Section 5.4: Non-Ideal TurboJet Operation
Idealized Turbojet Model and The Brayton Cycle

Idealized Assumptions:
1) Inlet and Diffuser are Isentropic
2) Compressor, Turbine ~ Isentropic
3) Burner @Low Mach Number, Constant Pressure
4) Turbine Work = Compressor Work
5) Nozzle is Isentropic
6) Ideally expanded nozzle where \( p_{exit} = p_{\infty} \)
How is this Idealized Model Unrealistic?

• Remember, everywhere there is an irreversibility, then we have entropy growth and a loss of stagnation pressure. Stagnation pressure losses limit the overall efficiency of the propulsion system,

\[ \eta = 1 - \left( \frac{P_A}{P_B} \right)^{\gamma - 1} \left( \frac{P_{0_B}}{P_{0_A}} \right)^{\gamma} \frac{T_C - \left( \frac{P_{0_B}}{P_{0_A}} \right)^{\gamma} T_B}{(T_C - T_B)} \]

• We have already studied two of most significant non-ideal mechanisms … inet shock waves and stagnation pressure losses across combustor
How is this Idealized Model Unrealistic? (2)

- Across shock wave(s) entropy increases and we get a resulting stagnation pressure loss, and limits overall system efficiency

\[ \frac{P_{02}}{P_{01}} = \frac{2}{(\gamma + 1) \left( \gamma \left( M_1 \sin \beta \right)^2 - \frac{(\gamma - 1)}{2} \right)^{\frac{1}{\gamma - 1}}} \left( \frac{\left( \frac{\gamma + 1}{2} \left( M_1 \sin \beta \right)^2 \right)^2}{\left( 1 + \frac{\gamma - 1}{2} \left( M_1 \sin \beta \right)^2 \right)^{\frac{\gamma}{\gamma - 1}}} \right) \]

- Other sources of stagnation pressure losses include nozzle stagnation pressure losses are associated with viscous skin friction.
- Stagnation pressure losses across the burner due to heat addition also cause \( \pi_b \) to be always less than one.
- Additional reduction of \( \pi_b \) occurs due to wall friction, nonzero burner exit Mach number, and injector drag to to Reynolds stresses (right angle injection into flow).
Combustor Losses and Inefficiencies

• Assuming Mean Values for $C_p, \gamma, M_w, R_g$, Conservation of Mass and Momentum across the combustor gives (MAE 5420 Lecture 5.4)

$$M_4^2 \cdot \left[1 + \frac{\gamma - 1}{2} M_4^2 \right] \frac{1}{\left[1 + \gamma M_4^2 \right]^2} = \frac{T_{04}}{T_{03}} \text{burner} \cdot \left(\frac{f+1}{f}\right)^2 \cdot M_3^2 \cdot \left[1 + \frac{\gamma - 1}{2} M_3^2 \right] \frac{1}{\left[1 + \gamma M_3^2 \right]^2}$$

• Conservation of Energy across combustor gives

$$C_p \cdot (\dot{m}_{air} + \dot{m}_f) \cdot T_{04} = C_p \cdot (\dot{m}_{air}) \cdot T_{03} + \dot{m}_f \cdot h_f \cdot \eta_{\text{burner}}$$

$$\frac{C_p \cdot (\dot{m}_{air} + \dot{m}_f)}{C_p \cdot (\dot{m}_{air})} \cdot \frac{T_{04}}{T_{03}} = 1 + \frac{\dot{m}_f \cdot h_f \cdot \eta_{\text{burner}}}{C_p \cdot (\dot{m}_{air} \cdot T_{03}} \rightarrow f = \frac{\dot{m}_{air}}{\dot{m}_f}$$

$$\frac{f+1}{f} \cdot \frac{T_{04}}{T_{03}} = 1 + \frac{1 \cdot \dot{m}_f \cdot h_f \cdot \eta_{\text{burner}}}{f \cdot C_p \cdot T_{03}} \rightarrow \frac{T_{04}}{T_{03}} = \left(\frac{f}{f+1}\right) \cdot \left(1 + \frac{1 \cdot h_f \cdot \eta_{\text{burner}}}{f \cdot C_p \cdot T_{03}}\right)$$
Combustor Losses and Inefficiencies (2)

• In addition to loss of stagnation pressure, also necessary to account for incomplete combustion, and radiation/conduction heat losses to combustor walls. Combustor efficiency is defined directly from energy balance across burner …

\[
\eta_c = \frac{\left( \frac{f + 1}{f} \right) \cdot h_{04} - h_{03}}{\frac{1}{f} h_f} = \frac{(f + 1) \cdot h_{04} - f \cdot h_{03}}{h_f}
\]

• Finally, second law of thermodynamic gives

\[
\frac{\Delta s_{burner}}{C_p} = \ln \left[ \frac{T_{04}}{T_{03}} \right] - \frac{\gamma - 1}{\gamma} \cdot \ln \left[ \frac{P_{04}}{P_{03}} \right] = \ln \left( \frac{M_4^2}{M_3^2} \left( \frac{1 + \gamma M_4^2}{1 + \gamma M_3^2} \right)^{\frac{\gamma+1}{\gamma}} \right)
\]

And …Solving for Stagnation Pressure Ratio

\[
\rightarrow \frac{P_{04}}{P_{03}} = \left[ \left( \frac{T_{04}}{T_{03}} \right) \cdot \left( \frac{M_3^2}{M_4^2} \left( \frac{1 + \gamma M_4^2}{1 + \gamma M_3^2} \right)^{\frac{\gamma+1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \right]
\]
Combustor Losses and Inefficiencies (3)

- Thus the collected conservation equations are

\[
\frac{P_{04}}{P_{03}} = \left(\frac{f}{f+1}\right) \cdot \left(1 + \frac{1}{f} \frac{m_f \cdot h_f \cdot \eta_{burner}}{C_p \cdot (m_{air}) \cdot T_{03}}\right) \cdot \left(\frac{M_3}{M_4} \left(\frac{1 + \gamma M_4^2}{1 + \gamma M_3^2}\right)^{\frac{\gamma+1}{\gamma}}\right)
\]

\[
\frac{T_{04}}{T_{03}} = \left(\frac{f}{f+1}\right) \cdot \left(1 + \frac{1}{f} \frac{h_f \cdot \eta_{burner}}{C_p \cdot T_{03}}\right)
\]

\[
M_4^2 \cdot \left[1 + \frac{\gamma-1}{2} M_4^2 \right] = \left[1 + \frac{\gamma-1}{2} M_4^2 \right] \cdot \left(\frac{T_{04}}{T_{03}}\right)_{\text{burner}} \cdot \left(\frac{f+1}{f}\right)^2 \cdot \left[1 + \frac{\gamma-1}{2} M_3^2 \right] \cdot \left[1 + \gamma M_3^2 \right]^2
\]

- Parametric equation set allows plot of stagnation pressure ratio as a function of compressor outlet Mach number (combustor inlet Mach number) \(M_3\)
Combustor Losses and Inefficiencies (4)

- Example

Freestream Conditions 2

<table>
<thead>
<tr>
<th>Freestream Mach Number</th>
<th>Fuel Enthalpy, Mkg</th>
<th>Gamma</th>
<th>Combustor efficiency</th>
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</thead>
<tbody>
<tr>
<td>0.8</td>
<td>40</td>
<td>1.4</td>
<td>0.9</td>
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<td>Altitude, km</td>
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<td>11</td>
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<td></td>
</tr>
<tr>
<td>Cp, J/kg-K</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1004.96</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Air/Fuel Ratio</td>
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<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compression Ratio</td>
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</tr>
<tr>
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</tbody>
</table>

Flight Parameters

Metric

<table>
<thead>
<tr>
<th>Qc, kPa</th>
<th>Qbar, kPa</th>
<th>Vtrue, m/sec</th>
<th>P0_inf, kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.87</td>
<td>10.14</td>
<td>236.06</td>
<td>34.50</td>
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</table>

<table>
<thead>
<tr>
<th>Pinf, kPa</th>
<th>Tinf, K</th>
<th>CpMax</th>
<th>T0_inf, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.63</td>
<td>216.65</td>
<td>1.1704</td>
<td>244.38</td>
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</tbody>
</table>

Combustor Thermal Results

<table>
<thead>
<tr>
<th>T0_3, K</th>
<th>Tau_b (T04/T03)</th>
<th>Tau_lambda (T04/Tinf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>575.16</td>
<td>2.20161</td>
<td>5.84483</td>
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</table>

<table>
<thead>
<tr>
<th>Delta Q, J/kg</th>
<th>T0_4 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2E+5</td>
<td>1266.28</td>
</tr>
</tbody>
</table>
Stagnation pressure losses across the burner due to heat addition cause $\pi_b$ to be always less than one …rule of thumb is

$$\pi_b = 1 - \text{constant} \times \gamma M_3^2$$
Compressor and Turbine Losses and Inefficiencies

• The shaft that connects the turbine and compressor is subject to frictional losses in the bearings that support the shaft and a shaft mechanical efficiency is defined using the work balance across the compressor and turbine. Typical shaft efficiencies are slightly less than unity.

\[
\eta_{c/t}^{Mech} = \frac{h_{03} - h_{02}}{\left(\frac{f + 1}{f}\right) \cdot (h_{04} - h_{05})}
\]

• For the ideal case we have analyzed the compression and turbine cycles are isentropic, i.e.

\[
\pi_c = \tau_c \gamma/(\gamma - 1) \quad \ldots \quad \pi_t = \tau_t \gamma/(\gamma - 1)
\]

• But with losses in these cycles, these relationships no longer strictly hold, and an adjustment is necessary to account for the losses

MAE 6530 - Propulsion Systems II
Compressor and Turbine Losses and Inefficiencies (2)

- Defining

\[
\eta_c = \frac{\text{The work needed to reach } \frac{P_{03}}{P_{02}} \text{ in an isentropic compression process}}{\text{The work needed to reach } \frac{P_{03}}{P_{02}} \text{ in the real compression process}}
\]

\[
\eta_c = \frac{h_{03} \big|_{\Delta s=0} - h_{02}}{h_{03} - h_{02}}
\]

- and

\[
\eta_e = \frac{\text{The work output in reaching } \frac{P_{05}}{P_{04}} \text{ in the real expansion process process}}{\text{The work output in reaching } \frac{P_{05}}{P_{04}} \text{ in an isentropic expansion process}}
\]

\[
\eta_e = \frac{h_{04} - h_{05}}{h_{04} - h_{05} \big|_{\Delta s=0}}
\]
Compressor and Turbine Losses and Inefficiencies (3)

- The efficiencies \( \{ \eta_{pc}, \eta_{pt} \} \) allow the relationship between compressor/turbine temperature and pressure ratios as a “polytropic process,” The polytropic process is a measure of the degree to which the compression process is isentropic,

\[
\frac{P_{03}}{P_{02}} = \pi_c = \tau_c \frac{\gamma}{(\gamma-1)} \eta_{pc}
\]

... \[ \pi_t = \frac{P_{05}}{P_{04}} = \tau_t \frac{\gamma}{(\gamma-1)} \eta_{pt} \]

Modern compressors are designed to have values of \( \eta_{pc} \) in the range 0.88 to 0.92.

\[
\eta_c = \frac{h_{03} |_{\Delta s=0} - 1}{h_{02}} \frac{1}{\left( \frac{P_{03}}{P_{02}} \right)^{\frac{1}{\gamma}}} - 1
\]

... \[ \eta_t = \frac{1 - \frac{h_{05}}{h_{04}}}{\left( \frac{P_{05}}{P_{04}} \right)^{\frac{\gamma-1}{\gamma}}} \frac{1}{\eta_{pt}} - 1 \]

Modern turbines are designed to have values of \( \eta_{pt} \) in the range 0.91 to 0.94.
Adjusted Brayton Cycle Plot for Non-Ideal TurboJet Operation

h-s path of a turbojet with non-ideal compressor and turbine.
Homework 5.4

- Engine operates at a free stream Mach number, \( M_\infty = 0.8 \).
- Cruise Altitude is in the stratosphere, 11 km so \( T_\infty = 216.65 \text{ K} \).
- The design turbine inlet temperature, \( T_{04} = 1944 \text{ K} \).
- The design compressor ratio, \( \pi_c = 20 \).
- Relevant area ratios are \( A_2/A_4^* = 10 \) and \( A_2/A_{1\text{throat}} = 1.2 \).
- Inlet throat area \( A_{1\text{throat}} = 20 \text{ cm}^2 \).
- Assume the compressor, burner and turbine all operate ideally.
- Nozzle is of a simple converging type with choked throat, \( A_8^* = A_{\text{exit}} \).
- Stagnation pressure losses due to wall friction in the inlet and nozzle are negligible.

\[ \frac{f + 1}{f} \approx 1 \]

\( \Rightarrow \) **CALCULATE**

- a) Correct Compressor massflow and \( M_2 \) at compressor face
- b) Normalized exit pressure thrust, momentum thrust, and total thrust
- c) Velocity ratio across Engine \( V_{\text{exit}}/V_\infty \)
- d) Mach number at diffuser throat, \( M_{\text{1throat}} \)
- e) Inlet capture area
Now allow an expandable Nozzle where, \( A_{exit} \geq A^*_8 \)

**CALCULATE**

a) Optimal expansion ratio for nozzle \( A_{exit}/A^*_8 \)

b) Velocity ratio across Engine \( V_{exit}/V_{\infty} \)

c) Thrust of optimal nozzle

d) Assuming the same combustor temperature and inlet throat area

- Plot the Compressor operating line \( \pi_c \) vs corrected massflow for \( 1 \leq \pi_c < 15 \)
- Plot the capture area \( A_{\infty} \) vs corrected massflow for \( 1 \leq \pi_c < 15 \)
Questions??