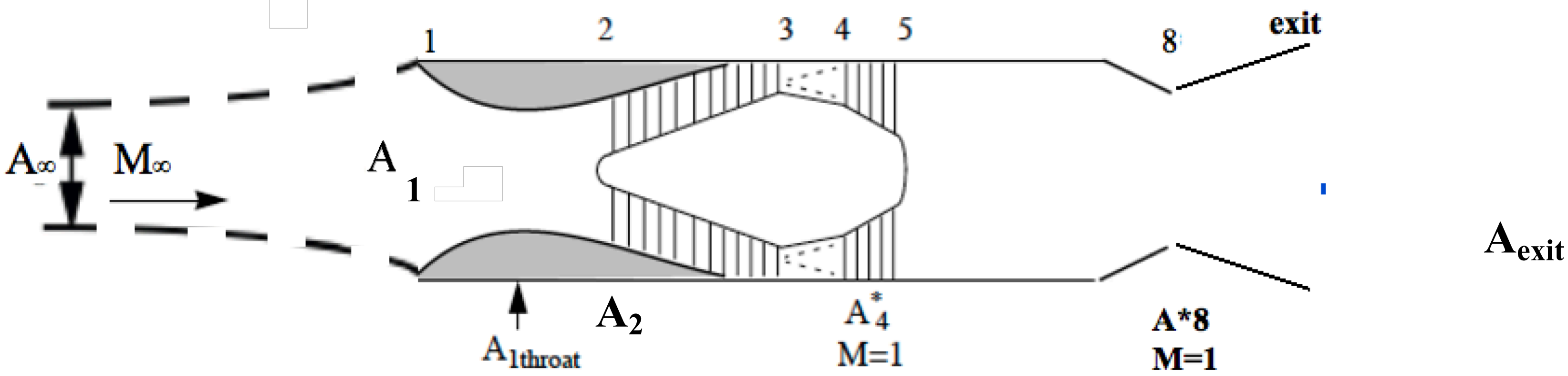
Turbojet, Matching Example



- Engine operates at a free stream Mach number, $M_\infty = 0.85$
- Cruise Altitude is in the stratosphere, 11 km so $T_\infty = 216.65$ K (11 km altitude).
- The design turbine inlet temperature, $T_{04} = 1750\ ^oC$
- The design compressor ratio, $\pi_c = 15$.
- Relevant area ratios are $A_2 / A^*_4 = 8$ and $A_2 / A_{1throat} = 1.5$.
- Inlet throat area $A_{1Throat} = 40\ cm^2$
- *Assume the compressor, burner and turbine all operate ideally.*
- *Converging/Diverging type Nozzle with choked throat*
- *Stagnation pressure losses due to wall friction in the inlet and nozzle are negligible.*

→ *CALCULATE*

→ *a) air fuel ratio, for $h_f = 50$ MJ/kg*

→ *Compressor Massflow and M_2 at compressor face*

→ *b) Optimal Nozzle Expansion Ratio, Normalized thrust and Isp*

→ *c) Velocity ratio across Engine V_{exit}/V_∞*

→ *d) Mach number at diffuser throat, $M_{1throat}$*

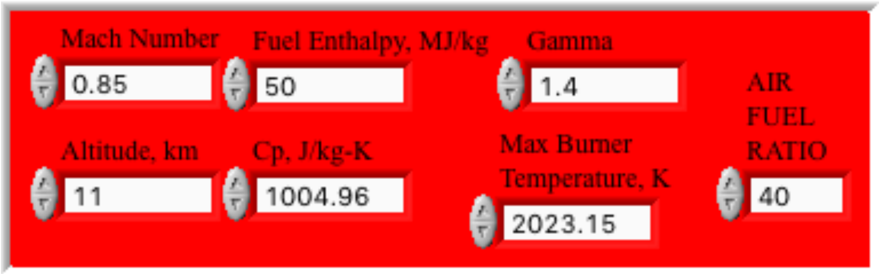
→ *e) Inlet capture area*

→ *f) Total Thrust, Isp, TSFC*

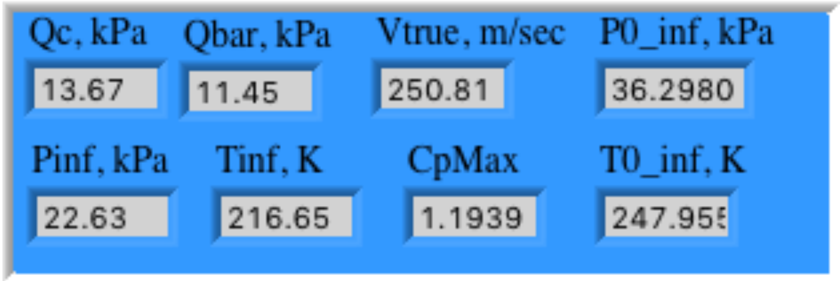
→ *Propulsive, Thermal, and Total Efficiency*

Freestream Conditions

Freestream Conditions



Flight Parameters
Metric



$$\rightarrow \left| \begin{array}{l} M_\infty = 0.85 \\ h = 11_{km} (36{,}090_{ft}) \end{array} \right. \rightarrow \left| \begin{array}{l} p_\infty = 22.632_{kPa} \\ T_\infty = 216.65_K \end{array} \right.$$

$$\rightarrow \left| \begin{array}{l} P_{0_\infty} = p_\infty \cdot \left(1 + \dfrac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}} = \\ T_{0_\infty} = T_\infty \cdot \left(1 + \dfrac{\gamma - 1}{2} M_\infty^2\right) = \end{array} \right.$$

$$22.632\left(1 + \frac{1.4 - 1}{2} 0.85^2\right)^{\frac{1.4}{(1.4-1)}} = 36.2976 \text{ kPa}$$

$$216.65\left(1 + \frac{1.4 - 1}{2} 0.85^2\right) = 247.956 \text{ Deg K}$$

Reference Conditions

Freestream Conditions

Mach Number	Fuel Enthalpy, MJ/kg	Gamma	AIR FUEL RATIO
0.85	50	1.4	

Altitude, km	Cp, J/kg-K	Max Burner Temperature, K	AIR FUEL RATIO
11	1004.96	2023.15	40

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1.1445$$

$$\tau_f = \frac{h_f}{h_\infty} = \frac{50 \cdot 10^6}{1004.96 \cdot 216.65} = 229.42$$

$$\tau_\lambda = \frac{T_{0_4}}{\tau_\infty} = \frac{(1750 + 273.15)}{216.65} = 9.33833$$

Temperature Ratios

t_l	t_c
9.33833	2.16783
t_r	**t_t**
1.1445	0.861191

AIR FUEL RATIO

32.128

TauF

229.648

$$\tau_c = \pi_c^{\frac{\gamma-1}{\gamma}} = 15^{\frac{(1.4-1)}{1.4}} = 2.16783$$

$$f = \frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r \cdot \tau_c} =$$

$$\frac{\left(\frac{50 \cdot 10^6}{1004.96 \cdot 216.65}\right) - 9.33833}{9.33833 - 1.1445 \cdot 2.16783} = 32.128$$

$$\tau_t = 1 - \frac{\tau_r \cdot (\tau_c - 1)}{\left(1 + \frac{1}{f}\right)\tau_\lambda} =$$

$$1 - \frac{1.1445\,(2.16783 - 1)}{\left(1 + \frac{1}{32.128}\right) 9.33833} = 0.861192$$

Compressor & Inlet Conditions

$$f(M_2) = \frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^{\,*}}{A_2} = \frac{32.128}{(32.128+1)}\left(\frac{15}{\left(\frac{9.33833}{1.1445}\right)^{0.5}}\right)\left(\frac{1}{8}\right)$$

$$= 0.636595$$

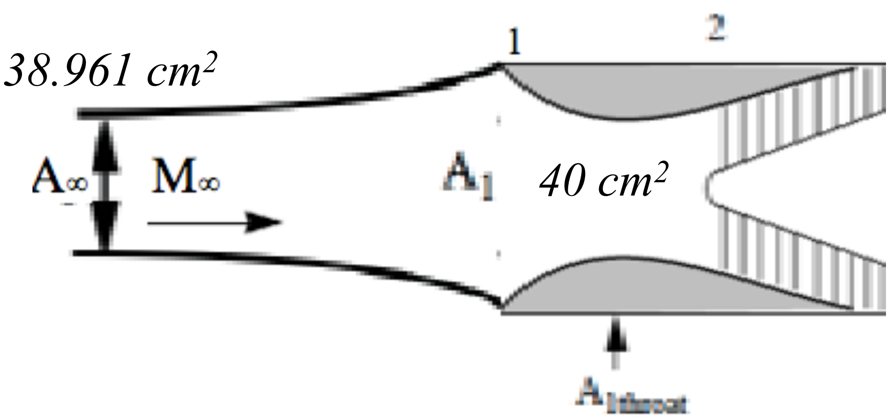
$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{1}{A_2/A_1^{\,*}} \rightarrow A_2/A_1^{\,*} = 1.57086 \rightarrow \boxed{M_2 = 0.4061}$$

$$\frac{A_{1throat}}{A_1^{\,*}} = \frac{A_2}{A_1^{\,*}} \Bigg/ \frac{A_2}{A_{1throat}} = \frac{1.57086}{1.5} = 1.04724 \rightarrow \boxed{M_{1_{throat}} = 0.7796}$$

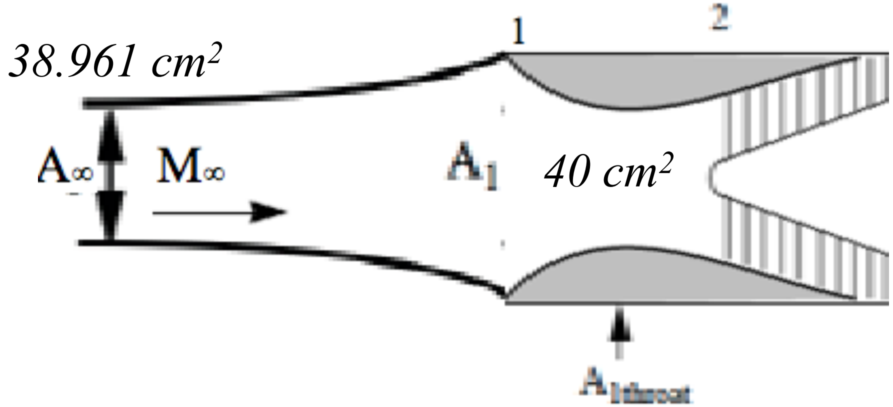
Compressor & Inlet Conditions (2)

$$\frac{A_\infty}{A_{1throat}} = \frac{A_2}{A_{1throat}}\frac{A_\infty}{A_2} = \frac{A_2}{A_{1throat}}\cdot\pi_d\cdot\frac{\dfrac{\sqrt{\gamma}\cdot M_2}{\left(1+\dfrac{\gamma-1}{2}M_2{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}{\dfrac{\sqrt{\gamma}\cdot M_\infty}{\left(1+\dfrac{\gamma-1}{2}M_\infty{}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}} = \frac{\left(\dfrac{1.4^{0.5}\,0.406061}{\left(1+\dfrac{1.4-1}{2}\left(0.406061^2\right)\right)^{\frac{1.4+1}{2(1.4-1)}}}\right)}{\dfrac{1.4^{0.5}\,0.85}{\left(1+\dfrac{1.4-1}{2}\left(0.85^2\right)\right)^{\frac{1.4+1}{2(1.4-1)}}}}\,1.5 = 0.974628$$

$$A_\infty = \frac{A_\infty}{A_{1throat}}\cdot A_{1throat} = 0.974628\cdot 40 = 38.961_{cm^2}$$



Compressor & Inlet Conditions (2)



$$\dot{m}_{air} = \rho_\infty \cdot V_\infty \cdot A_\infty = \frac{p_\infty}{R_g \cdot T_\infty} \cdot V_\infty \cdot A_\infty = \frac{p_\infty}{\sqrt{R_g \cdot T_\infty}} \cdot \frac{\sqrt{\gamma} \cdot V_\infty}{\sqrt{\gamma \cdot R_g \cdot T_\infty}} \cdot A_\infty = \sqrt{\frac{\gamma}{R_g \cdot T_\infty}} \cdot p_\infty \cdot M_\infty \cdot A_\infty =$$

$$\left(\frac{1.4}{287.06 \cdot 216.65}\right)^{0.5} 22.63 \cdot 1000 \cdot 0.85 \, \frac{38.9851}{10000} \, 1000 = 355.834 \text{ g/sec}$$

$$\dot{m}_{fuel} = \frac{\dot{m}_{air}}{f} = \frac{355.834}{32.128} = 11.0755 \text{ g/sec}$$

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End-to-End Conditions

$$\left(\frac{V_{exit}}{V_\infty}\right) = \sqrt{\left(\frac{(\tau_r \cdot \tau_c \cdot \tau_t) - 1}{(\tau_r - 1)}\right)\left(\frac{\tau_\lambda}{\tau_c \tau_r}\right)} =$$

$$\left(\left(\left(\frac{1.1445 \cdot 2.16783 \cdot 0.861191 - 1}{(1.1445 - 1)}\right)\frac{9.33833}{2.16783 \cdot 1.1445}\right)^{0.5}\right) = 5.44128$$

- Calculate the Stagnation Pressure Ratio Across Engine

$$\frac{P_{0_{exit}}}{P_{0_\infty}} = \pi_d \cdot \pi_c \cdot \pi_t = \quad 1 \cdot 15 \cdot 0.861191^{\left(\frac{1.4}{1.4 - 1}\right)} = 8.89077$$

- Calculate the Exit Stagnation Pressure

$$P_{0_{exit}} = \frac{P_{0_{exit}}}{P_{0_\infty}} \cdot p_\infty \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{(\gamma-1)}} = \quad 22.63 \left(1 + \frac{1.4 - 1}{2} 0.85^2\right)^{\frac{1.4}{(1.4 - 1)}} 8.89077 = 322.685$$

kPa

End-to-End Conditions

- Calculate the Exit Plane Mach Number (Optimal Nozzle, $p_{exit} = p_\infty$)

$$M_{exit} = \sqrt{\frac{2}{\gamma - 1}\left(\left(\frac{P_{0_{exit}}}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)} = \left(\frac{2}{1.4-1}\left(\left(\frac{322.685}{22.63}\right)^{\frac{1.4-1}{1.4}} - 1\right)\right)^{0.5} = 2.384$$

- Calculate the Stagnation Temperature Ratio Across Engine

$$\frac{T_{0_{exit}}}{T_{0_\infty}} = \frac{\tau_\lambda}{\tau_r} \cdot \tau_t = \frac{9.33833 \cdot 0.861191}{1.1445} = 7.02672$$

- Calculate the Temperature Ratio Across Engine

$$\frac{T_{exit}}{T_\infty} = \frac{1}{T_\infty}\frac{T_{0_{exit}}}{\left(1+\frac{\gamma-1}{2}M_{exit}^{\;2}\right)} = \frac{1}{T_\infty}\frac{T_{0_\infty}\cdot\frac{T_{0_{exit}}}{T_{0_\infty}}}{\left(1+\frac{\gamma-1}{2}M_{exit}^{\;2}\right)} = \frac{\left(1+\frac{\gamma-1}{2}M_\infty^{\;2}\right)}{\left(1+\frac{\gamma-1}{2}M_{exit}^{\;2}\right)}\cdot\frac{T_{0_{exit}}}{T_{0_\infty}} = \frac{\left(1+\frac{1.4-1}{2}0.85^2\right)}{1+\frac{1.4-1}{2}(2.38397)^2}7.02672 = 3.76385$$

End-to-End Conditions

$$\left(\frac{V_{exit}}{V_\infty}\right) = \sqrt{\left(\frac{(\tau_r \cdot \tau_c \cdot \tau_t) - 1}{(\tau_r - 1)}\right)\left(\frac{\tau_\lambda}{\tau_c \tau_r}\right)} =$$

$$\left[\left(\left(\frac{1.1445 \cdot 2.16783 \cdot 0.861191 - 1}{(1.1445 - 1)}\right)\frac{9.33833}{2.16783 \cdot 1.1445}\right)^{0.5}\right] = 5.44128$$

$$\frac{A_4^*}{A_8} = (\tau_t)^{\frac{\gamma+1}{2(\gamma-1)}} = 0.861191^{\frac{(1.4+1)}{2\,(1.4-1)}} = 0.638702$$

Thrust

- Normalized Thrust

$$\mathbb{T} = \frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r-1\right)\cdot\left[\left(\frac{f+1}{f}\right)\cdot\sqrt{\left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\cdot\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)}-1\right] = \frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r-1\right)\cdot\left[\left(\frac{V_{exit}}{V_\infty}\right)-1\right] =$$

$$\frac{2\cdot 1.4}{1.4-1}\,(1.1445-1)\left(\frac{(32.128+1)}{32.128}\,5.44128-1\right) = 4.66363$$

- Total Thrust

$$F = \mathbb{T}\cdot p_{\infty}\cdot A_\infty =$$

$$\frac{2\cdot 1.4}{1.4-1}\,(1.1445-1)\left(\frac{(32.128+1)}{32.128}\,5.44128-1\right)22.63\cdot 1000\left(\frac{38.9851}{10000}\right) = 411.485\ \text{N}$$

Specific Impulse

- Normalized Isp

$$\mathbb{I} = \mathbb{T} \cdot \frac{f}{\gamma \cdot M_\infty} = \frac{4.66366 \cdot 32.128}{1.4 \cdot 0.85} = 125.911$$

- Total Isp

$$I_{sp} = \mathbb{I} \cdot \frac{c_\infty}{g_0} = \frac{4.66366 \cdot 32.128}{1.4 \cdot 0.85} \frac{(1.4 \cdot 287.056 \cdot 216.65)^{0.5}}{9.8067} = 3788.5 \text{ seconds}$$

- Thrust Specific Fuel Consumption

$$TSFC = \frac{1}{I_{sp} \cdot g_0} = \left(\frac{4.66366 \cdot 32.128}{1.4 \cdot 0.85} (1.4 \cdot 287.056 \cdot 216.65)^{0.5} \right)^{-1} = 2.69159\text{e-}05 \text{ kg/N-sec}$$

$$1 \frac{kg}{N - \sec} \cdot 2.204 \frac{lbm}{kg} \cdot \frac{4.44951}{} \frac{N}{lbf} \cdot 3600 \frac{\sec}{hr} = 35{,}304.1 \frac{lbm}{lbf - hr}$$

$$\rightarrow TSFC = \frac{1}{I_{sp} \cdot g_0} \times 35{,}304.1 \frac{lbm}{lbf - hr} = (9.8067)\ 3600 \cdot 2.69159 \cdot 10^{-5} = 0.950242 \frac{lbm}{lbf - hr}$$

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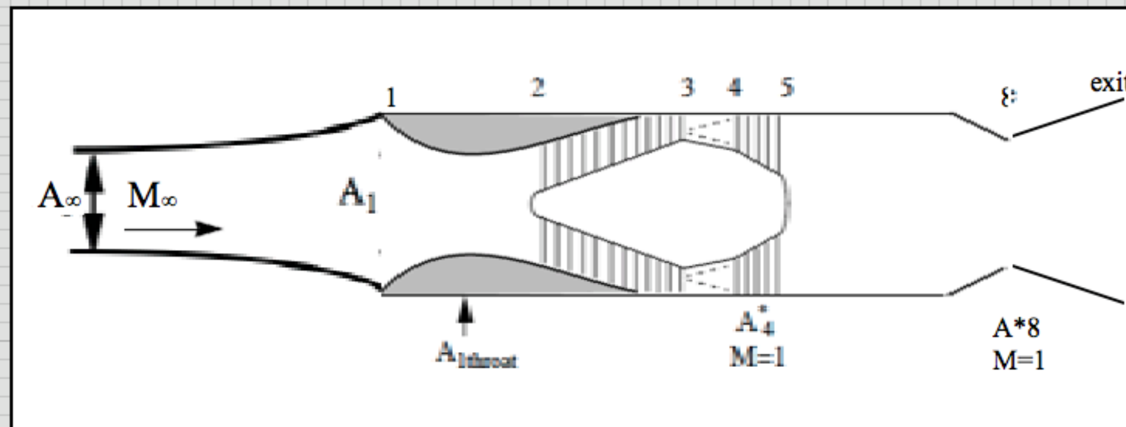
Summary of Results



Freestream Conditions

Mach Number	Fuel Enthalpy, MJ/kg	Gamma
0.85	50	1.4

Altitude, km	Cp, J/kg-K	Max Burner Temperature, K
11	1004.96	2023.15

Flight Parameters Metric

Qc, kPa	Qbar, kPa	Vtrue, m/sec	P0_inf, kPa
13.67	11.45	250.81	36.2980

Pinf, kPa	Tinf, K	CpMax	T0_inf, K
22.63	216.65	1.1939	247.955

AIR FUEL RATIO: 32.127993

TauF: 229.648

Thrust Data

Ainf, cm^2	Aexit/Ainf	Aexit, cm^2	Norm Total Thrust	Total Thrust, N	TSFC kg/N-s
38.9851	0.713243	27.8059	4.66366	411.485	2.69159E-5

Normalized Momentum Thrust	Norm Pressure Thrust	Norm Isp	True Isp, sec	TSFClbm/lbf-hr
4.66366	0.00000169(	125.911	3788.5	0.950243

Design Parameters

Pic	A2/ A*4	A2/ A1 t	Aexit/A*8	A1_throat, cm^2
15.00	8	1.5	2.36796	40

Temperature Ratios

tl	tc
9.338334	2.167834

tr	tt
1.144500	0.861191

Area Ratios

A2/A*2	A*4/A8	Ainf/A1 t	Mexit
1.57086	0.638703	0.974628	2.384

FM2	M2	Te/inf	Pe/PINF
0.636595	0.406061	3.76381	1

A1t/A*2	M1 t	P*e/PINF	Ve/VINF
1.04724	0.779603	7.53288	5.44128

Massflow data

mdot/, g/-sec	P0inf/Psl	Ainf, cm^2
355.834	0.358234	38.9851

T0inf/Tsl	Corrected Mdot/Ainf kg/m^2-sec	mdot_COR g/SEC
0.86051	236.352	921.422

Efficiencies

$$\eta_{prop} = \frac{\left(\left(\frac{f+1}{f}\right)\cdot V_{exit} - V_{\infty}\right)\cdot V_{\infty}}{\left(\frac{1}{2}\left(\frac{f+1}{f}\right)\cdot V^2{}_{exit} - V^2{}_{\infty}\right)} = \frac{\left(\left(\frac{f+1}{f}\right)\cdot V_{exit} - V_{\infty}\right)\cdot V_{\infty} \Big/ V^2{}_{\infty}}{\left(\frac{1}{2}\left(\frac{f+1}{f}\right)\cdot V^2{}_{exit} - V^2{}_{\infty}\right) \Big/ V^2{}_{\infty}} = \frac{\left(\left(\frac{f+1}{f}\right)\cdot \frac{V_{exit}}{V_{\infty}} - 1\right)}{\left(\frac{1}{2}\left(\frac{f+1}{f}\right)\cdot\left(\frac{V_{exit}}{V_{\infty}}\right)^2 - 1\right)} =$$

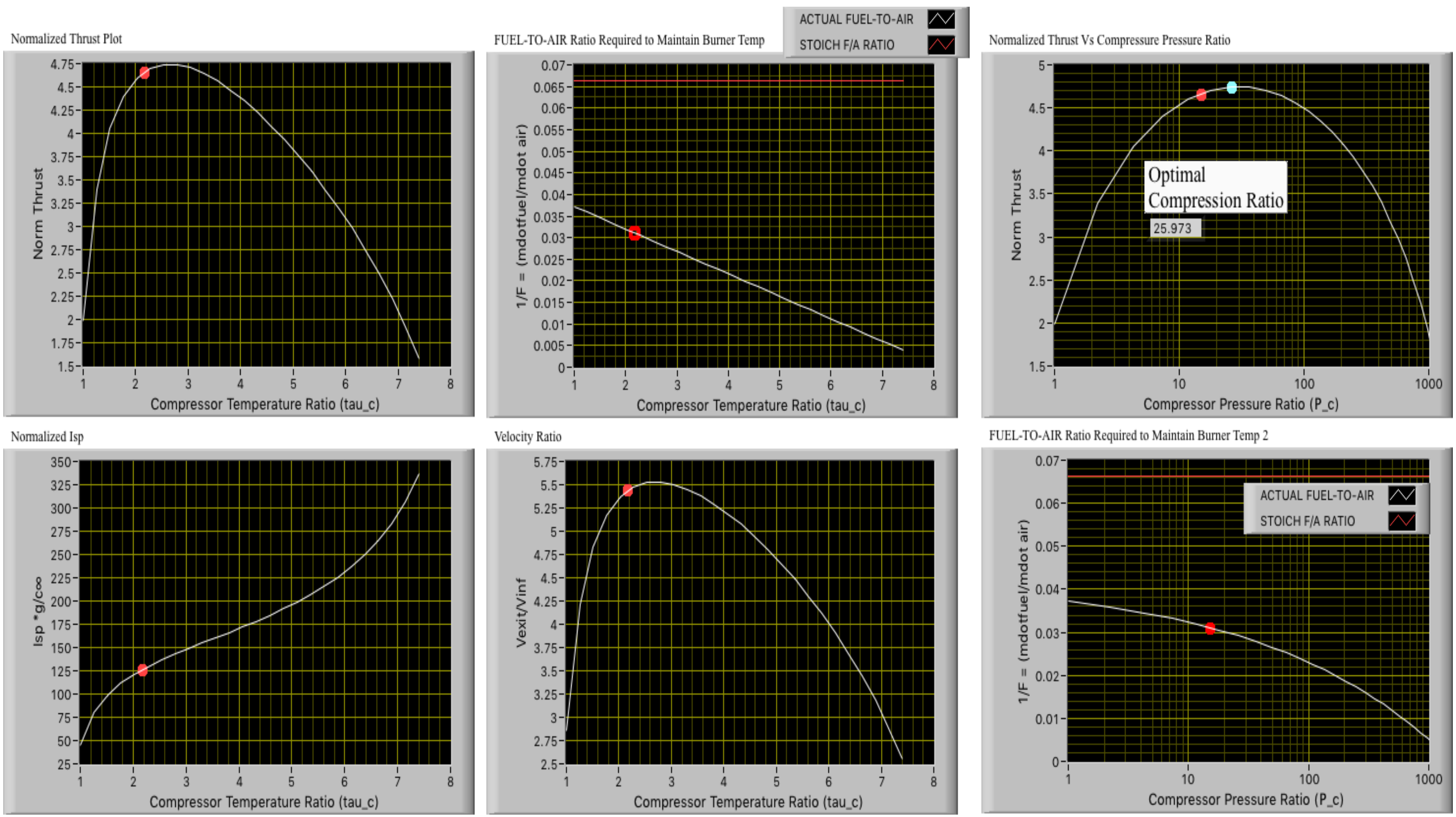
$$\frac{\frac{32.128+1}{32.128}(5.44125) - 1}{\frac{1}{2}\,\frac{32.128+1}{32.128}\left(5.44125^2\right) - 1} = 0.32323$$

$$\eta_{total} = \eta_{prop}\cdot\eta_{thermal} = 0.32323\cdot 0.5766 = 0.18637$$

$$\eta_{thermal} = \frac{\left(\frac{1}{2}\left(\frac{f+1}{f}\right)\cdot V^2{}_{exit} - V^2{}_{\infty}\right)}{\frac{1}{f}\cdot h_{fuel}} = \frac{(250.81)^2\left(\frac{1}{2}\,\frac{32.128+1}{32.128}\left(5.44125^2\right) - 1\right)}{\frac{50\cdot 10^6}{32.128}} = 0.576575$$

→ *Assuming the same combustor outlet temperature and inlet throat area as previous example*

→ *At what compressor demand π_c do we get the optimal Thrust level*




Questions??