Section 3
Rocket Science Review 103:
Estimating the Launch Vehicle Drag Coefficient

Newton's Laws as Applied to "Rocket Science"

... its not just a job ... its an adventure

Newtonian Flow Analysis

Fin Leading Edge Drag

- Stagnation Pressure Coefficient calculated based on Mach number Normal to leading edge of fins
- Scaled by leading edge area, *W•t*
- *Assumed fin thickness, t*

$$C_{p_{max}} = \frac{q_c - p_\infty}{\bar{q}} = \frac{p_\infty \cdot \left(1 + \frac{\gamma-1}{2} M_\perp^2\right)^{\frac{\gamma}{\gamma-1}} - p_\infty}{\frac{\gamma}{2} p_\infty M_\perp^2} = \frac{\left(1 + \frac{\gamma-1}{2} \cdot \left(M_\infty \cdot \cos\theta_{L.E.}\right)^2\right)^{\frac{\gamma}{\gamma-1}} - 1}{\frac{\gamma}{2} \cdot \left(M_\infty \cdot \cos\theta_{L.E.}\right)^2}$$

$$\left(C_{D_{L.E.}}\right)_{\substack{total \\ fins}} = \sum_{i=1}^{N_{fins}} \left(\frac{W_i \cdot t_i}{A_{ref}}\right) \cdot \left\{\left(C_{P_{max}}\right)_{subsonic}\right\}_i = \sum_{i=1}^{N_{fins}} \left(\frac{W_i \cdot t_i}{A_{ref}}\right) \cdot \left\{\frac{\left(1 + \frac{\gamma-1}{2} \cdot \left(M_\infty \cdot \cos\theta_{L.E.}\right)_i^2\right)^{\frac{\gamma}{\gamma-1}} - 1}{\frac{\gamma}{2} \cdot \left(M_\infty \cdot \cos\theta_{L.E.}\right)_i^2}\right\}_i$$

Tends to Over-predict drag, Model can be refined using Newtonian Flow Theory

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Newtonian Flow Analysis

• Newton had an Often unrecognized original contribution to Fluid Mechanics …. Propositions 34 and 35 in *"Principia…"*
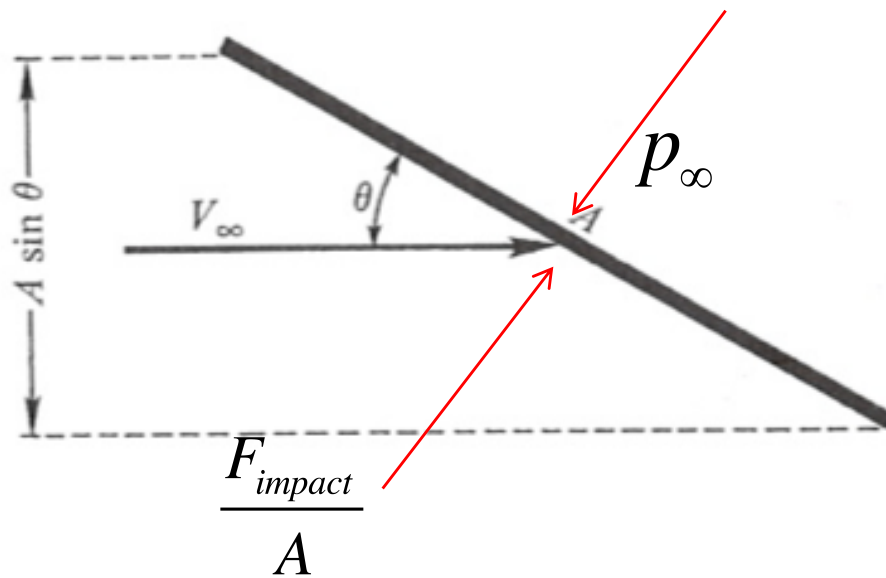
• *"Impact Theory"*
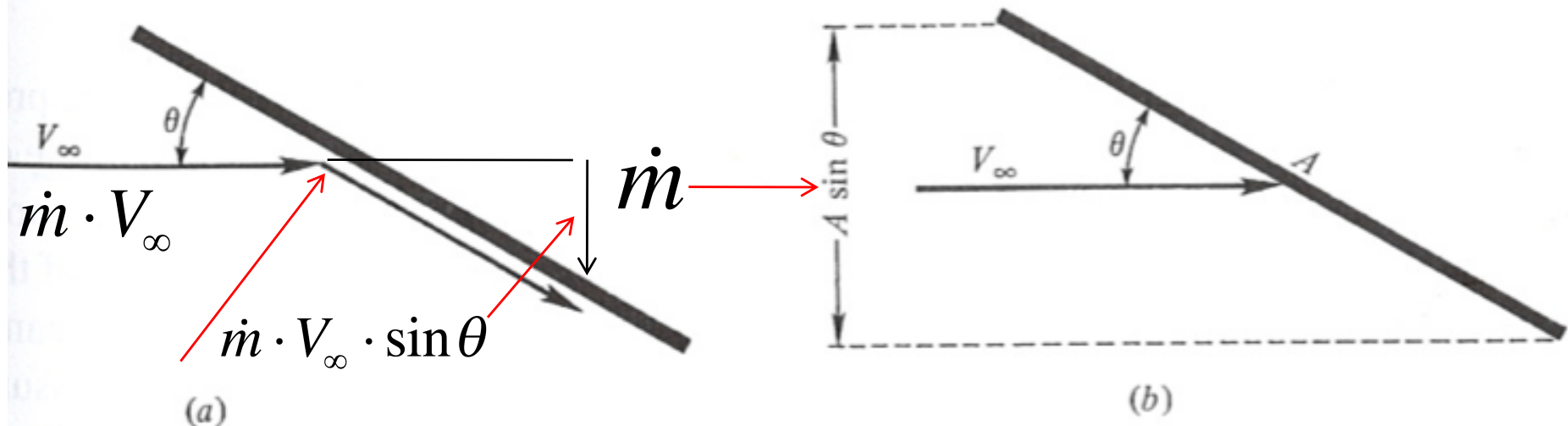
$$F_{impact} = change\ in\ momentum = \dot{m} \cdot V_\infty \cdot \sin\theta$$

$$\rightarrow F_{impact} = \rho_\infty \cdot V_\infty \cdot (A\sin\theta) \cdot V_\infty \cdot \sin\theta = \rho_\infty \cdot V_\infty^2 \cdot A \cdot \sin^2\theta$$

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Newtonian Flow Analysis (2)

$$F_{impact} = \rho_\infty \cdot V_\infty^2 \cdot A \cdot \sin^2 \theta \rightarrow \boxed{p(\theta) = \frac{F_{impact}}{A} + p_\infty = \rho_\infty \cdot V_\infty^2 \cdot \sin^2 \theta + p_\infty}$$
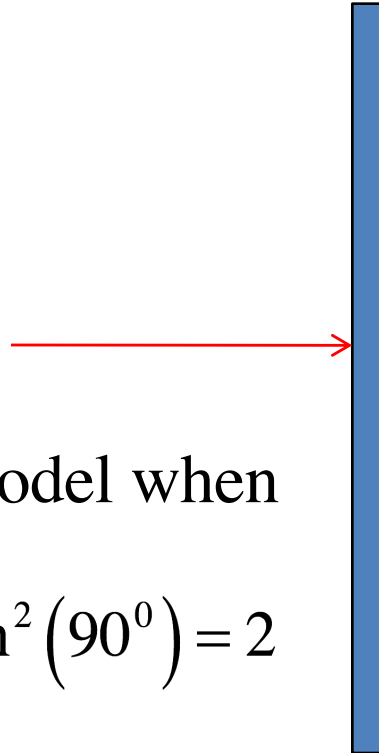

Newton Ignores the random motion of the molecules and Considers only the linear or translational motion

$$\boxed{C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = \frac{\rho_\infty \cdot V_\infty^2 \cdot \sin^2 \theta + p_\infty - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = 2 \cdot \sin^2 \theta}$$

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Newtonian Flow Analysis (3)

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = 2 \cdot \sin^2 \theta$$

What happens in Newtonian Model when

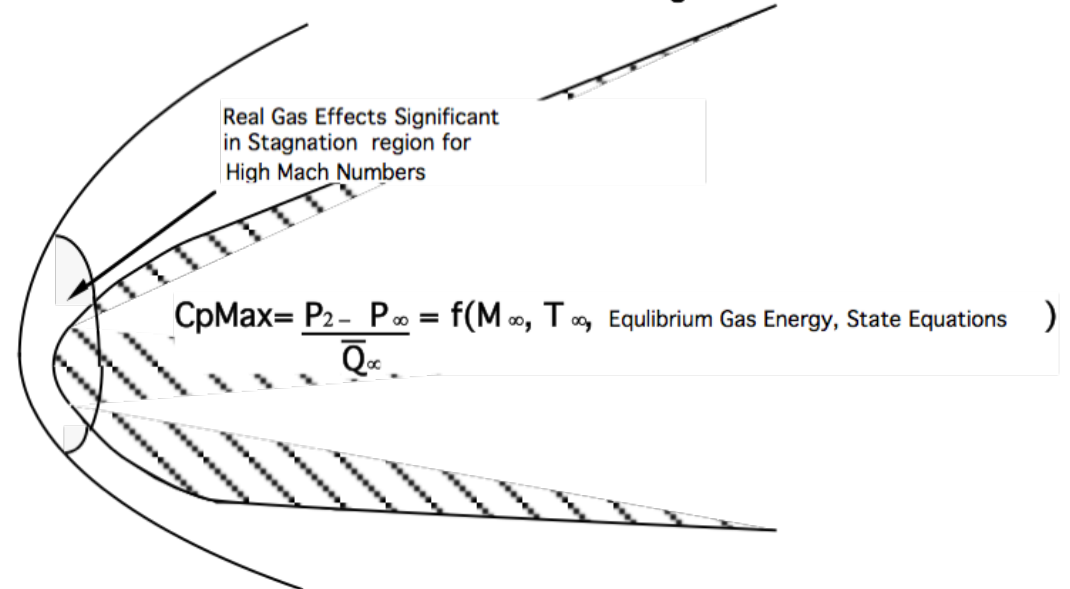$$\theta = 90^0 \quad ? \quad C_p(90^0) = 2 \cdot \sin^2(90^0) = 2$$

But for direct impact …

$$p(90^0) = P_0 \rightarrow C_p(90^0) = \frac{P_0 - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} \equiv C_{p_{max}}!$$

Newtonian Flow Analysis (4)

$$C_p\left(90^0\right) = \frac{P_0 - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\,2}} \equiv C_{p\,max}$$

Schematic of the Bow Shock near the Stagnation Point

Real Gas Effects Significant
in Stagnation region for
High Mach Numbers

P_∞, T_∞

Q_∞, Pt_∞,

$M_\infty \gg 1$

$CpMax = \dfrac{P_2 - P_\infty}{\overline{Q_\infty}} = f(M_\infty,\ T_\infty,\ \text{Equlibrium Gas Energy, State Equations}\ )$

$$Cp_{max} = \left[\frac{(P_{02} - p_\infty)}{\frac{1}{2}\rho_\infty V_\infty^{\,2}}\right] = \frac{(P_{02} - p_\infty)}{\frac{\gamma}{2}p_\infty M_\infty^{\,2}} = \frac{1}{\frac{\gamma}{2}M_\infty^{\,2}}\left\{\frac{\left(\frac{\gamma+1}{2}M_\infty^{\,2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)}M_\infty^{\,2} - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}} - 1\right\}$$

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Newtonian Flow Analysis (5)
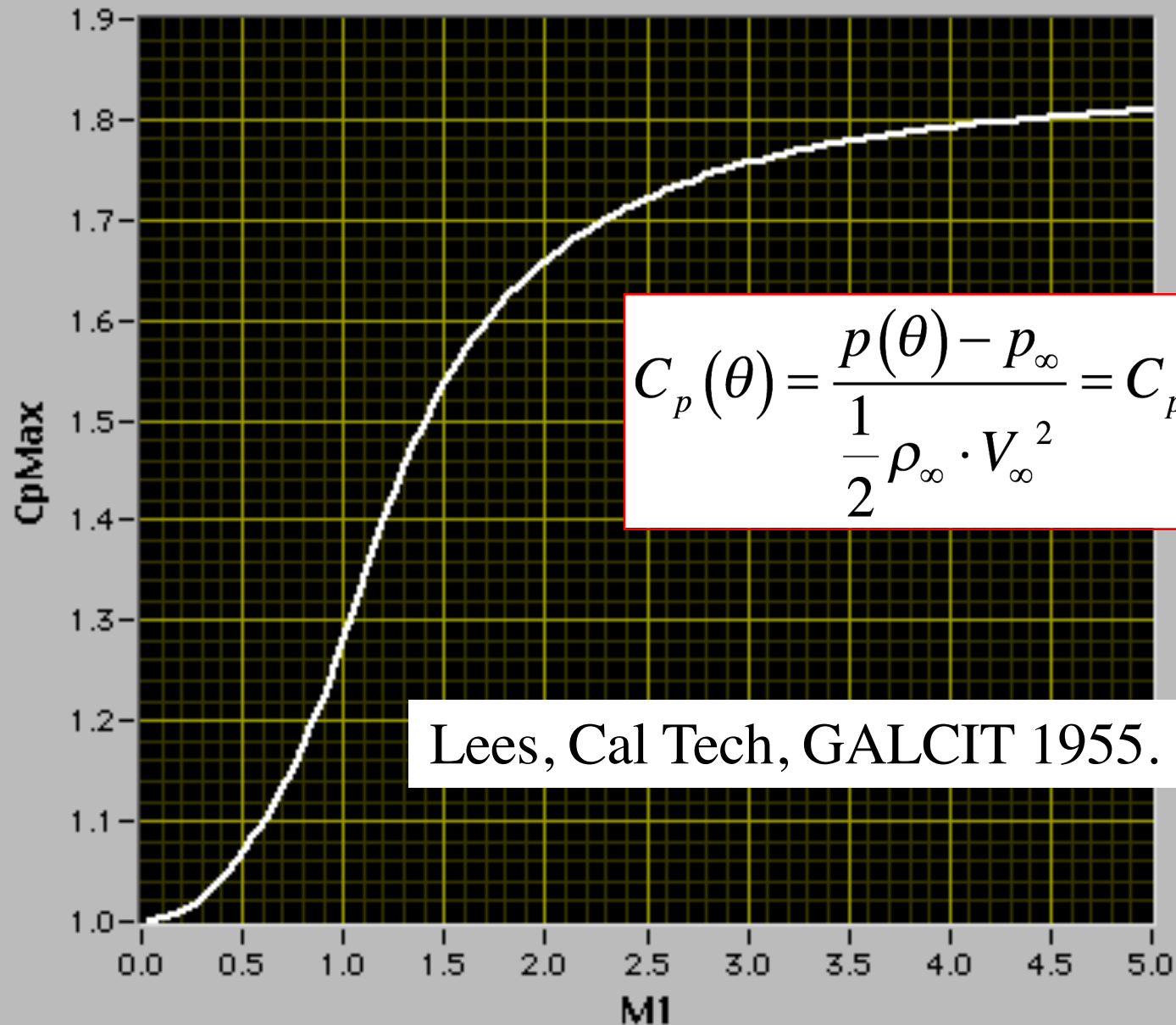
$$C_{p_{max}} = \frac{1}{\frac{\gamma}{2} M_\infty^2} \left\{ \frac{\left(\frac{\gamma+1}{2} M_\infty^2\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)} M_\infty^2 - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}} - 1 \right\}$$

$$C_p\left(90^0\right) = \frac{P_0 - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} \equiv C_{p_{max}}$$

< 2 always for $\gamma = 1.4$

7
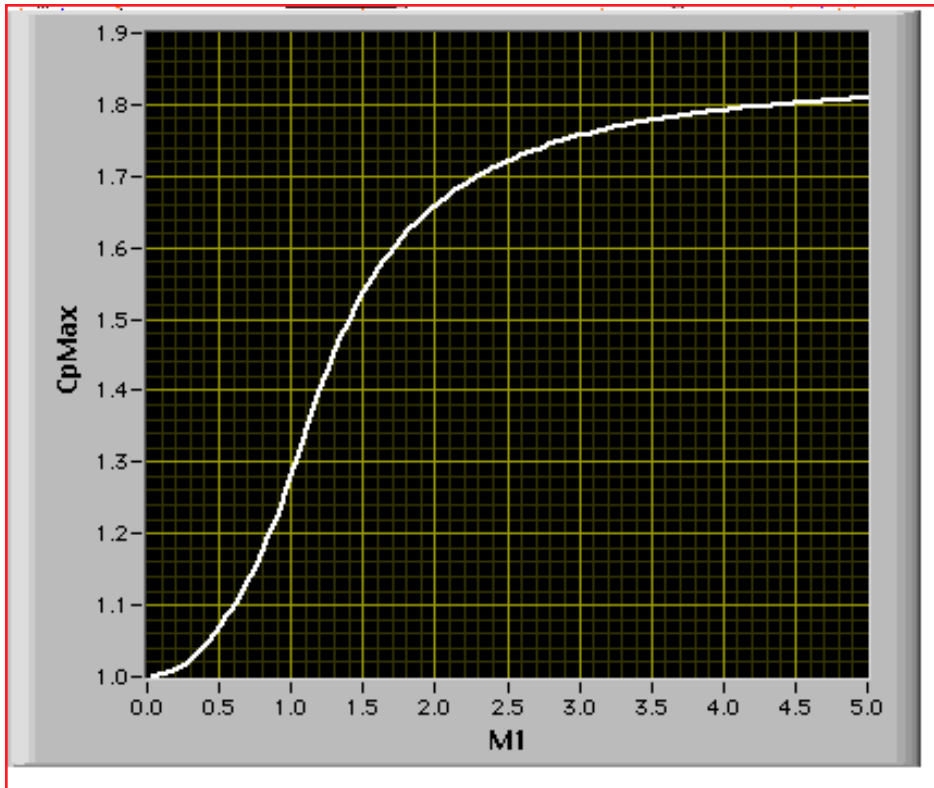
"Modified Newtonian Flow"

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = C_{p_{max}} \cdot \sin^2\theta$$

Lees, Cal Tech, GALCIT 1955.

8

Modified Newtonian Flow (2)

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = C_{p_{max}} \cdot \sin^2 \theta$$
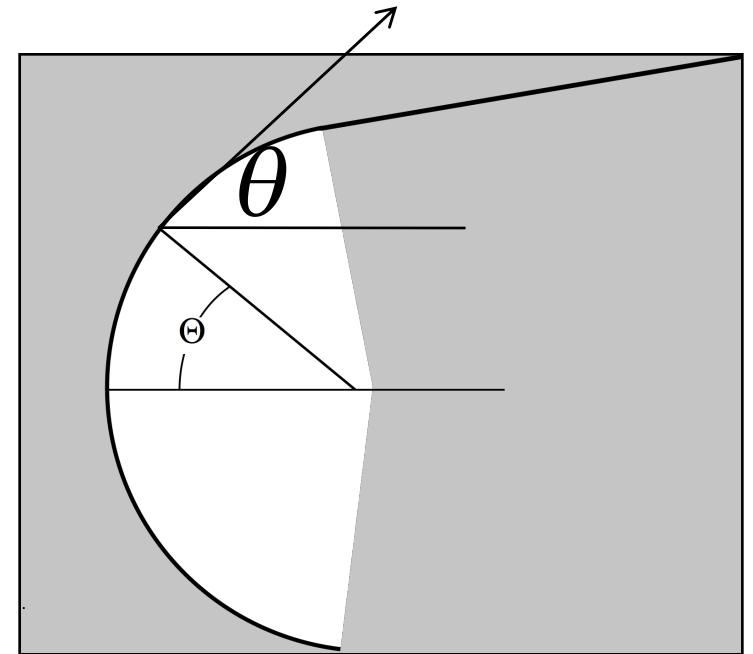
Often convenient to express model in terms of a Polar surface coordinate

$$\Theta = 90^o - \theta$$

$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = C_{p_{max}} \cdot \sin^2(90^0 - \Theta) =$$

$$C_{p_{max}} \cdot \left[\sin(90^0) \cdot \cos(\Theta) - \sin(\Theta) \cdot \cos(90^0)\right]^2$$

$$\Rightarrow C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = C_{p_{max}} \cdot \cos^2(\Theta)$$
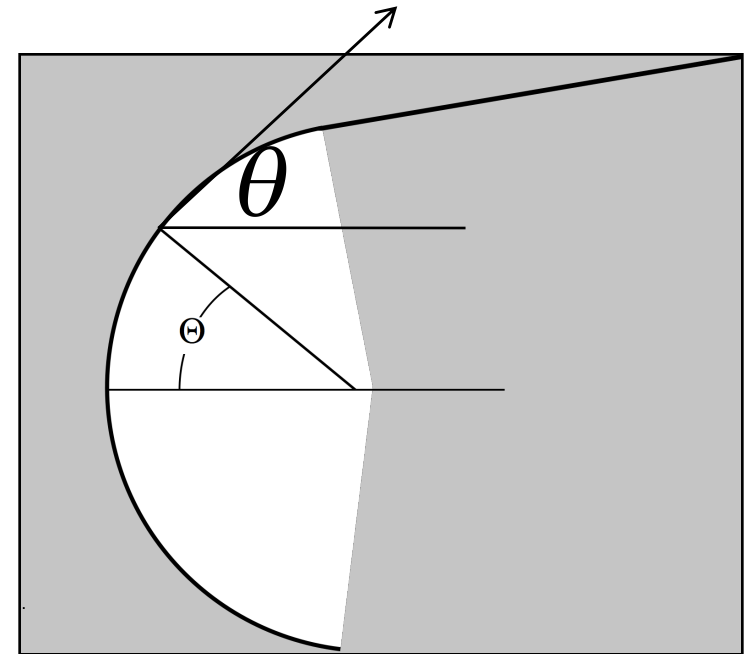

θ

Θ

Modified Newtonian Flow (3)

Expressed in terms of pressure ratio

$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = C_{p_{max}} \cdot \cos^2(\Theta) \Rightarrow \frac{p(\Theta) - p_\infty}{\frac{\gamma}{2} p_\infty \cdot M_\infty^2} = C_{p_{max}} \cdot \cos^2(\Theta)$$

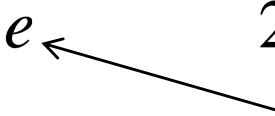$$\boxed{\frac{p(\Theta)}{p_\infty} = 1 + \frac{\gamma}{2} \cdot M_\infty^2 \cdot C_{p_{max}} \cdot \cos^2(\Theta)}$$

$$\Theta = 90^o - \theta$$

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Modified Newtonian Flow (4)

$$\Rightarrow C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = C_{p_{max}} \cdot \cos^2(\Theta)$$

For Newtonian model … Newton Ignores the random motion of the molecules and Considers only the linear or translational motion

"equivalent to infinite Mach number assumption"

• As derived in section 3

$$\frac{V^2/2}{e} = \frac{\gamma(\gamma - 1)}{2} M^2$$

Mach number is a measure of the ratio of the fluid Kinetic energy to the fluid internal energy (direct motion To random thermal motion of gas molecules) -- Fundamental Parameter of Compressible Flow --

Example 3

- Example: Cylindrically Blunted 2-D wedge, $M_\infty=10.0$, 1-cm Radius

- *Map conditions at three points along shock*
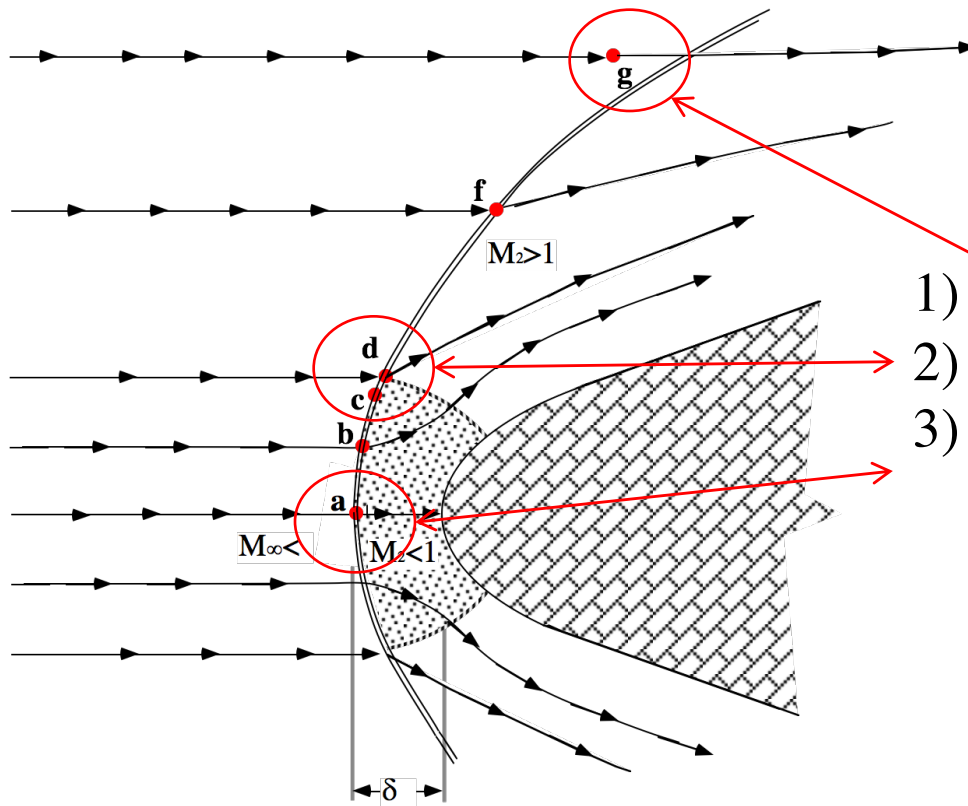
1) Far field (*g*)… "tangent wedge"
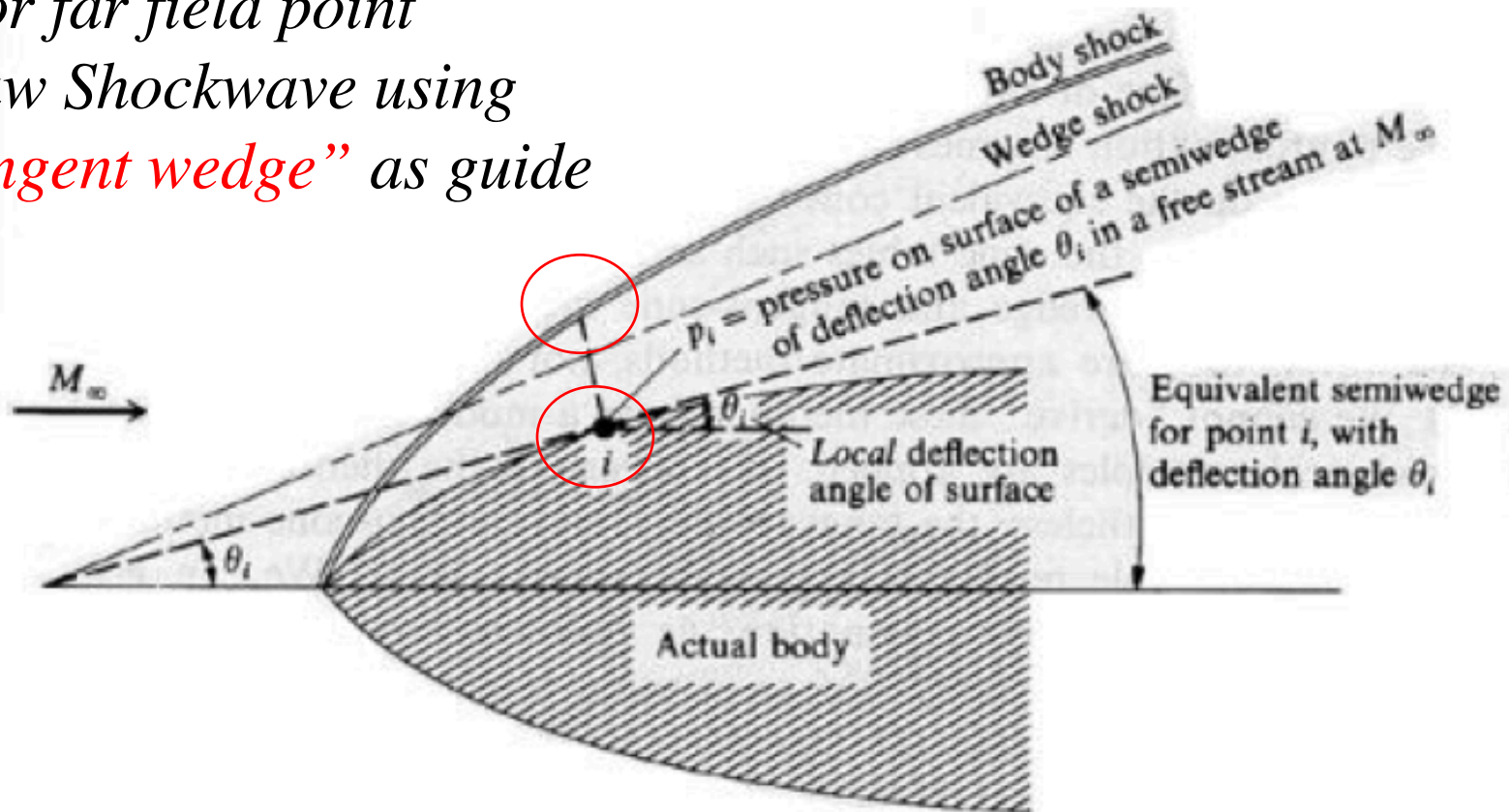2) Sonic Point (*d*)
3) Stagnation Point (*a*)

Example 3 (2)

- Example Cylindrically Blunted 2-D wedge, $M_\infty = 10.0$ 1-cm Radius

- *For far field point*
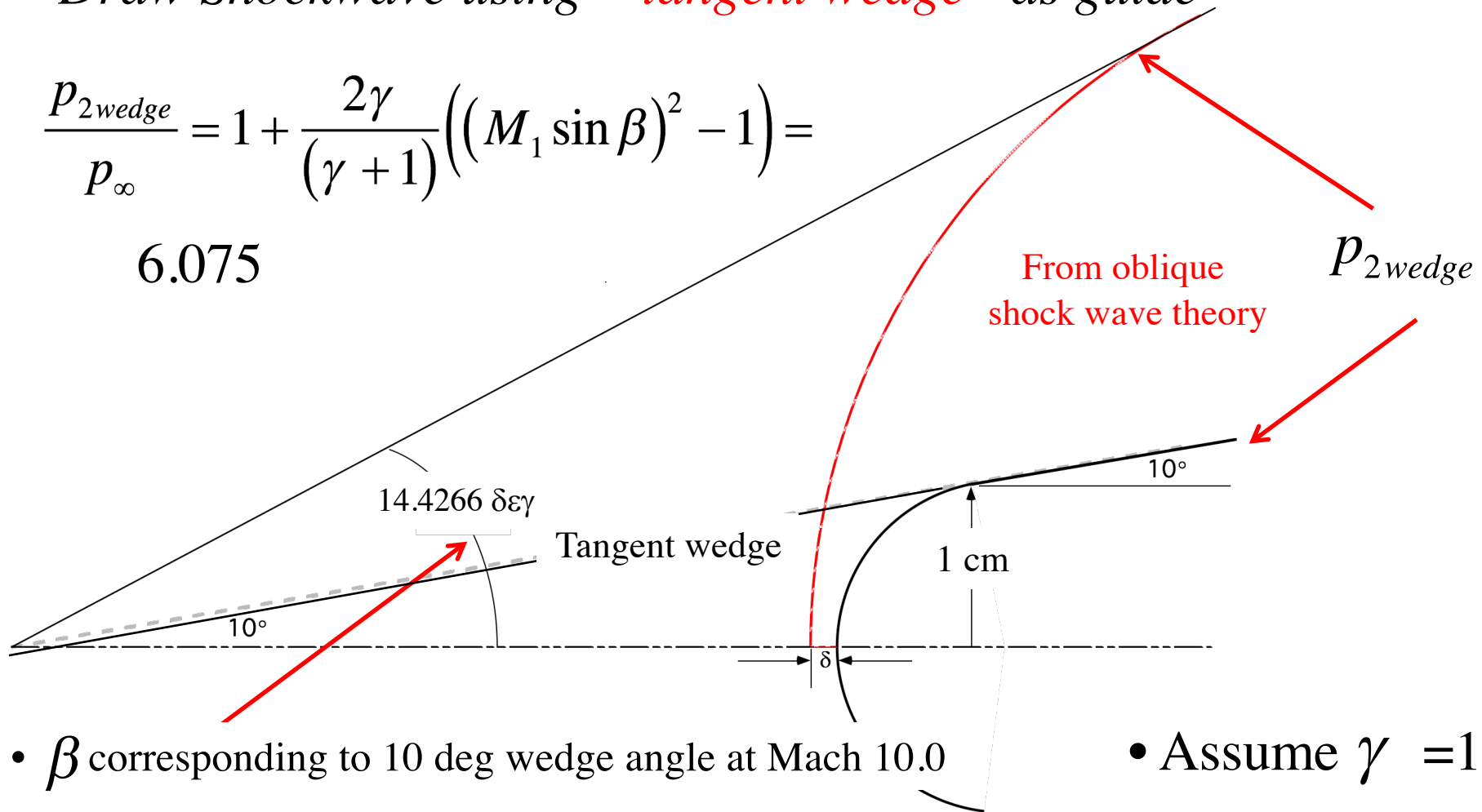 *Draw Shockwave using*
  "tangent wedge" as guide

M_∞

Body shock

Wedge shock

p_i = pressure on surface of a semiwedge of deflection angle θ_i in a free stream at M_∞

θ_i

Local deflection angle of surface

i

Equivalent semiwedge for point i, with deflection angle θ_i

θ_i

Actual body

Example 3 (3)

- Example Cylindrically ⸀ Blunted 2-D wedge, $M_\infty = 10.0$ 1-cm Radius

- *Draw Shockwave using "tangent wedge" as guide*

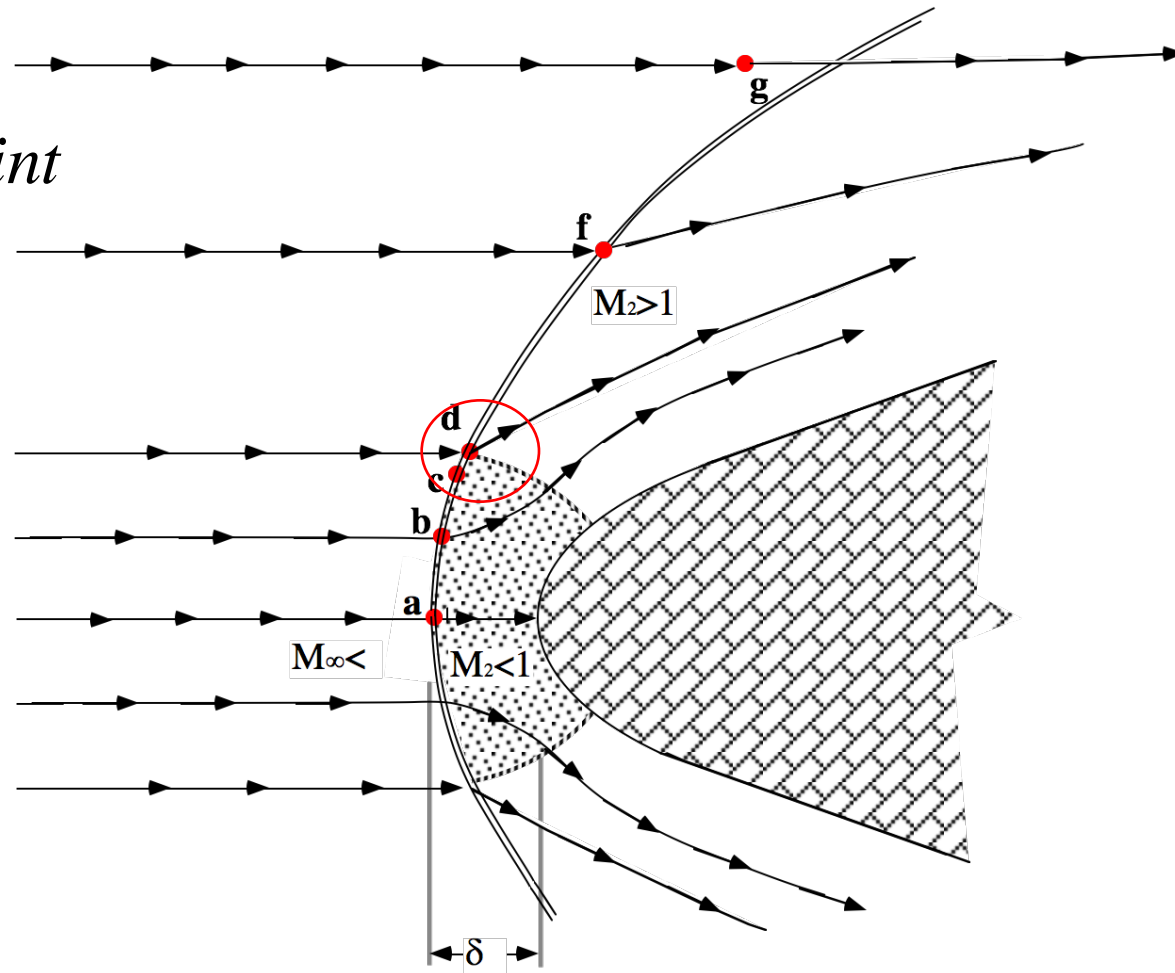$$\frac{p_{2wedge}}{p_\infty} = 1 + \frac{2\gamma}{(\gamma + 1)}\left((M_1 \sin\beta)^2 - 1\right) =$$

6.075

From oblique
shock wave theory

p_{2wedge}

10°

1 cm

14.4266 δεγ

Tangent wedge

10°

δ

- β corresponding to 10 deg wedge angle at Mach 10.0

- Assume $\gamma = 1.4$

Example 3: Sonic Point Surface Conditions behind detached shock wave: (5)

Sonic Point

Example 3 (6)

- Compute Standoff Distance

$$\frac{\delta}{D} = 0.193 \cdot e^{\frac{4.67}{M_\infty^2}} = 2 \cdot 1 \cdot 0.193 \exp\left(\frac{4.67}{1.75^2}\right) = 0.4044 \text{ cm}$$

- Compute Sonic Point on Shock

$$\beta_{sonic} = \frac{180}{\pi} \sin^{-1}\sqrt{\frac{(\gamma-3)M_1^2 + (\gamma+1)M_1^4 + \sqrt{16\gamma M_1^4 + \left[(\gamma-3)M_1^2 + (\gamma+1)M_1^4\right]^2}}{4\gamma M_1^4}}$$

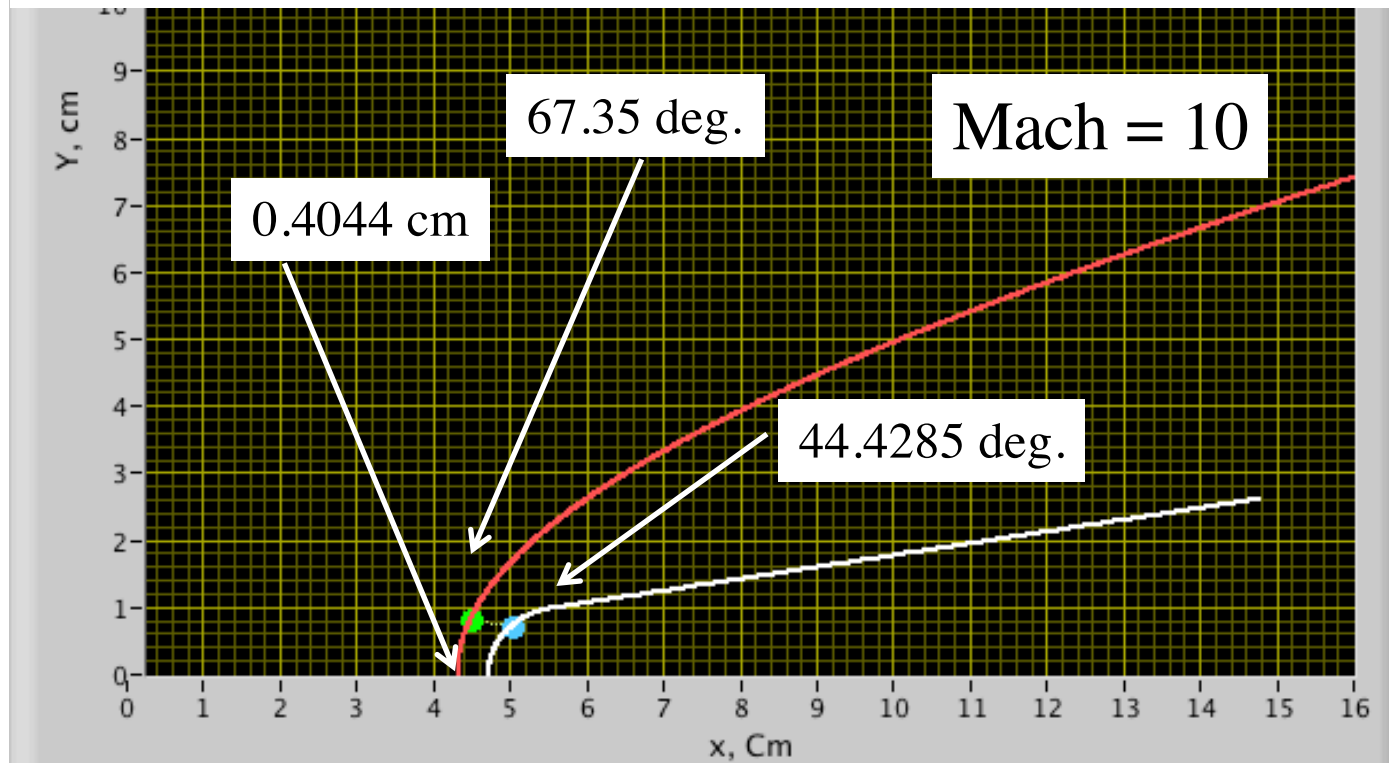$$= 67.335 \text{ deg.}$$

- Compute Sonic Point on Surface

$$\tan(\theta_{sonic}) = \frac{2\left\{M_1^2 \sin^2(\beta_{sonic}) - 1\right\}}{\tan(\beta_{sonic})\left[2 + M_1^2\left[\gamma + \cos(2\beta_{sonic})\right]\right]} = \quad 44.428 \text{ deg.}$$

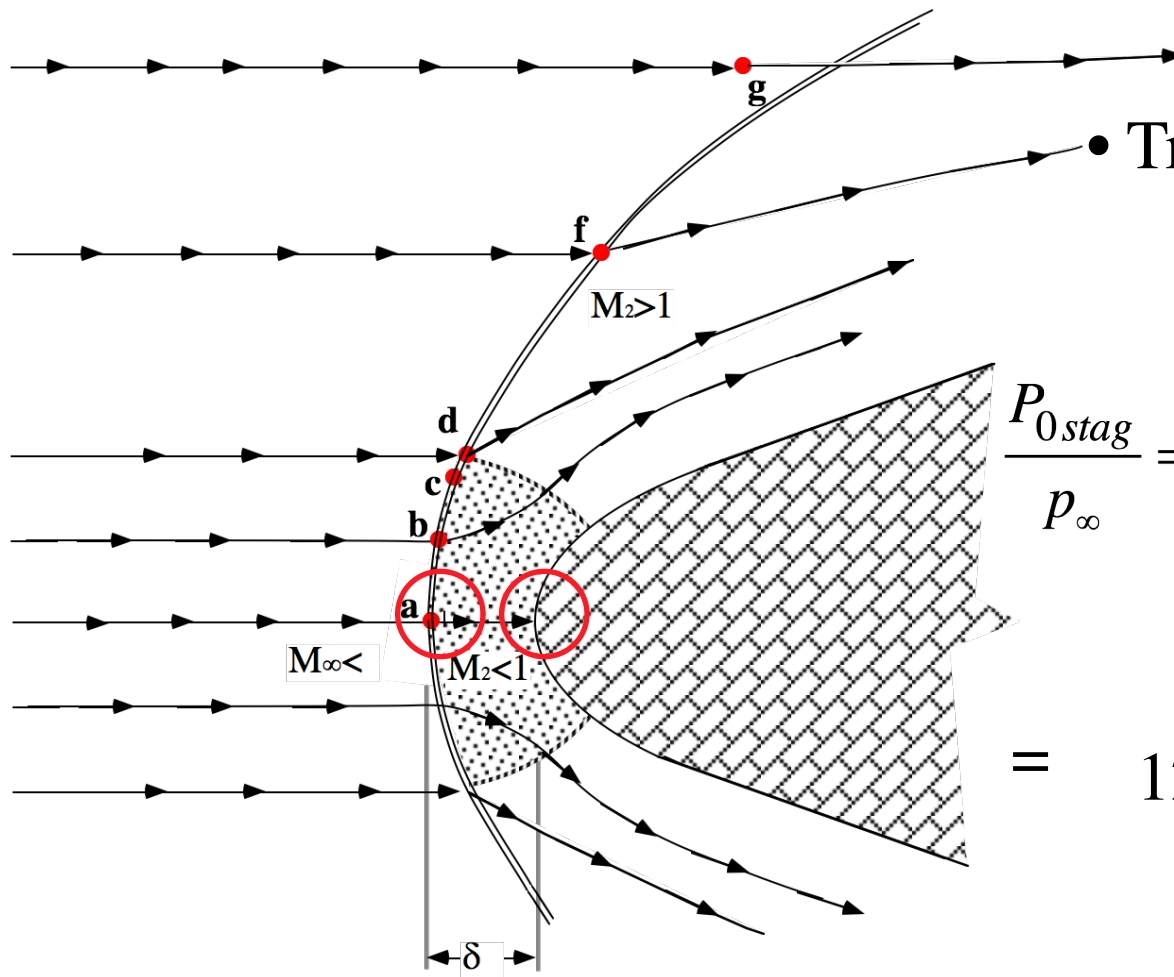*See for derivation: http://mae-nas.eng.usu.edu/MAE_5420_Web/section8/section8.3.pdf*

Example 3 (cont'd)

$$
\frac{p_{sonic}}{p_\infty} = \frac{p_{sonic}}{P_{0sonic}} \frac{P_{0sonic}}{p_\infty} = \frac{1}{\left(1+\frac{\gamma+1}{2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}} \left\{ \frac{\left(\frac{\gamma+1}{2}\left[M_\infty \sin\left(\beta_{sonic}\right)\right]^2\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)}\left[M_\infty \sin\left(\beta_{sonic}\right)\right]^2 - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}} \right\} =
$$

$$
0.52828 \times 110.098512 = 64.16
$$

67.35 deg.

0.4044 cm

Mach = 10

44.4285 deg.

Surface Conditions behind
detached shock wave: *stagnation point* (cont'd)
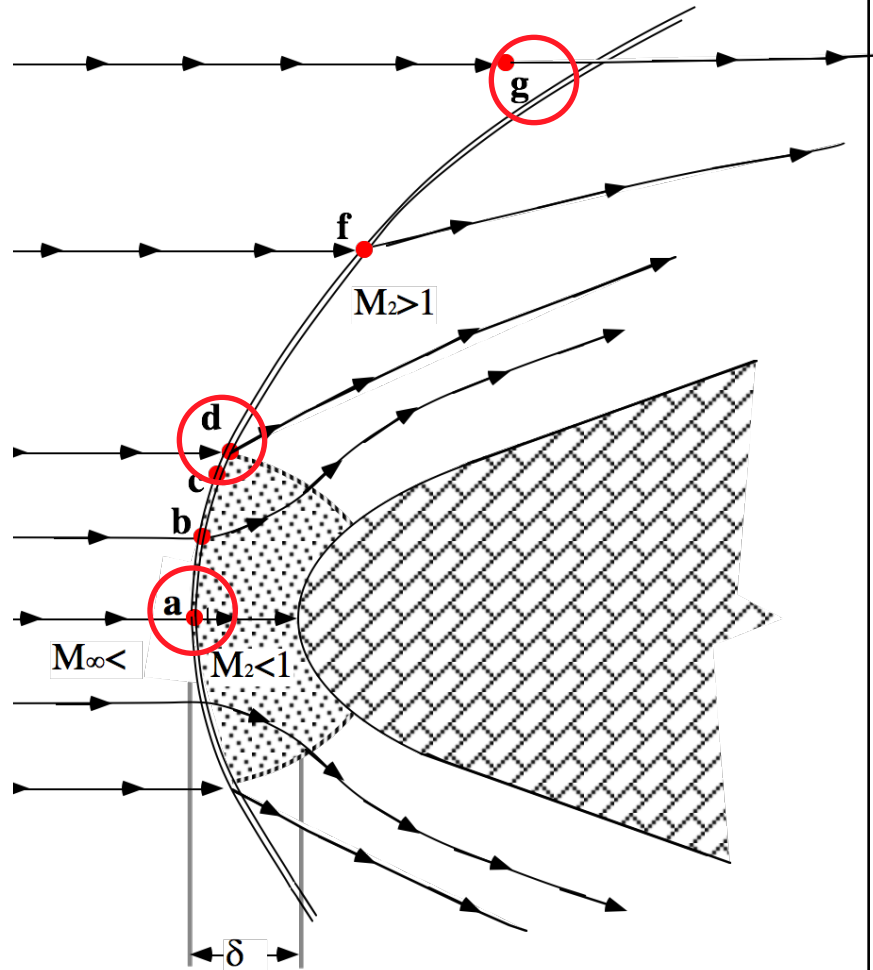

- Treat as normal shockwave

$$\frac{P_{0\,stag}}{p_\infty} = \left\{ \frac{\left(\frac{\gamma+1}{2} M_\infty^{\,2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)} M_\infty^{\,2} - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}} \right\}$$
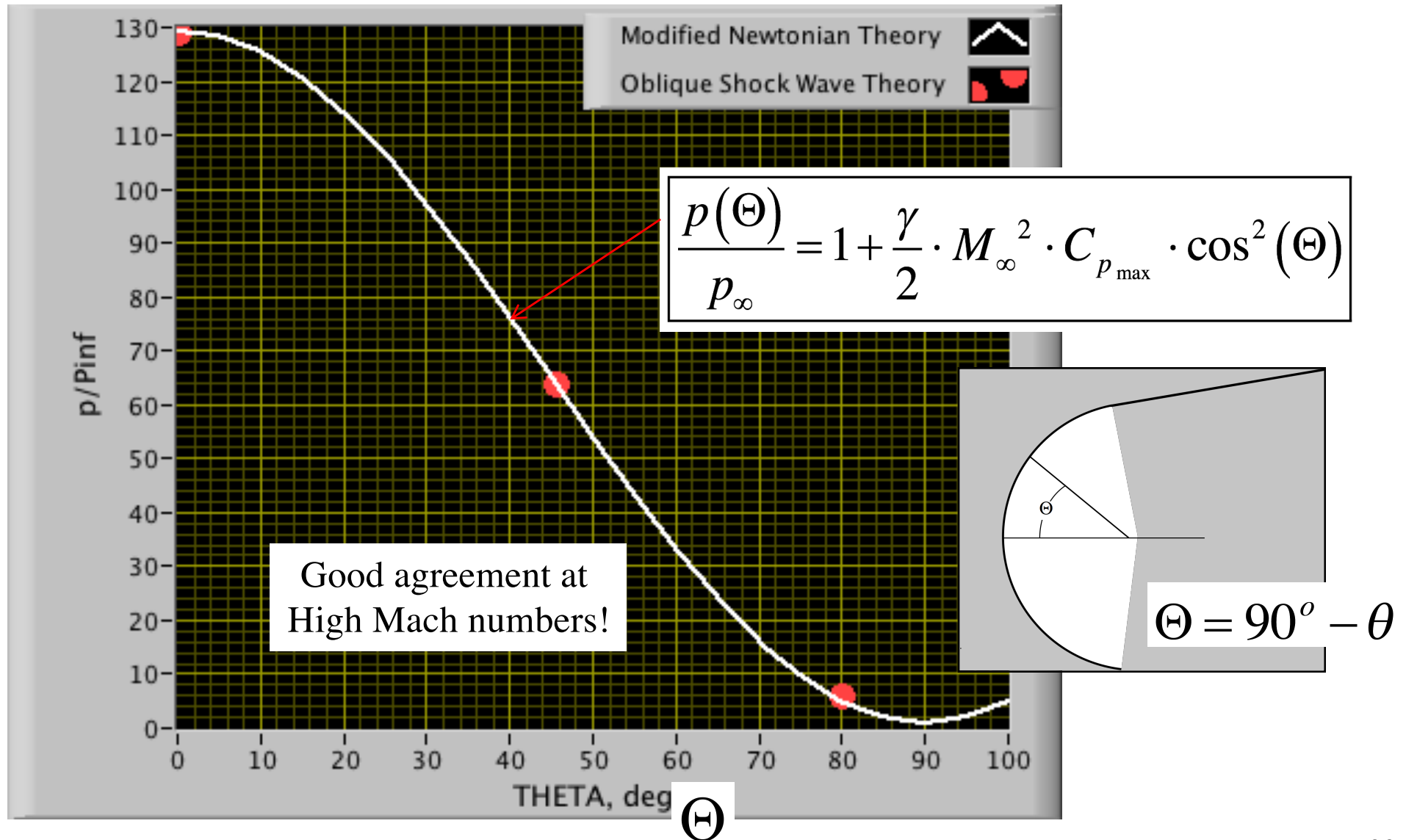
$M_2 > 1$

$M_\infty <$ $M_2 < 1$

$\leftarrow \delta \rightarrow$

$= \quad 129.21$

- $M_\infty = 10.0$

Plot Conditions along detached shock

summary $M_\infty = 10.0$

Point on Shock	beta, deg β	theta, deg θ	90°- theta Θ	$\dfrac{p_\theta}{p_\infty}$
a	90°	90°	0°	129.21
d	67.34°	44.43°	45.57°	64.16
g	14.43°	10°	80°	6.075

Surface Conditions behind detached shock wave: Mach 10.0

$$\frac{p(\Theta)}{p_\infty} = 1 + \frac{\gamma}{2} \cdot M_\infty^{\,2} \cdot C_{p_{max}} \cdot \cos^2(\Theta)$$

Legend:
- Modified Newtonian Theory
- Oblique Shock Wave Theory

Good agreement at High Mach numbers!

$$\Theta = 90^o - \theta$$

Axis labels: p/Pinf (vertical), THETA, deg Θ (horizontal)

Repeat Oblique Shock Calculations at

$$M_\infty = 3$$
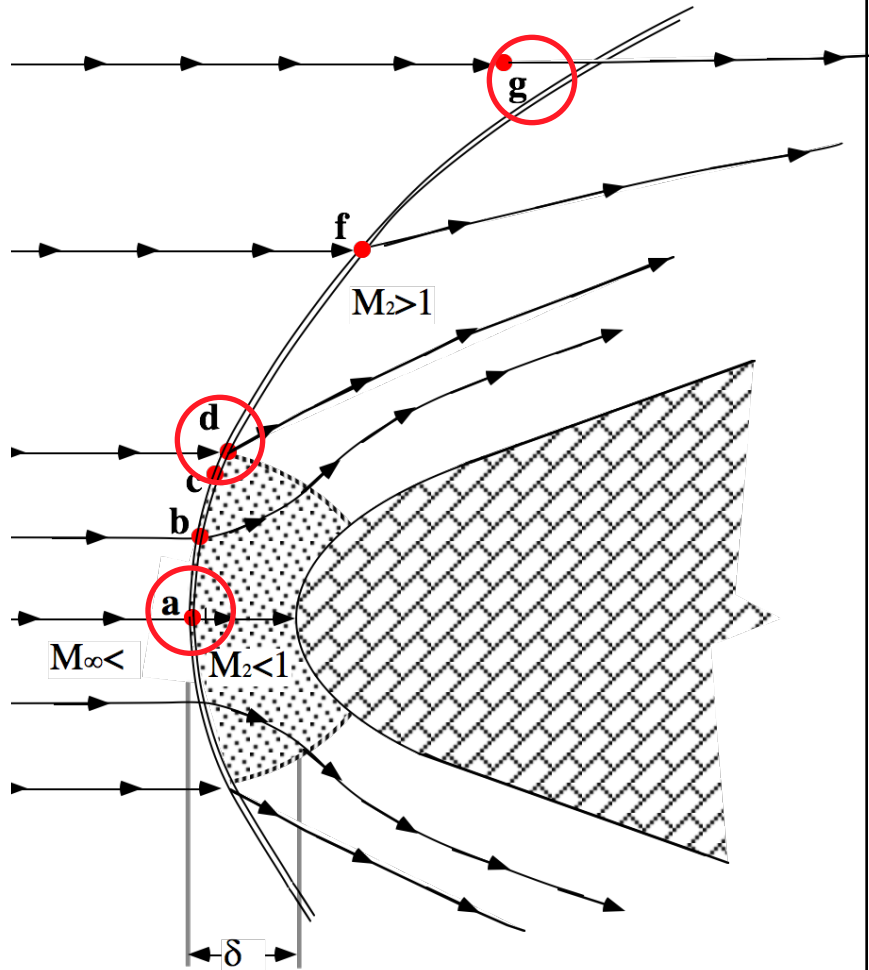
Point on Shock	beta, deg β	theta, deg θ	90°- theta Θ	$\dfrac{p_\theta}{p_\infty}$
a	90°	90°	0°	12.06
d	63.77°	34.01°	55.99°	5.18
g	27.38°	10°	80°	2.05

Surface Conditions behind detached shock wave: *Mach 3.0*

$M_\infty = 3$

$\Theta = 90^o - \theta$

Comparison is not nearly as good? Why?

Θ

Surface Conditions behind detached shock wave: *Mach 3*

$M_\infty = 3$

Modified Newtonian Theory

Oblique Shock Wave Theory

"Calibrated" Newtonian Theory

$\varepsilon = 0.074$

$\varepsilon = 0.00$

"calibrated" flow With curve fit
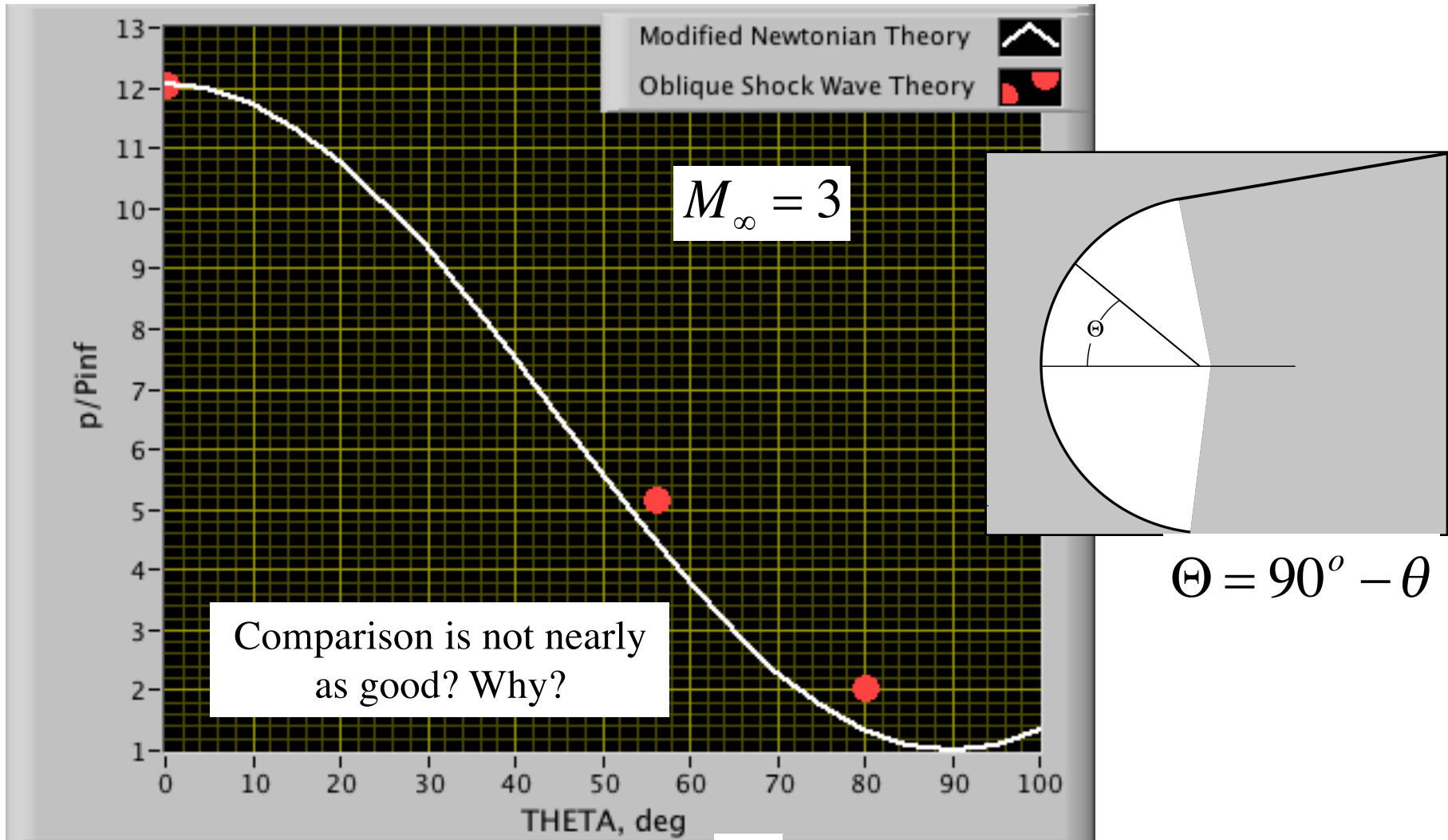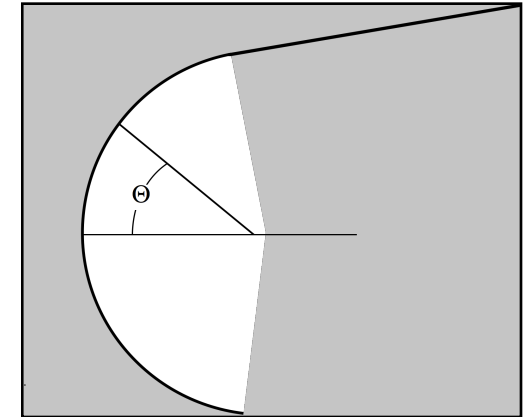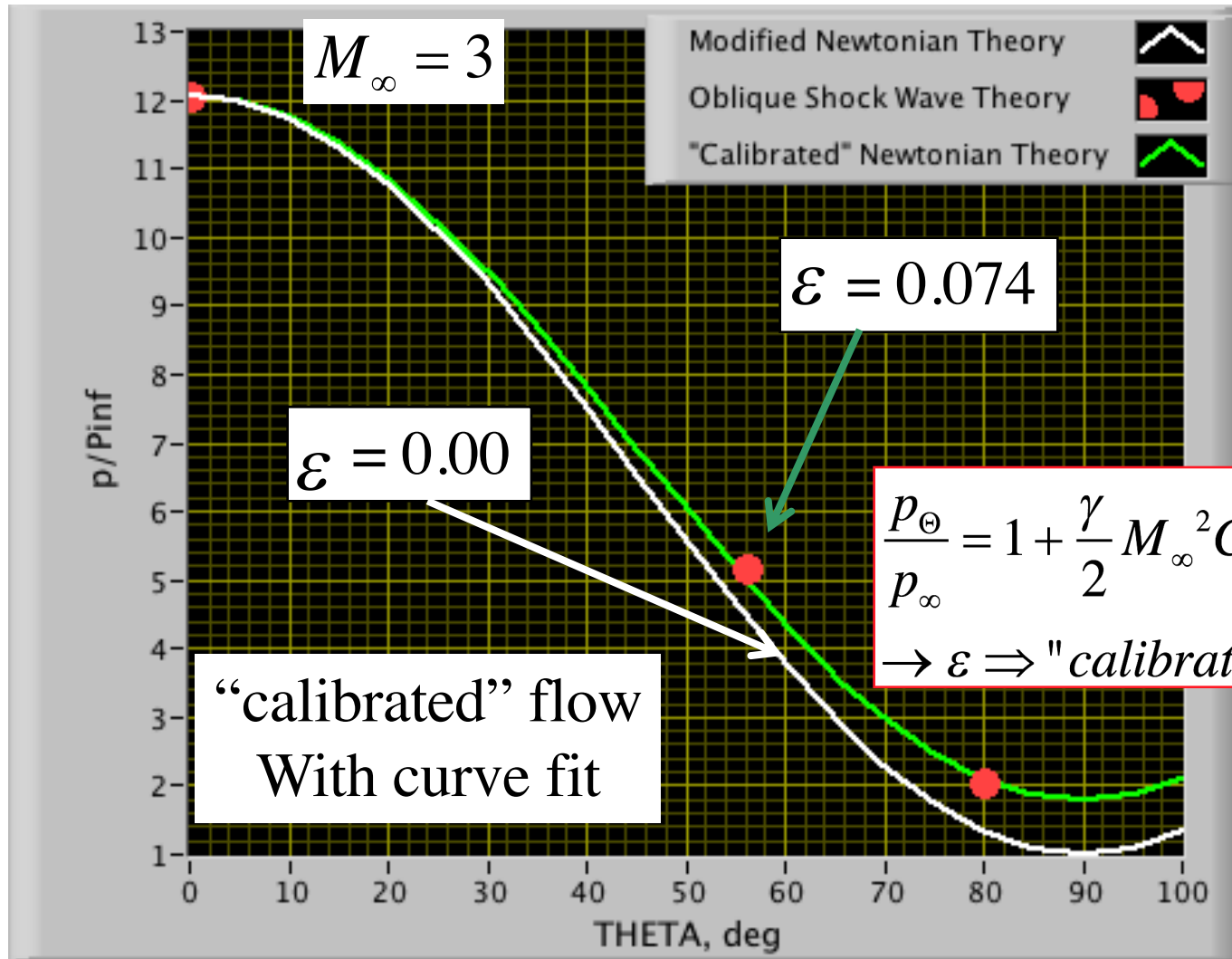
$$\frac{p_\Theta}{p_\infty} = 1 + \frac{\gamma}{2} M_\infty^2 Cp_{Max} \left[\cos^2 \Theta + \varepsilon \sin^2 \Theta \right]$$

$$\rightarrow \varepsilon \Rightarrow "calibration" \approx 0.074$$

p/Pinf

THETA, deg

Surface Conditions behind detached shock wave: *Surface plot*

• In limit as M->infinity Modified Newtonian flow model is
A good descriptor of the local surface pressures

$$\frac{p_\Theta}{p_\infty} = 1 + \frac{\gamma}{2} M_\infty^2 C_{p_{Max}} \cos^2 \Theta$$

• At lower Mach numbers we include a "calibration" term

$$\frac{p(\Theta)}{p_\infty} = 1 + \frac{\gamma}{2} \cdot M_\infty^2 \cdot C_{p_{max}} \cdot \left[\cos^2(\Theta) + \varepsilon \cdot \sin^2(\Theta) \right]$$

Modified Newtonian Flow, Concluded

- Writing in terms of pressure coefficient

$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{\gamma}{2} \cdot p_\infty \cdot M_\infty^2} = C_{p_{max}} \cdot \left[\cos^2(\Theta) + \varepsilon \cdot \sin^2(\Theta)\right]$$

$$C_{p_{max}} \cdot \left[\cos^2\Theta + \varepsilon \cdot \left(1 - \cos^2\Theta\right)\right] = C_{p_{max}} \cdot \left[(1-\varepsilon) \cdot \cos^2\Theta + \varepsilon\right]$$

$or\ldots$

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{\gamma}{2} \cdot p_\infty \cdot M_\infty^2} = C_{p_{max}} \cdot \left[\sin^2(\theta) + \varepsilon \cdot \cos^2(\theta)\right] = C_{p_{max}} \cdot \left[(1-\varepsilon) \cdot \sin^2(\theta) + \varepsilon\right]$$

- *"Calibrated Newtonian Flow"*
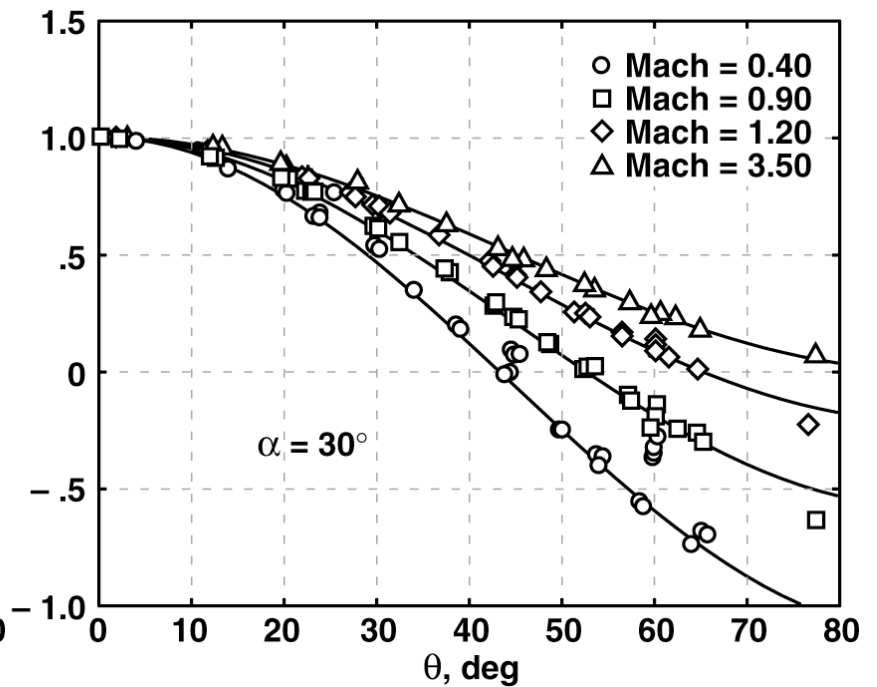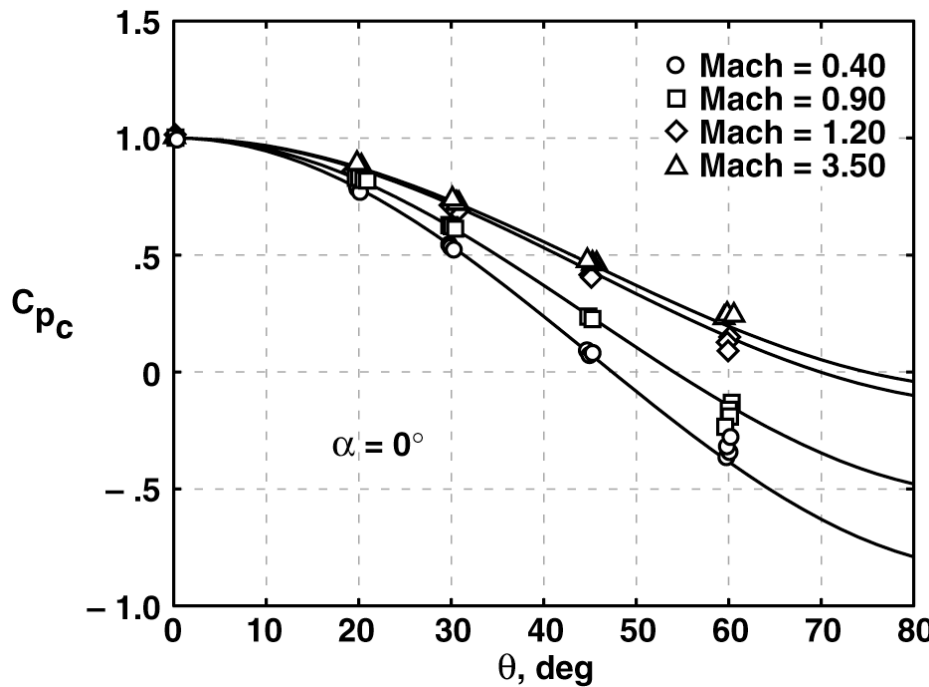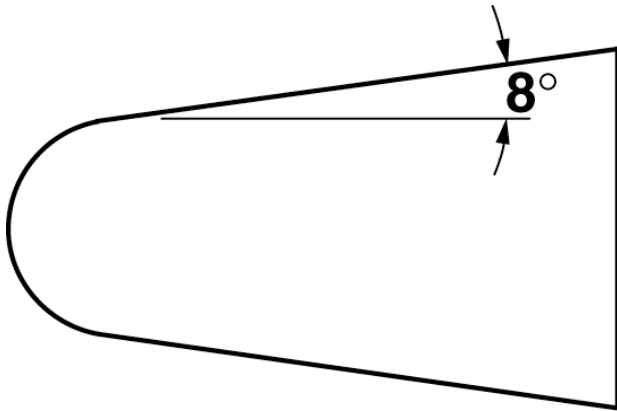 Semi-empirical model
  Valid for very high speeds, 3-D
  And 2-D blunt bodies … accurate
  For both sonic and supersonic regions
  Behind detached shock wave
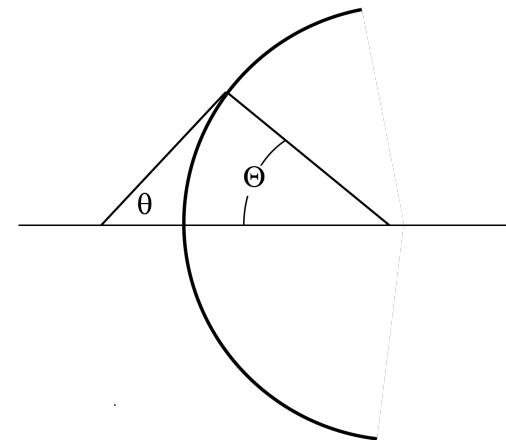
Calibration Examples: 8-deg Cone
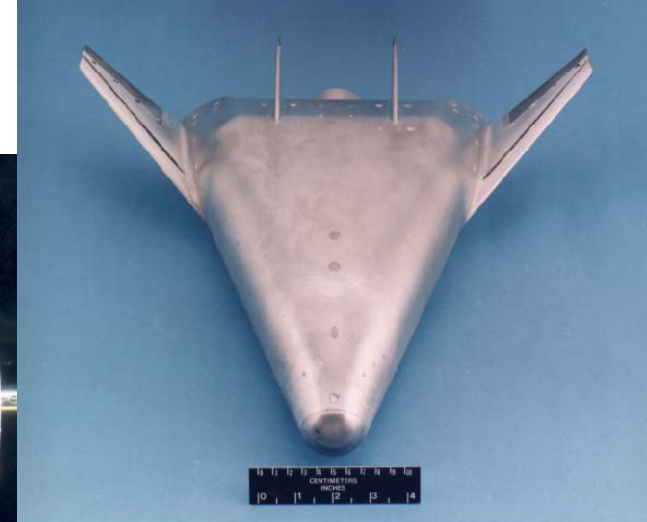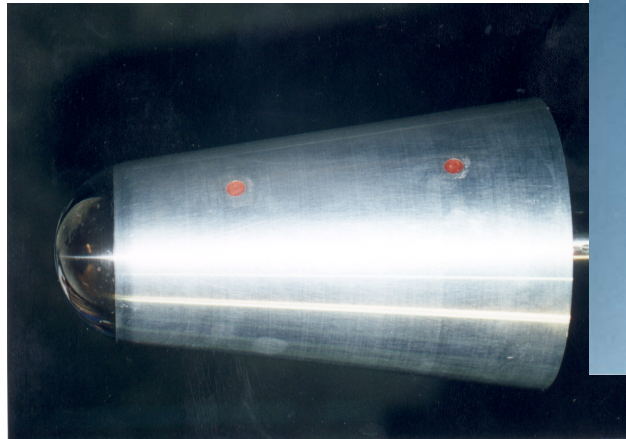
More Calibration Examples

Analytical

- Potential solutions: sphere, cylinder, arbitrary ellipsoid
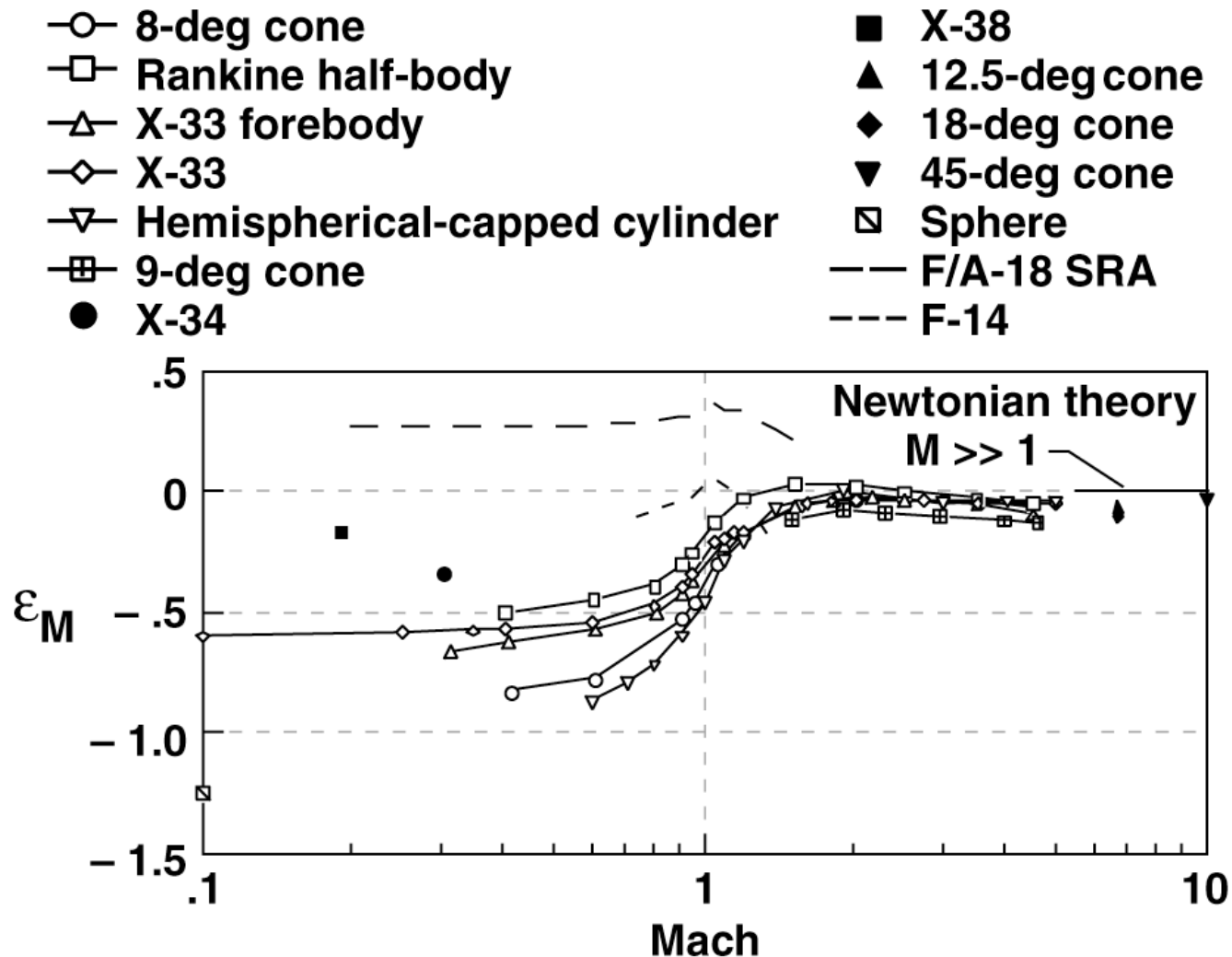
Wind Tunnel

- **8-deg sphere-cone, Rankine half-body, X-33 forebody (M=.3 - 4.75)**
- **9, 12.5, 18, and 45-deg sphere cones**
- **X-33**
- **X-34**
- **X-38**
- **Sphere**

Flight Data

- F/A-18 SRA
- F-14

Mach Effect on Calibration (ε):

Bottom Line

- Even though flow across detached shock wave is extremely complex and is difficult to model theoretically, it turns out that simple engineering models do a good job with the pressure distribution

- Allow the drag due to detached shock wave to be estimated using very simple tools

Homework 7.1, Blunt Edge Drag

- Consider a 2° angle (δ) diamond-wedge wing, 2 meter chord (c), With A spherically blunted leading edge with a 1 cm radius (R)
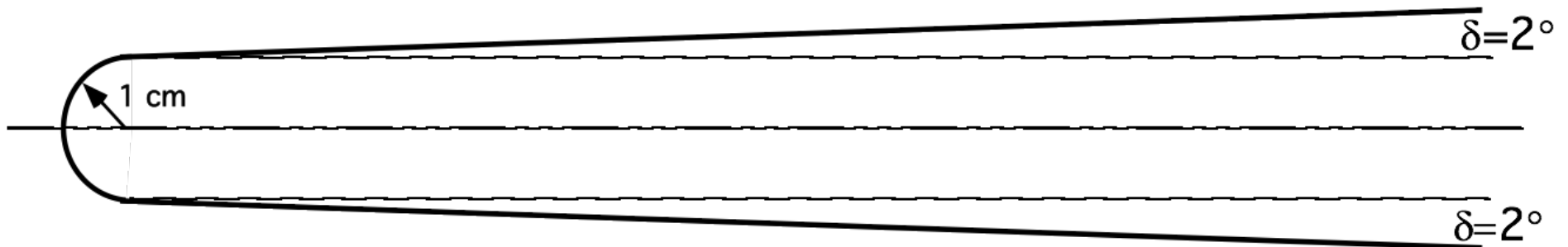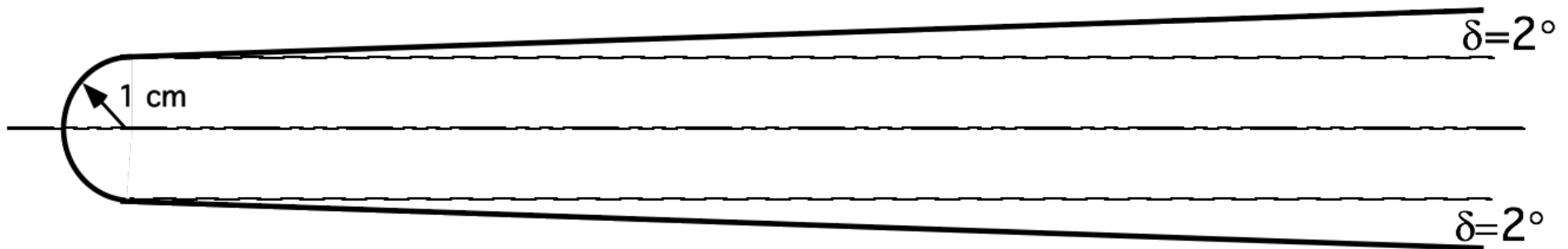

- Show that in general, the drag coefficient due to the the blunt leading edge is:

$$C_{D_{LE}} = \int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} Cp(\Theta)\cos(\Theta)\frac{R}{c}d\Theta$$

Homework 7.1, Blunt Edge Drag (cont'd)

- Use the calibrated Newtonian Model

$$Cp(\Theta) = Cp_{\max}\left[\cos^2(\Theta) + \varepsilon\sin^2(\Theta)\right] = C_{p_{\max}} \cdot \left[(1-\varepsilon)\cdot\cos^2\Theta + \varepsilon\right]$$

$\delta = 2°$

1 cm

$\delta = 2°$

… calculate the leading edge drag coefficient
at $M_\infty = 3.0$ ($\varepsilon\sim0.074$) AND $M_\infty = 10.0$ ($\varepsilon\sim0.00$)

- Compare to wave drag coefficient and
  skin drag coefficient (referenced to plan area)
  for the same conditions at 25 km altitude

Questions??