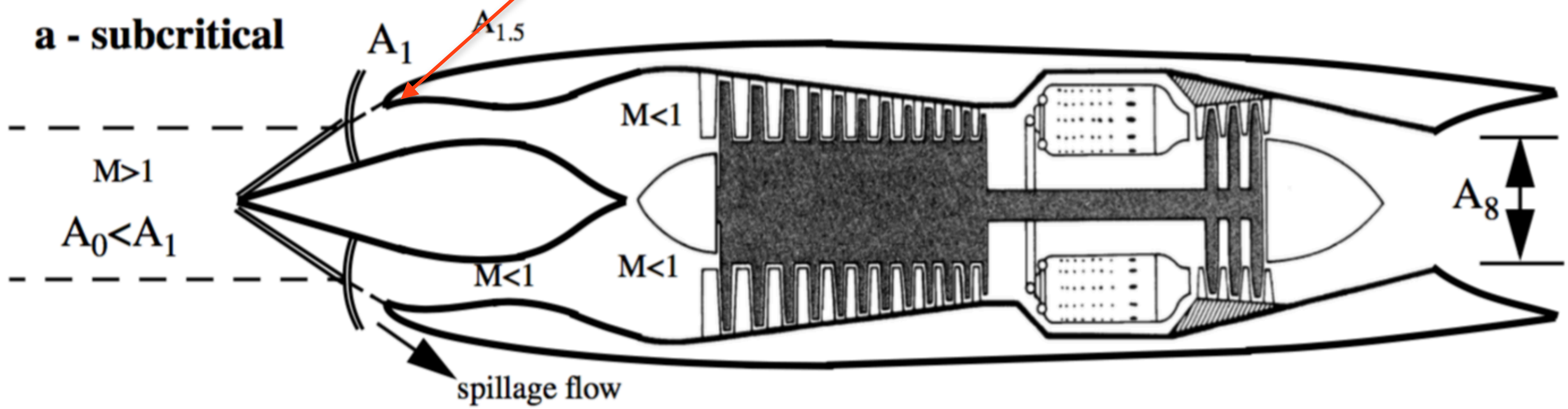
Supersonic Inlet Flow (2)

• For case *below,* Mach number at station 2 is sufficiently low that Mach number at station 1 is less than Mach number behind the normal shock ahead of the inlet as results from A_2 / A_1.

• Inlet operation is said to ***be sub-critical*** and flow into the inlet is all subsonic after system of oblique and normal shocks over the center-body.

• Inlet is not-isentropic and Inlet pressure ratio π_d is less than one due to the oblique and normal shocks ahead of the inlet .

• *Sub-critical operation* is characterized by a normal shock that is detached from the inlet cowl. Flow

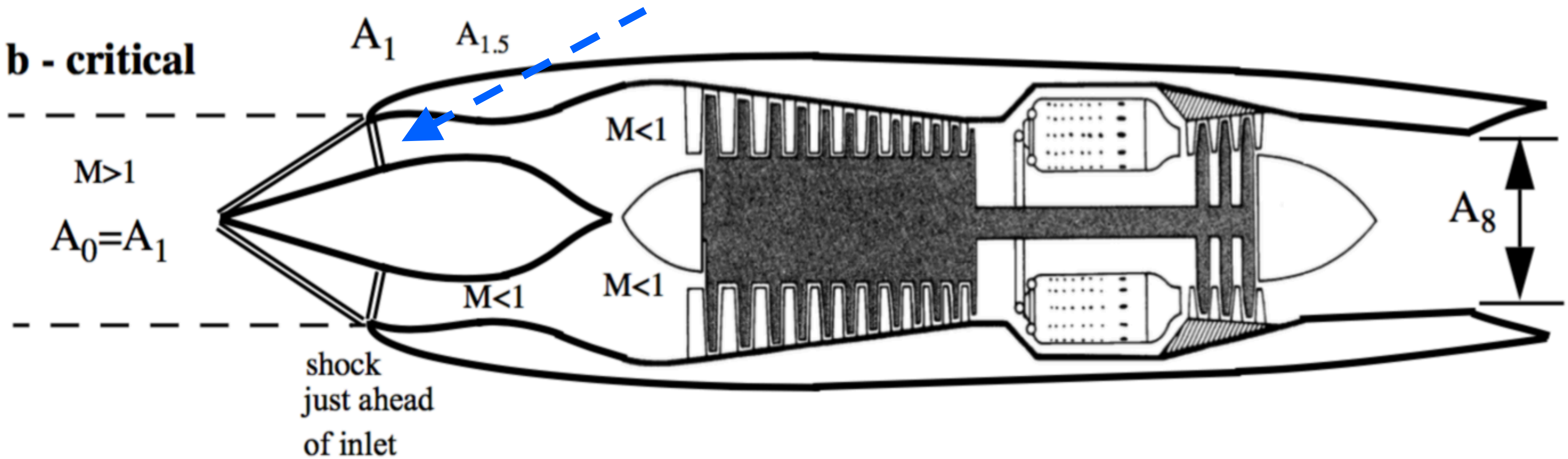


Supersonic Inlet Flow (3)

• When Mach number at station 2 has increased such that Mach number at station 1 is just slightly less than Mach number behind normal shock, normal shock will be positioned just ahead of inlet lip and inlet operation is said *to be critical*.

• Mach number between stations 1 and 2 is all subsonic.

• The normal shock wave is attached to the cowl tip, and no flow spillage occurs.



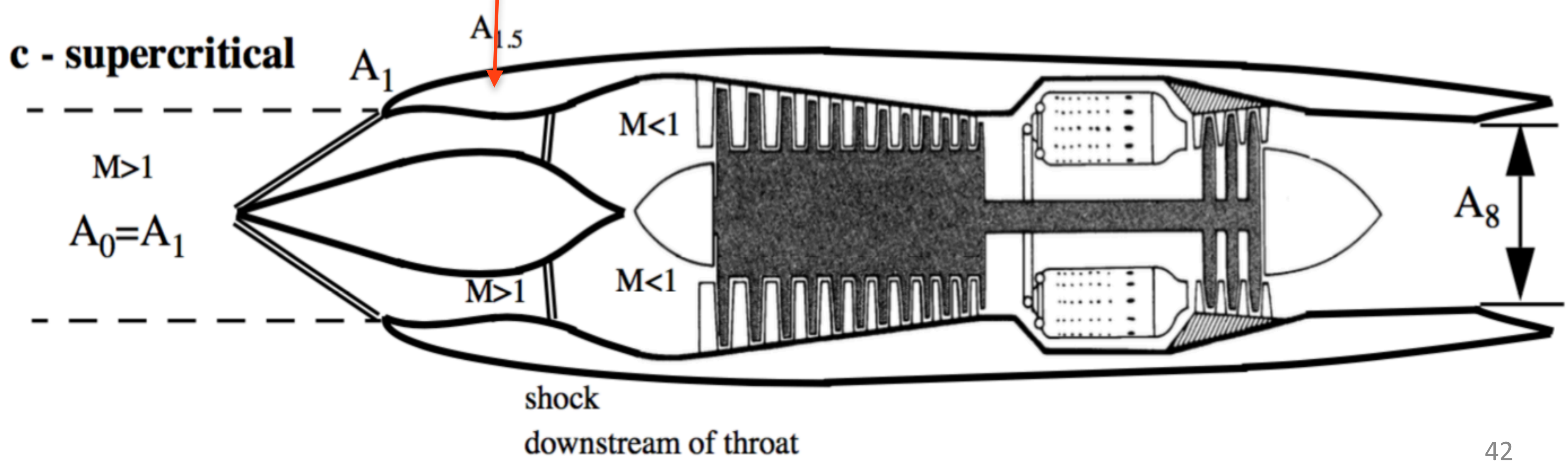
Supersonic Inlet Flow (4)

• Further increasing A^*_8 leads to "starting" of inlet flow and shock formation downstream of *the inlet throat*.

• If nozzle is opened up still more engine will demand increasing values of corrected massflow [$f(M_2)$].

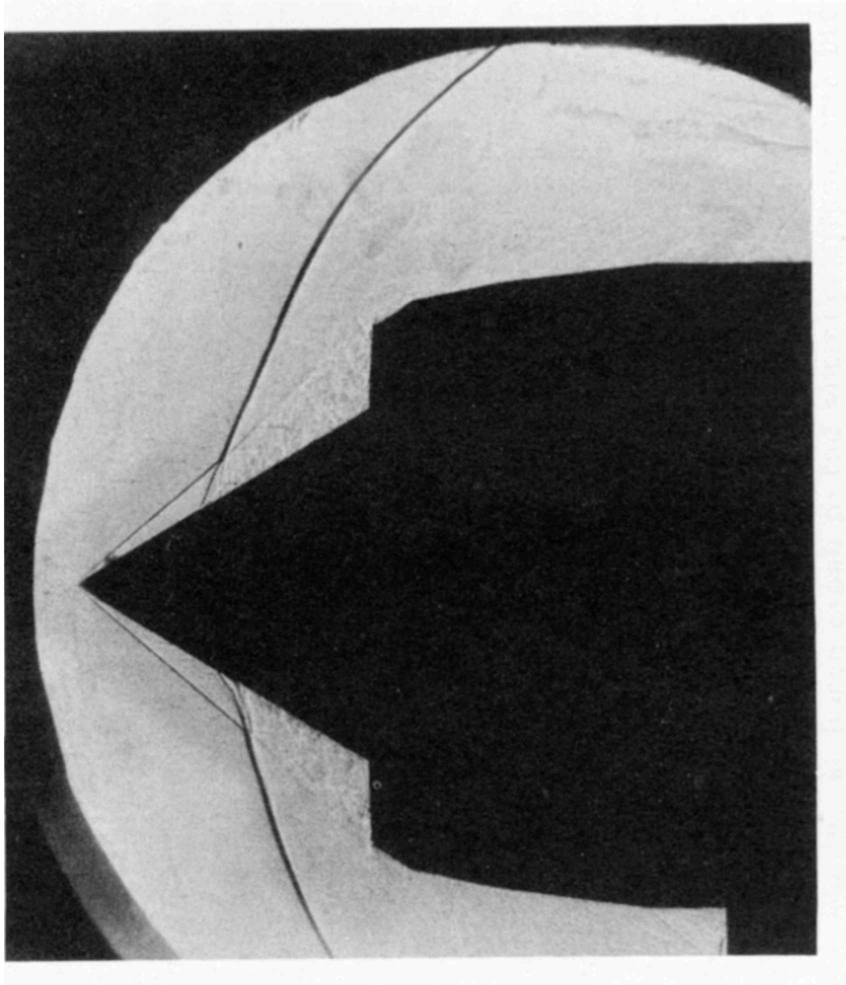
• In this case inlet mass flow can no longer increase and mass flow balance between the freestream and compressor face is satisfied through decreasing values of π_d (super- critical operation) due to downstream movement of the shock to higher shock Mach numbers.

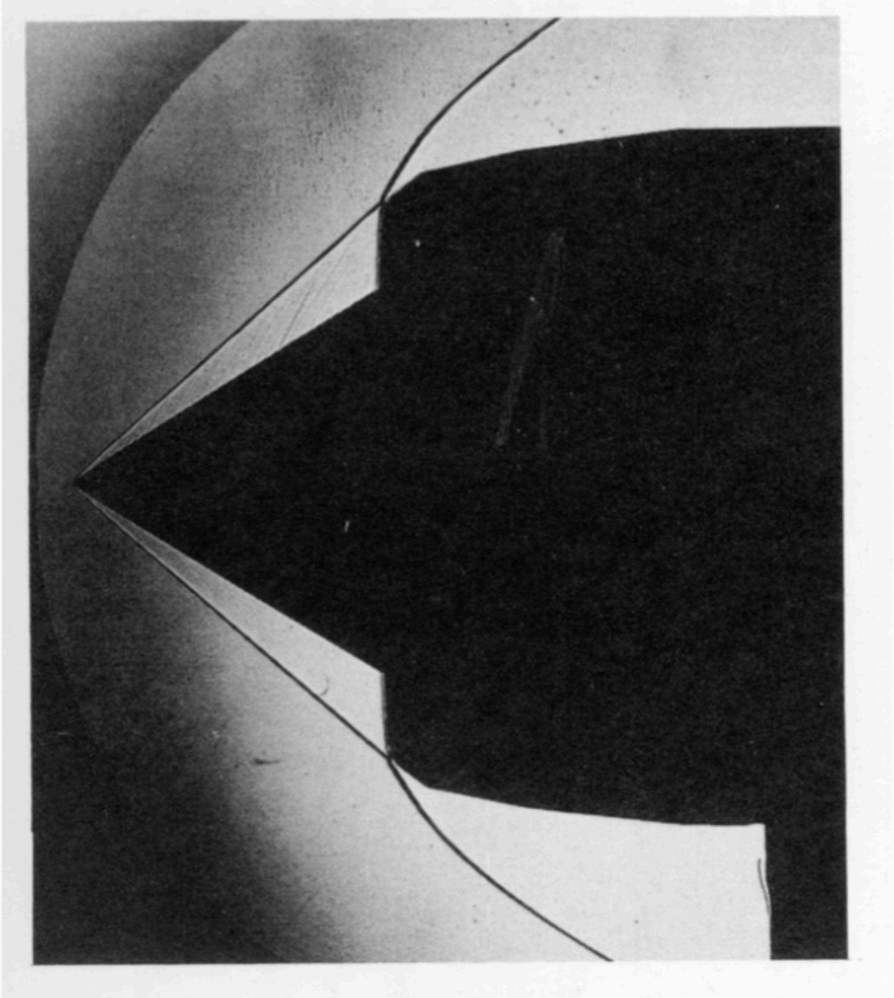
• Positioning of the shock wave at the diffuser throat is considered to be the "design" point. This approach allows the convergent section of the inlet to isentropically decelerate the flow to near Mach 1 before the normal shock wave forms .. And minimizes stagnation pressure losses.



Supersonic Inlet Flow (5)

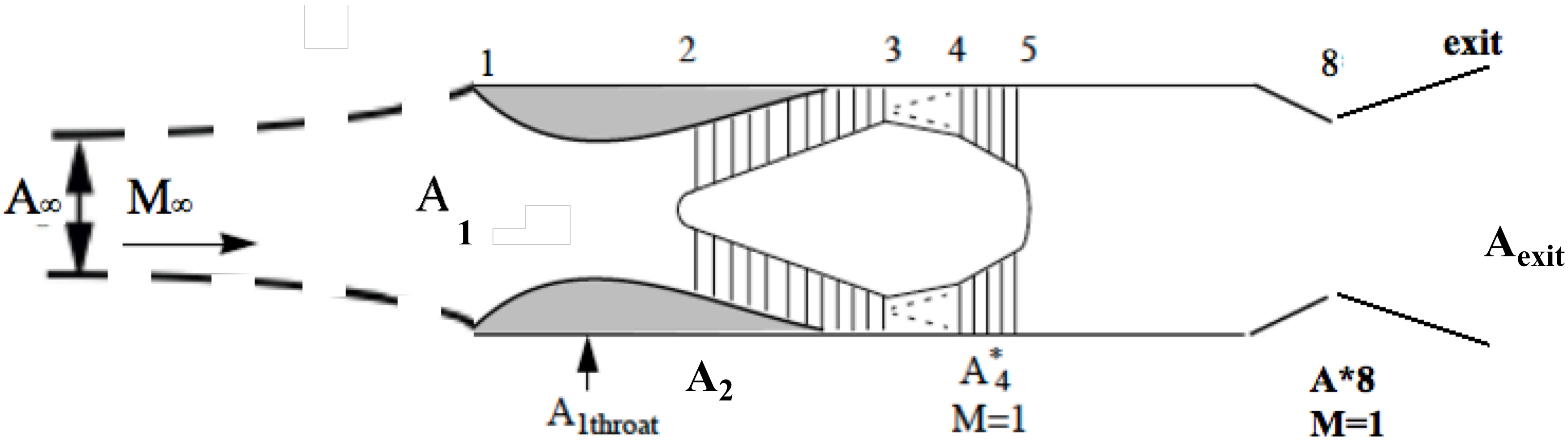
- The shadowgraph photos below illustrate sub and supercritical flow on an axisymmetric spike inlet. Similar inlet behavior occurs with fuel throttling.





Flow over a Mach 3 spike inlet, left photo subcritical behavior, right photo supercritical behavior.

Example Calculation



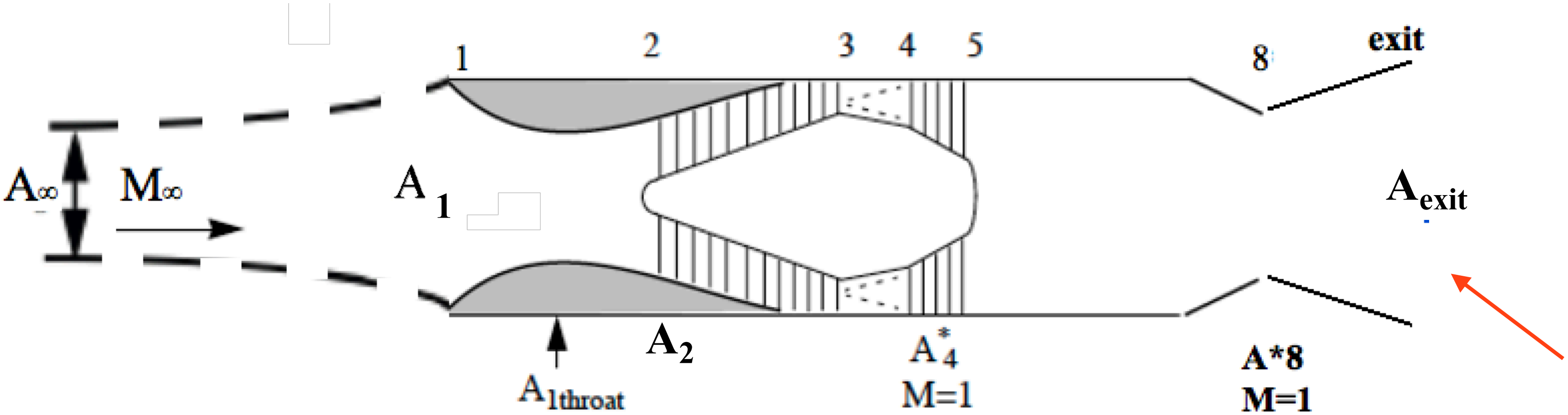
- Engine operates at a free stream Mach number, $M_\infty = 0.85$.
- Cruise Altitude is in the stratosphere, 11 km so T_∞ = 216.65 K.
- The design turbine inlet temperature, $T_{04} = 1750\ ^oC$
- The design compressor ratio, $\pi_c = 15$.
- Relevant area ratios are $A_2 / A^*_4 = 8$ and $A_2/A_{1throat} = 1.5$.
- Inlet throat area $A_{1Throat} = 10\ cm^2$, $A^*_8/A_{exit} = \mathbf{0.5}$
- *Assume the compressor, burner and turbine all operate ideally.*
- *Stagnation pressure losses due to wall friction in the inlet and nozzle are negligible.*

$$f = 40$$

$$\pi_b = 1$$

→ *CALCULATE*
→ *a) Correct*ed *Compressor massflow and M_2 at compressor face*
→ *b) Normalized exit pressure thrust, momentum thrust, and total thrust, normalized specific impulse*
→ *c) Velocity ratio across Engine V_{exit}/V_∞*
→ *d) Mach number at diffuser throat, $M_{1throat}$*
→ *e) Inlet capture area*
→ *f) True Thrust, Isp, TSFC*
→ *g) Plot the Compressor operating line → π_c vs corrected massflow, $F(M_2)$ for $2 \le \pi_c < \pi_{c\ \_inlet\ choke}$ → $M1_{throat} \sim 1$*
→ *h) the Inlet capture area A_∞ vs corrected massflow, $F(M_2)$ for $1 \le \pi_c < \pi_{c\ \_inlet\ choke}$ → $M1_{throat} \sim 1$*

Example Calculation (2)



REFERENCE CONDITIONS

Freestream Conditions

Mach Number	Fuel Enthalpy, J/kg	Gamma	
0.85	4.947E+7	1.4	
Altitude, km	**Cp, J/kg-K**	**Max Burner Temperature, C**	**AIR FUEL RATIO**
11	1004.96	1750	40

Design Parameters

πc	A2/ A*4	A2/ A1 t	Aexit/A*8	A1_throat, cm^2
15.0000	8	1.5	2	10

$$\tau_\lambda = \frac{T_{0_4}}{T_\infty} = \frac{2023.15}{216.65} = 9.33833$$

$$\tau_c = (\pi_c)^{\frac{\gamma-1}{\gamma}} = (15)^{\frac{1.4-1}{1.4}} = 2.16783$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = \frac{247.96}{216.65} = 1.1445$$

$$\tau_t = 1 - \frac{f}{f+1}\cdot(\tau_c - 1)\cdot\frac{\tau_r}{\tau_\lambda} =$$

$$1 - \left(\frac{40}{40+1}\right)\cdot(2.16783 - 1)\cdot\frac{1.1445}{9.33833} =$$

0.860362

Turbine Power Matching

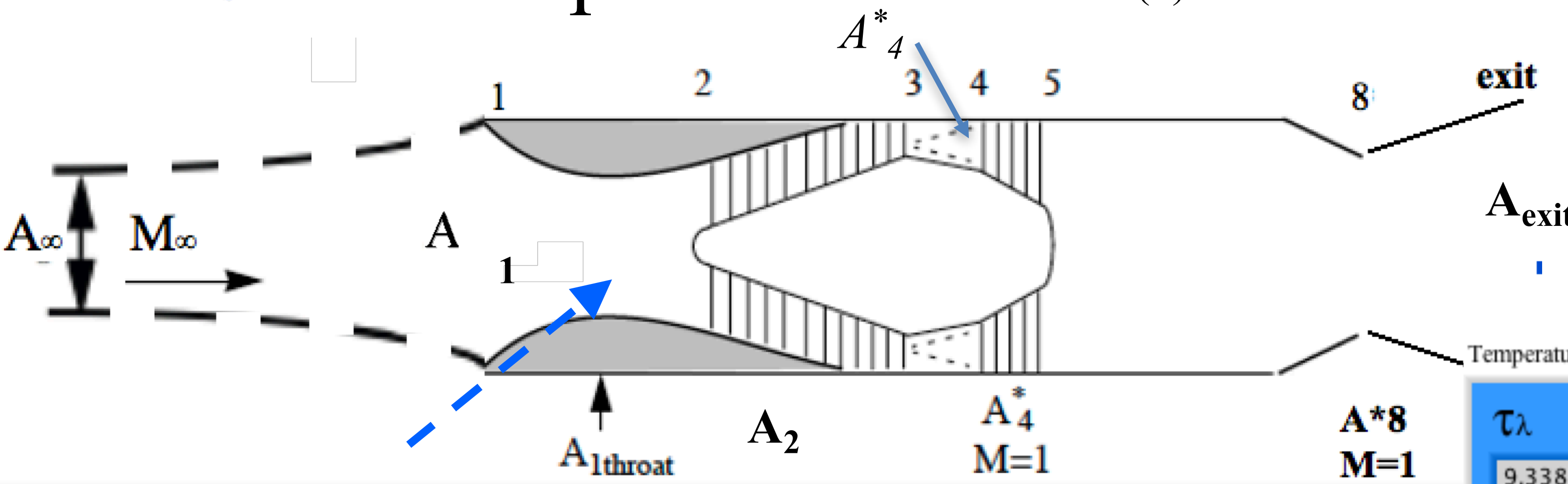
Flight Parameters Metric

Qc, kPa	Qbar, kPa	Vtrue, m/sec	P0_inf, kPa
13.67	11.45	250.81	36.30
Pinf, kPa	**Tinf, K**	**CpMax**	**T0_inf, K**
22.63	216.65	1.1939	247.96

Temperature Ratios

τλ	τc
9.33833	2.16783
τr	**τt**
1.1445	0.860362

Example Calculation (3)



Temperature Ratios

τ_λ	τ_c
9.33833	2.16783
τ_r	τ_t
1.1445	0.860362

Compressor Demand (Massflow) Matching

$$\frac{1}{A_2/A_2^*} = F(M_2) = \frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^2\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c\cdot\pi_b)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^*}{A_2} = \left(\frac{40}{40+1}\right)\frac{(15\cdot 1)}{\left(\frac{9.33833}{1.1445}\right)^{0.5}}\left(\frac{1}{8}\right) = 0.640399$$

$\rightarrow M_2 = 0.40907$

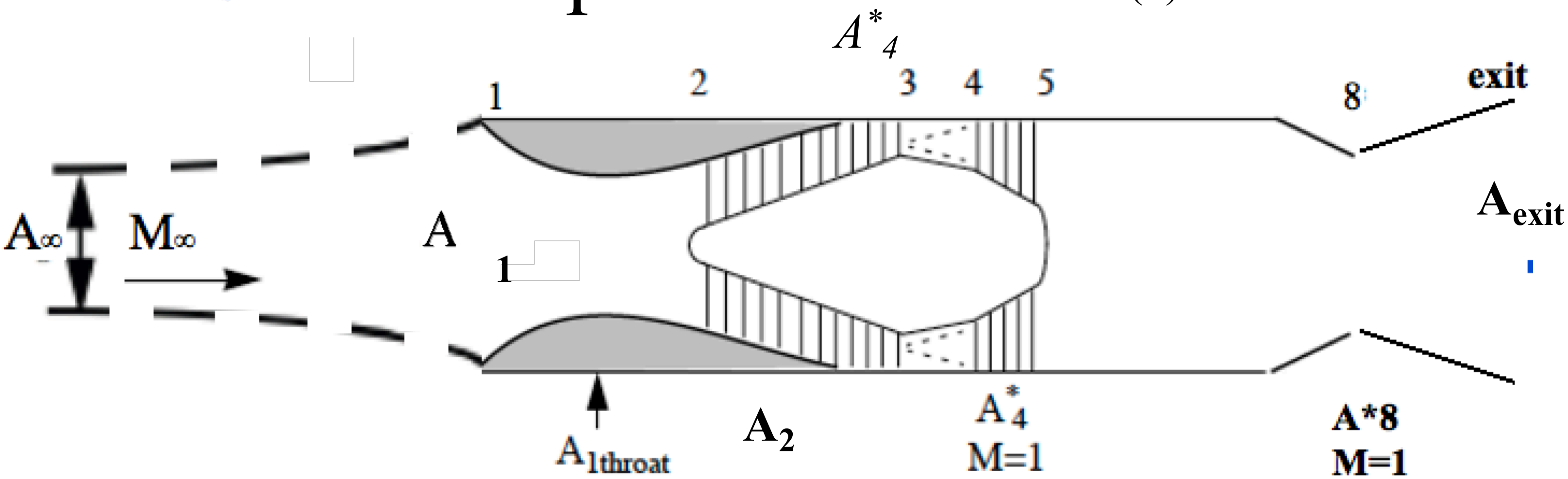
Nozzle Turbine (Massflow) Matching

$$\frac{A_4^*}{A_8} = (\tau_t)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} = 0.860362^{0.5\left(\frac{1.4+1}{1.4-1}\right)} = 0.63686$$

$$\frac{A_{1_{throat}}}{A_2^*} = \left(\frac{A_2}{A_2^*}\right)\bigg/\left(\frac{A_2}{A_{1_{throat}}}\right) = \left(\frac{A_2}{A_2^*}\right)\cdot\left(\frac{A_{1_{throat}}}{A_2}\right) =$$

$$\frac{1}{0.640388}\,\frac{1}{1.5} = 1.04104$$

Example Calculation (4)

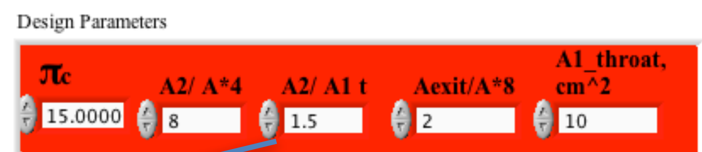


Inlet MassFlow Matching

$$\frac{1}{\pi_d}\frac{A_\infty}{A_2}\frac{M_\infty}{\left(\left(1+\frac{\gamma-1}{2}M_\infty^{\ 2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}=\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\ 2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

→ $M_2 = 0.40907$

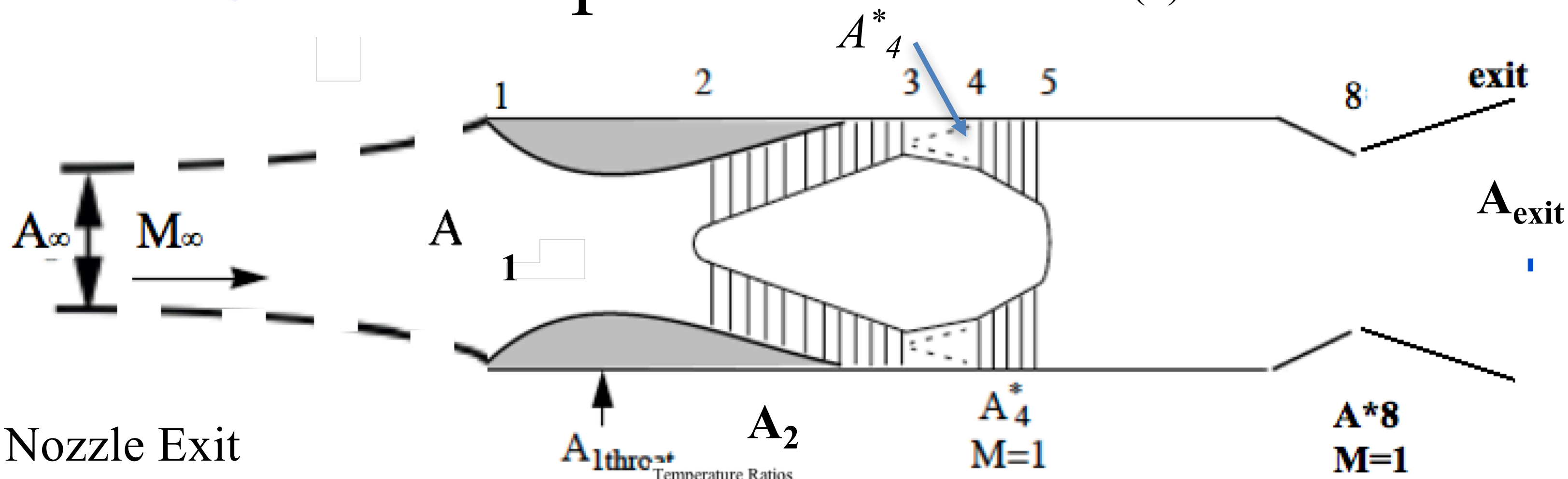
→ $M_\infty = 0.85$



$\boxed{\pi_d = 1}$

$$\frac{A_\infty}{A_{1_{throat}}}=\frac{A_\infty}{A_2}\cdot\frac{A_2}{A_{1_{throat}}}=\pi_d\left[\frac{\dfrac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\ 2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}{\dfrac{M_\infty}{\left(\left(1+\frac{\gamma-1}{2}M_\infty^{\ 2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}\right]\cdot\frac{A_2}{A_{1_{throat}}}= \quad 0.980453$$

Example Calculation (5)



- Nozzle Exit

$$\frac{A_{exit}}{A^*_8} = 2 \rightarrow M_{exit} = 2.1972$$

Temperature Ratios

τλ	τc
9.33833	2.16783
τr	τt
1.1445	0.860362

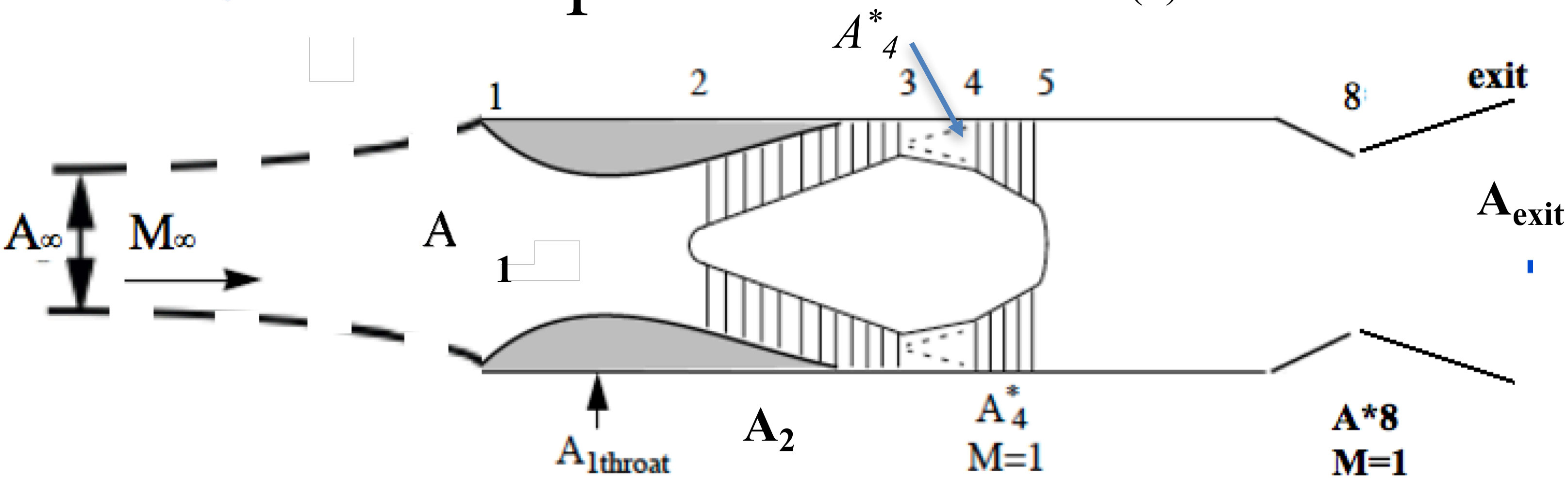
→ $M_2 = 0.40907$
→ $M_\infty = 0.85$

$$\frac{T_{exit}}{T_\infty} = \cdot\left(\frac{T_{0_\infty}}{T_\infty}\cdot\frac{T_{0_1}}{T_{0_\infty}}\cdot\frac{T_{0_2}}{T_{0_1}}\cdot\frac{T_{0_3}}{T_{0_2}}\cdot\frac{T_{0_4}}{T_{0_3}}\cdot\frac{T_{0_5}}{T_{0_4}}\cdot\frac{T_{0_{exit}}}{T_{0_5}}\cdot\frac{T_{exit}}{T_{0_{exit}}}\right) = \left(\frac{\tau_\lambda\cdot\tau_t}{\tau_r}\right)\cdot\frac{\left(1+\frac{\gamma-1}{2}M_\infty^{\ 2}\right)}{\left(1+\frac{\gamma-1}{2}M_{exit}^{\ 2}\right)} = 4.08761$$

πc 15.0000

$$\frac{p_{exit}}{p_\infty} = \left(\frac{P_{0_\infty}}{p_\infty}\cdot\frac{P_{0_1}}{P_{0_\infty}}\cdot\frac{P_{0_2}}{P_{0_1}}\cdot\frac{P_{0_3}}{P_{0_2}}\cdot\frac{P_{0_4}}{P_{0_3}}\cdot\frac{P_{0_5}}{P_{0_4}}\cdot\frac{P_{0_8}}{P_{0_5}}\cdot\frac{P_{0_{exit}}}{P_{0_8}}\cdot\frac{p_{exit}}{P_{0_{exit}}}\right) = \pi_c\cdot\left(\tau_t\cdot\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M_{exit}^2}\right)^{\frac{\gamma}{\gamma-1}} = 1.3349$$

Example Calculation (6)



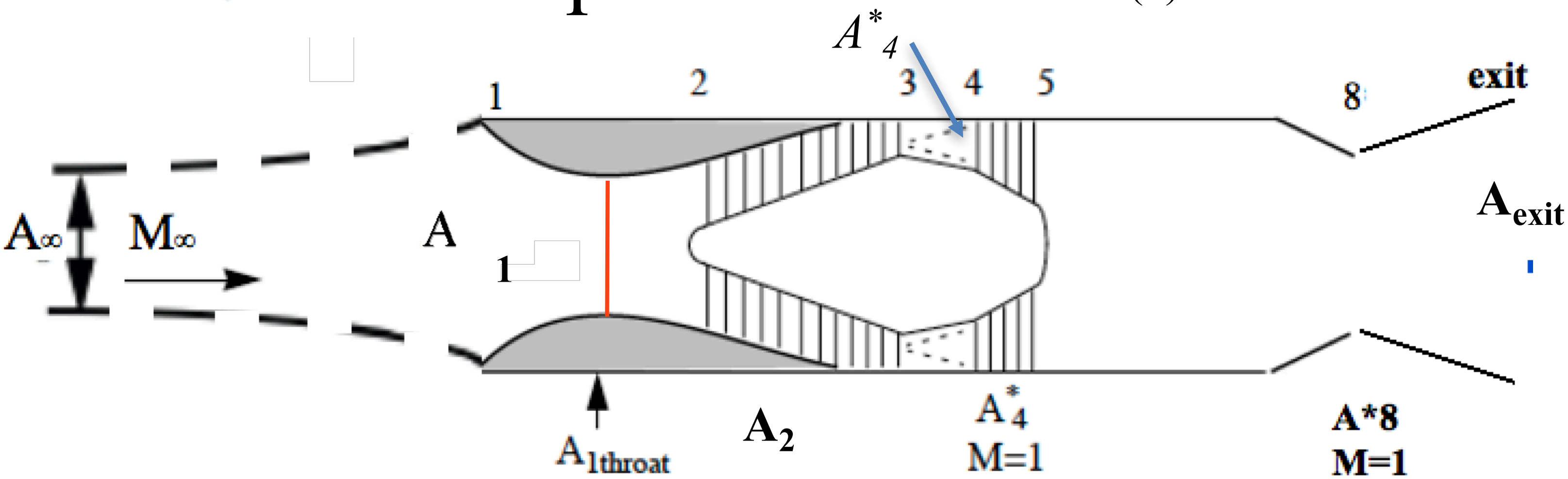
- Pressure at Nozzle Choke Point

$$\frac{p^*_8}{p_\infty} = \pi_c \cdot \left(\tau_t \cdot \frac{1 + \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} 1^2} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1 + \frac{1.4-1}{2} 0.85^2}{1 + \frac{1.4-1}{2} 1^2} 0.860362 \right)^{\frac{1.4}{1.4-1}} 15 = 7.50752$$

- Velocity Ratio Across Engine

$$\frac{V_{exit}}{V_\infty} = \frac{M_{exit}}{M_\infty} \left(\sqrt{\frac{T_{exit}}{T_\infty}} \right) = \frac{2.1972}{0.85} 4.088761^{0.5} = 5.22693$$

Example Calculation (7)

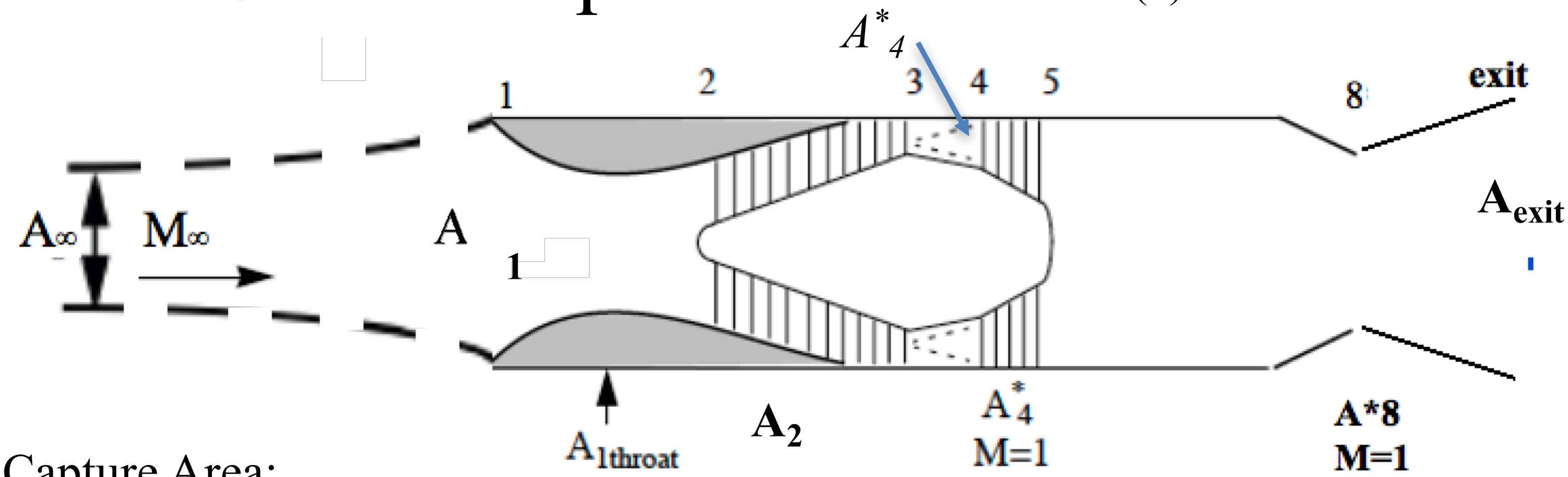


- Summary of State Calculations

Area Pressure, Temperature, Velocty Ratios, Mach Function, and Mach Number

A2/A*2	A*4/A8	A∞/A1 t	Mexit
1.56153	0.636859	0.980453	2.1972
FM2	**M2**	**Te/T∞**	**Pe/P∞**
0.640399	0.40907	4.08761	1.3349
A1t/A*2	**M1 t**	**P*e/P∞**	**Ve/V∞**
1.04102	0.793409	7.50752	5.22619

Example Calculation (8)



- Capture Area:

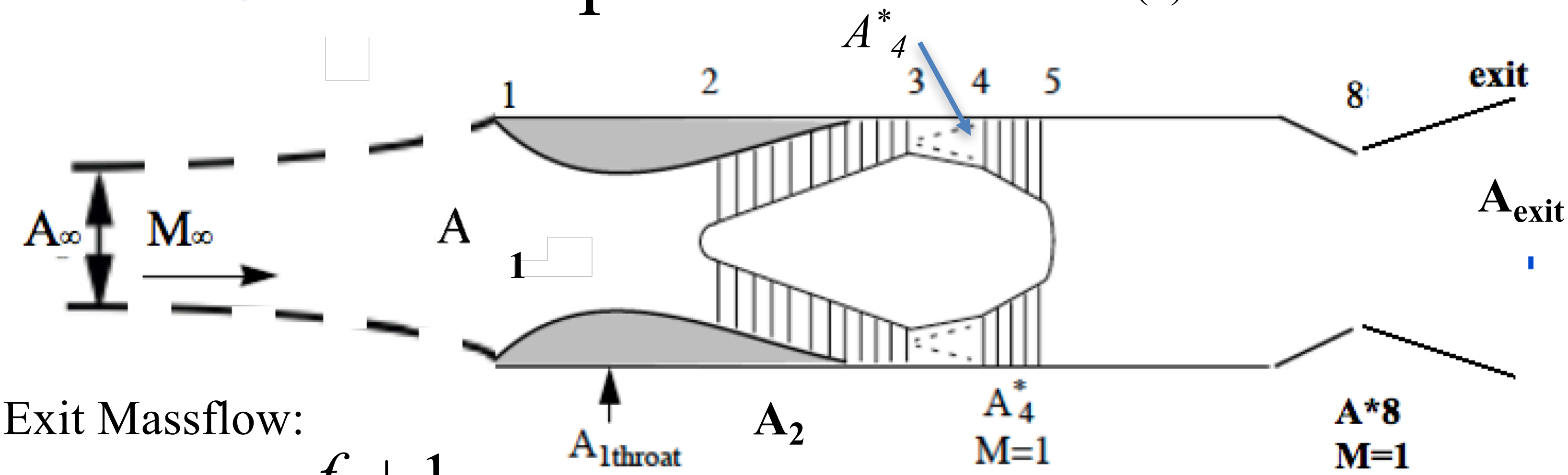
$$A_\infty = \frac{A_\infty}{A_{1_{throat}}} \cdot A_{1_{throat}} = 0.980453 \cdot 10 \quad = 9.80453 \text{ cm^2}$$

- Inlet Massflow:

$$\dot{m}_\infty = \rho_\infty \cdot A_\infty \cdot V_\infty = \frac{p_\infty}{R_g \cdot T_\infty} \cdot A_\infty \cdot V_\infty = \frac{\gamma \cdot p_\infty}{\gamma \cdot R_g \cdot T_\infty} \cdot A_\infty \cdot V_\infty = \sqrt{\frac{\gamma}{R_g \cdot T_\infty}} p_\infty \cdot A_\infty \cdot M_\infty =$$

$$\left(\frac{1.4}{287.056 \cdot 216.65}\right)^{0.5} (22.63 \cdot 10^3)\, 0.85 \frac{9.80453}{100^2} 1000 = 89.490 \text{ g/sec}$$

Example Calculation (9)



- Exit Massflow:

$$\dot{m}_{exit} = \dot{m}_{\infty} \cdot \frac{f+1}{f} =$$

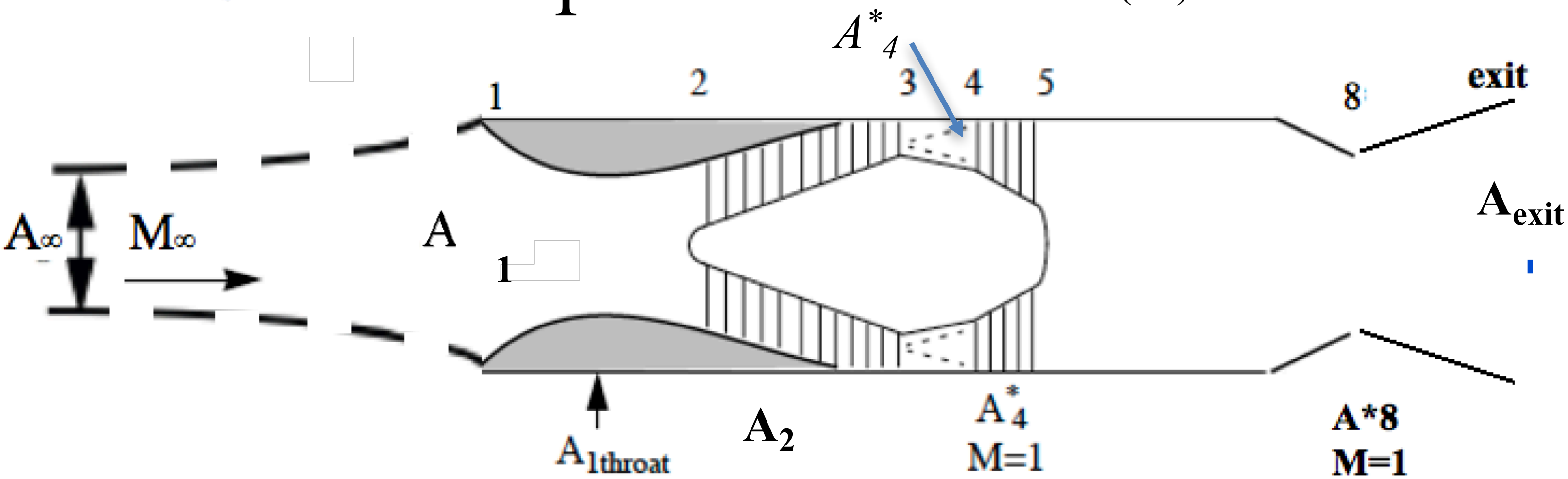
$$\left(\left(\frac{1.4}{287.056 \cdot 216.65}\right)^{0.5} (22.63 \cdot 10^3)\ 0.85\ \frac{9.80453}{100^2}\ 1000\right)\left(\frac{40+1}{40}\right) = 91.7182 \text{ g/sec}$$

- Corrected (Inlet) Massflow:

$$\dot{m}_{cor} = \dot{m}_{\infty} \cdot \sqrt{\frac{T_0}{T_{SL}}} \Big/ \frac{P_0}{P_{SL}} =$$

$$\left(\left(\frac{1.4}{287.056 \cdot 216.65}\right)^{0.5} (22.63 \cdot 10^3)\ 0.85\ \frac{9.80453}{100^2}\ 1000\right) \frac{\left(\frac{247.96}{288.15}\right)^{0.5}}{\frac{36.30}{101.325}} = 231.698 \text{ g/sec}$$

Example Calculation (10)



- Normalized Momentum Thrust:

$$\mathbb{T}_{mom} = \frac{F_{thrust\,mom}}{p_\infty \cdot A_0} = \gamma \cdot M^2_\infty \left(\frac{f+1}{f} \cdot \frac{V_{exit}}{V_\infty} - 1\right) = \quad 1.4 \cdot 0.85^2 \left(\left(\frac{40+1}{40}\right) 5.22619 - 1\right) \quad = 4.40695$$

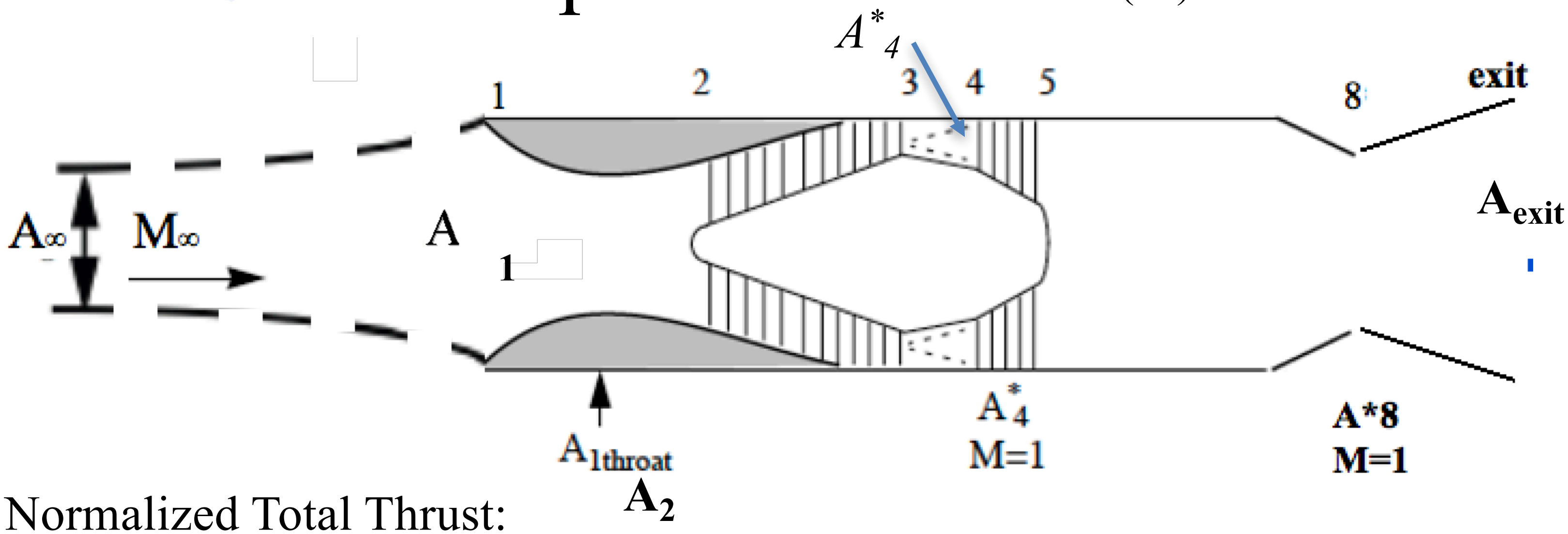
- Normalized Pressure Thrust:

$$\mathbb{T}_{press} = \frac{\left(p_{exit} - p_\infty\right) \cdot A_{exit}}{p_\infty \cdot A_0} = \left(\frac{p_{exit}}{p_\infty} - 1\right) \cdot \frac{A_{exit}}{A_0} = \quad (1.3349 - 1)\, 0.600566 \quad = 0.20113$$

- Normalized Total Thrust:

$$\mathbb{T}_{total} = \gamma \cdot M^2_\infty \left(\frac{f+1}{f} \cdot \frac{V_{exit}}{V_\infty} - 1\right) + \left(\frac{p_{exit}}{p_\infty} - 1\right) \cdot \frac{A_{exit}}{A_0} = \quad 4.60807$$

Example Calculation (11)



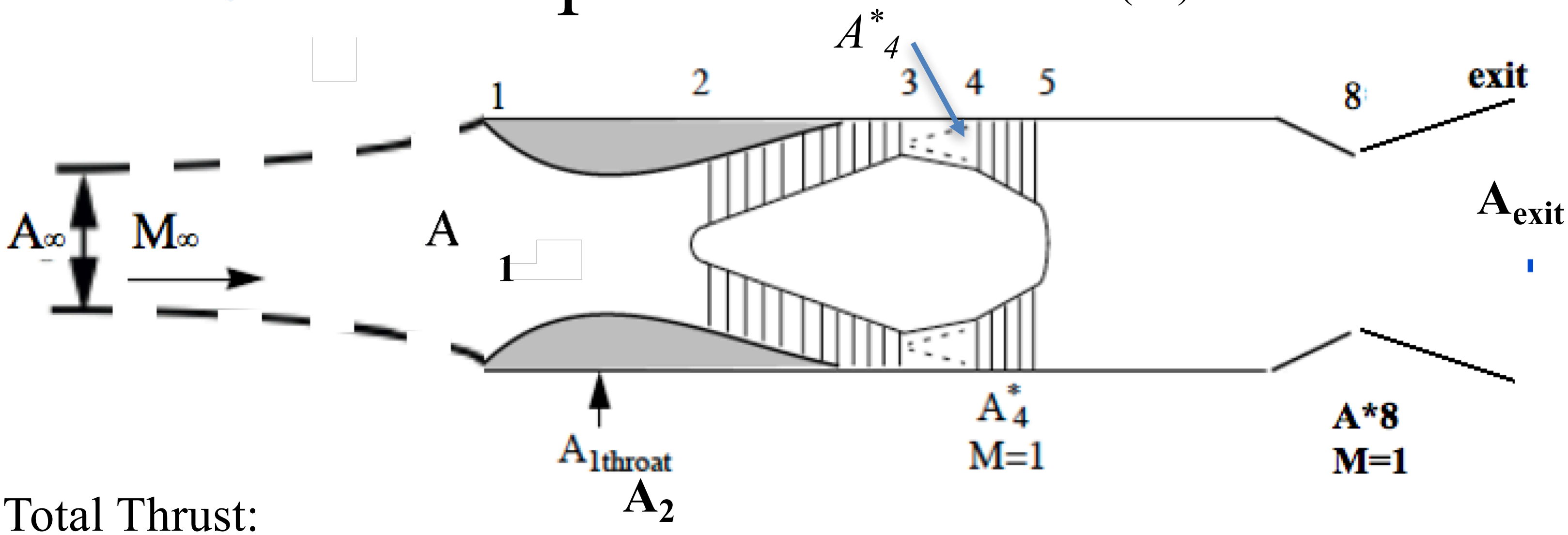
• Normalized Total Thrust:

$$\mathbb{T}_{total} = \gamma \cdot M^2_{\infty}\left(\frac{f+1}{f}\cdot\frac{V_{exit}}{V_{\infty}} - 1\right) + \left(\frac{p_{exit}}{p_{\infty}} - 1\right)\cdot\frac{A_{exit}}{A_0} = \quad 4.60807$$

• Normalized Specific Impulse:

$$\mathbb{I} = \frac{\mathbb{T}_{total}\cdot f}{\gamma \cdot M_{\infty}} = 154.893$$

Example Calculation (12)



• Total Thrust:

$$Thrust = \mathbb{T}_{total} \cdot p_\infty \cdot A_\infty = \; 4.60807 \frac{9.80453}{100^2} 22.63 \cdot 1000$$

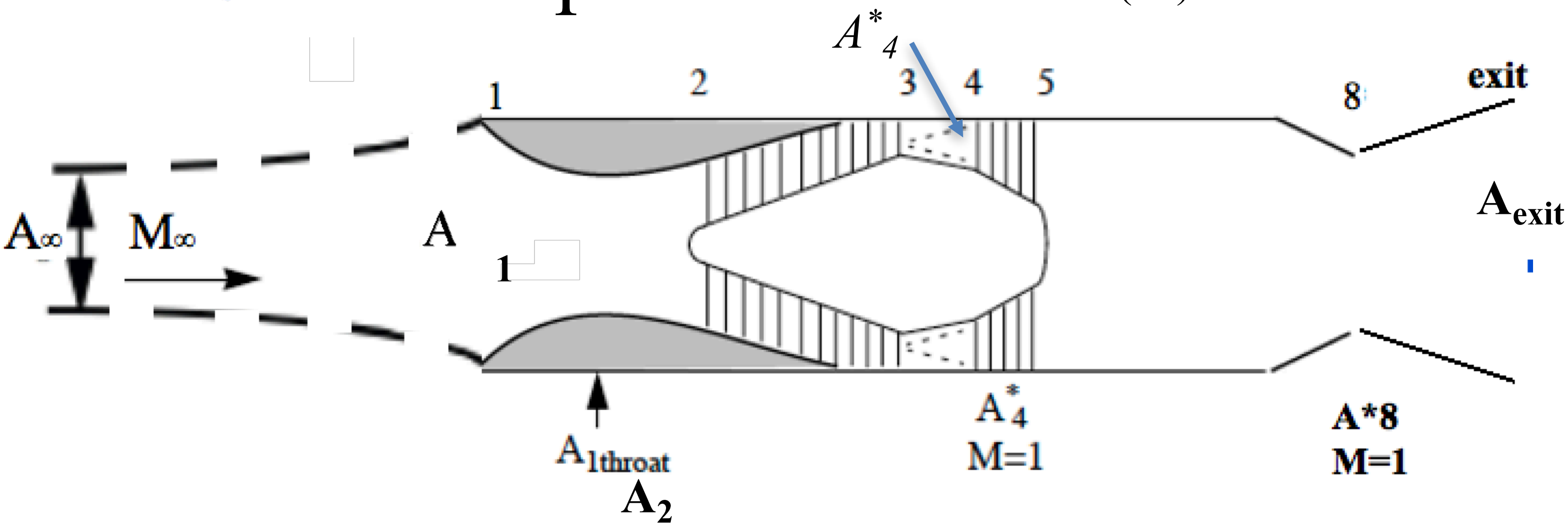
$= 102.252$ N

• Total Specific Impulse:

$$I_{sp} = \mathbb{I} \cdot p_\infty \cdot A_\infty = \; 154.893 \cdot 22.63 \cdot 1000 \frac{9.80453}{100^2}$$

$= 3437.06$ sec

Example Calculation (13)



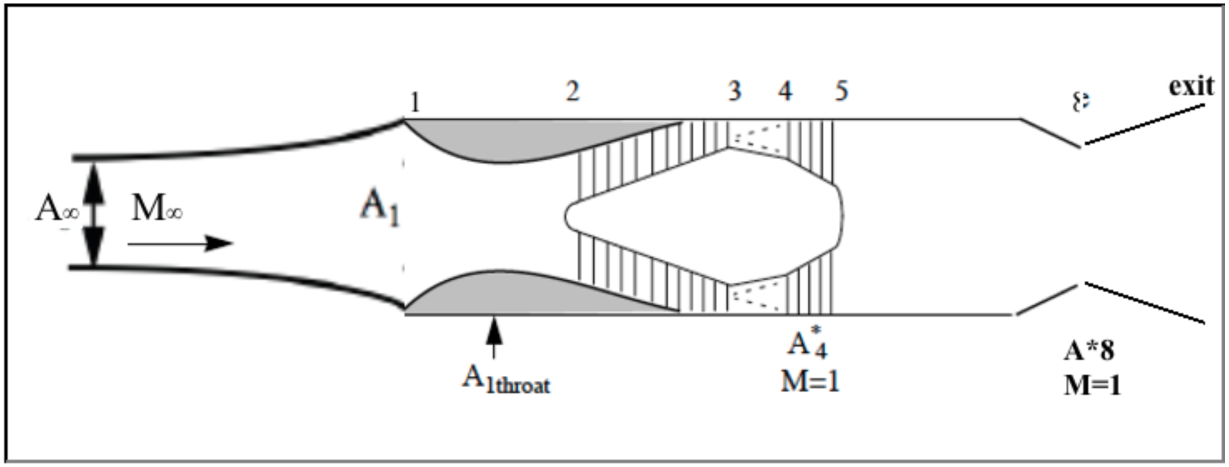
- Thrust Specific Fuel Consumption (TSFC)

$$TSFC = \frac{\dot{m}_f}{F_{total}} = \frac{1}{g_0 \cdot I_{sp}} = \frac{1}{9.8067_{\frac{m}{\sec^2}} \cdot 3437.06_{\sec}} \times 3600_{\frac{\sec}{hr}} = 0.10681_{\frac{kg/hr}{N}}$$

$$TSFC = 0.10681_{\frac{kg/hr}{N}} \times 2.204_{\frac{lbm}{kg}} \times 4.4495_{\frac{N}{lbf}} = 1.04741_{\frac{lbm/hr}{lbf}}$$

Example Calculation (14)

End-to-End Analysis



Freestream Conditions

Mach Number	Fuel Enthalpy, J/kg	Gamma	AIR FUEL RATIO
0.85	4.947E+7	1.4	

Altitude, km	Cp, J/kg-K	Max Burner Temperature, C	AIR FUEL RATIO
11	1004.96	1750	40

Flight Parameters Metric

Qc, kPa	Qbar, kPa	Vtrue, m/sec	P0_inf, kPa
13.67	11.45	250.81	36.30

Pinf, kPa	Tinf, K	CpMax	T0_inf, K
22.63	216.65	1.1939	247.96

Design Parameters

π_c	A2/ A*4	A2/ A1 t	Aexit/A*8	A1_throat, cm^2
15.0000	8	1.5	2	10

Thrust and Performance Data

Ainf, cm^2	Aexit/A∞	Aexit, cm^2	Norm Total Thrust	Total Thrust, N	TSFC/hr
9.80453	0.600566	5.88827	4.60807	102.252	1.04741

Normalized Momentum Thrust	Norm Pressure Thrust	Norm Isp	True Isp, sec
4.40695	0.201127	154.893	3437.06

Temperature Ratios

τ_λ	τ_c
9.33833	2.16783

τ_r	τ_t
1.1445	0.860362

Area Pressure, Temperature, Velocty Ratios, Mach Function, and Mach Number

A2/A*2	A*4/A8	A∞/A1 t	Mexit
1.56153	0.636859	0.980453	2.1972

FM2	M2	Te/T∞	Pe/P∞
0.640399	0.40907	4.08761	1.3349

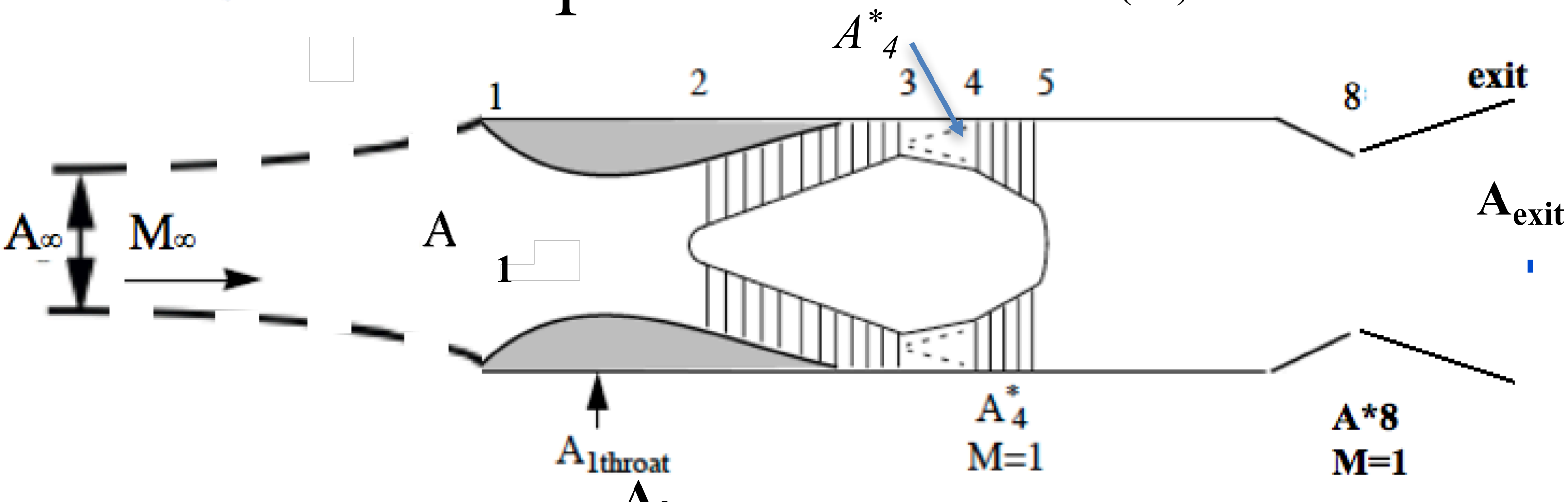
A1t/A*2	M1 t	P*e/P∞	Ve/V∞
1.04102	0.793409	7.50752	5.22619

Massflow, Corrected massflow data

mdot/, g/-sec	P0∞/Psl	A∞, cm^2
89.49	0.358234	9.80453

T0∞/Tsl	Corrected Mdot/A∞ kg/m^2-sec	mdot_COR KG/SEC
0.86051	236.352	231.732

Example Calculation (15)



- Plot Compressor Operating Line vs $F(M_2)$, Corrected massflow

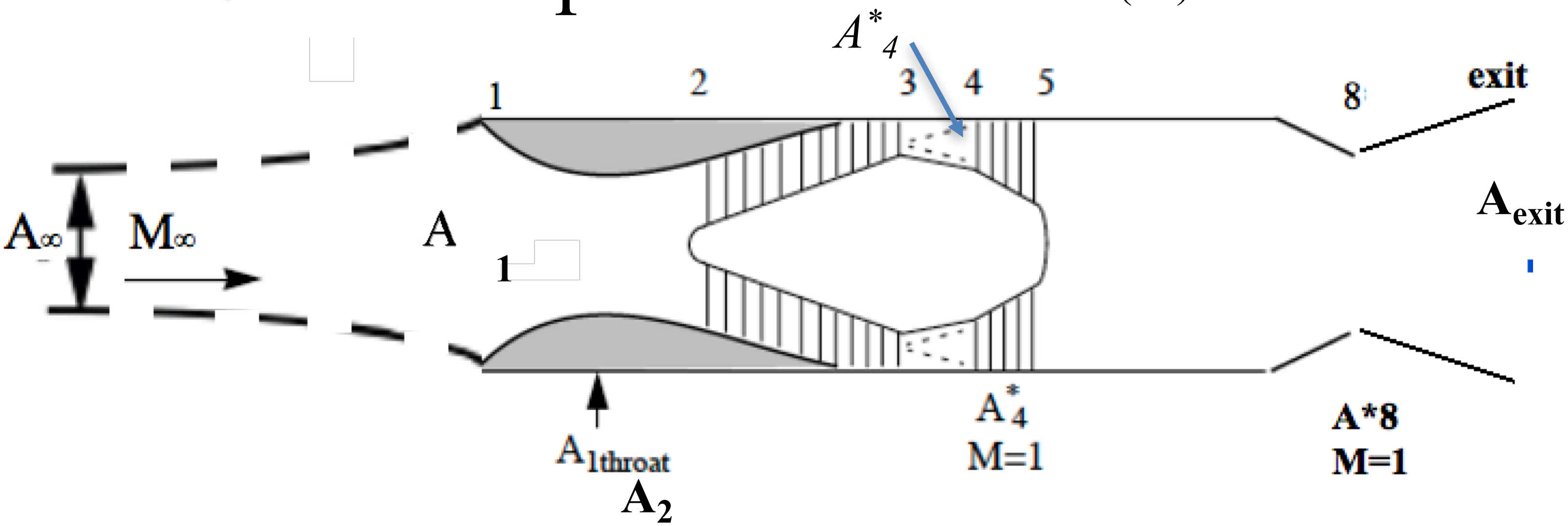
$$\frac{\pi_c}{\sqrt{\pi_c^{\frac{\gamma-1}{\gamma}}-1}} = \sqrt{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right)} \cdot \left(\frac{A_2}{A_4^*}\cdot\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\right)$$

Massflow, Corrected massflow data

mdot/, g/-sec	P0∞/Psl	A∞, cm^2
89.49	0.358234	9.80453
T0∞/Tsl	**Corrected Mdot/A∞ kg/m^2-sec**	**mdot_COR KG/SEC**
0.86051	236.352	231.732

$$F(M_2) = \frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}} \cdot \frac{1}{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{\dot{m}_c}{A_2 \cdot p_{sl}\sqrt{\frac{\gamma}{R_g \cdot T_{sl}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}}$$

Example Calculation (16)



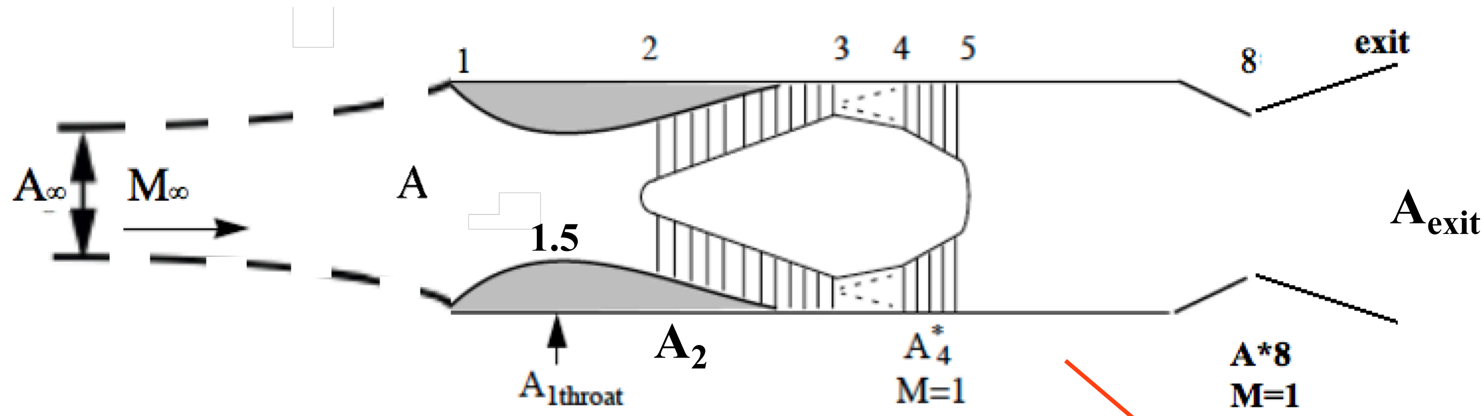
- Plot Capture Area vs $F(M_2)$, Corrected massflow, Compression ratio

$$F(M_2) = \frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{\dot{m}_c}{A_2 \cdot p_{sl}\sqrt{\frac{\gamma}{R_g \cdot T_{sl}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}}$$

$$A_\infty = \pi_d \cdot A_2 \cdot \frac{M_2}{M_\infty}\left(\frac{1+\frac{\gamma-1}{2}M_\infty^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

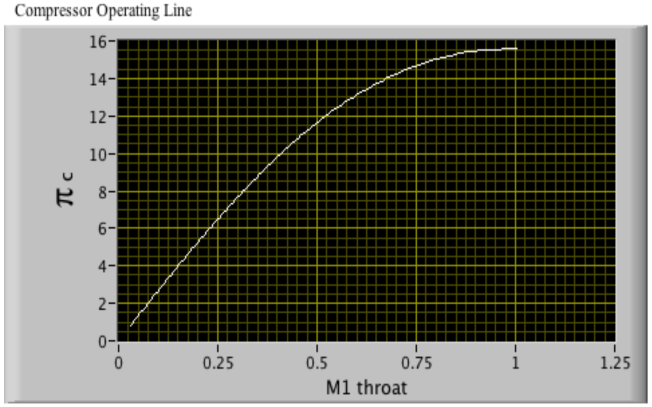
$$\frac{\pi_c}{\sqrt{\pi_c^{\frac{\gamma-1}{\gamma}}-1}} = \sqrt{\left(\frac{f+1}{f}\right)\left(\frac{1}{1-\left(\frac{A_4^*}{A_8}\right)^{2\left(\frac{\gamma-1}{\gamma+1}\right)}}\right)} \cdot \left(\frac{A_2}{A_4^{\,*}} \cdot \frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\right)$$

Example Calculation (17)

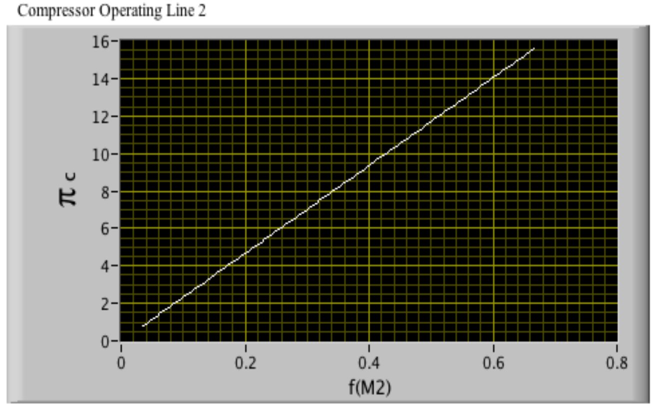


- Plot Compressor Operating Line vs $F(M_2)$, Corrected Massflow

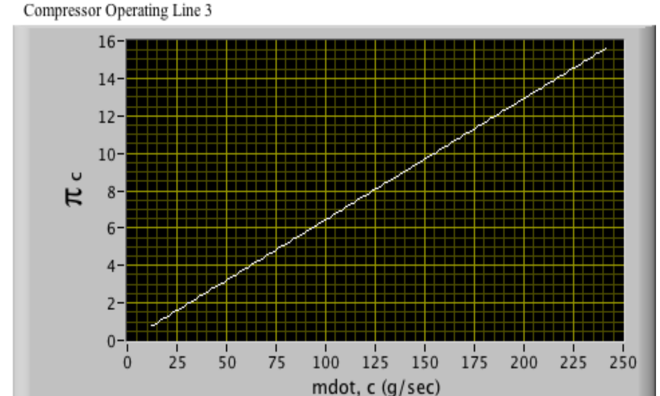
Compressor Operating Line



Compressor Operating Line 2

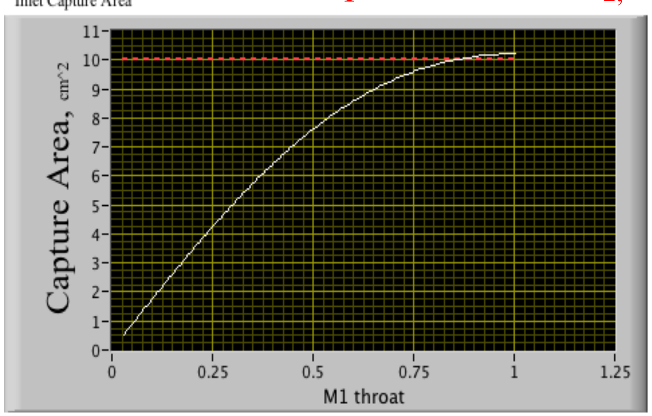


Compressor Operating Line 3

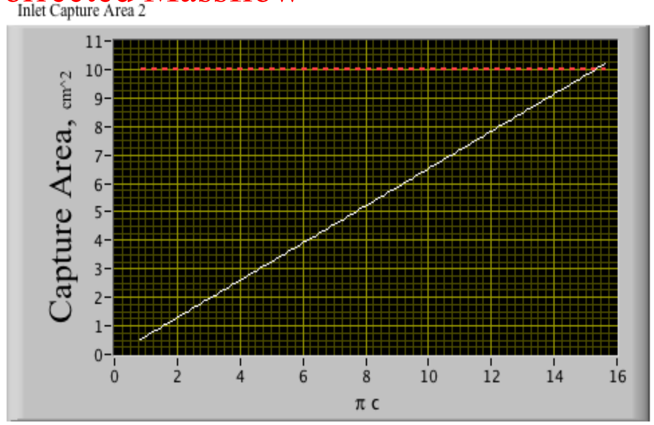


- Plot Inlet Capture Area vs M_2, Corrected Massflow

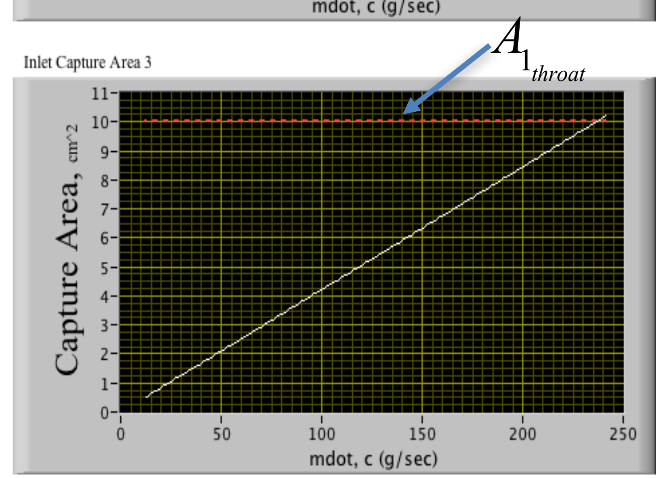
Inlet Capture Area



Inlet Capture Area 2



Inlet Capture Area 3




Questions??