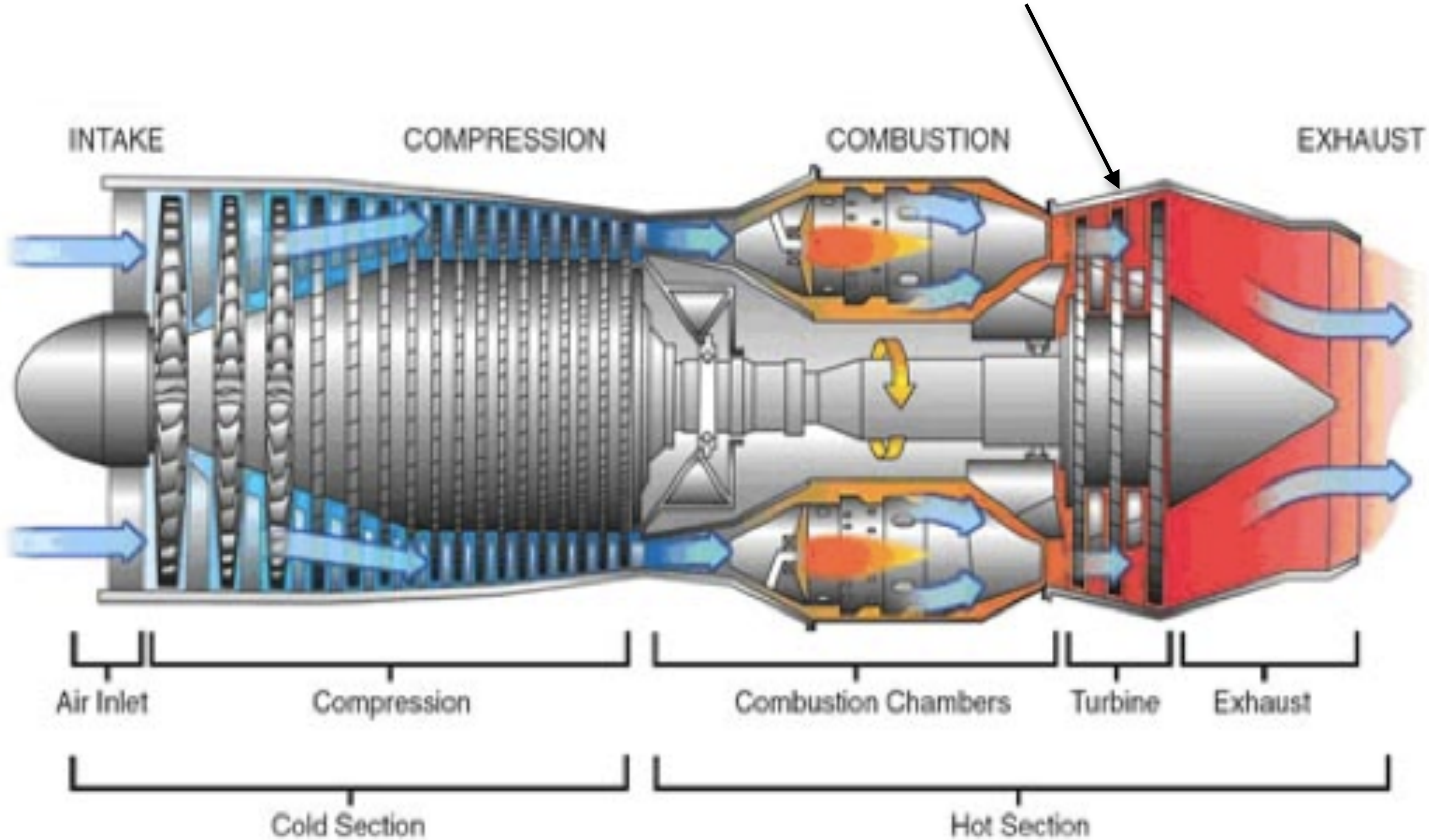
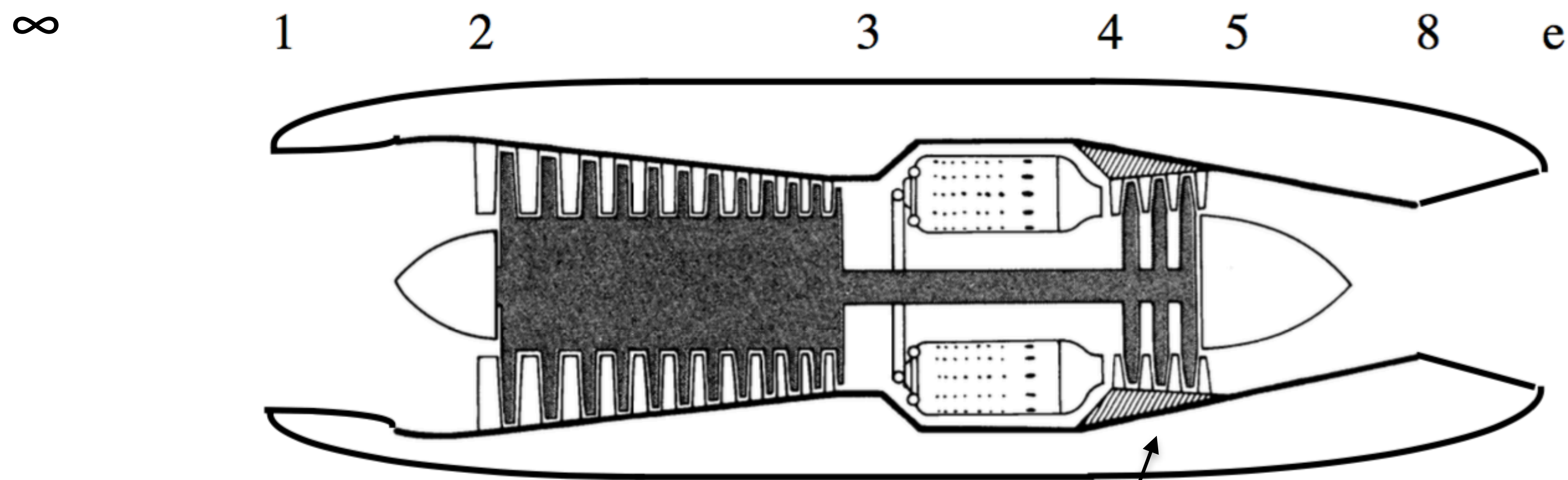
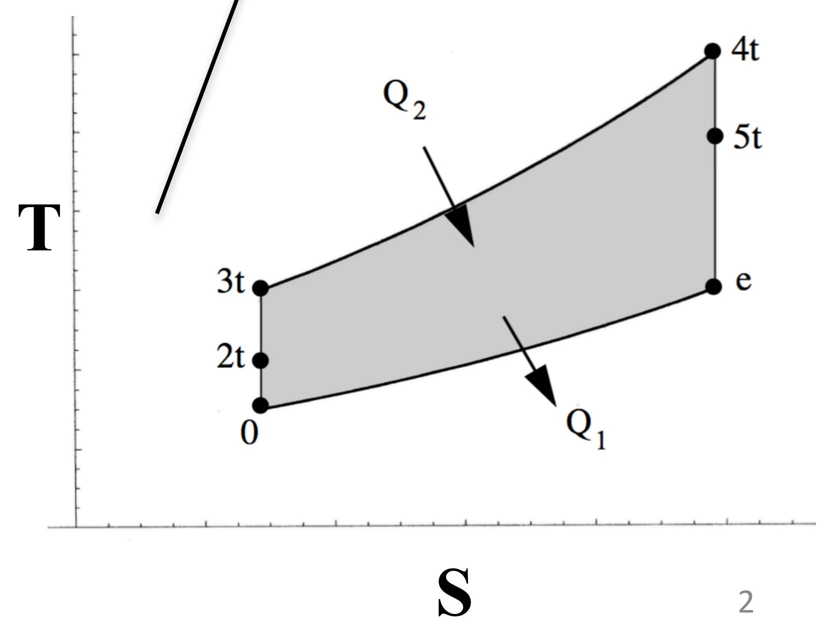
Section 5.4: Non-Ideal TurboJet Operation



Idealized Turbojet Model and The Brayton Cycle



Idealized Assumptions:
1) Inlet and Diffuser are Isentropic
2) Compressor, Turbine ~ Isentropic
3) Burner @Low Mach Number, Constant Pressure
4) Turbine Work = Compressor Work
5) Nozzle is Isentropic
6) Ideally expanded nozzle where $p_{exit} = p_\infty$

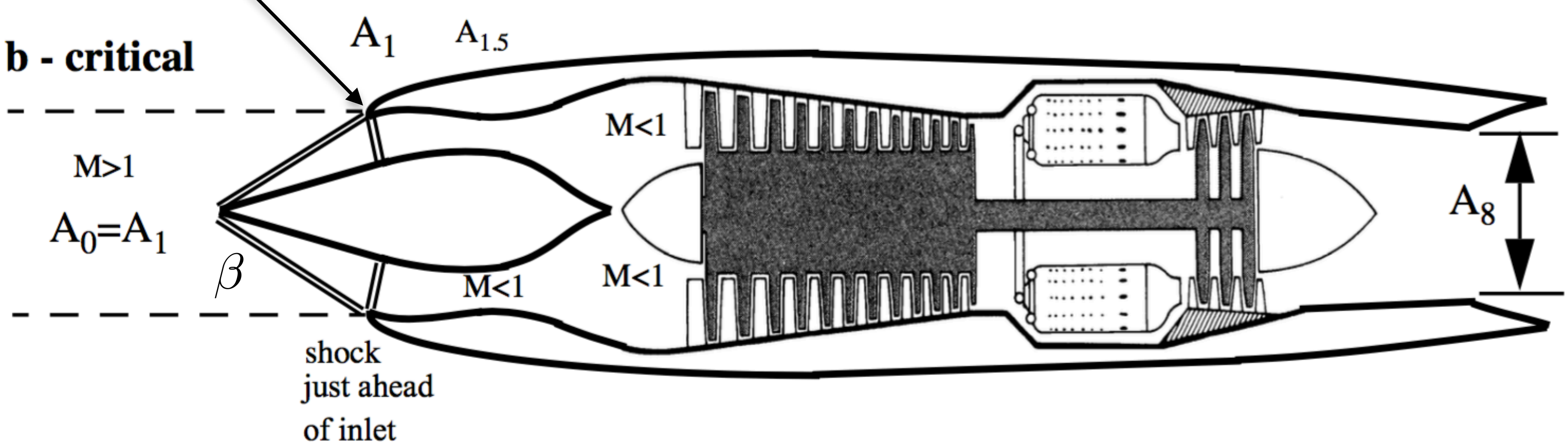


How is this Idealized Model Unrealistic?

• Remember, everywhere there is an irreversibility, then we have entropy growth and a loss of stagnation pressure. Stagnation pressure losses limit the overall efficiency of the propulsion system,

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{\left(T_C - T_B\right)}$$

• We have already studied two of most significant non-ideal mechanisms … inet shock waves and stagnation pressure losses across combustor



How is this Idealized Model Unrealistic? (2)

• Across shock wave(s) entropy increases and we get a resulting stagnation pressure loss, and limits overall system efficiency

$$\frac{P_{02}}{P_{01}} = \frac{2}{(\gamma+1)\left(\gamma\left(M_1 \sin\beta\right)^2 - \frac{(\gamma-1)}{2}\right)^{\frac{1}{\gamma-1}}}\left(\frac{\left[\frac{(\gamma+1)}{2}\left(M_1\sin\beta\right)\right]^2}{\left(1+\frac{\gamma-1}{2}\left(M_1\sin\beta\right)^2\right)}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

• Other sources of stagnation pressure losses include nozzle stagnation pressure losses are associated with viscous skin friction.

• Stagnation pressure losses across the burner due to heat addition also cause π_b to be always less than one.

• Additional reduction of π_b occurs due to wall friction, nonzero burner exit Mach number, and injector drag to to Reynolds stresses (right angle injection into flow).

Combustor Losses and Inefficiencies

- Assuming Mean Values for C_p, γ, M_w, R_g, Conservation of Mass and Momentum across the combustor gives (MAE 5420 Lecture 5.4)

$$\frac{M_4^{\,2}\cdot\left[1+\frac{\gamma-1}{2}M_4^{\,2}\right]}{\left[1+\gamma M_4^{\,2}\right]^2}=\left[\frac{T_{04}}{T_{03}}\right]_{burner}\cdot\left(\frac{f+1}{f}\right)^2\cdot\frac{M_3^{\,2}\cdot\left[1+\frac{\gamma-1}{2}M_3^{\,2}\right]}{\left[1+\gamma M_3^{\,2}\right]^2}$$

- Conservation of Energy across combustor gives Gives

$$C_p\cdot\left(\dot{m}_{air}+\dot{m}_f\right)\cdot T_{04}=C_p\cdot\left(\dot{m}_{air}\right)\cdot T_{03}+\dot{m}_f\cdot h_f\cdot\eta_{burner}$$

$$\frac{C_p\cdot\left(\dot{m}_{air}+\dot{m}_f\right)}{C_p\cdot\left(\dot{m}_{air}\right)}\frac{T_{04}}{T_{03}}=1+\frac{\dot{m}_f\cdot h_f\cdot\eta_{burner}}{C_p\cdot\left(\dot{m}_{air}\right)\cdot T_{03}}\rightarrow f=\frac{\dot{m}_{air}}{\dot{m}_f}$$

$$\frac{f+1}{f}\cdot\frac{T_{04}}{T_{03}}=1+\frac{1}{f}\frac{\dot{m}_f\cdot h_f\cdot\eta_{burner}}{C_p\cdot T_{03}}\rightarrow\boxed{\frac{T_{04}}{T_{03}}=\left(\frac{f}{f+1}\right)\cdot\left(1+\frac{1}{f}\frac{h_f\cdot\eta_{burner}}{C_p\cdot T_{03}}\right)}$$

Combustor Losses and Inefficiencies (2)

• In addition to loss of stagnation pressure, also necessary to account for incomplete combustion, and radiation/conduction heat losses to combustor walls. Combustor efficiency is defined directly from energy balance across burner …

$$\eta_c = \frac{\left(\frac{f+1}{f}\right)\cdot h_{0_4} - h_{0_3}}{\frac{1}{f} h_f} = \frac{(f+1)\cdot h_{0_4} - f\cdot h_{0_3}}{h_f}$$

• Finally, second law of thermodynamic gives

$$\frac{\Delta s_{burner}}{C_p} = \ln\left[\frac{T_{04}}{T_{03}}\right] - \frac{\gamma-1}{\gamma}\cdot\ln\left[\frac{P_{04}}{P_{03}}\right] \quad = \ln\left(\frac{M_4{}^2}{M_3{}^2}\left(\frac{1+\gamma M_3{}^2}{1+\gamma M_4{}^2}\right)^{\frac{\gamma+1}{\gamma}}\right)$$

And …Solving for Stagnation Pressure Ratio

$$\rightarrow \frac{P_{04}}{P_{03}} = \left[\left(\frac{T_{04}}{T_{03}}\right)\cdot\left(\frac{M_3{}^2}{M_4{}^2}\left(\frac{1+\gamma M_4{}^2}{1+\gamma M_3{}^2}\right)^{\frac{\gamma+1}{\gamma}}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Combustor Losses and Inefficiencies (3)

- Thus the collected conservation equations are

$$\frac{P_{04}}{P_{03}} = \left(\frac{f}{f+1}\right)\cdot\left(1+\frac{1}{f}\frac{\dot{m}_f\cdot h_f\cdot\eta_{burner}}{C_p\cdot\left(\dot{m}_{air}\right)\cdot T_{03}}\right)\cdot\left(\frac{M_3^{\,2}}{M_4^{\,2}}\left(\frac{1+\gamma M_4^{\,2}}{1+\gamma M_3^{\,2}}\right)^{\frac{\gamma+1}{\gamma}}\right)$$

$$\frac{T_{04}}{T_{03}} = \left(\frac{f}{f+1}\right)\cdot\left(1+\frac{1}{f}\frac{h_f\cdot\eta_{burner}}{C_p\cdot T_{03}}\right)$$

$$\frac{M_4^{\,2}\cdot\left[1+\frac{\gamma-1}{2}M_4^{\,2}\right]}{\left[1+\gamma M_4^{\,2}\right]^2} = \left[\frac{T_{04}}{T_{03}}\right]_{burner}\cdot\left(\frac{f+1}{f}\right)^2\cdot\frac{M_3^{\,2}\cdot\left[1+\frac{\gamma-1}{2}M_3^{\,2}\right]}{\left[1+\gamma M_3^{\,2}\right]^2}$$

- Parametric equation set allows plot of stagnation pressure ratio as a function of compressor outlet Mach number (combustor inlet Mach number) M_3

Combustor Losses and Inefficiencies (4)

- Example

Freestream Conditions 2

Freestream Mach Number	Fuel Enthalpy, Mkg	Gamma	Combustor efficiency
0.8	40	1.4	0.9
Altitude, km	**Cp, J/kg-K**	**Air/Fuel Ratio**	**Compression Ratio**
11	1004.96	50	20

Combustor Inlet Mach Number Range from 0 to 0.5 …

Flight Parameters Metric

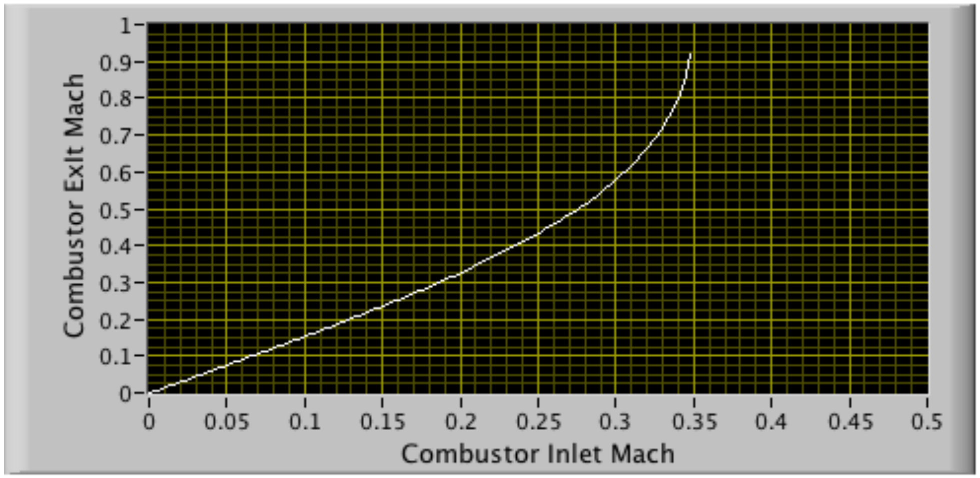
Qc, kPa	Qbar, kPa	Vtrue, m/sec	P0_inf, kPa
11.87	10.14	236.06	34.50
Pinf, kPa	**Tinf, K**	**CpMax**	**T0_inf, K**
22.63	216.65	1.1704	244.38

Combustor Thermal Results

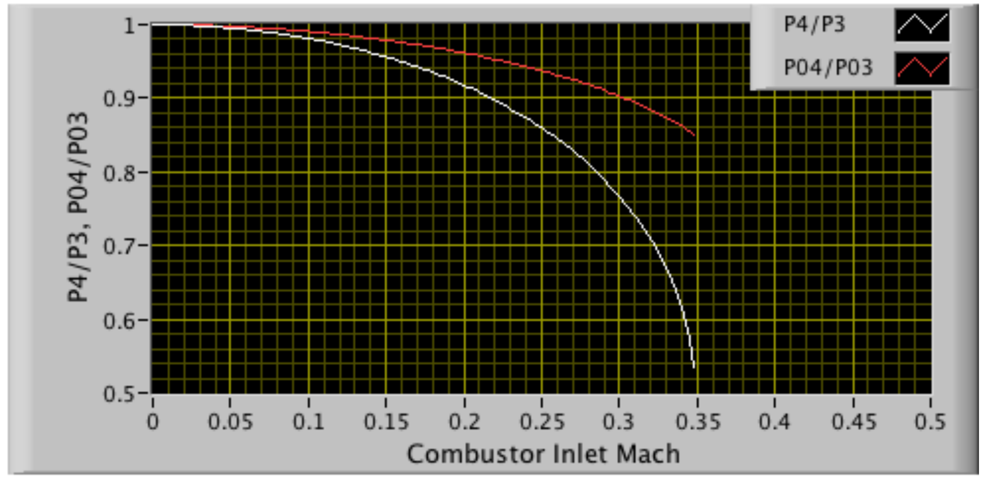
T0_3, K	Tau_b (T04/T03)	Tau_lambda (T04/Tinf)
575.16	2.20161	5.84483
Delta Q, J/kg	**T0_4 K**	
7.2E+5	1266.28	

Combustor Losses and Inefficiencies(4)

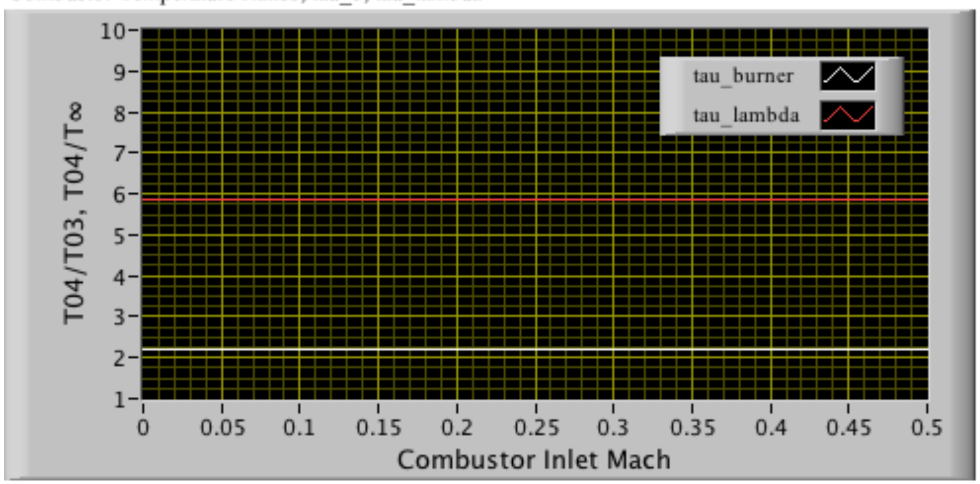
Combustor Exit Mach Number



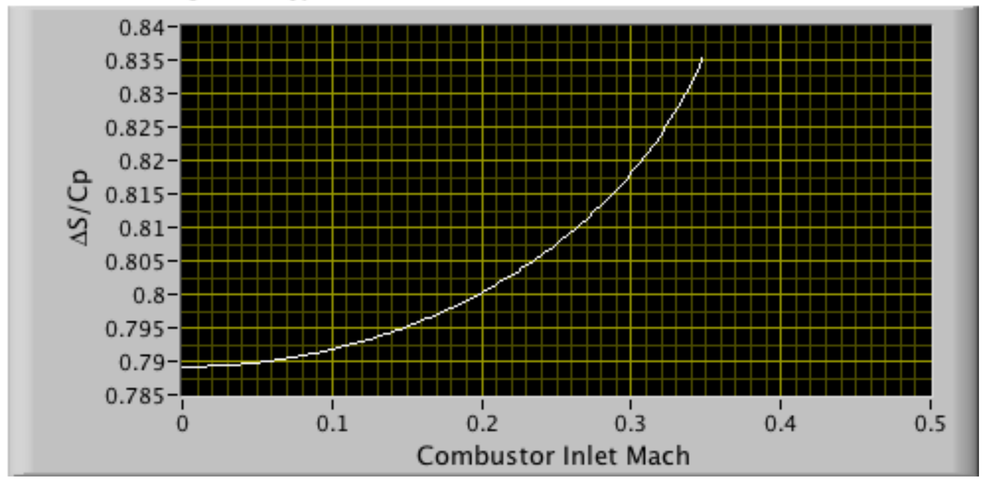
Combustor PRESSURE Ratio, P4/ P3 & P04/P03



Combustor Temperature Ratios, tau_b, tau_lambda



Normalized Change in Entropy



- Stagnation pressure losses across the burner due to heat addition cause π_b to be always less than one …rule of thumb is

$$\pi_b = 1 - constant \times \gamma M_3^{\,2}$$

Compressor and Turbine Losses and Inefficiencies

• The shaft that connects the turbine and compressor is subject to frictional losses in the bearings that support the shaft and a shaft mechanical efficiency is defined using the work balance across the compressor and turbine. Typical shaft efficiencies are slightly less than unity.

$$\eta_{\substack{c/t \\ Mech}} = \frac{h_{0_3} - h_{0_2}}{\left(\frac{f+1}{f}\right) \cdot \left(h_{0_4} - h_{0_5}\right)}$$

• For the ideal case we have analyzed the compression and turbine cycles are isentropic, i.e.

$$\pi_c = \tau_c^{\gamma/(\gamma-1)} \quad . \ . \ . \quad \pi_t = \tau_t^{\gamma/(\gamma-1)}$$

• But with losses in these cycles, these relationships no longer strictly hold, and an adjustment is necessary to account for the losses

Compressor and Turbine Losses and Inefficiencies (2)

- Defining

$$\eta_c = \frac{\textit{The work needed to reach } P_{0_3}/P_{0_2} \textit{ in an isentropic compression process}}{\textit{The work needed to reach } P_{0_3}/P_{0_2} \textit{ in the real compression process}} =$$

$$\eta_c = \frac{h_{0_3}|_{\Delta s=0} - h_{0_2}}{h_{0_3} - h_{0_2}}$$

- and

$$\eta_t = \frac{\textit{The work output in reaching } P_{0_5}/P_{0_4} \textit{ in the real expansion process process}}{\textit{The work output in reaching } P_{0_5}/P_{0_4} \textit{ in an isentropic expansion process}} =$$

$$\eta_t = \frac{h_{0_4} - h_{0_5}}{h_{0_4} - h_{0_5}|_{\Delta s=0}}$$

Compressor and Turbine Losses and Inefficiencies (3)

- The efficiencies { η_c, η_t } allow the relationship between compressor/turbine temperature and pressure ratios as a *"polytropic process," The polytropic process is a* measure of the degree to which the compression process is isentropic,

$$\frac{P_{03}}{P_{02}} = \pi_c = \tau_c^{\frac{\gamma}{(\gamma-1)}\cdot\eta_{pc}} \quad \ldots \quad \pi_t = \frac{P_{05}}{P_{04}} = \tau_t^{\frac{\gamma}{(\gamma-1)}\cdot\eta_{pt}}$$

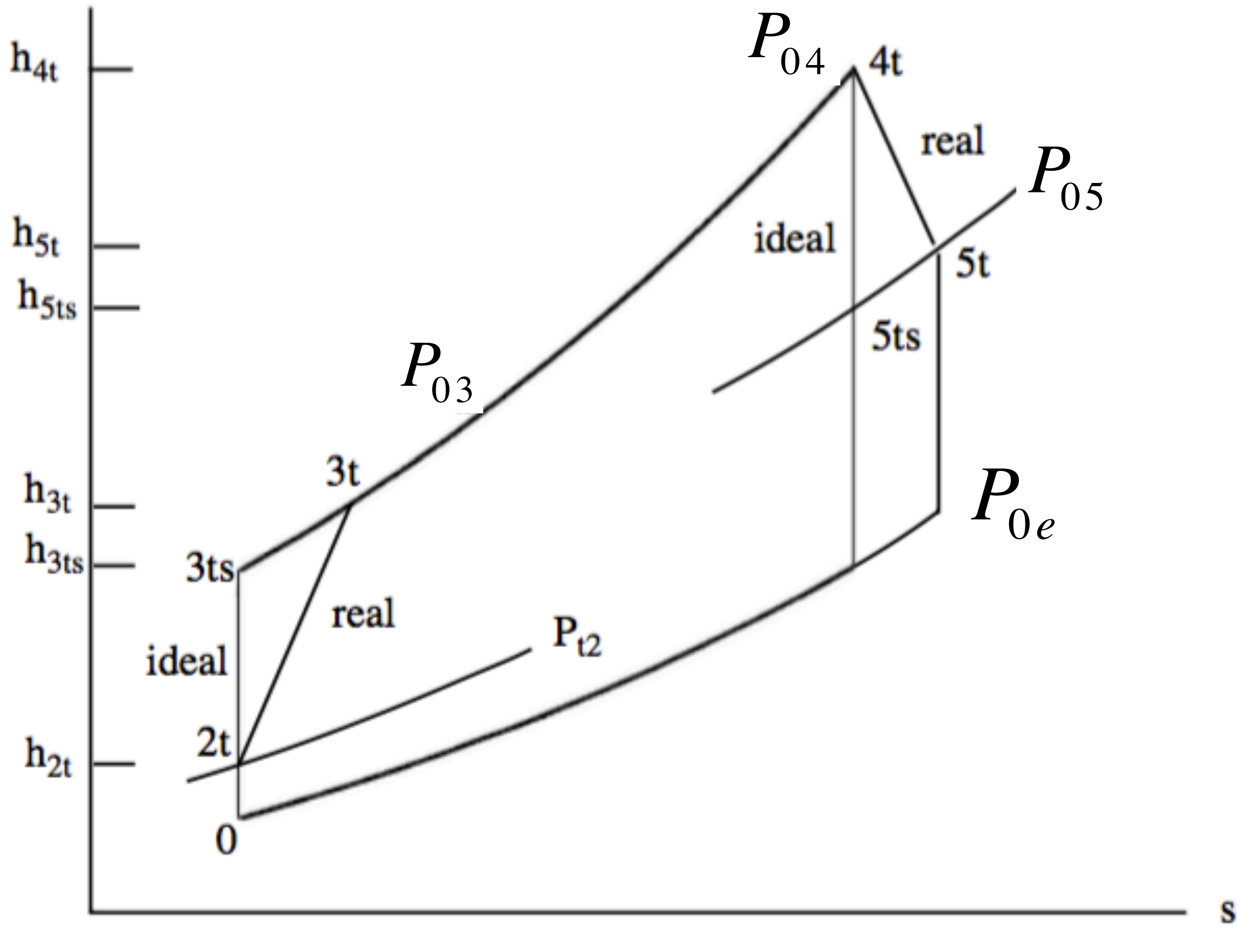
Modern compressors are designed to have values of η_c in the range 0.88 to 0.92.

and ...

$$\eta_c = \frac{\frac{h_{0_3}}{h_{0_2}}\Big|_{\Delta s=0} - 1}{\frac{h_{0_3}}{h_{0_2}} - 1} == \frac{\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}\cdot\frac{1}{\eta_{pc}}} - 1} \quad \ldots \quad \eta_t = \frac{1 - \frac{h_{0_5}}{h_{0_4}}}{1 - \frac{h_{0_5}\big|_{\Delta s=0}}{h_{0_4}}} = \frac{\left(\frac{P_{05}}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}\cdot\frac{1}{\eta_{pt}}} - 1}{\left(\frac{P_{05}}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

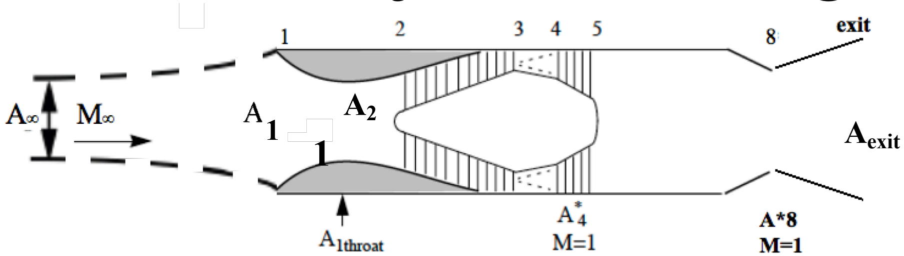
Modern turbines are designed to values of η_t in the range 0.91 to 0.94.

Adjusted Brayton Cycle Plot for Non-Ideal TurboJet Operation



h-s path of a turbojet with non-ideal compressor and turbine.

Turbojet, Matching Example (2)



- Engine operates at a free stream Mach number, $M_\infty = 0.80$
- Cruise Altitude is in the stratosphere, 11 km so $T_\infty = 216.65$ *K*, $p_\infty = 22.63$ *kPa*.
- The design turbine inlet temperature, $T_{04} = 2000$ *K (1726.85* ^oC*)*
- The design compressor ratio range, $\pi_c = 2\text{-}10$.
- Relevant area ratios are $A_2 / A^*_4 = 9.65$ and $A_2/A_{1throat} = 1.45$.
- Inlet throat area $A_{1Throat} = 2000\ cm^2$ *(50.463 cm, 19.87" diameter)*
- *Assume the compressor, burner and turbine all operate ideally.*
- *Converging/Diverging type Nozzle with choked throat*
- *Stagnation pressure losses due to wall friction in the inlet and nozzle are negligible.*
- *Octane (Gasoline) Fuel,* $h_f = 49.47$ *MJ/kg*

Note: the Nozzle Throat Area for this Problem must Vary to Accommodate the Changes in Compression Ratio

Part 1. Assume sonic nozzle exit, $\pi_c = 2$ **.. 10** ***CALCULATE***

a) *Compressor Operating Line, Plot* π_c *vs. Corrected massflow,* $\pi_c = 2 \rightarrow 10$

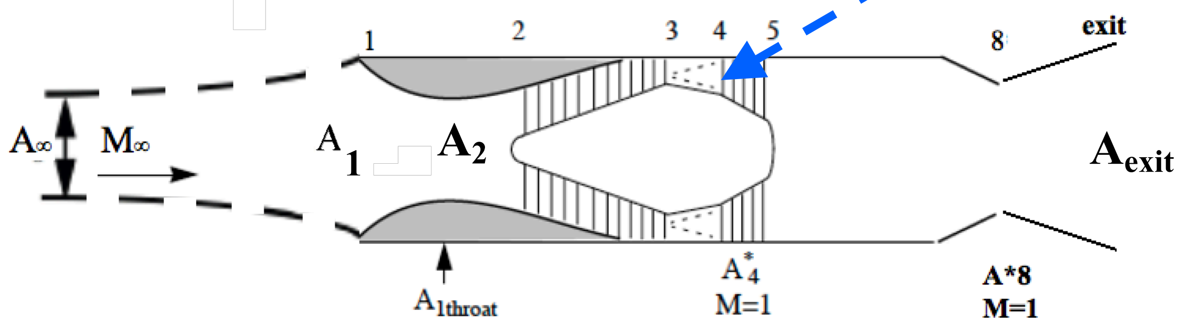
b) *Overlay Operating Line on J-85 Compressor Map*
   - *You can manually plot Operating line on Map Image or Use .xls file link for Compressor map*
   - *What is the Design Operating Condition at 100% Rotor Speed*
     - *(corrected massflow, compression ratio)*
   - *Plot the Engine Surge and Choke Margins as a function of % Rotor Speed*
     - *Surge Margin =*
     - *Choke margin =*

$$100\% \times \left(\frac{\dot{m}_w - \dot{m}_{surge,\, choke}}{\dot{m}_w} \right)$$

c) *Plot Diffuser Throat, Compressor face Mach numbers, AND Inlet Capture Area vs* π_c

d) *Plot Fuel-to-Air Ratio (1/f) vs.* π_c*, as required to maintain* T_{04} *at 2000 K*

Turbojet, Matching Example (3)



Incoming Air to Turbojet (@ to station 3)

• Molecular weight	=	28.96443 *kg/kg-mole*
• γ	=	1.40
• R_g	=	287.058 *J/kg-K*
• T_∞	=	230 K
• p_∞	=	26 kPa
• V_∞	=	220 *m/sec*

• Universal Gas Constant: R_u = 8314.4612 *J/kg-K*

Calorically Perfect Gas

$$\gamma = \frac{c_p}{c_v}$$

$$R_g = c_p - c_v$$

$$c_p = \frac{\gamma}{\gamma - 1} \cdot R_g$$

$$c_v = \frac{1}{\gamma - 1} \cdot R_g$$

Assume γ, c_p, M_w are constant across engine

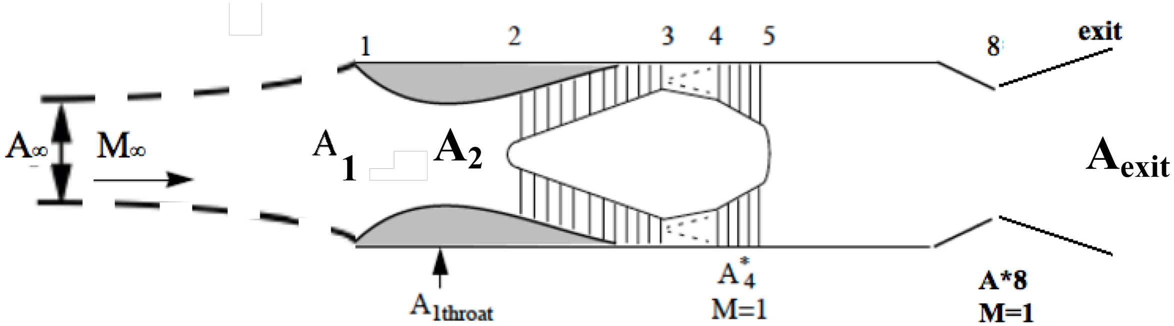
For …Isentropic Conditions →

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Ideal Gas

$$p = \rho \cdot R_g \cdot T$$

Turbojet, Matching Example (4)



Parameter Definitions

$$\{\tau_r, \tau_c, \tau_\lambda, \tau_f, \gamma\} \rightarrow$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \rightarrow \text{Freestream Mach number reference conditions}$$

$$\tau_c = \frac{T_{0_3}}{T_{0_2}} \rightarrow \text{Compressor stagnation temperature ratio measure of compressor work input}$$

$$\tau_\lambda = \frac{T_{0_4}}{T_\infty} \rightarrow \text{Combustor flame temperature...Optimized up to Material limits of combustor, turbine}$$

$$\tau_f = \frac{h_f}{h_\infty} \rightarrow \text{Fuel enthalpy of combustion relative to incoming air stream total enthalpy}$$

$$\gamma = \frac{C_p}{C_v} \rightarrow \text{Ratio of specific heats}$$

Choice of Fuel


Questions??