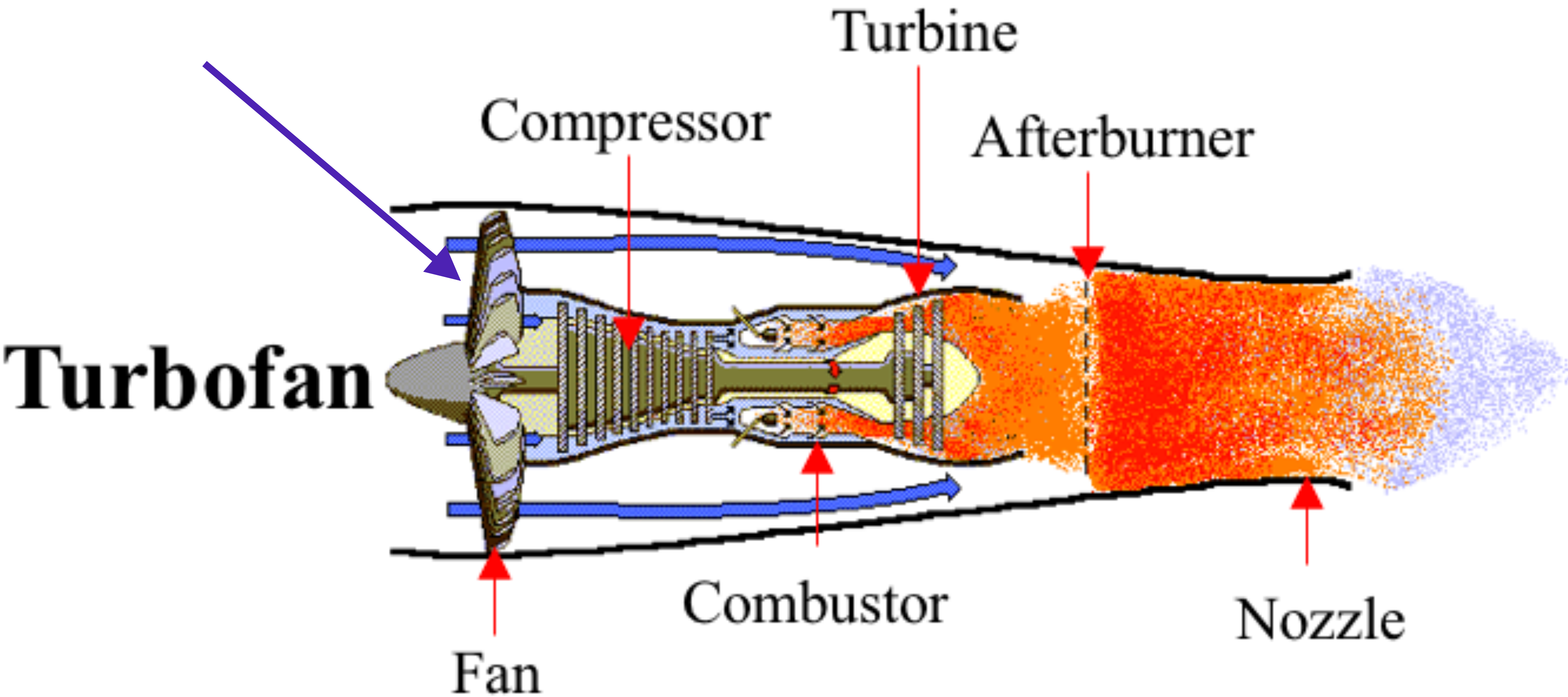
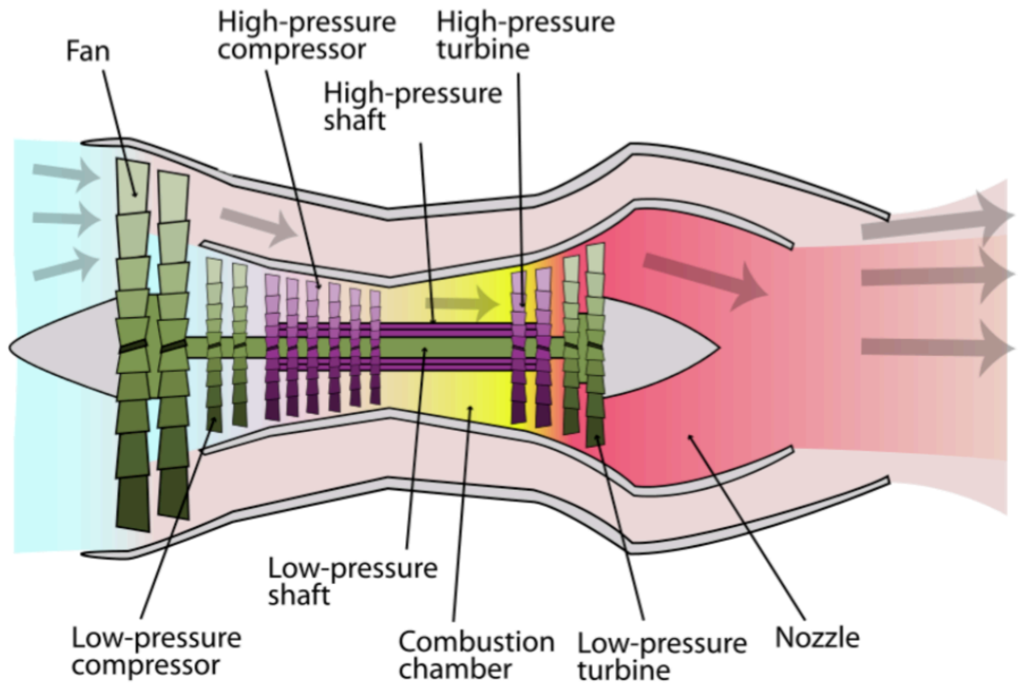
Section 6.1: The TurboFan Propulsion Cycle



Overview

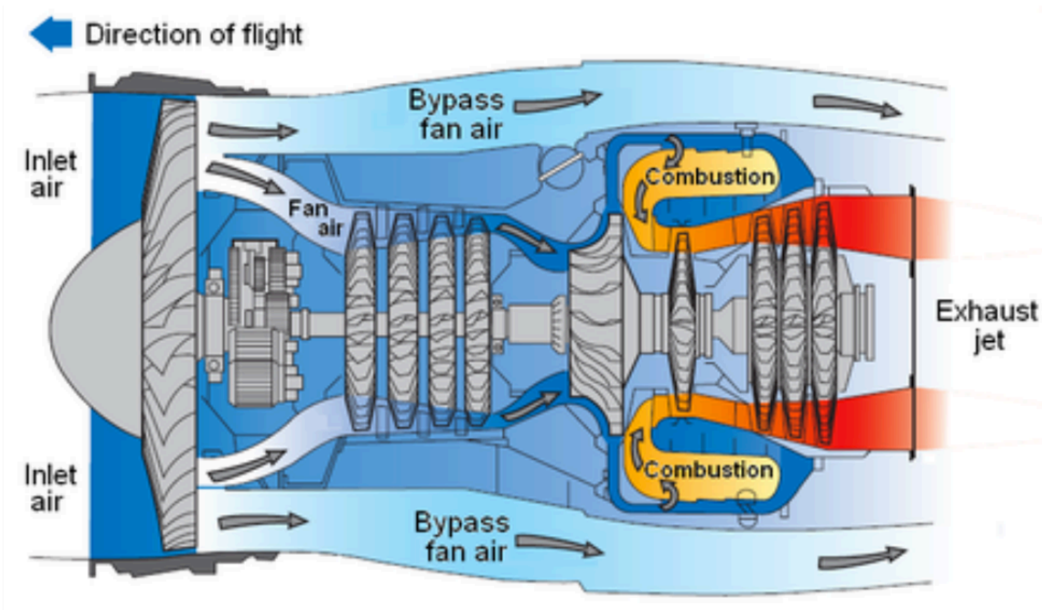
Turbofan. Turbine primarily drives a fan at the front of the engine. Most engines drive the fan directly from the turbine. Part of the air enters the turbine section of the engine, and the rest is bypassed around the engine. In high-bypass engines, most of the air only goes through the fan and bypasses the rest of the engine and providing most of the thrust.

A turbofan thus can be thought of as a turbojet being used to drive a ducted fan, with both of those contributing to the thrust. The ratio of the mass-flow of air bypassing the engine core compared to the mass-flow of air passing through the core is referred to as the bypass ratio.

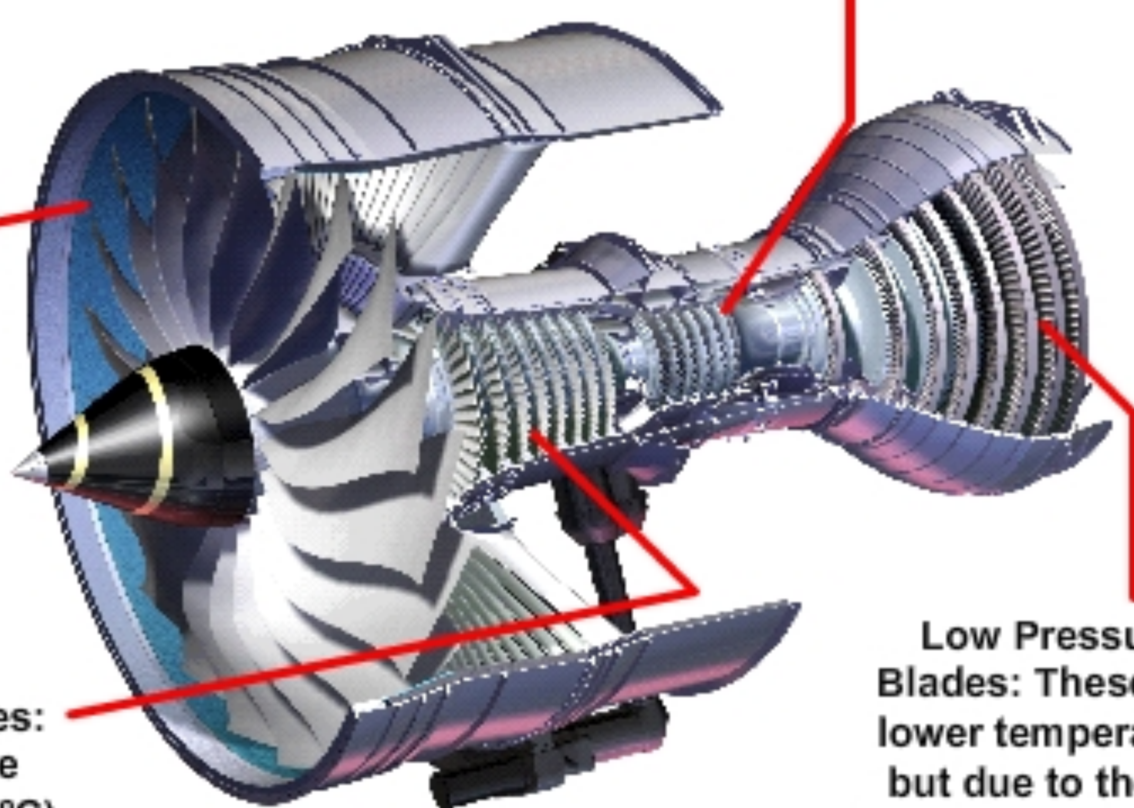


Overview (2)

- **Turbofan** engine is the most modern variation of the basic gas turbine engine.
- As with other gas turbines, there is a core engine, whose parts and operation are nearly identical to the turbojet operation.
- In the **turbofan** engine, the core engine is surrounded by a fan in the front and an additional turbine at the rear.



Overview (3)



High Pressure Turbine Blades: Work at very high temperatures temperatures (1390 °C) Cooled nickel super-alloy must be used.

Fan Blades: Work at low temperature, lightweight titanium can be used

Compressor Blades: Work at moderate temperatures (490 °C) Titanium may be used.

Low Pressure Turbine Blades: These operate at a lower temperature (600 °C) but due to their large size should have as low a density as possible. Inter-metallic alloys of Titanium Aluminide are being developed for this role.

Overview(4)

- Turbofan engines power now all civil transports flying at transonic speeds up to Mach 0.9.
- Several advantages to turbofan engines over both propeller-driven and turbojet engines
- By enclosing fan inside a duct or cowling, aerodynamics are better controlled.
- Fan is not as large as propeller, so increase of speeds along blade is less, and there is less chance of tip stall and shock wave development.
- Turbofan can suck in more total air massflow than a turbojet, thus offer potential for generating more thrust.
- Turbofan has low fuel consumption compared with turbojet.

Jet Engine Performance Efficiency (revisited)

Propulsive Efficiency

Ratio of Power Developed from Engine (desired beneficial output) Thrust to Change in Kinetic Energy of the Moving Airstream (cost of propulsion)

Propulsive power

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{\dot{m}_{air} \cdot \left(\left(\frac{f+1}{f}\right)V_{exit} - V_{\infty}\right) \cdot V_{\infty}}{\dot{m}_{air} \cdot \left(\frac{1}{2}\left(\frac{f+1}{f}\right)V^2_{exit} - \frac{1}{2}V^2_{\infty}\right)}$$

Kinetic energy production rate

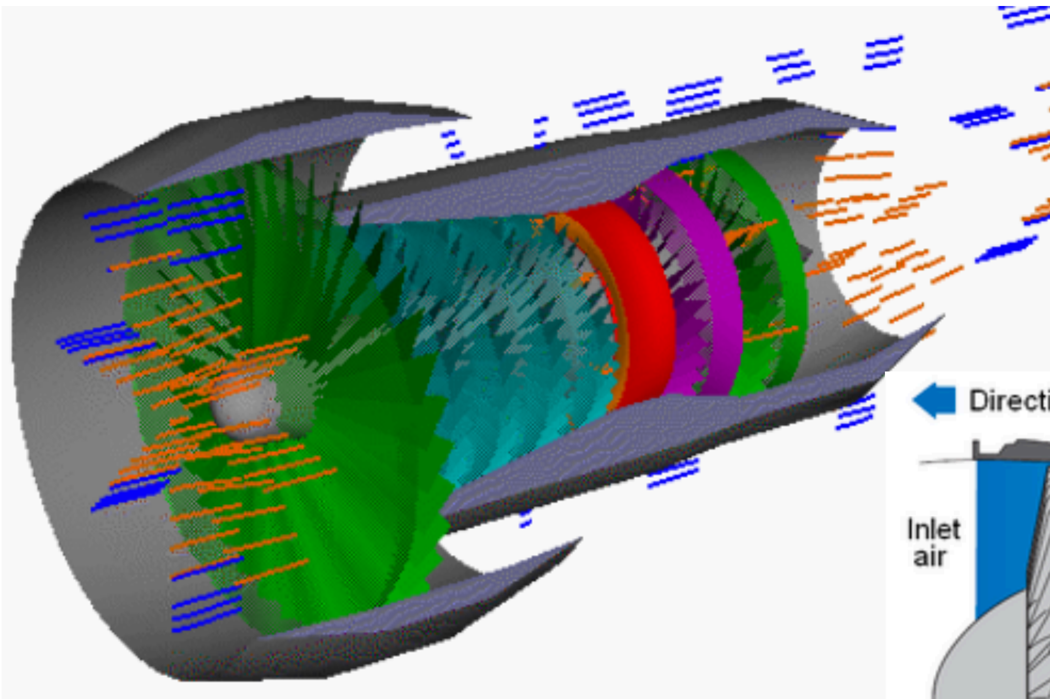
$$\text{assuming } \dot{m}_{air} >> \dot{m}_{fuel} \rightarrow f << 1$$

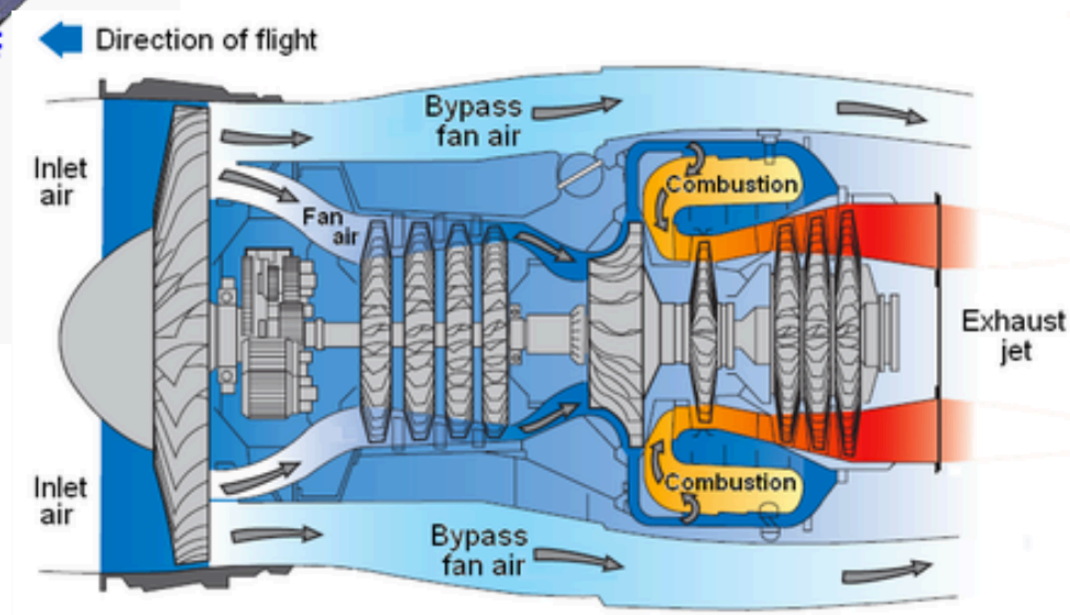
$$\eta_{propulsive} = \frac{2 \cdot \left(V_{exit} - V_{\infty}\right) \cdot V_{\infty}}{\left(V_{exit} + V_{\infty}\right) \cdot \left(V_{exit} - V_{\infty}\right)} = \frac{2 \cdot V_{\infty}}{\left(V_{exit} + V_{\infty}\right)} = \frac{2}{\left(1 + V_{exit}/V_{\infty}\right)}$$

Maximum propulsive efficiency achieved by generating thrust moving as much air as possible with as little a change in velocity across the engine as possible.

Jet Engine Performance Efficiency (2)

- **Maximum propulsive efficiency achieved by generating thrust moving as much air as possible with as little a change in velocity across the engine as possible. .. Fan provides that function**

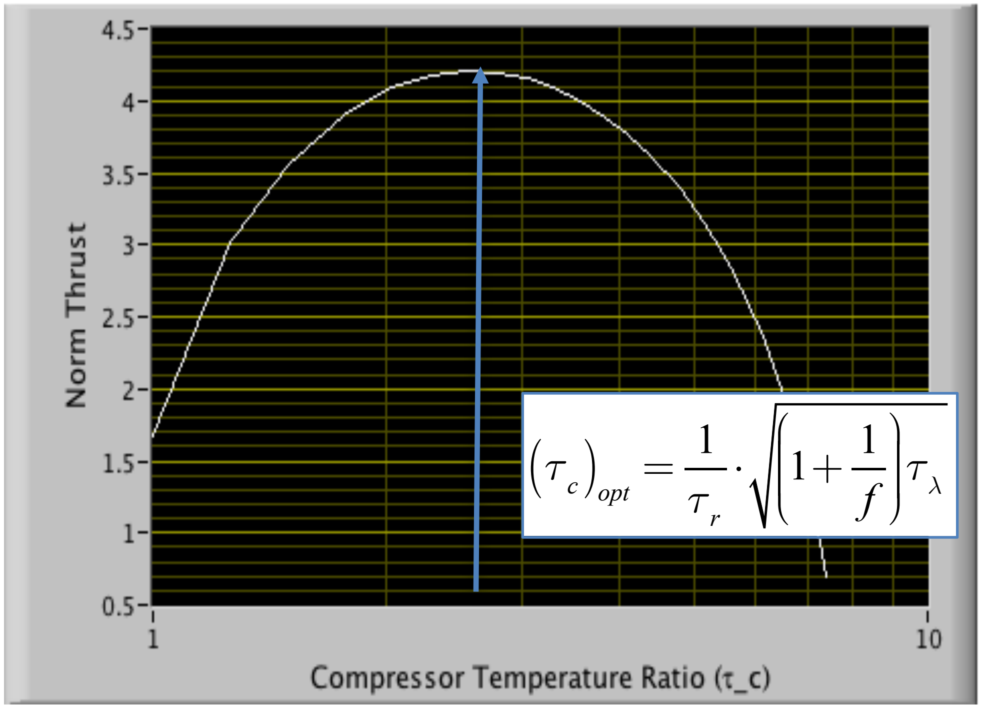




Jet Engine Performance Efficiency (3)

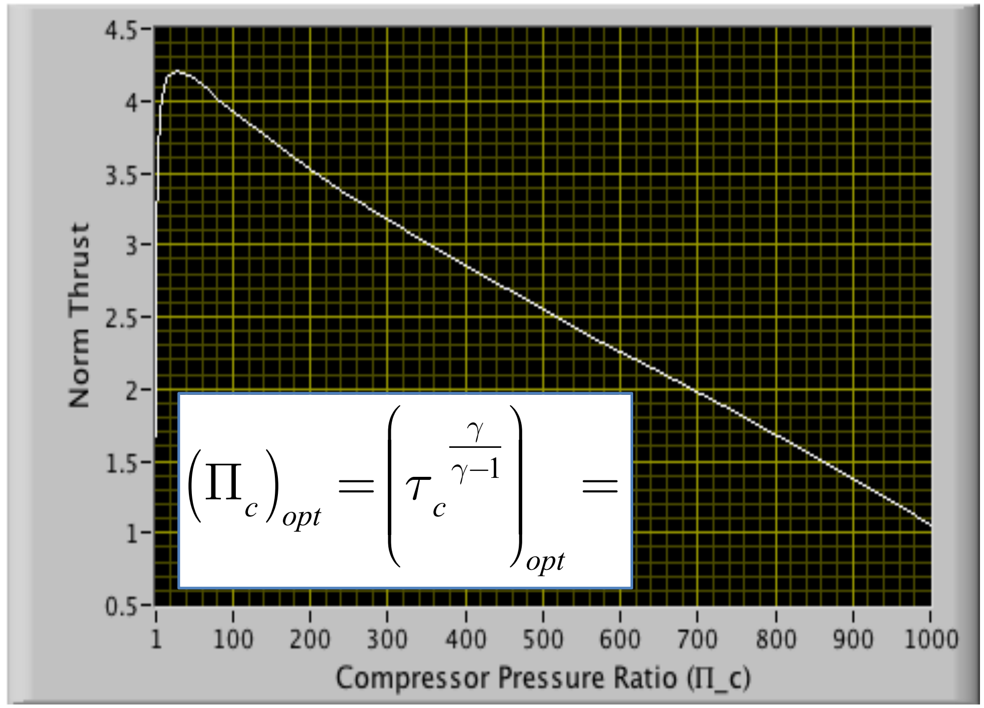
• Recall that optimal thrust level of a turbo jet is characterized by

Normalized Thrust Plot, vs Compressor Temperaure Ratio



$$\left(\tau_c\right)_{opt} = \frac{1}{\tau_r}\cdot\sqrt{\left(1+\frac{1}{f}\right)\tau_\lambda}$$

Normalized Thrust Plot, vs Compressor Pressure Ratio



$$\left(\Pi_c\right)_{opt} = \left(\tau_c^{\frac{\gamma}{\gamma-1}}\right)_{opt} =$$

• As supersonic flight Mach become larger, compression goes down until

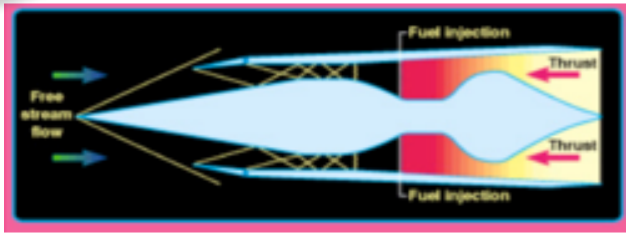
$$1 = \frac{1}{\tau_r}\cdot\sqrt{\left(1+\frac{1}{f}\right)\tau_\lambda} \rightarrow \tau_\lambda = \frac{f}{f+1}\tau_r^{\ 2}$$

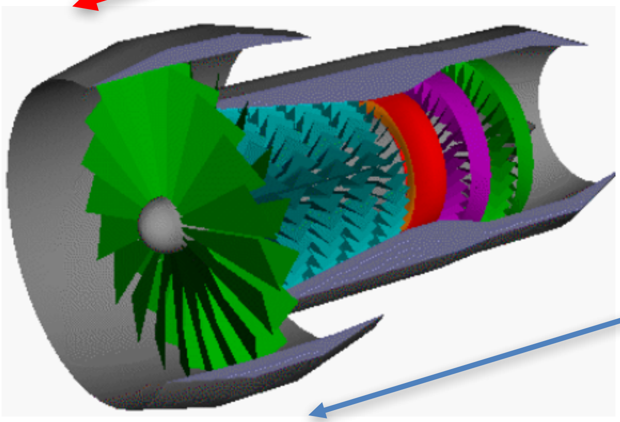
…. optimal solution gets rid of compressor all together, and best engine becomes a ramjet!

Jet Engine Performance Efficiency (4)

- **We are going to show that similar effect occurs as Mach drops significantly below Mach 1**

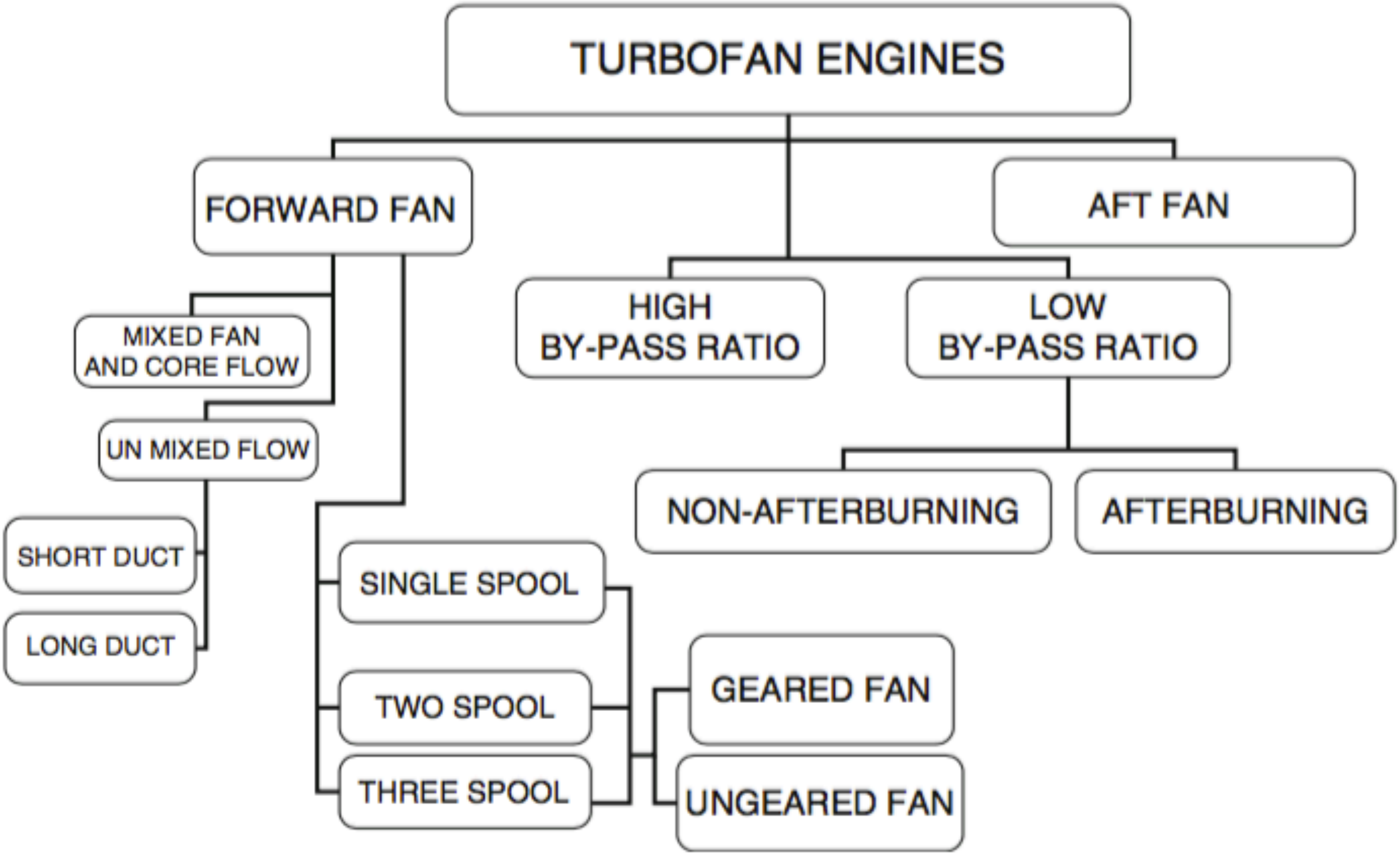
- **Trend emerges that replaces turbojet with Turbofan at lower Mach numbers**





Decreasing Mach Number

Classification of Turbofan Engines



Classification of Turbofan Engines (2)

• Turbofan engines may be classified based on fan location as either forward or aft fan.

• Based on a number of spools, engine may be classified as single, double, or triple spools.

•Based on a bypass ratio, engine may be categorized as either low- or high- bypass ratio.

• Fan may be geared or ungeared to its driving low-pressure turbine.

• Mixed-flow types (where flow merges in nozzle) may be fitted with afterburner.

Classification of Turbofan Engines (3)

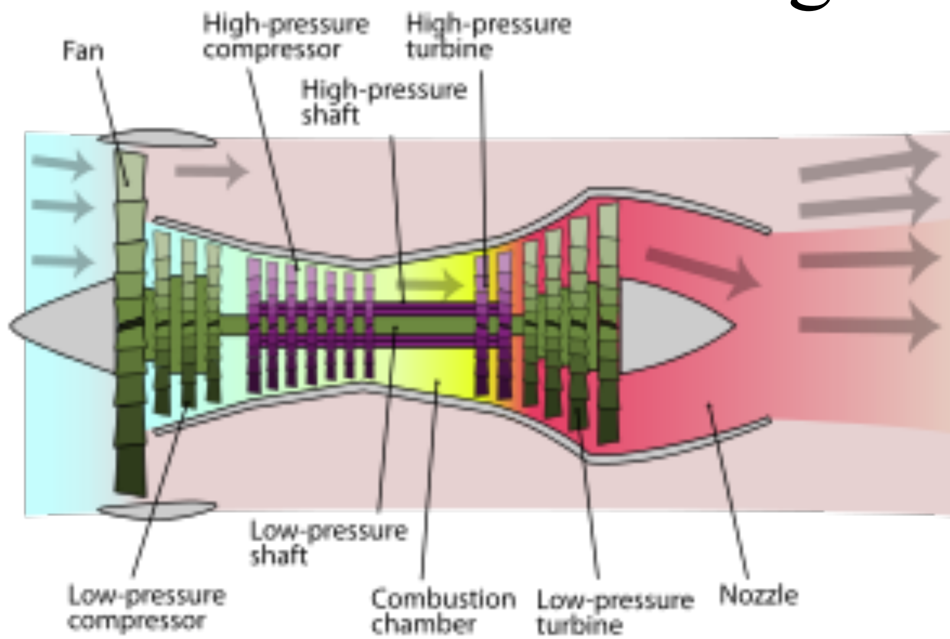
- High-bypass ratio turbofan ($\beta > 5$) achieves 75 % of thrust from bypass air
  - Ideal for subsonic transport aircraft. e.g.
  - Rolls-Royce Trent series (Airbus A330, A340, A350, A380),
  - Pratt & Whitney PW1000 G (geared) (Airbus A320neo, Bombardier CSeries, Embraer E2, Mitsubishi Regional Jet MC-21)
  - General Electric GE90 powering Boeing 777-300ER, 747.

- Low-bypass ratio turbofan $\beta \sim 1$ achieves approximately equal thrust distribution between gas generator and bypass duct
  - Well suited to high-speed military applications. e.g.
  - Rolls-Royce RB199 in the Tornado
  - Pratt & Whitney F100-PW-200 in F-16A/B and F-15
  - EuroJet Turbo GmbH EJ200 powering the Typhoon Fighter

Classification of Turbofan Engines (4)

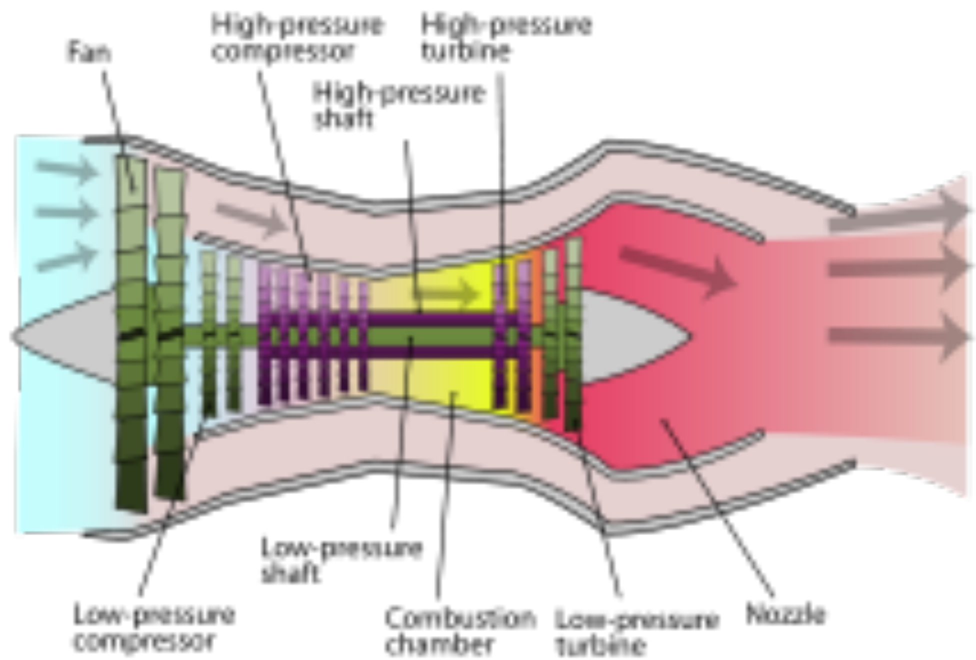
- Unmixed Bypass Flow Turbofan

2-Spool High-bypass turbofan engine with a unmixed exhaust, showing the fan (pink) and core (red) exhaust streams



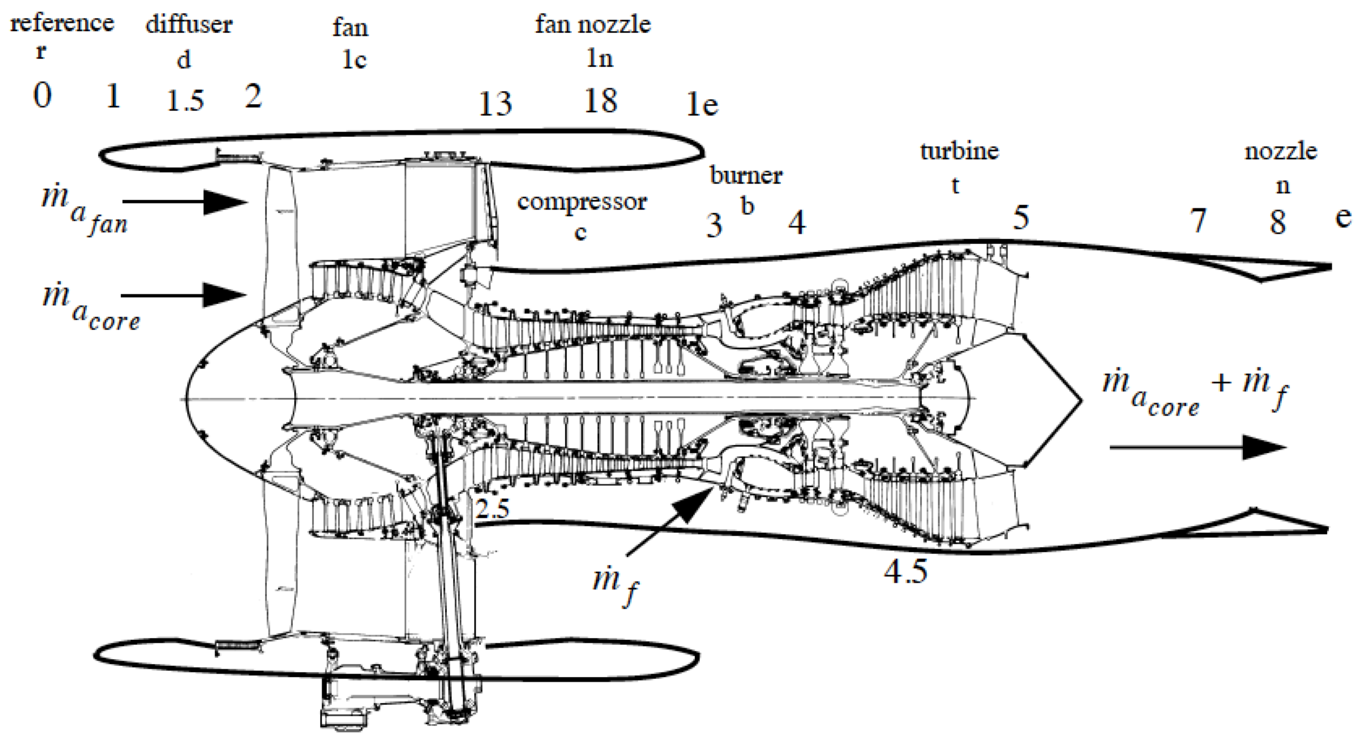
- Mixed Bypass Flow Turbofan

2-spool, low-bypass turbofan engine with a mixed exhaust, showing the low-pressure (green) and high-pressure (purple) spools.



Classification of Turbofan Engines (5)

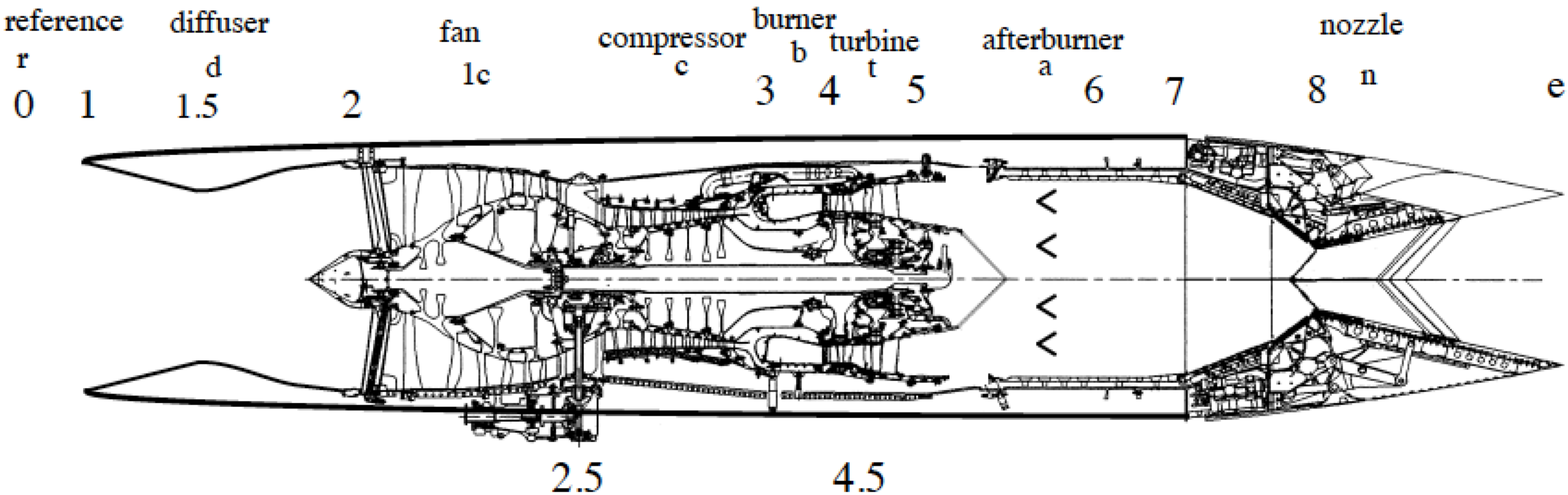
High Bypass Ratio Commercial Turbofan



- Modern high bypass ratio engine designed for long distance cruise at subsonic Mach numbers around 0.83.
- Fan uses a single stage composed of a large diameter fan (rotor) with wide chord blades followed by a single nozzle stage (stator).
- Bypass ratio is ~6 and the fan pressure ratio is ~2.

Classification of Turbofan Engines (6)

Low Bypass Ratio Military Turbofan

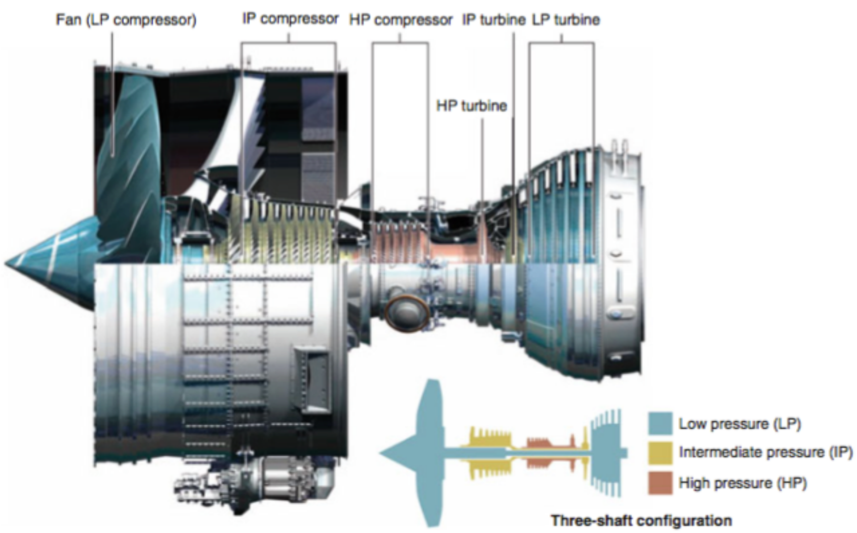


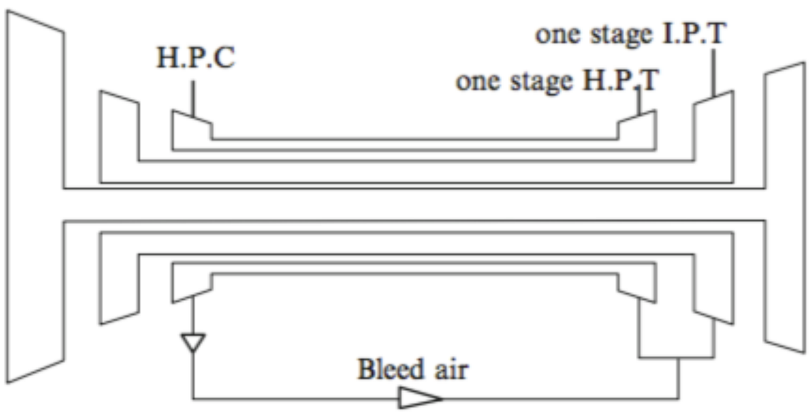
• Military turbofan designed for high performance at supersonic Mach numbers in the range of 1.1 to 1.5.

• Fan has three stages with an overall pressure ratio of about 6 and a bypass ratio of only about 0.6.

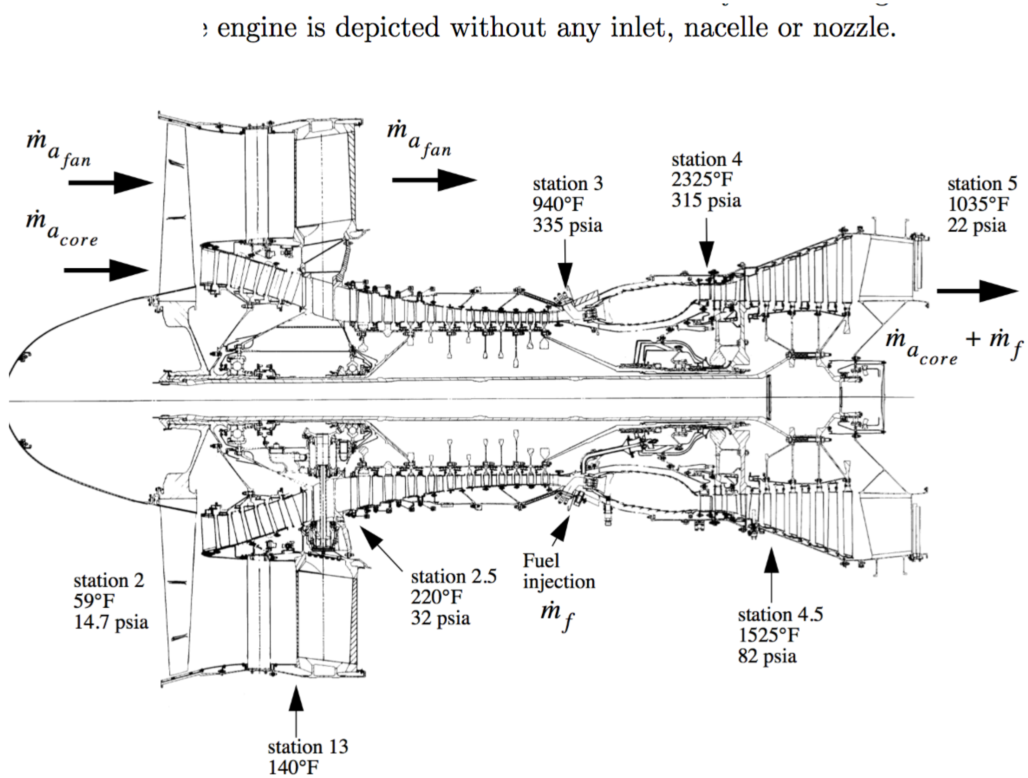
• Let's investigate to understand why these engines look so different due to the the differences in the design flight condition.

Classification of Turbofan Engines (7)



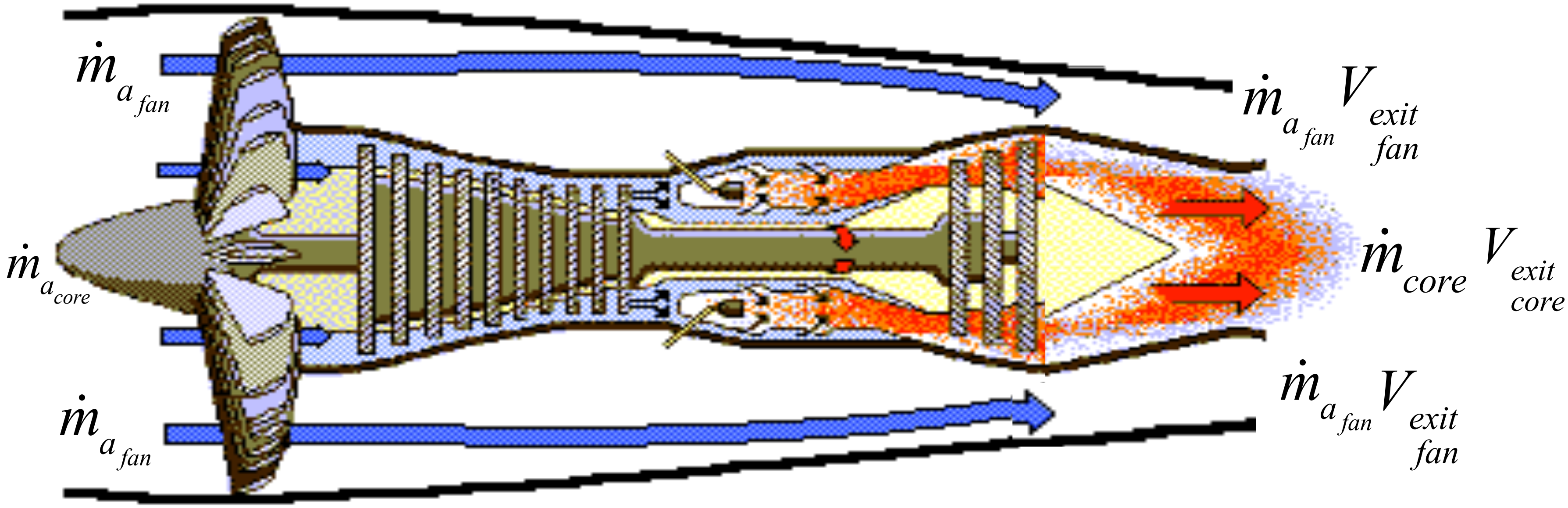


Layout of unmixed three-spool engine (Trent 700)



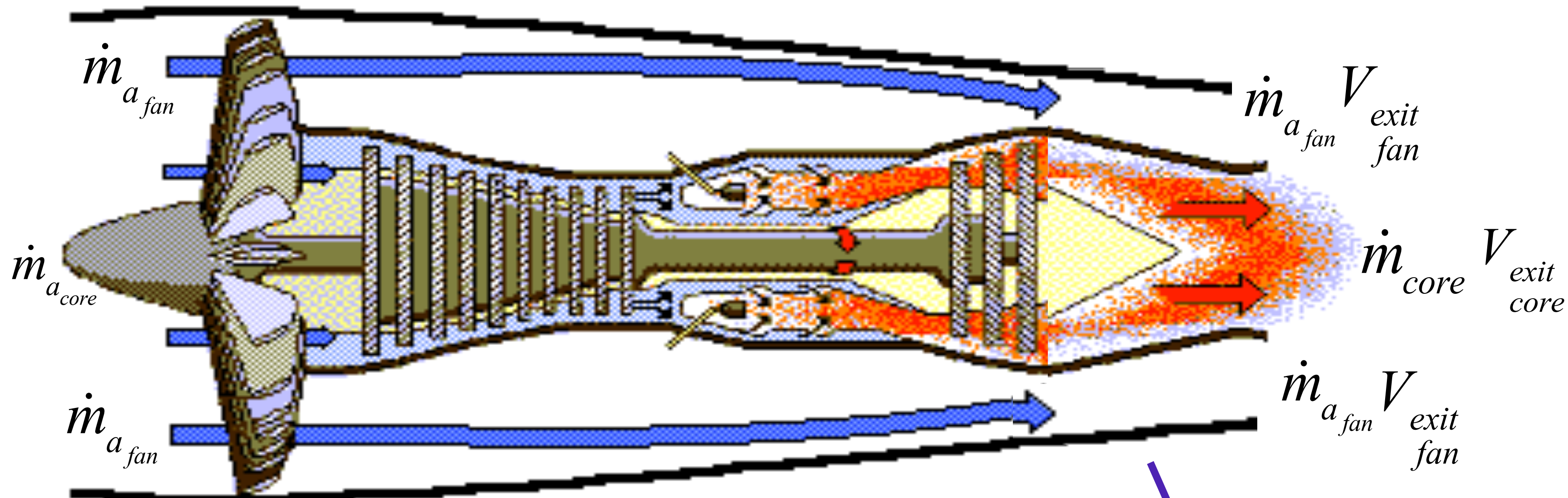
Cross-section of the Pratt and Whitney JT9D-7 turbofan engine

Thrust of a TurboFan Engine



- **4-Primary Components of TurboFan Thrust**
  - *Bypass Momentum Thrust*
  - *Core Momentum Thrust*
  - *Fan Pressure Thrust*
  - *Core Pressure Thrust*

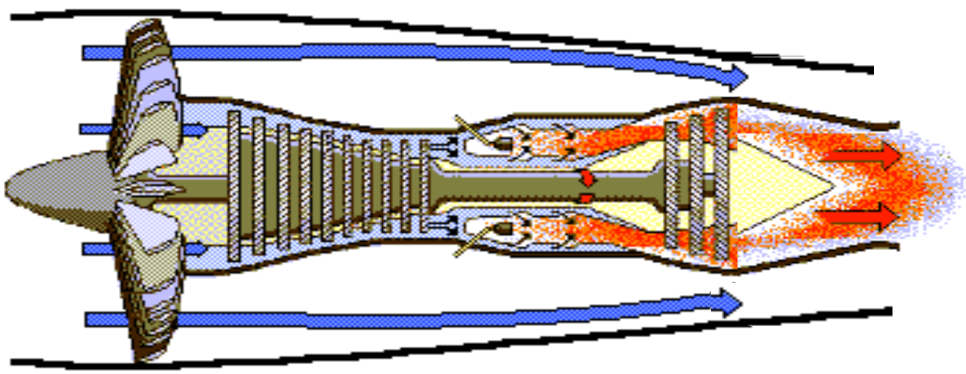
Thrust of a TurboFan Engine (2)



$$F_{thrust} = \dot{m}_{a_{core}} \cdot \left(V_{exit\,core} - V_\infty\right) + \dot{m}_{fuel} \cdot V_{exit\,core} + \dot{m}_{a_{fan}} \cdot \left(V_{exit\,fan} - V_\infty\right) + \left(p_{exit\,core} - p_\infty\right) \cdot A_{exit\,core} + \left(p_{exit\,fan} - p_\infty\right) \cdot A_{exit\,fan}$$

Core Momentum Thrust | Bypass Momentum Thrust | Core Pressure Thrust | Bypass Pressure Thrust

Thrust of a TurboFan Engine (3)



Total Air Massflow

$$\dot{m}_a = \dot{m}_{a_{core}} + \dot{m}_{a_{fan}} \rightarrow$$

Air − to − fuel Ratio

$$f = \frac{\dot{m}_a}{\dot{m}_{fuel}}$$

Bypass Ratio

$$\beta = \frac{\dot{m}_{a_{fan}}}{\dot{m}_{a_{core}}} = \frac{B}{1-B}$$

Bypass Fraction

$$B = \frac{\dot{m}_{a_{fan}}}{\dot{m}_a} = \frac{\beta}{1+\beta}$$

$$F_{thrust} = \dot{m}_a \left[\frac{\dot{m}_{a_{core}}}{\dot{m}_a} \cdot \left(V_{\substack{exit \\ core}} - V_\infty \right) + \frac{\dot{m}_{fuel}}{\dot{m}_a} \cdot V_{\substack{exit \\ core}} + \frac{\dot{m}_{a_{fan}}}{\dot{m}_a} \cdot \left(V_{\substack{exit \\ fan}} - V_\infty \right) \right] + \left(p_{exit} - p_\infty \right) \cdot A_{exit} + \left(p_{1exit} - p_\infty \right) \cdot A_{\substack{exit \\ fan}}$$

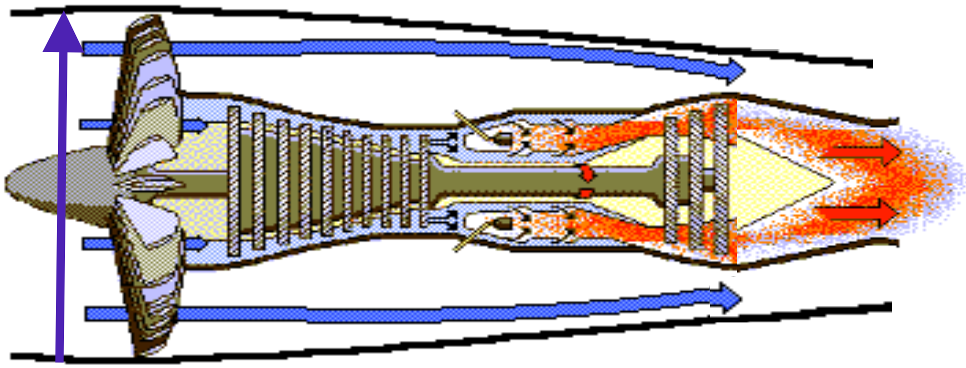
$$\frac{\dot{m}_{a_{core}}}{\dot{m}_a} = \frac{\dot{m}_{a_{core}}}{\dot{m}_{a_{core}} + \dot{m}_{a_{fan}}} = \frac{1}{1 + \dfrac{\dot{m}_{a_{fan}}}{\dot{m}_{a_{core}}}} = \frac{1}{1+\beta}$$

$$\frac{\dot{m}_{a_{core}}}{\dot{m}_a} = \frac{\dot{m}_a - \dot{m}_{a_{fan}}}{\dot{m}_a} = 1 - B$$

$$\frac{\dot{m}_{a_{fan}}}{\dot{m}_a} = B = \frac{\beta}{1+\beta}, \quad \frac{\dot{m}_{fuel}}{\dot{m}_a} = \frac{1}{f}$$

$$\rightarrow F_{thrust} = \dot{m}_a \left[\frac{1}{1+\beta} \cdot \left(V_{\substack{exit \\ core}} - V_\infty \right) + \frac{1}{f} \cdot V_{\substack{exit \\ core}} + \frac{\beta}{1+\beta} \cdot \left(V_{\substack{exit \\ fan}} - V_\infty \right) \right] + \left(p_{\substack{exit \\ core}} - p_\infty \right) \cdot A_{\substack{exit \\ core}} + \left(p_{\substack{exit \\ fan}} - p_\infty \right) \cdot A_{\substack{exit \\ fan}}$$

Thrust of a TurboFan Engine (4)



- Similar to the Previous Discussion for the TurboJet, Normalized Thrust is

$$\left(\mathbb{T}\right)_{turbofan} = \frac{F_{thrust}}{p_\infty A_0} = \frac{\dot{m}_a V_\infty}{p_\infty A_0}\left[\frac{1+f+\beta}{f\left(1+\beta\right)}\cdot\frac{V_{exit_{core}}}{V_\infty} + \frac{\beta}{1+\beta}\cdot\frac{V_{exit_{fan}}}{V_\infty} - 1\right] + \left(\frac{p_{exit_{core}}}{p_\infty}\cdot\frac{A_{exit_{core}}}{A_0} + \frac{p_{exit_{fan}}}{p_\infty}\frac{A_{exit_{fan}}}{A_0} - \frac{A_{exit_{core}} + A_{exit_{fan}}}{A_0}\right) =$$

$$\frac{\dot{m}_a V_\infty}{p_\infty A_0} = \frac{\rho_\infty \cdot A_0 \cdot V_\infty \cdot V_\infty}{p_\infty A_0} = \frac{\gamma V_\infty^{\ 2}}{\gamma \cdot R_g \cdot T_\infty} = \gamma \cdot M_\infty^2$$

$$\left(\mathbb{I}\right)_{turbofan} = \frac{I_{sp}\cdot g_0}{c_\infty} = \left(\mathbb{T}\right)_{turbofan}\cdot\frac{f}{\gamma\cdot M_\infty}$$

$$A_{exit_{core}} + A_{exit_{fan}} = A_{exit} \rightarrow$$

$$\left(\mathbb{T}\right)_{turbofan} = \gamma\cdot M_\infty^2\left[\frac{1+\frac{1}{f}\left(1+\beta\right)}{\left(1+\beta\right)}\cdot\frac{V_{exit_{core}}}{V_\infty} + \frac{\beta}{1+\beta}\cdot\frac{V_{exit_{fan}}}{V_\infty} - 1\right] + \left(\frac{A_{exit}}{A_0}\right)\left(\frac{p_{exit}}{p_\infty}\cdot\frac{A_{exit_{core}}}{A_{exit}} + \frac{p_{exit_{fan}}}{p_\infty}\frac{A_{exit_{fan}}}{A_{exit}} - 1\right)$$

Thrust of a TurboFan Engine (5)

- Fully expanded nozzle & $f >> 1$
- Inlet, fan, compressor, turbine, and fan /core nozzles are isentropic
- Combustor heat addition is as constant pressure and Low Mach

$$p_{\underset{fan}{exit}} = p_{\underset{core}{exit}} = p_\infty \rightarrow \pi_d = \pi_b = \pi_{n_{core}} = \pi_{n_{fan}} = 1$$

$$\pi_{c_{core}} = \left(\tau_{c_{core}}\right)^{\frac{\gamma}{\gamma-1}}, \pi_{c_{fan}} = \left(\tau_{c_{fan}}\right)^{\frac{\gamma}{\gamma-1}}, \pi_t = \left(\tau_t\right)^{\frac{\gamma}{\gamma-1}}.$$

$$\left(\mathbb{T}_{optimal}\right)_{turbofan} = \gamma \cdot M_\infty^2 \left[\left(\frac{1}{1+\beta}\right) \cdot \frac{V_{\underset{core}{exit}}}{V_\infty} + \left(\frac{\beta}{1+\beta}\right) \cdot \frac{V_{\underset{fan}{exit}}}{V_\infty} - 1\right] =$$

$$= \gamma \cdot M_\infty^2 \left[\left(\frac{1}{1+\beta}\right) \cdot \left(\frac{V_{\underset{core}{exit}}}{V_\infty} - 1\right) + \left(\frac{\beta}{1+\beta}\right) \cdot \left(\frac{V_{\underset{fan}{exit}}}{V_\infty} - 1\right)\right]$$

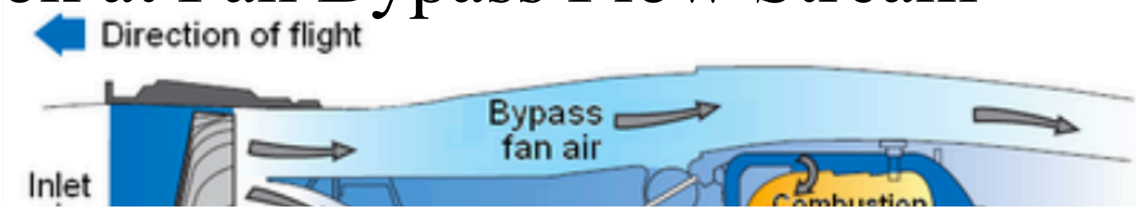
$$\left(\mathbb{I}_{optimal}\right)_{turbofan} = \left(\mathbb{T}_{optimal}\right)_{turbofan} \cdot \frac{f}{\gamma \cdot M_\infty}$$

The Ideal TurboFan Cycle

• Too lengthy to analyze all types of turbofan engines. So, only a simple single spool fan, compressor, turbine cycle will be analyzed

• Look at Fan Bypass Flow Stream

V_∞



$\dot{m}_{a_{fan}} \; V_{\substack{exit\\fan}}$

• Look at by pass velocity ratio $\dfrac{V_{\substack{exit\\fan}}}{V_\infty} = \dfrac{M_{\substack{exit\\fan}}}{M_\infty} \cdot \sqrt{\dfrac{T_{\substack{exit\\fan}}}{T_\infty}}$

• First calculate bypass exit stagnation pressure

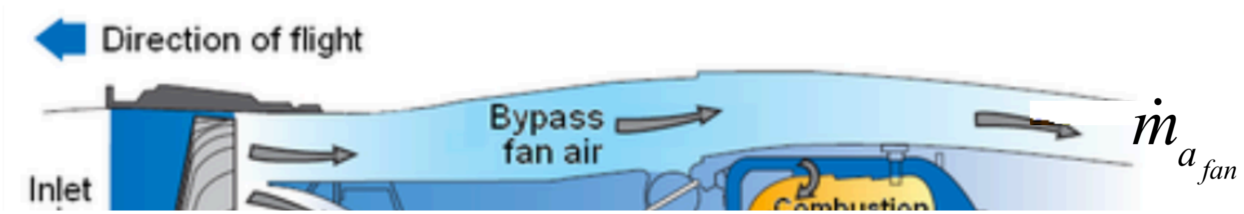
$$\rightarrow P_{0_{\substack{exit\\fan}}} = \frac{P_{0_\infty}}{p_\infty} \cdot \frac{P_{0_1}}{P_{0_\infty}} \cdot \frac{P_{0_2}}{P_{0_1}} \cdot \frac{P_{0_{\substack{exit\\fan}}}}{P_{0_2}} \cdot p_\infty \rightarrow ideal\ cycle \rightarrow \pi_d = \frac{P_{0_1}}{P_{0_\infty}} \cdot \frac{P_{0_2}}{P_{0_1}} = 1$$

$$\rightarrow P_{0_{\substack{exit\\fan}}} = \pi_r \cdot \pi_{c_{fan}} \cdot p_\infty = p_{\substack{exit\\fan}} \cdot \left(1 + \frac{\gamma+1}{2} M^2_{\substack{exit\\fan}}\right)^{\frac{\gamma}{\gamma-1}}$$

The Ideal TurboFan Cycle (2)

- Look at Fan Bypass Flow Stream

V_∞



$\dot{m}_{a_{fan}}$ $V_{exit_{fan}}$

$$\text{fully expanded nozzle} \rightarrow \left\{ p_{exit_{fan}} = p_\infty \right\} \rightarrow \pi_r \cdot \pi_{c_{fan}} = \left(1 + \frac{\gamma+1}{2} M^2_{exit_{fan}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\text{Isentropic fan} \rightarrow \pi_{c_{fan}} = \left(\tau_{c_{fan}}\right)^{\frac{\gamma}{\gamma-1}} \rightarrow \boxed{\tau_r \cdot \tau_{c_{fan}} = \left(1 + \frac{\gamma+1}{2} M^2_{exit_{fan}}\right)}$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty}\left(1 + \frac{\gamma+1}{2} M^2_\infty\right) \rightarrow \frac{\gamma+1}{2} M^2_\infty = \tau_r - 1$$

$$\rightarrow \frac{\gamma+1}{2} M^2_{exit_{fan}} = \tau_r \cdot \tau_{c_{fan}} - 1 \rightarrow \frac{\frac{\gamma+1}{2} M^2_{exit_{fan}}}{\frac{\gamma+1}{2} M^2_\infty} = \frac{\tau_r \cdot \tau_{c_{fan}} - 1}{\tau_r - 1} \rightarrow \boxed{\frac{M^2_{exit_{fan}}}{M^2_\infty} = \frac{\tau_r \cdot \tau_{c_{fan}} - 1}{\tau_r - 1}}$$

The Ideal TurboFan Cycle (3)

- Look at Fan Bypass Flow Stream

Substitute

$$\frac{V_{exit_{fan}}}{V_\infty} = \frac{M_{exit_{fan}}}{M_\infty}\cdot\sqrt{\frac{T_{exit_{fan}}}{T_\infty}} = \sqrt{\left(\frac{\tau_r\cdot\tau_{c_{fan}}-1}{\tau_r-1}\right)}\cdot\sqrt{\frac{T_{exit_{fan}}}{T_\infty}} \rightarrow isentropic\ fan \rightarrow \left(p_{exit_{fan}}\right)^{\frac{\gamma-1}{\gamma}} = \left(p_\infty\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_{exit_{fan}} = T_\infty$$

$$\boxed{\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2 = \left(\frac{\tau_r\cdot\tau_{c_{fan}}-1}{\tau_r-1}\right)}$$

Fan Thrust

$$\rightarrow \left(\mathbb{T}\right)_{fan} = \gamma\cdot M_\infty^2\left[\left(\frac{\beta}{1+\beta}\right)\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}-1\right)\right] = \gamma\cdot M_\infty^2\left[\left(\frac{\beta}{1+\beta}\right)\cdot\left(\sqrt{\frac{\tau_r\cdot\tau_{c_{fan}}-1}{\tau_r-1}}-1\right)\right]$$

The Ideal TurboFan Cycle (4)

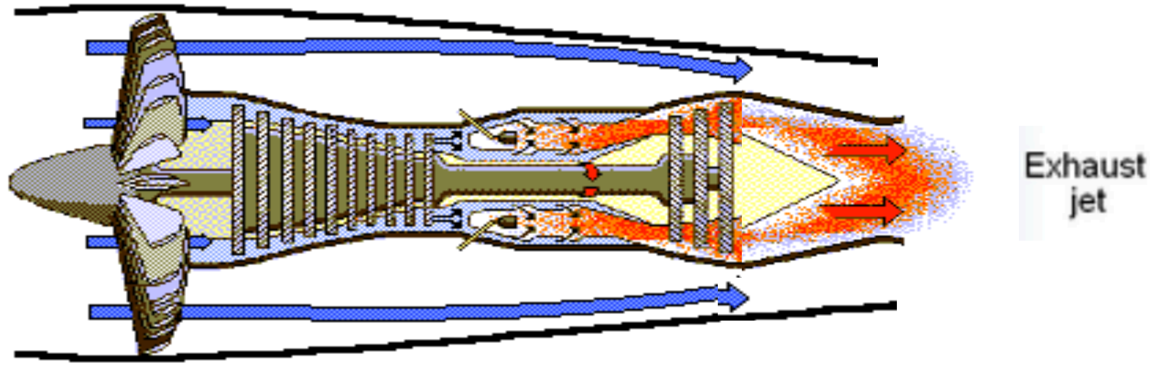
- Look at Fan Bypass Flow Stream

- ***Fan Thrust***

$$\left(\mathbb{T}\right)_{fan} = \gamma \cdot M_{\infty}^{2}\left[\left(\frac{\beta}{1+\beta}\right)\cdot\left(\frac{V_{exit\ fan}}{V_{\infty}} - 1\right)\right] = \gamma \cdot M_{\infty}^{2}\left[\left(\frac{\beta}{1+\beta}\right)\cdot\left(\sqrt{\frac{\tau_{r}\cdot\tau_{c\ fan} - 1}{\tau_{r} - 1}} - 1\right)\right]$$

The Ideal TurboFan Cycle (5)

- Now look at Core Flow Stream



$\dot{m}_{core}$ $\quad V_{\substack{exit\\core}}$

- Look at by core velocity ratio

$$\frac{V_{\substack{exit\\core}}}{V_\infty} = \frac{M_{\substack{exit\\core}}}{M_\infty} \cdot \sqrt{\frac{T_{\substack{exit\\core}}}{T_\infty}}$$

- First calculate core exit stagnation pressure

$$\rightarrow P_{0_{\substack{exit\\core}}} = \frac{P_{0_\infty}}{p_\infty} \cdot \frac{P_{0_1}}{P_{0_\infty}} \cdot \frac{P_{0_2}}{P_{0_1}} \cdot \frac{P_{0_{3_{core}}}}{P_{0_2}} \cdot \frac{P_{0_{4_{core}}}}{P_{0_{3_{core}}}} \cdot \frac{P_{0_{5_{core}}}}{P_{0_{4_{core}}}} \, p_\infty \rightarrow ideal\ cycle \rightarrow$$

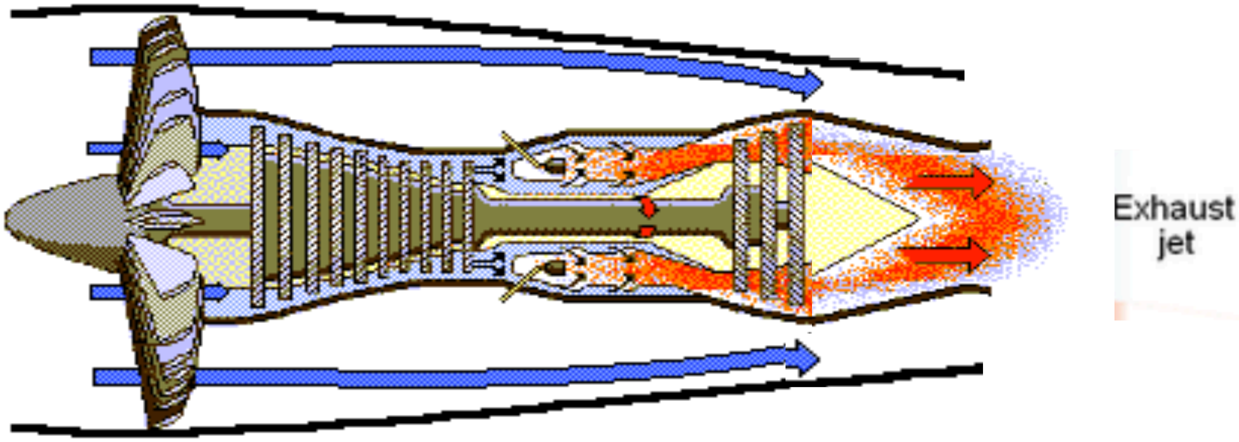
$$\pi_d = \frac{P_{0_1}}{P_{0_\infty}} \cdot \frac{P_{0_2}}{P_{0_1}} = 1$$

$$\pi_b = 1$$

$$p_{\substack{exit\\core}} = p_\infty$$

The Ideal TurboFan Cycle (6)

- Now look at Core Flow Stream



$V_{\substack{exit \\ core}}$

Substitute

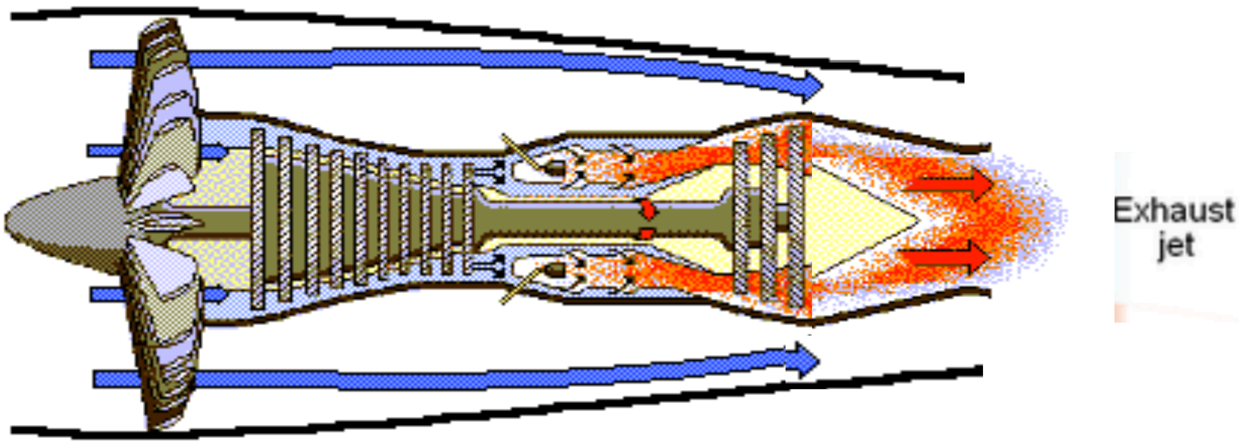
$$\rightarrow P_{0_{\substack{exit \\ core}}} = \pi_r \cdot \pi_c \cdot \pi_t \cdot p_\infty = p_{\substack{exit \\ core}} \cdot \left(1 + \frac{\gamma+1}{2} M^2_{\substack{exit \\ core}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$Isentropic\ compressor \rightarrow \pi_c = \left(\tau_c\right)^{\frac{\gamma}{\gamma-1}} \rightarrow \boxed{\tau_r \cdot \tau_c \cdot \tau_t = \left(1 + \frac{\gamma+1}{2} M^2_{\substack{exit \\ core}}\right)}$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty}\left(1 + \frac{\gamma+1}{2} M^2_\infty\right) \rightarrow \frac{\gamma+1}{2} M^2_\infty = \tau_r - 1$$

The Ideal TurboFan Cycle (7)

- Now look at Core Flow Stream



$V_{\substack{exit\\core}}$

Solve for exit Mach

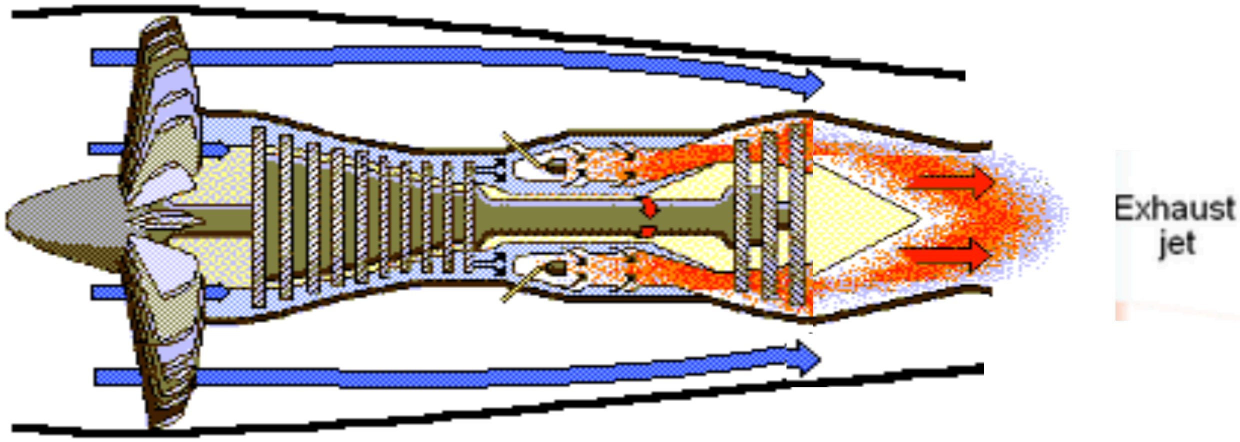
$$\rightarrow \frac{\gamma+1}{2} M^2_{\substack{exit\\core}} = \tau_r \cdot \tau_c \cdot \tau_t - 1 \rightarrow \frac{\frac{\gamma+1}{2} M^2_{\substack{exit\\core}}}{\frac{\gamma+1}{2} M^2_{\infty}} = \frac{\tau_r \cdot \tau_c \cdot \tau_t - 1}{\tau_r - 1} \rightarrow \boxed{\frac{M^2_{\substack{exit\\core}}}{M^2_{\infty}} = \frac{\tau_r \cdot \tau_c \cdot \tau_t - 1}{\tau_r - 1}}$$

Substitute

$$\frac{V_{\substack{exit\\core}}}{V_\infty} = \frac{M_{\substack{exit\\core}}}{M_\infty} \cdot \sqrt{\frac{T_{\substack{exit\\core}}}{T_\infty}} = \sqrt{\left(\frac{\tau_r \cdot \tau_c \cdot \tau_t - 1}{\tau_r - 1}\right)} \cdot \sqrt{\frac{T_{\substack{exit\\core}}}{T_\infty}}$$

The Ideal TurboFan Cycle (8)

- Now look at Core Flow Stream



$V_{\substack{exit\\core}}$

Calculate exit Stagnation Temperature

$$Calculate \rightarrow T_{0_{\substack{exit\\core}}} = T_{\infty} \cdot \left(\frac{T_{0_{\infty}}}{T_{\infty}} \cdot \frac{T_{0_{2_{core}}}}{T_{0_{\infty}}} \cdot \frac{T_{0_{3_{core}}}}{T_{0_{2_{core}}}} \cdot \frac{T_{0_{4_{core}}}}{T_{0_{3_{core}}}} \cdot \frac{T_{0_{5_{core}}}}{T_{0_{4_{core}}}} \cdot \frac{T_{0_{\substack{exit\\core}}}}{T_{0_{5_{core}}}} \right) = T_{\infty} \cdot \left(\tau_r \cdot \tau_d \cdot \tau_c \cdot \tau_t \cdot \tau_n \right)$$

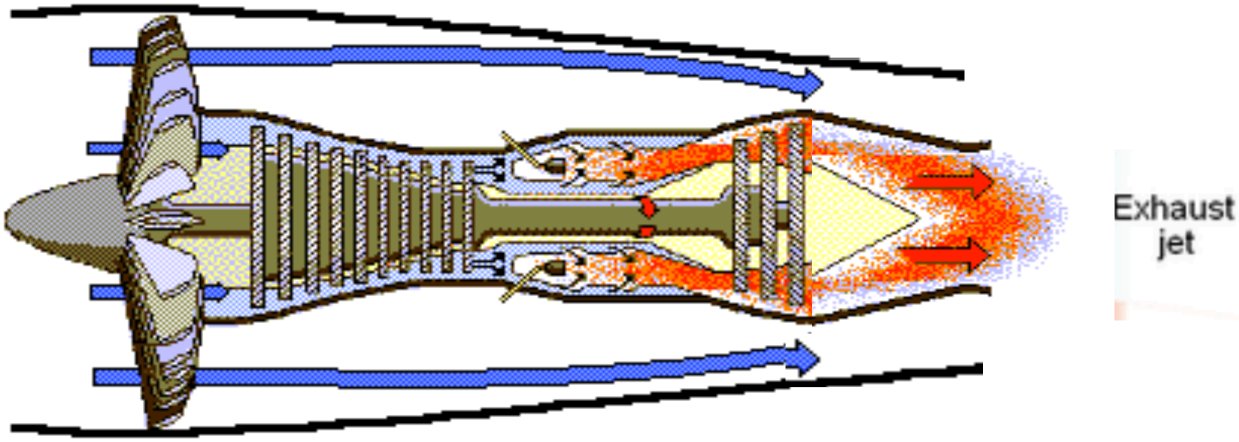
$$Isentropic\ nozzle \rightarrow T_{0_{\substack{exit\\core}}} = T_{\substack{exit\\core}} \cdot \left(1 + \frac{\gamma+1}{2} M^2_{\substack{exit\\core}} \right)$$

$$From\ previous \rightarrow \left(1 + \frac{\gamma+1}{2} M^2_{\substack{exit\\core}} \right) = \tau_r \cdot \tau_c \cdot \tau_t$$

$$\rightarrow T_{0_{\substack{exit\\core}}} = T_{\substack{exit\\core}} \cdot \left(\tau_r \cdot \tau_c \cdot \tau_t \right)$$

The Ideal TurboFan Cycle (9)

- Now look at Core Flow Stream



$V_{\substack{exit\\core}}$

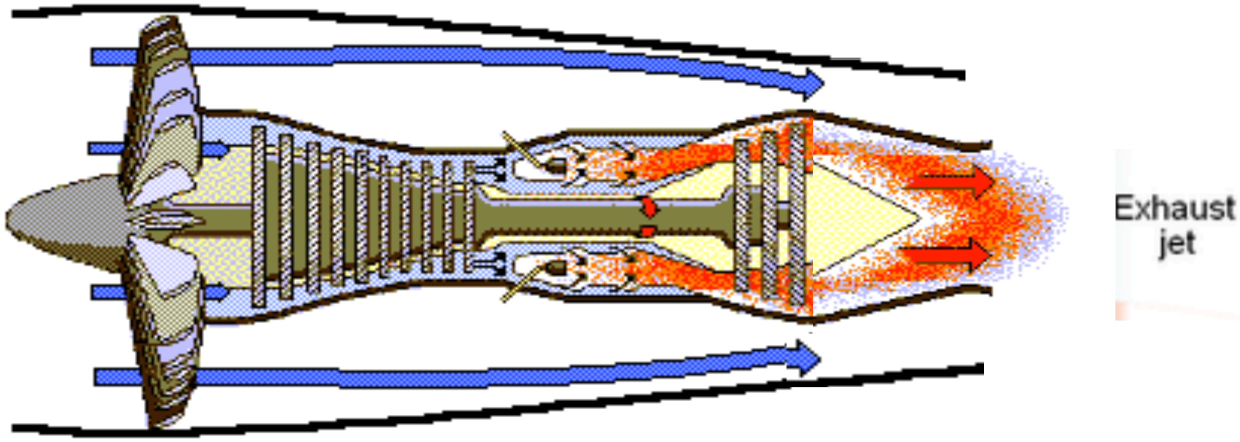
Write in terms of maximum temperature ratio

$$\frac{T_{\substack{exit\\core}}}{T_\infty} = \frac{T_{0_{4_{core}}}\left(1+\frac{\gamma+1}{2}M^2_{4_{core}}\right)}{T_{0_{3_{core}}}\left(1+\frac{\gamma+1}{2}M^2_{3_{core}}\right)} \rightarrow M_{4_{core}} \approx M_{3_{core}} \approx 0 \rightarrow \frac{T_{\substack{exit\\core}}}{T_\infty} = \frac{T_{0_{4_{core}}}}{T_{0_{3_{core}}}} \cdot \frac{T_{0_{2_{core}}}}{T_{0_\infty}} \cdot \frac{T_\infty}{T_\infty} = \frac{T_{0_{4_{core}}}}{T_\infty} \cdot \frac{T_{0_{2_{core}}}}{T_{0_{3_{core}}}} \cdot \frac{T_\infty}{T_{0_\infty}} = \frac{\tau_\lambda}{\tau_c \cdot \tau_r}$$

$$\frac{T_{\substack{exit\\core}}}{T_\infty} = \frac{\tau_\lambda}{\tau_c \cdot \tau_r} \rightarrow \frac{V_{\substack{exit\\core}}}{V_\infty} = \sqrt{\left(\frac{\tau_r \cdot \tau_c \cdot \tau_t - 1}{\tau_r - 1}\right)} \cdot \sqrt{\frac{\tau_\lambda}{\tau_c \cdot \tau_r}} \rightarrow \left(\frac{V_{\substack{exit\\core}}}{V_\infty}\right)^2 = \left(\frac{\tau_r \cdot \tau_c \cdot \tau_t - 1}{\tau_r - 1}\right) \cdot \left(\frac{\tau_\lambda}{\tau_c \cdot \tau_r}\right)$$

The Ideal TurboFan Cycle (10)

- Now look at Core Flow Stream



$V_{exit\ core}$

- ***Core Thrust***

$$\left(\mathbb{T}\right)_{core} = \gamma \cdot M_\infty^2 \left[\frac{1+\frac{1}{f}\left(1+\beta\right)}{\left(1+\beta\right)} \cdot \left(\sqrt{\left(\frac{\tau_r \cdot \tau_c \cdot \tau_t - 1}{\tau_r - 1}\right) \cdot \left(\frac{\tau_\lambda}{\tau_c \cdot \tau_r}\right)} - 1\right)\right]$$

The Ideal TurboFan Cycle (11)

Fan/Core/Pressure Thrust Summary

- Fan

$$\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2 = \left(\frac{\tau_r \cdot \tau_{c_{fan}} - 1}{\tau_r - 1}\right)$$

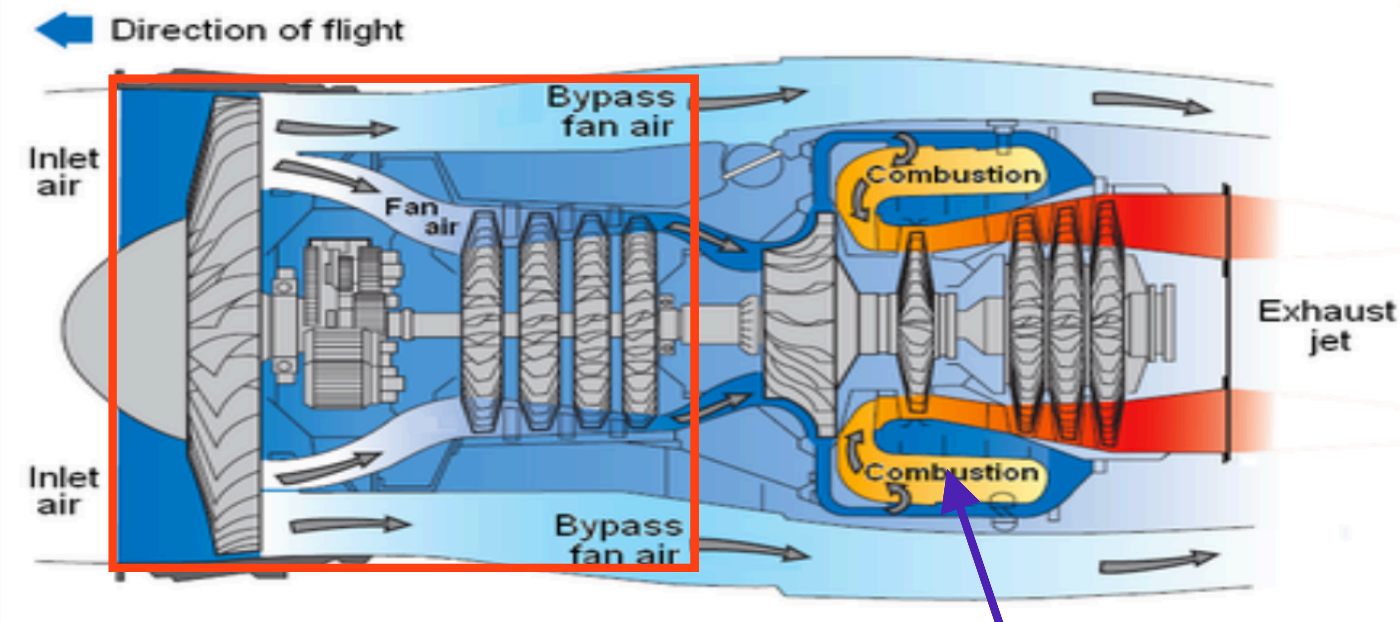
$$\left(\mathbb{T}\right)_{fan} = \gamma \cdot M_\infty^2 \left[\left(\frac{\beta}{1+\beta}\right)\cdot\left(\frac{V_{exit_{fan}}}{V_\infty} - 1\right)\right] = \gamma \cdot M_\infty^2 \left[\left(\frac{\beta}{1+\beta}\right)\cdot\left(\sqrt{\frac{\tau_r \cdot \tau_{c_{fan}} - 1}{\tau_r - 1}} - 1\right)\right]$$

- Core

$$\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2 = \left(\frac{\tau_r \cdot \tau_c \cdot \tau_t - 1}{\tau_r - 1}\right)\cdot\left(\frac{\tau_\lambda}{\tau_c \cdot \tau_r}\right)$$

$$\left(\mathbb{T}\right)_{core} = \gamma \cdot M_\infty^2 \left[\frac{1 + \frac{1}{f}(1+\beta)}{(1+\beta)}\cdot\left(\sqrt{\left(\frac{\tau_r \cdot \tau_c \cdot \tau_t - 1}{\tau_r - 1}\right)\cdot\left(\frac{\tau_\lambda}{\tau_c \cdot \tau_r}\right)} - 1\right)\right]$$

Turbine-Compressor-Fan Work Matching



- Work taken from flow by high and low pressure turbine drives both compressor and fan,

$$\dot{W}_{turbine} = \left(\dot{m}_{a_{core}} + \dot{m}_{fuel}\right)\cdot\left(h_{0_4} - h_{0_5}\right) = \left(\dot{m}_{a_{core}}\right)\cdot\left(h_{0_3} - h_{0_2}\right) + \left(\dot{m}_{a_{fan}}\right)\cdot\left(h_{0_{3_{fan}}} - h_{0_2}\right)$$

- $Approximating\ Cp \approx const, factoring\ out\ \dot{m}_a C_p T_\infty$

$$\dot{m}_a C_p T_\infty \left[\left(\frac{\dot{m}_{a_{core}}}{\dot{m}_a} + \frac{\dot{m}_{fuel}}{\dot{m}_a}\right)\cdot\left(\frac{T_{0_4} - T_{0_5}}{T_\infty}\right) = \left(\frac{\dot{m}_{a_{core}}}{\dot{m}_a}\right)\cdot\left(\frac{T_{0_{3_{core}}} - T_{0_2}}{T_\infty}\right) + \left(\frac{\dot{m}_{a_{fan}}}{\dot{m}_a}\right)\cdot\left(\frac{T_{0_{3_{fan}}} - T_{0_2}}{T_\infty}\right)\right]$$

Turbine-Compressor-Fan Work Matching (2)

→ *Simplify*

$$\left(1-B+\frac{1}{f}\right)\cdot\left(\frac{T_{0_4}}{T_\infty}-\frac{T_{0_5}}{T_{0_4}}\frac{T_{0_4}}{T_\infty}\right)=\left(1-B\right)\cdot\left(\frac{T_{0_2}}{T_\infty}\frac{T_{0_{3_{core}}}}{T_{0_2}}-\frac{T_{0_2}}{T_\infty}\right)+\left(\frac{\beta}{1+\beta}\right)\cdot\left(\frac{T_{0_2}}{T_\infty}\frac{T_{0_{3_{fan}}}}{T_{0_2}}-\frac{T_{0_2}}{T_\infty}\right)$$

→ *Substitute*

$$\left[\begin{array}{ccc} \tau_\lambda=\frac{T_{0_4}}{T_\infty} & \tau_t=\frac{T_{05}}{T_{04}} & \tau_r=\frac{T_{0_\infty}}{T_\infty}=\frac{T_{0_2}}{T_\infty} \\ & \frac{T_{0_{3_{core}}}}{T_{0_2}}=\tau_{c_{core}} & \frac{T_{0_{3_{fan}}}}{T_{0_2}}=\tau_{c_{fan}} \end{array}\right]\rightarrow\left(1-B+\frac{1}{f}\right)\cdot\left(\tau_\lambda-\tau_t\tau_\lambda\right)=\left(1-B\right)\cdot\left(\tau_r\tau_{c_{core}}-\tau_r\right)+\left(\frac{\beta}{1+\beta}\right)\cdot\left(\tau_r\tau_{c_{fan}}-\tau_r\right)$$

→ *Solve for* τ_t

$$\tau_t=1-\frac{\left(1-B\right)\cdot\frac{\tau_r}{\tau_\lambda}\cdot\left(\tau_{c_{core}}-1\right)+\left(\frac{\beta}{1+\beta}\right)\cdot\frac{\tau_r}{\tau_\lambda}\cdot\left(\tau_{c_{fan}}-1\right)}{\left(1-B+\frac{1}{f}\right)}$$

Turbine-Compressor-Fan Work Matching (3)

$$\rightarrow let \quad B = \frac{\beta}{1+\beta} \rightarrow \tau_t = 1 - \left(\frac{\tau_r}{\tau_\lambda}\right) \frac{\left(1 - \frac{\beta}{1+\beta}\right)\cdot\left(\tau_{c_{core}} - 1\right) + \left(\frac{\beta}{1+\beta}\right)\cdot\left(\tau_{c_{fan}} - \tau_r\right)}{\left(1 - \frac{\beta}{1+\beta} + \frac{1}{f}\right)}$$

$\rightarrow$ *Simplify*

$$\tau_t = 1 - \left(\frac{\tau_r}{\tau_\lambda}\right) \frac{\left(\frac{1}{1+\beta}\right)\cdot\left(\tau_{c_{core}} - 1\right) + \left(\frac{\beta}{1+\beta}\right)\cdot\frac{\tau_r}{\tau_\lambda}\cdot\left(\tau_{c_{fan}} - 1\right)}{\left(\frac{1}{1+\beta} + \frac{1}{f}\right)} = 1 - \left(\frac{\tau_r}{\tau_\lambda}\right) \frac{\left(\tau_{c_{core}} - 1\right) + \beta\cdot\left(\tau_{c_{fan}} - 1\right)}{\left(1 + \frac{1+\beta}{f}\right)}$$

$$\boxed{\tau_t = 1 - \left(\frac{\tau_r}{\tau_\lambda}\right) \frac{\left(\tau_{c_{core}} - 1\right) + \beta\cdot\left(\tau_{c_{fan}} - 1\right)}{\left(1 + \frac{1+\beta}{f}\right)}}$$

$$\rightarrow for\ \dot{m}_{air} >> \dot{m}_{fuel} \rightarrow \frac{1}{f} = 0 \rightarrow \boxed{\tau_t = 1 - \left(\frac{\tau_r}{\tau_\lambda}\right)\cdot\left[\left(\tau_{c_{core}} - 1\right) + \beta\cdot\left(\tau_{c_{fan}} - 1\right)\right]}$$

Air to Fuel Ratio

$\rightarrow$ *Rearrange*

$$\dot{m}_{fuel}\cdot\left(h_{fuel}-h_{0_4}\right)=\dot{m}_{a_{core}}\cdot\left(h_{0_4}-h_{0_{3_{core}}}\right)$$

$\rightarrow$ *factor out* $\dot{m}_a C_p \cdot T_\infty$

$$\dot{m}_a C_p\cdot T_\infty\left[\frac{\dot{m}_{fuel}}{\dot{m}_a}\cdot\left(\frac{h_{fuel}}{C_p\cdot T_\infty}-\frac{T_{0_4}}{T_\infty}\right)=\frac{\dot{m}_{a_{core}}}{\dot{m}_a}\cdot\left(\frac{T_{0_4}}{T_\infty}-\frac{T_{0_{2_{core}}}}{T_\infty}\cdot\frac{T_{0_{3_{core}}}}{T_{0_{2_{core}}}}\right)\right]$$

$\rightarrow$ *Simplify*

$$\frac{1}{f}\cdot\left(\tau_{fuel}-\tau_\lambda\right)=\frac{1}{1+\beta}\cdot\left(\tau_\lambda-\tau_r\cdot\tau_{c_{core}}\right)\rightarrow$$

$$\frac{1}{f}=\frac{1}{1+\beta}\cdot\left(\frac{\tau_\lambda-\tau_r\cdot\tau_{c_{core}}}{\tau_{fuel}-\tau_\lambda}\right)$$

$$f=\left(1+\beta\right)\cdot\left(\frac{\tau_{fuel}-\tau_\lambda}{\tau_\lambda-\tau_r\cdot\tau_{c_{core}}}\right)$$

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Collected TurboFan Matching Equations

- Turbine Work

$$\tau_t = 1 - \left(\frac{\tau_r}{\tau_\lambda}\right) \frac{\left(\tau_{c_{core}} - 1\right) + \beta \cdot \left(\tau_{c_{fan}} - 1\right)}{\left(1 + \frac{1+\beta}{f}\right)}$$

- Fuel/Air Flow

$$\frac{1}{f} = \frac{1}{1+\beta} \cdot \left(\frac{\tau_\lambda - \tau_r \cdot \tau_{c_{core}}}{\tau_{fuel} - \tau_\lambda}\right)$$

$$f = \frac{\dot{m}_a}{\dot{m}_{fuel}}$$

$$f = (1+\beta) \cdot \left(\frac{\tau_{fuel} - \tau_\lambda}{\tau_\lambda - \tau_r \cdot \tau_{c_{core}}}\right)$$

If the bypass ratio goes to zero the matching condition reduces to the usual turbojet formula.


Questions??