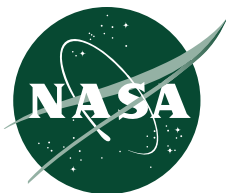NASA/CR—2012–217475

Missile Aerodynamics for Ascent and Re-entry

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November 2012

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Acknowledgment

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LIST OF ACRONYMS AND ABBREVIATIONS

B frame   traditional body coordinate frame (body-fixed) with origin at the missile center-of-mass, X-axis forward, Y-axis starboard, and Z-axis completing the right-handed system

DOF   degrees-of-freedom

MAVERIC   Marshall Aerospace Vehicle Representation in "C," a 6-DOF digital flight simulation used at MSFC

M frame   missile coordinate frame with origin at the MRP, X-axis forward along the missile centerline, and X-Z plane oriented to contain the wind-relative velocity vector

MRP   moment reference point for missile aerodynamics on the missile centerline

P frame   MRP coordinate frame (body-fixed) with origin at the MRP and all three axes parallel to those of the B frame

NOMENCLATURE

C_{Am}	axial force coefficient in the M frame
C_{lm}	rolling moment coefficient in the M frame
C_{lmd}	roll damping moment coefficient in the M frame
C_{lpm}	roll damping derivative in the M frame
C_{mm}	pitching moment coefficient in the M frame
C_{mmd}	pitch damping moment coefficient in the M frame
C_{mqm}	pitch damping derivative in the M frame
C_{Nm}	normal force coefficient in the M frame
C_{Ym}	side force coefficient in the M frame
C_{yawm}	yawing moment coefficient in the M frame
C_{yawmd}	yaw damping moment coefficient in the M frame
C_{yawrm}	yaw damping derivative in the M frame
D_{ref}	missile aerodynamic reference length (cylinder diameter)
$\vec{F}_b$	resultant aerodynamic force vector in the B frame
$\vec{F}_m$	resultant aerodynamic force vector in the M frame
$\vec{F}_p$	resultant aerodynamic force vector in the P frame
$\vec{M}_b$	resultant aerodynamic moment vector in the B frame
$\vec{M}_m$	resultant aerodynamic moment vector in the M frame
$\vec{M}_p$	resultant aerodynamic moment vector in the P frame
$\overrightarrow{MRP}_b$	vector from the center of mass to the MRP
p_m, q_m, r_m	components of the missile angular rate in the M frame (radians/s)

$\bar{Q}$	missile dynamic pressure ($\frac{1}{2}\,\rho V_R{}^2$)
S_{ref}	missile aerodynamic reference area (cylinder cross-sectional area)
V_R	magnitude of the wind-relative velocity vector at center-of-mass
$\vec{V}_R$	wind-relative velocity vector of the missile center-of-mass
α_{tot}	total angle-of-attack, measured in the X-Z plane of the M frame
ϕ_A	aerodynamic roll angle (rotation angle between the M and P frames)
ρ	density of the atmosphere at the missile's altitude

CONTRACTOR REPORT

MISSILE AERODYNAMICS FOR ASCENT AND RE-ENTRY

INTRODUCTION

This document presents equations for aerodynamic forces and moments to be used in 6-DOF missile simulations such as MAVERIC. The Missile coordinate frame (M frame) and a frame parallel to the M frame were used for aerodynamics to allow convenient simulation of either the ascent phase of flight or a tumbling re-entry. The missile configuration chosen as an example is a cylinder with fixed fins and a nose cone. The equations include both the static aerodynamic coefficients and the aerodynamic damping derivatives. The inclusion of aerodynamic damping is essential for simulating a tumbling re-entry in which large angular rates may occur. The missile aerodynamic reference area and length are the cylinder cross-sectional area (S_{ref}) and diameter (D_{ref}), respectively.

Appendix A presents a mathematical equivalency for damping derivatives to give insight into aerodynamic damping. Appendix B provides additional insight into aerodynamic damping by presenting an analytical technique for calculating the damping moment of a cylinder at its center-of-mass in cross-flow conditions.

STATIC COEFFICIENT MODEL

The static coefficients in this document are *dimensionless* and are defined in the M frame. The M frame, shown in Figure 1, can greatly simplify the missile aerodynamics, as will be discussed later in the document. The M frame naturally fits the situation of large angular excursions experienced by a tumbling missile and allows an analyst to more easily visualize the missile rotational dynamics. The M-frame origin is at the moment reference point (MRP) on the missile centerline with the X-axis pointing forward along the centerline. For purposes of illustrating the M frame, the wind-relative velocity vector, $\vec{V}_R$, has been translated from the missile center-of-mass to the MRP. The Y-axis of the M frame is pointed in the direction of $\vec{V}_R \times \hat{\imath}$, where $\hat{\imath}$ is a unit vector along the X-axis of the M frame. In other words, $\vec{V}_R$ (after being translated to the MRP) remains in the X-Z plane of the M frame regardless of the roll position of the missile. The total angle-of-attack, α_{tot}, shown in Figure 1, is measured in the X-Z plane of the M frame and has a range of 0° to $+180^o$.
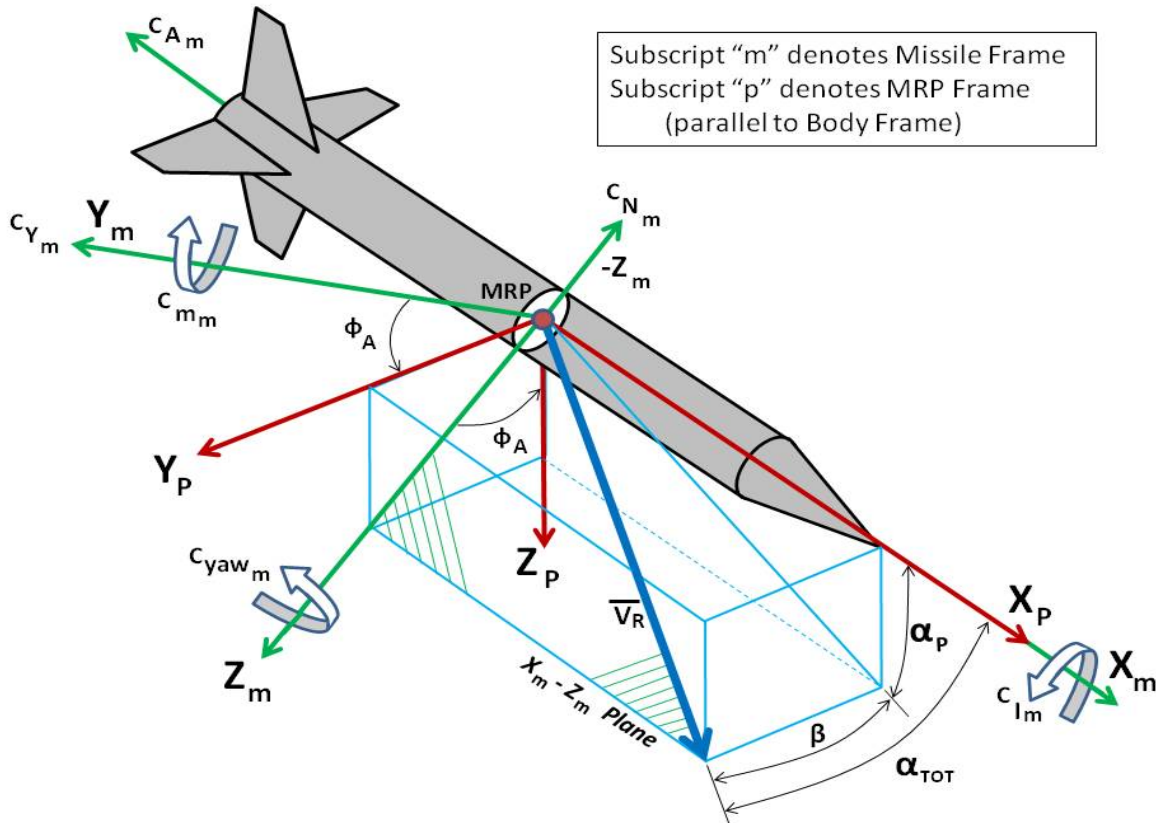

Figure 1. M frame, P frame, and static coefficients.

The six static coefficients, shown in Figure 1, are listed below. The final subscript "*m*" denotes the M frame.

$$C_{Am} \quad - \quad \text{axial force coefficient}$$
$$C_{Ym} \quad - \quad \text{side force coefficient}$$
$$C_{Nm} \quad - \quad \text{normal force coefficient}$$
$$C_{lm} \quad - \quad \text{rolling moment coefficient}$$
$$C_{mm} \quad - \quad \text{pitching moment coefficient}$$
$$C_{yawm} \quad - \quad \text{yawing moment coefficient}$$

The axial force coefficient, C_{Am} , and the normal force coefficient, C_{Nm} , are positive in a direction opposite to that of the X-axis and Z-axis, respectively. The three moment coefficients are positive by the right-hand rule.

C_{Am} , C_{Nm} , and C_{mm} are called the pitch plane (or longitudinal) coefficients. C_{Ym} , C_{lm} , and C_{yawm} are called the lateral-directional (or lateral) coefficients and can be set equal to zero in the M frame for a missile that is axisymmetric about the centerline. The reason that the lateral-directional coefficients can be set equal to zero for an axisymmetric missile (leaving only the axial force, normal force, and pitching moment) is that the X-Z plane of the M frame is a plane of symmetry containing the $\vec{V}_R$ vector.

Figure 1 also shows a body-fixed P frame, which is called the MRP frame, with origin at the MRP and with axes parallel to those of a traditional Body frame (B frame, not shown). The origin of the B frame is at the missile center-of-mass with the X-axis forward, the Y-axis starboard, and the Z-axis completing the right-handed system.

The aerodynamic roll angle, φ_A , is the rotation angle between the M and P frames and will be defined mathematically in the next paragraph. The P-frame angle-of-attack, α_p, and sideslip angle, β , in Figure 1, are not used in the formulation of static coefficients in the M frame.

Static coefficients for a missile are, in general, a function of α_{tot}, φ_A , and Mach number. For a missile that is axisymmetric about the centerline, the static coefficients in the M frame are only a function of α_{tot} and Mach, because the aerodynamic properties do not change with φ_A . The equations for these three variables are:

$$\alpha_{tot} = \cos^{-1}\left(\frac{V_{R_x}}{V_R}\right) \tag{1}$$

$$\varphi_A = \tan^{-1}\left(\frac{V_{R_y}}{V_{R_z}}\right) \tag{2}$$

$$MACH = \frac{V_R}{V_{sound}} \tag{3}$$

where V_{R_x} , V_{R_y} , V_{R_z} are components of $\vec{V}_R$ in the B frame, translated to the MRP

V_{sound} is the speed of sound at the missile's altitude

α_{tot} has a range of 0° to $+180^\circ$ and is never negative

φ_A has a range of -180° to $+180^\circ$

It is imperative to note that singularities exist in the φ_A equation at $\alpha_{tot} = 0^\circ$ and 180°, and these must be handled by specifying a value of φ_A (such as 0°) for these two values of α_{tot}.

If aerodynamic uncertainties are to be modeled in a simulation, the dispersions should be applied to the static coefficients in the M frame. The dispersed static coefficients will then be used to calculate aerodynamic forces and moments on the missile in the M frame as described later in this document.

AERODYNAMIC DAMPING MODEL

The missile aerodynamic damping model is based on traditional damping derivatives that are *dimensionless* and are used to calculate aerodynamic moments caused by angular rates of the missile. See Appendix A for mathematical insight into damping derivatives. Missile damping derivatives are typically negative and, therefore, tend to reduce the angular rates.

Damping derivatives should be developed at (or near) the missile center-of-mass rather than at the MRP. See Appendix B for an explanation of the effects of longitudinal center-of-mass location upon damping. Damping derivatives should also be developed in a coordinate frame parallel to the M frame to properly separate the M-frame pitch and yaw damping effects. The coordinate frame that will be used for calculating missile damping derivatives is parallel to the M frame and has its origin on the centerline at the same X-axis location as the center-of-mass. In using such a coordinate frame on the centerline, the effects upon damping of small lateral center-of-mass offsets from the centerline are ignored.

Damping derivatives represent pure couples at the center-of-mass, and these couples may be applied at any other point. After being calculated on the centerline at the same X-axis location as the center-of-mass, the damping derivatives will be applied at the MRP in the M frame. Damping derivatives will be given names as though they were originally established in the M frame. Damping derivatives in this document are C_{lpm}, C_{mqm}, and C_{yawrm} for roll, pitch, and yaw, respectively. These damping derivatives are positive in the M frame by the right-hand rule, and their mathematical definitions are:

$$C_{lpm} = \frac{\partial C_{lm}}{\partial \left(\frac{p_m D_{ref}}{2V_R} \right)} \tag{4}$$

$$C_{mqm} = \frac{\partial C_{mm}}{\partial \left(\frac{q_m D_{ref}}{2V_R} \right)} \tag{5}$$

$$C_{yawrm} = \frac{\partial C_{yawm}}{\partial \left(\frac{r_m D_{ref}}{2V_R} \right)} \tag{6}$$

The dimensionless terms in the denominators of equations (4), (5), and (6) do seem a bit strange, but their mathematical basis can be found in a close examination of the damping equations in Appendix A.

These damping derivatives use M-frame angular rates (p_m, q_m, r_m), which are obtained by transforming the body rates from the B frame to the M frame by a negative rotation ($-\varphi_A$) about the centerline. For more complicated vehicle configurations such as airplanes, aerodynamicists sometimes use additional damping moment terms that are not discussed in this document. There also might be a change in an *aerodynamic force* due to an angular rate, such as the change in normal force caused by a pitch angular rate, but this type force will not be considered at this time.

At $\alpha_{tot} = 0^o$, the pitch damping derivative for the missile is assumed to be equal to the yaw damping derivative (also applies for $\alpha_{tot} = 180^o$) because the missile shape approximates a vehicle that is axisymmetric about the longitudinal axis. The yaw damping derivative for the missile at $\alpha_{tot} = 90^o$ (cross-flow conditions) is assumed to be much less than the corresponding pitch damping derivative because of the difference in the orientations of the pitch and yaw axes with respect to the wind-relative velocity vector.

If aerodynamic uncertainties are to be modeled in a simulation, the dispersions should be applied to the damping derivatives in the M frame. The dispersed damping derivatives should then be converted into the form of *dimensionless damping moment coefficients* (C_{lmd} , C_{mmd} , C_{yawmd}) in the M frame by assuming a linear relationship between coefficients and angular rates:

$$C_{lmd} = \frac{p_m \, D_{ref}}{2 V_R} \cdot C_{lpm} \tag{7}$$

$$C_{mmd} = \frac{q_m \, D_{ref}}{2 V_R} \cdot C_{mqm} \tag{8}$$

$$C_{yawmd} = \frac{r_m \, D_{ref}}{2 V_R} \cdot C_{yawrm} \tag{9}$$

The damping moment coefficients in equations (7), (8), and (9) can be interpreted as the changes in the rolling, pitching, and yawing moment coefficients caused by damping. Damping moment coefficients will be used with the six static coefficients to calculate the aerodynamic forces and moments.

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AERODYNAMIC FORCES AND MOMENTS

Aerodynamic forces and moments will first be calculated in the M frame at the MRP using the six static coefficients and three damping moment coefficients described in this document. The vector equations for the aerodynamic forces and moments in the M frame are:

$$\vec{F}_m = \begin{pmatrix} F_{Xm} \\ F_{Ym} \\ F_{Zm} \end{pmatrix} = \overline{Q} \; S_{ref} \begin{pmatrix} -C_{Am} \\ C_{Ym} \\ -C_{Nm} \end{pmatrix} \tag{10}$$

$$\vec{M}_m = \begin{pmatrix} M_{Xm} \\ M_{Ym} \\ M_{Zm} \end{pmatrix} = \overline{Q} \; S_{ref} \; D_{ref} \begin{pmatrix} C_{lm} + C_{lmd} \\ C_{mm} + C_{mmd} \\ C_{yawm} + C_{yawmd} \end{pmatrix} \tag{11}$$

where $\overline{Q}$ is the missile dynamic pressure. These aerodynamic forces and moments must be transformed from the M frame to the P frame by a rotation about the centerline through the aerodynamic roll angle, φ_A . These transformations, in matrix notation, are:

$$\begin{pmatrix} F_{Xp} \\ F_{Yp} \\ F_{Zp} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\varphi_A) & sin(\varphi_A) \\ 0 & -sin(\varphi_A) & cos(\varphi_A) \end{pmatrix} \begin{pmatrix} F_{Xm} \\ F_{Ym} \\ F_{Zm} \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} M_{Xp} \\ M_{Yp} \\ M_{Zp} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\varphi_A) & sin(\varphi_A) \\ 0 & -sin(\varphi_A) & cos(\varphi_A) \end{pmatrix} \begin{pmatrix} M_{Xm} \\ M_{Ym} \\ M_{Zm} \end{pmatrix} \tag{13}$$

These P-frame aerodynamic forces and moments must finally be transformed to the B frame at the missile center-of-mass, taking into account the location of the moment reference point $(\overrightarrow{MRP_b}$) relative to the center-of-mass. The vector equations for the aerodynamic forces and moments in the B frame are:

$$\vec{F}_b = \begin{pmatrix} F_{X_b} \\ F_{Y_b} \\ F_{Z_b} \end{pmatrix} = \begin{pmatrix} F_{Xp} \\ F_{Yp} \\ F_{Zp} \end{pmatrix} \tag{14}$$

$$\vec{M}_b = \begin{pmatrix} M_{Xb} \\ M_{Yb} \\ M_{Zb} \end{pmatrix} = \begin{pmatrix} M_{Xp} \\ M_{Yp} \\ M_{Zp} \end{pmatrix} + \begin{pmatrix} MRP_{Xb} \\ MRP_{Yb} \\ MRP_{Zb} \end{pmatrix} \times \begin{pmatrix} F_{Xp} \\ F_{Yp} \\ F_{Zp} \end{pmatrix} \qquad (15)$$

Carrying out the cross product in equation (15), the moment equations in the B frame become:

$$M_{Xb} = M_{Xp} + MRP_{Yb}\, F_{Zp} - MRP_{Zb}\, F_{Yp} \qquad (16)$$

$$M_{Yb} = M_{Yp} + MRP_{Zb}\, F_{Xp} - MRP_{Xb}\, F_{Zp} \qquad (17)$$

$$M_{Zb} = M_{Zp} + MRP_{Xb}\, F_{Yp} - MRP_{Yb}\, F_{Xp} \qquad (18)$$

The force and moment equations in this section should be used to incorporate the aerodynamic models in this document into a 6-DOF simulation.

APPENDIX A—

MATHEMATICAL EQUIVALENCY FOR DAMPING DERIVATIVES

Nomenclature

C_{fin}	normal force coefficient of fin based on fin projected area, S_{fin}
C_{mqm}	pitch damping derivative
$\vec{F}_{fin}$	normal force on fin
$\vec{M}_d$	damping moment caused by fin
$\vec{M}_{fin}$	total pitching moment caused by fin
Q_d	dynamic pressure for fin damping calculation
Q_{fin}	dynamic pressure at fin
q_m	pitch angular rate (radians/s)
V_R	magnitude of the wind-relative velocity vector at center-of-mass

This Appendix provides insight into aerodynamic damping by showing that a pitch damping derivative, C_{mqm}, for a missile is equivalent to the negative of a normal force coefficient for a fin on the missile. The example used to develop the mathematics for this coefficient equivalency is a simple missile (not necessarily realistic) with a horizontal fin at the rear to provide all the pitch damping (Figure A.1). The distance from the fin to the missile center-of-mass will be set equal to the missile reference length, D_{ref}, solely for the purpose of obtaining a final result without inserting additional constants. For this derivation the M, P, and B frames (not shown) are assumed to be coincident. All three frames have their origins on the centerline at the center-of-mass with their Z-axes pointed downward. The equations are simplified by assuming cross-flow conditions.
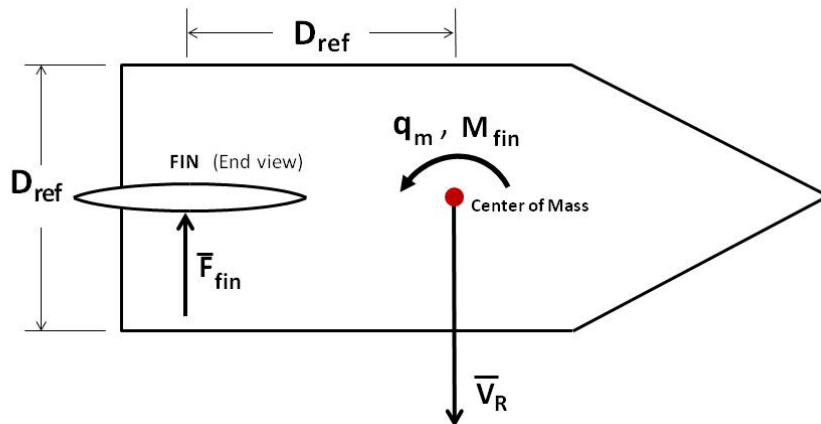

Figure A.1 Example missile for damping derivative equivalency.

By inspection of Figure A.1, the dynamic pressure at the fin is:

$$Q_{fin} = \frac{1}{2} \rho \left(V_R^2 + 2 V_R D_{ref}\, q_m + D_{ref}^2\, q_m^2 \right) \qquad (A.1)$$

The magnitude of the third term in parentheses in equation (A.1) is typically less than 1% of the magnitude of the second term, so the third term will be ignored. The fin dynamic pressure now becomes:

$$Q_{fin} = \frac{1}{2} \rho \left(V_R^2 + 2\, V_R D_{ref}\, q_m \right) \qquad (A.2)$$

The fin normal force is simply:

$$F_{fin} = Q_{fin}\, C_{fin}\, S_{fin} \qquad (A.3)$$

where S_{fin} is the fin projected area and C_{fin} is the normal force coefficient of the fin based on the fin projected area. The total pitching moment caused by the fin is the negative of the fin normal force times the moment arm, D_{ref}, as follows:

$$M_{fin} = -Q_{fin}\, C_{fin}\, S_{fin}\, D_{ref} \qquad (A.4)$$

It will now be assumed that the fin projected area, S_{fin}, is 1/4 of the missile reference area, S_{ref}. (This assumption permits the final result to be obtained without inserting additional constants). Plugging this 1/4 fin area relationship into equation (A.4) produces:

$$M_{fin} = -\frac{1}{4} Q_{fin}\, C_{fin}\, S_{ref}\, D_{ref} \qquad (A.5)$$

Going back to equation (A.1) for the dynamic pressure at the fin, the first term inside the parentheses is for the static moment, so only the second term is needed for damping calculations. The dynamic pressure to be used for fin damping is therefore:

$$Q_d = \rho\, V_R\, D_{ref}\, q_m \qquad (A.6)$$

Substituting the right side of equation (A.6) for the Q_{fin} term in equation (A.5) produces the fin damping moment as follows:

$$M_d = -\frac{1}{4} \rho\, V_R\, q_m\, C_{fin}\, S_{ref}\, D_{ref}^2 \qquad (A.7)$$

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The fin damping moment will now be expressed in a generic form using the following traditional damping moment coefficient equation:

$$M_d = \frac{1}{2} \rho \, V_R^2 \, S_{ref} \, D_{ref} \, C_{mmd} \tag{A.8}$$

where C_{mmd} is the pitch damping moment coefficient, which represents the change in the pitching moment coefficient caused by fin damping in this example. Combining equations (A.7) and (A.8) to eliminate M_d produces the following equation that shows a linear relationship between C_{mmd} and q_m for the simple example chosen:

$$C_{mmd} = -\frac{q_m D_{ref}}{2V_R} \cdot C_{fin} \tag{A.9}$$

C_{mmd} can also be expressed, in a general sense, as the following partial differential equation:

$$C_{mmd} = \frac{\partial C_{mm}}{\partial q_m} \cdot q_m \tag{A.10}$$

Combining equations (A.9) and (A.10) to eliminate C_{mmd}, and then solving for the normal force coefficient of the fin, C_{fin}, produces the following relationship:

$$C_{fin} = -\frac{\partial C_{mm}}{\partial \left(\dfrac{q_m D_{ref}}{2 V_R}\right)} \tag{A.11}$$

The traditional equation that defines the pitch damping derivative, C_{mqm}, is:

$$C_{mqm} = \frac{\partial C_{mm}}{\partial \left(\dfrac{q_m D_{ref}}{2 V_R}\right)} \tag{A.12}$$

The identical expressions on the right side of equations (A.11) and (A.12) illustrate that for purposes of missile damping, a pitch damping derivative is equivalent to the negative of a normal force coefficient for a horizontal fin.

APPENDIX B—

PITCH DAMPING MOMENT FOR A CROSS-FLOW CYLINDER

Nomenclature

C	cylinder cross-flow drag coefficient (The reference area for C is always the cylinder cross-flow projected area being analyzed, whether that area is an incremental area or the total area; C is assumed to not vary with the size of the cylinder or the size of any incremental projected area)
cg	distance from center-of-mass to forward end of cylinder, expressed as a fraction of cylinder length (forward end is right side of cylinder in figure B.1)
D	cylinder diameter (total projected area in cross-flow conditions = L D)
dx	incremental distance along cylinder length (incremental projected area = $dx\,D$)
$\vec{F}_i$	incremental aerodynamic force on incremental projected area ($\vec{F}_i = Q\,C\,dx\,D$)
L	cylinder length
$\vec{M}_d$	total damping moment about center-of-mass
$\vec{M}_i$	incremental moment about center-of-mass
Q	dynamic pressure at incremental projected area
q_m	pitch angular rate (radians/s)
V_R	magnitude of the wind-relative velocity vector at center-of-mass
X	distance from center-of-mass to incremental projected area (positive forward)

This Appendix provides additional insight into aerodynamic damping by presenting a derivation of the pitch damping moment for a cylinder in cross-flow conditions (Figure B.1). The effects of the longitudinal location of the center-of-mass are included and are very important. The cylinder is assumed to have the aerodynamic properties of an infinite cylinder (no end flow). For this derivation, the M, P and B frames (not shown) are assumed to be coincident. All three frames have their origins on the centerline at the center-of-mass with their Z-axes pointed downward.
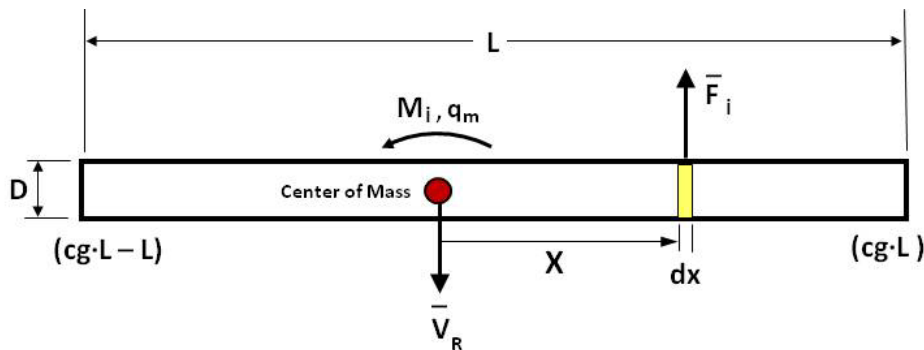

Figure B.1 Cross-flow cylinder.

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By inspection of Figure B.1, the dynamic pressure at the incremental projected area is:

$$Q = \frac{1}{2}\rho(V_R^2 - 2V_R q_m X + q_m^2 X^2) \qquad (B.1)$$

Typically, the magnitude of the third term in parentheses in equation (B.1) is approximately 1% of the magnitude of the second term, so the third term will be ignored. The dynamic pressure now becomes:

$$Q = \frac{1}{2}\rho(V_R^2 - 2V_R q_m X) \qquad (B.2)$$

The incremental aerodynamic force on the incremental projected area is:

$$F_i = \frac{1}{2}\rho(V_R^2 - 2V_R q_m X)\, C\, D\, dx \qquad (B.3)$$

The incremental moment about the center-of-mass caused by the incremental aerodynamic force is:

$$M_i = \frac{1}{2}\rho(V_R^2 - 2V_R q_m X)\, C\, X\, D\, dx \qquad (B.4)$$

The first term in parentheses of equation (B.4) is for calculating the incremental static moment and is not needed in this derivation. The second term, $-2V_R q_m X$, is the term of interest in this example because it determines the incremental damping moment, M_{di}, as follows:

$$M_{di} = -\rho V_R q_m\, C\, X^2\, D\, dx \qquad (B.5)$$

Integrating equation (B.5) from the aft end to the forward end of the cylinder (from left to right in Figure B.1) produces the following equation for the total damping moment about the center-of-mass:

$$M_d = -\frac{1}{3}\rho V_R q_m\, C\, D\, L^3(1 - 3cg + 3cg^2) \qquad (B.6)$$

Equation (B.6) for the total damping moment is the final result for this example, and it shows the significant sensitivity of pitch damping to the longitudinal center-of-mass location and to the cylinder length.

1. REPORT DATE *(DD-MM-YYYY)* 01–11–2012	2. REPORT TYPE Contractor Report	3. DATES COVERED *(From - To)* October 2011–September 2012

4. TITLE AND SUBTITLE Missile Aerodynamics for Ascent and Re-entry	5a. CONTRACT NUMBER NNM12AA41C
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) G.L. Watts* and J.W. McCarter**	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) George C. Marshall Space Flight Center, Huntsville, AL 35812 Jacobs ESSSA Group, Huntsville, AL	8. PERFORMING ORGANIZATION REPORT NUMBER M–1351
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546–0001	10. SPONSORING/MONITOR'S ACRONYM(S)
	11. SPONSORING/MONITORING REPORT NUMBER NASA/CR—2012–217475

12. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified-Unlimited
Subject Category 02
Availability: NASA CASI (443–757–5802)

13. SUPPLEMENTARY NOTES
Prepared for the Spacecraft & Vehicle Systems Dept., Engineering Directorate
COTR: Kimball Y. Ibrahim
*Qualis Corporation **DESE Research

14. ABSTRACT

Aerodynamic force and moment equations are developed for 6-DOF missile simulations of both the ascent phase of flight and a tumbling re-entry. The missile coordinate frame (M frame) and a frame parallel to the M frame were used for formulating the aerodynamic equations. The missile configuration chosen as an example is a cylinder with fixed fins and a nose cone. The equations include both the static aerodynamic coefficients and the aerodynamic damping derivatives. The inclusion of aerodynamic damping is essential for simulating a tumbling re-entry. Appended information provides insight into aerodynamic damping.

15. SUBJECT TERMS
missile aerodynamics, aerodynamic damping, damping derivatives, missile coordinate frame, missile re-entry, 6 degrees-of-freedom

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON STI Help Desk at email: help@sti.nasa.gov
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