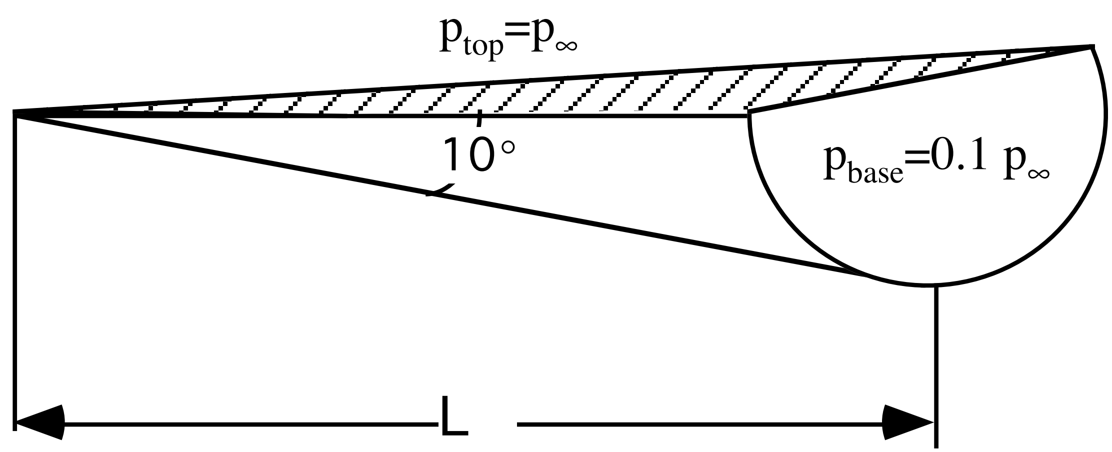
Problem 4

- Consider a hypersonic wave rider constructed from a 10° half-angle cone, cut horizontally in half along centerline



- Compute L/D at zero angle of attack, Mach=15
- Assume inviscid flow, and γ=1.325, 30 km altitude

Problem 4 Solution

$$\left\{M_\infty = 15, \theta_{cone} = 10^o\right\} \to \beta_{shock} = 11.413^o$$

$$\text{behind shock} \to \frac{p_{shock}}{p_\infty} = 9.902,\ M_{shock} = 9.600$$

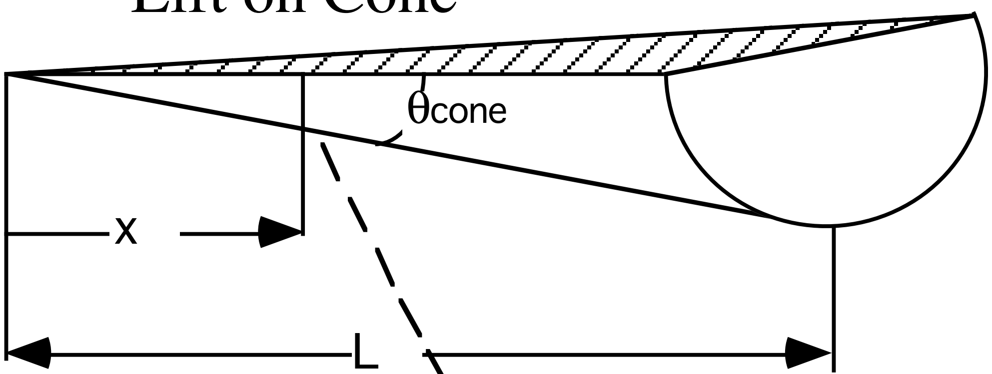
Taylor-Maccol Solution

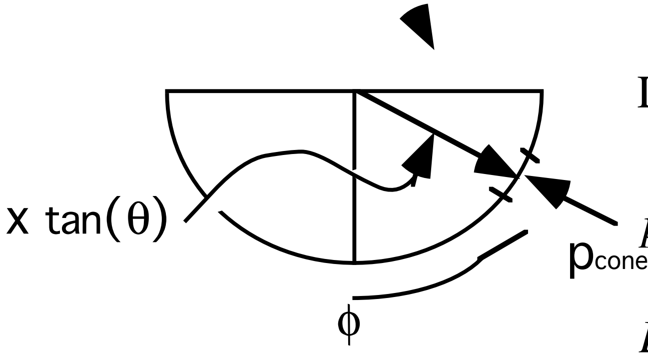
$$M_{cone} = 9.5166 \to \frac{p_{cone}}{p_\infty} = \frac{p_{shock}}{p_\infty}\left[\frac{1+\frac{\gamma-1}{2}M_{shock}^2}{1+\frac{\gamma-1}{2}M_{cone}^2}\right]^{\frac{\gamma}{\gamma-1}} =$$

$$\frac{p_{cone}}{p_\infty} = 9.90197\left(\frac{1+\frac{1.325-1}{2}(9.60049^2)}{1+\frac{1.325-1}{2}(9.5166^2)}\right)^{\frac{1.325}{1.325-1}} = 10.5888$$

Problem 4 Solution

- Lift on Cone





Vertical Pressure Force on Lower Cone Surface

$$L_{lower} = \int_0^L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(p_{cone} x \tan(\theta_{cone}) \cos(\phi) d\phi \right) \cos(\theta_{cone}) \frac{dx}{\cos(\theta_{cone})} =$$

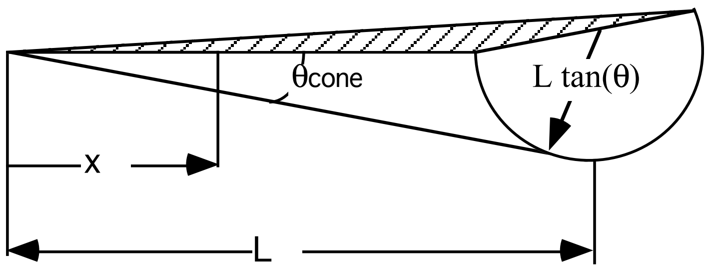
$$p_{cone} \int_0^L 2x \tan(\theta_{cone}) dx = p_{cone} L^2 \tan(\theta_{cone})$$

$$L^2 \tan(\theta_{cone}) = \frac{1}{2} L \times 2L \tan(\theta_{cone}) = \text{planform area of cone}$$

$$\rightarrow \text{Lift} = (p_{cone} - p_\infty) L^2 \tan(\theta_{cone})$$

Problem 4 Solution

- Drag on Cone



Axial Pressure force on Lower Cone Surface

$$D_{lower} = \int_0^L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(p_{cone} x \tan(\theta_{cone}) d\phi \right) \sin(\theta_{cone}) \frac{dx}{\cos(\theta_{cone})} =$$

$$p_{cone} \int_0^L \pi x \tan^2(\theta_{cone}) dx = \frac{\pi}{2} p_{cone} L^2 \tan^2(\theta_{cone}) = p_{cone} \frac{1}{2} \pi \left[L \tan(\theta_{cone}) \right]^2$$

$$\frac{1}{2} \pi \left[L \tan(\theta_{cone}) \right]^2 = \textit{base} \text{ area of cone}$$

$$\rightarrow \text{Drag} = \left(p_{cone} - p_{base} \right) \frac{1}{2} \pi \left[L \tan(\theta_{cone}) \right]^2 = \left(p_{cone} - 0.1 p_\infty \right) \frac{1}{2} \pi \left[L \tan(\theta_{cone}) \right]^2$$

Problem 4 Solution

- Lift to drag ratio

$$\frac{\text{Lift}}{\text{Drag}} = \frac{\left(p_{cone} - p_{\infty}\right) L^2 \tan\left(\theta_{cone}\right)}{\left(p_{cone} - 0.1 p_{\infty}\right)\frac{1}{2}\pi\left[L\tan\left(\theta_{cone}\right)\right]^2} =$$

$$\frac{2}{\pi}\frac{\left(p_{cone} - p_{\infty}\right)}{\left(p_{cone} - 0.1 p_{\infty}\right)\tan\left(\theta_{cone}\right)} = \frac{2}{\pi}\frac{\left(\frac{p_{cone}}{p_{\infty}} - 1\right)}{\left(\frac{p_{cone}}{p_{\infty}} - 0.1\right)\tan\left(\theta_{cone}\right)}$$

$$\frac{\frac{2}{\pi}(10.5888 - 1)}{(10.5888 - 0.1)\tan\left(\frac{\pi}{180}10\right)} = 3.3007$$