Section 7.2
Rocket Science Review 102:
Launch Energy Management

Utah State University

Mechanical & Aerospace Engineering

Newton's Laws as Applied to "Rocket Science"
... its not just a job ... its an adventure

RS 101: Summary and Terminology

• **Rocket Thrust Equation**

$$F_{thrust} = \dot{m} V_{exit} + A_{exit}(P_{exit} - P_{atmosphere})$$

• **Specific Impulse ..**   **Total**   **…. Instantaneous)**

$$I_{sp} = \frac{I_{mpulse}}{g_0 M_{propellant}} = \frac{\int\limits_0^{t} F_{thrust} dt}{g_0 \int\limits_0^{t} \dot{m}_{propellant} dt} = \frac{F_{thrust}}{g_0 \dot{m}_{propellant}}$$

• **Rocket Equation**

$$\Delta V_{Burn} = g_0 I_{sp} \ln\left[1 + \frac{m_{propellant}}{M_{final}} \right]$$

• **Propellant Mass Budget Equation**

$$M_{fuel + oxidizer} = \left[M_{dry} + M_{payload} \right] \left[e^{\left[\frac{\Delta V_{burn}}{g_0 I_{sp}} \right]} - 1 \right]$$

• **Load Mass Fraction**

$$L_{mf} \equiv \frac{M_{propellant}}{M_{initial}} = \frac{M_{propellant}}{M_{final} + M_{initial}} 1 - e^{-\left(\frac{\Delta V}{g_0 \cdot I_{sp}} \right)}$$

Summary and Terminology (2)

Available ΔV

$$\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \left[\ln\left(1 + P_{mf}\right) \right] -$$

$$\textit{"combustion } \Delta V\text{"}$$

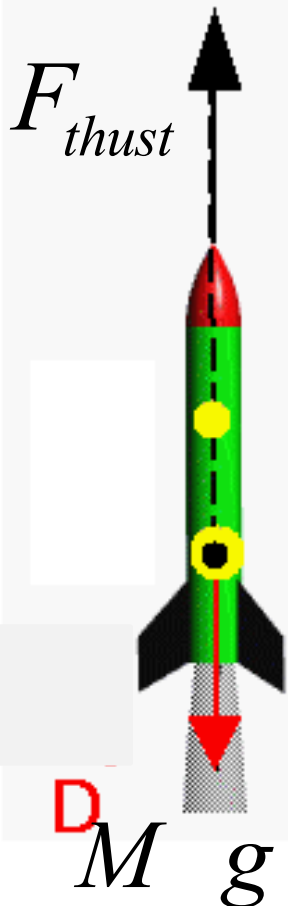$$\int^{t_{burn}} g_{(t)} \cdot t \cdot \sin\theta_{(t)} dt - \sqrt{\int_0^{t_{burn}} \frac{\rho V^3}{\beta} dt}$$

$$\beta = \frac{M}{C_D \cdot A_{ref}}$$

Path dependent losses

$$\textit{"gravity loss"} \qquad \textit{"drag loss"}$$

ΔV for a Vertically Accelerating Vehicle

Flight Direction

F_{thust}

$$\Delta V_{t_{burn}} = g_0 \cdot I_{sp} \left[\ln\left(1 + P_{mf}\right) \right] - g_0 \cdot t_{burn}$$

$\rightarrow$

$$P_{mf} = \frac{M_{propellant}}{M_{final}}$$

$$t_{burn} = \frac{g_0 \cdot I_{sp} \cdot m_{propellamt}}{F_{thrust}}$$

D

M g

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ΔV for a Vertically Accelerating Vehicle (2)

•*Calculate burnout altitude*

Instantaneously :

$$\frac{dh}{dt} = \left(g_0 \cdot I_{sp} \ln\left(\frac{M_{initial}}{M_{initial} - \dot{m} \cdot t} \right) - g_0 \cdot t \right)$$

at Burnout : (Above ground level − AGL)

$$h_{t_{burn}} = \int_0^{t_{burn}} \left(g_0 \cdot I_{sp} \ln\left(\frac{M_{initial}}{M_{initial} - \dot{m} \cdot \tau} \right) - g_0 \cdot \tau \right) \cdot d\tau$$

•*After a lot of arithmetic!*

$$h_{t_{burn}} = -\frac{g_0 \cdot t_{burn}^2}{2} + \left(g_0 \cdot I_{sp} \right) \cdot \left(\frac{M_{initial}}{\dot{m}} \cdot \ln\left(\frac{M_{final}}{M_{initial}} \right) + t_{burn} \cdot \left(1 + \ln\left(\frac{M_{initial}}{M_{final}} \right) \right) \right)$$

... ignoring aerodynamic drag

ΔV for a Vertically Accelerating Vehicle (3)

•*Collecting terms and simplifying*

$$h_{t_{burn}} = -\frac{g_0 \cdot t_{burn}^2}{2} + \left(g_0 \cdot I_{sp}\right) \cdot \left(\frac{M_{initial}}{\dot{m}} \cdot \ln\left(\frac{M_{final}}{M_{initial}}\right) + t_{burn} \cdot \left(1 + \ln\left(\frac{M_{initial}}{M_{final}}\right)\right)\right)$$

$$\frac{M_{initial}}{\dot{m}} = \frac{M_{final} + m_{propellant}}{m_{propellant}/t_{burn}}$$

$$\rightarrow \quad h_{t_{burn}} = g_0 \cdot t_{burn} \left\{ I_{sp} \cdot \left(1 - \frac{\ln\left(1 + P_{mf}\right)}{P_{mf}}\right) - \frac{t_{burn}}{2} \right\}$$

... ignoring aerodynamic drag

•*Summary*

Flight Direction

F_{thust}

$M \; g$

$$h_{t_{burn}} = g_0 \cdot t_{burn} \left\{ I_{sp} \cdot \left(1 - \frac{\ln\left(1 + P_{mf}\right)}{P_{mf}} \right) - \frac{t_{burn}}{2} \right\}$$

$$V_{t_{burn}} = g_0 \cdot I_{sp} \left[\ln\left(1 + P_{mf}\right) \right] - g_0 \cdot t_{burn}$$

$$\rightarrow \quad P_{mf} = \frac{M_{propellant}}{M_{final}}$$

$$t_{burn} = \frac{g_0 \cdot I_{sp} \cdot m_{propellamt}}{F_{thrust}}$$

... ignoring aerodynamic drag

How High will my Rocket go? (2)

Solving for **h$_{apogee}$**

$$E_{mech} = M_{final}\left(\frac{\left(V_{apogee}\right)^2}{2} + g_0 \cdot h_{apogee}\right) \approx M_{final} \cdot g_0 \cdot h_{apogee}$$
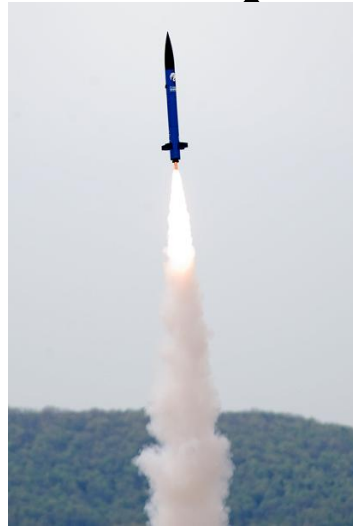
$$h_{apogee} \approx \frac{E_{mech}}{M_{final} \cdot g_0} = \frac{\left(V_{burnout}\right)^2}{2 \cdot g_0} + h_{burnout}$$

$$h_{apogee} \approx$$
$$\frac{g_0}{g_0}\left[\frac{1}{2}\left(I_{sp}\left[\ln\left(1+P_{mf}\right)\right] - g_0 \cdot t_{burn}\right)^2 + t_{burn}\left\{I_{sp} \cdot \left(1 - \frac{\ln\left(1+P_{mf}\right)}{P_{mf}}\right) - \frac{t_{burn}}{2}\right\}\right]$$

... ignoring aerodynamic drag

Example Energy Calculation

2009 USLI Rocket

AMW L777 Motor

"Dry" vehicle mass : *11.2451 kg*, Propellant mass: 1.7623 kg
Propellant I_{sp}: 181.49sec, Mean Motor Thrust: 774.475 Newtons

$$P_{mf} = \frac{m_{propellant}}{M_{final}} = \frac{1.7623}{11.2451} = 0.156717$$

$$t_{burn} = \frac{g_0 \, I_{sp} \, M_{propellamt}}{F_{thrust}} = \frac{9.8067 \cdot 181.49 \cdot 1.7623}{774.475} = 4.04993 \text{ sec}$$

Example Energy Calculation (2)

$$P_{mf} = \frac{m_{propellant}}{M_{final}} = 0.156717 \qquad t_{burn} = \frac{g_0 \ I_{sp} \ M_{propellamt}}{F_{thrust}} = 4.04993 \ \text{sec}$$

$$h_{t_{burn}} = g_0 \cdot t_{burn} \left\{ I_{sp} \cdot \left[1 - \frac{\ln\left(1 + P_{mf}\right)}{P_{mf}} \right] - \frac{t_{burn}}{2} \right\} =$$

$$9.8067 \cdot 4.04933 \left(181.49 \left(1 - \frac{\ln\left(1 + 0.156717\right)}{0.156717} \right) - \frac{4.04993}{2} \right) \text{ = 431.5 meters}$$

$$V_{t_{burn}} = g_0 \cdot I_{sp} \left[\ln\left(1 + P_{mf}\right) \right] - g_0 \cdot t_{burn} =$$

= 219. 5 m/sec

$$9.8067 \cdot 181.49 \left(\ln\left(1 + 0.156717\right) \right) - 9.8067 \cdot 4.004993$$

$$h_{apogee} \qquad \frac{E_{mech}}{M_{final} \ g_0} = \frac{\left(V_{burnout}\right)^2}{2 \ g_0} + h_{burnout} = \frac{219.5^2}{2 \cdot 9.8067} + 431.5$$

= 2888 meters

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Compare to Simulation Results

We will build this simulation later

$$h_{apogee} = 2888.71\ m$$

$$V_{burnout} = 219.34\ m/sec$$

Analytical Solution
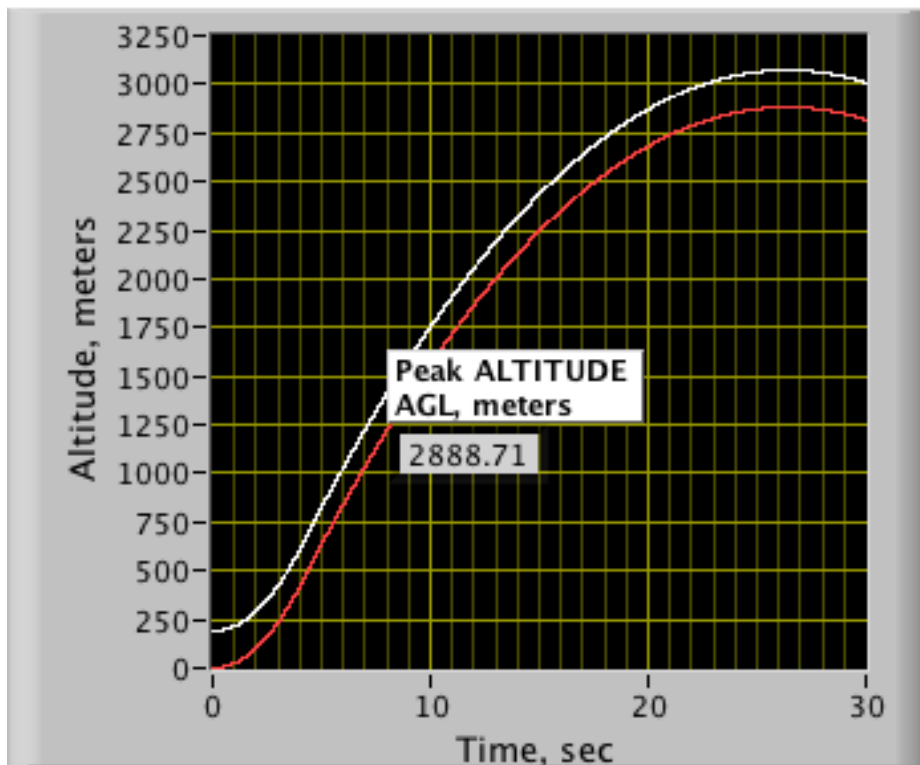
=> 2888 meters

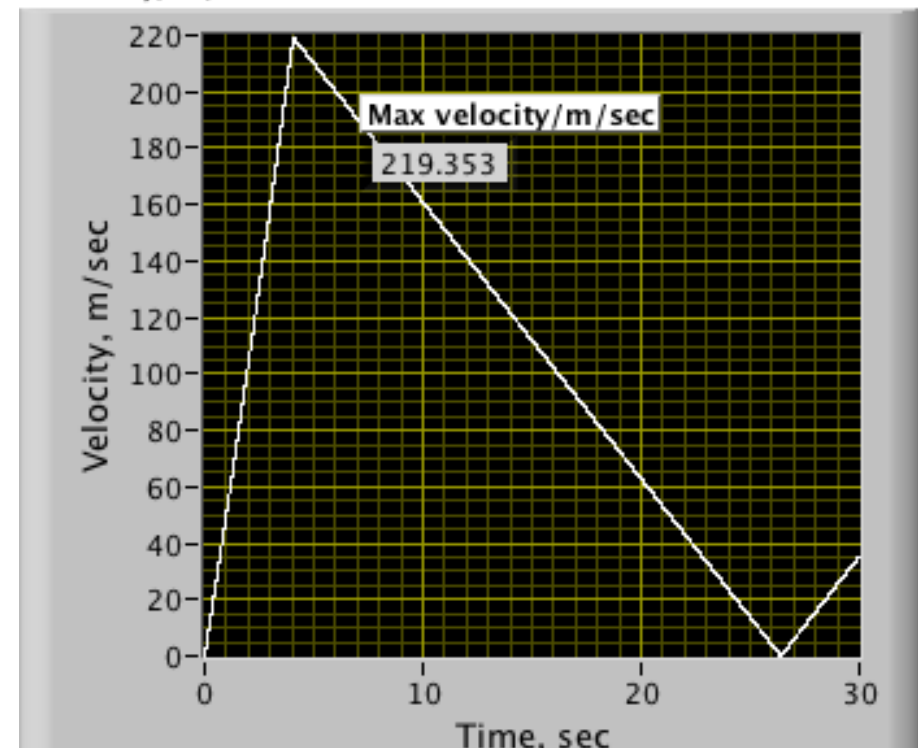*= 219. 5 m/sec*

Ignoring drag for now!

Altitude

Peak ALTITUDE AGL, meters

2888.71

Velocity, m/sec

Max velocity/m/sec

219.353

Better than 0.056%

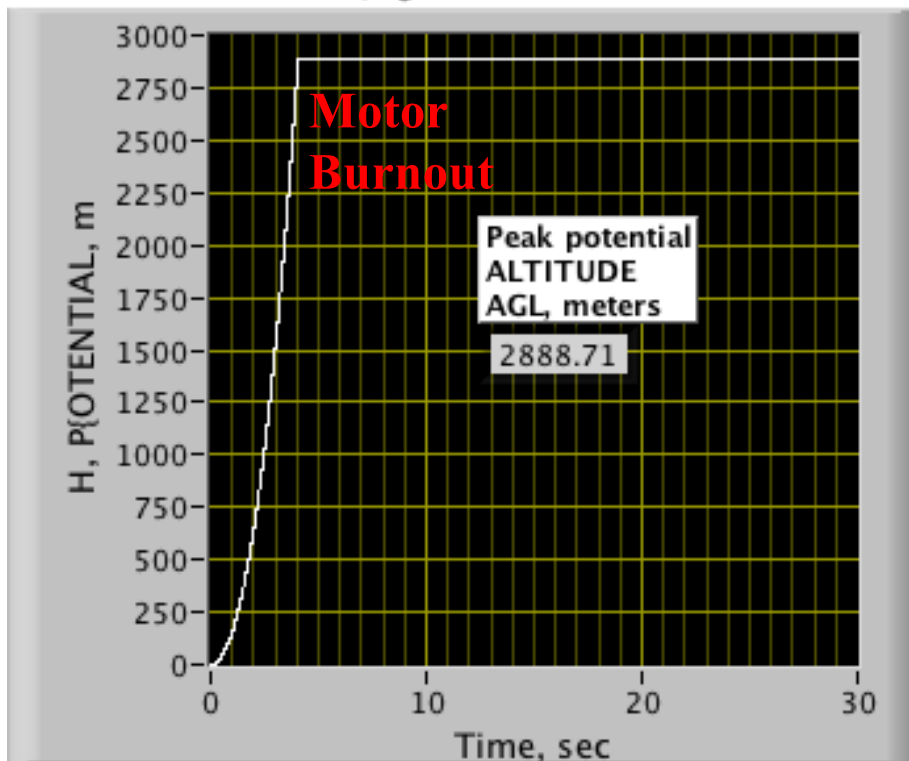# Compare to Simulation Results (2)

$$h_{apogee} = 2888.71 \ m$$

$$V_{burnout} = 219.34 \ m/sec$$

Ignoring drag for now!

POTENTIAL ALTITUDE, agl

Motor Burnout

Peak potential ALTITUDE AGL, meters

2888.71

$$h_{apogee} \longleftarrow h_{potential} =$$

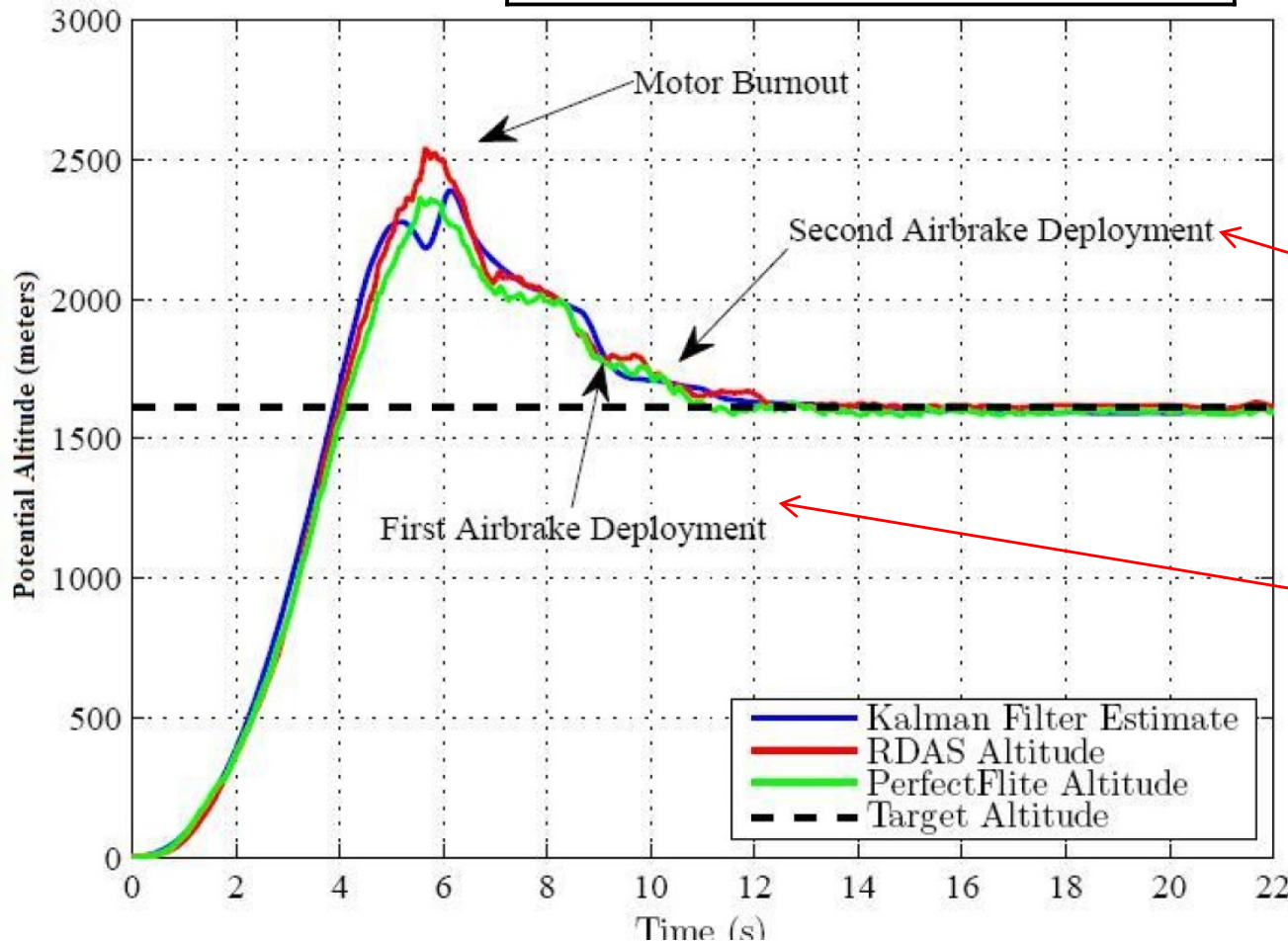$$\frac{E_{mech}}{M_{final} \ g_0} = \frac{(V_{burnout})^2}{2 \ g_0} + h_{burnout}$$

Compare to Fight Data

$$h_{apogee} = 2888.71 \ m$$

$$V_{burnout} = 219.34 \ m/sec$$

Why the difference?
Primarily …. Drag!

~ 15% error in peak altitude

$$h_{potential} = h + \frac{V^2}{2 \ g_0}$$

Compare to Fight Data (2)

Why the difference? We have ignored Drag!

$$\frac{\left(V_{apogee}\right)^2}{2} = \frac{E_{mech}}{M_{final}} \qquad V_{apogee} = \sqrt{\frac{2\, E_{mech}}{M_{final}}} = \sqrt{2\, g_0\, h_{potential}}$$

$$\frac{\left(V_{apogee}\right)_{calc} - \left(V_{apogee}\right)_{flight}}{\left(V_{apogee}\right)_{calc} + \left(V_{apogee}\right)_{flight}} \cdot 100_{\%} =$$

$$\frac{(2888.71 \cdot 2 \cdot 9.8067)^{0.5} - (2500 \cdot 2 \cdot 9.8067)^{0.5}}{(2888.71 \cdot 2 \cdot 9.8067)^{0.5} + (2500 \cdot 2 \cdot 9.8067)^{0.5}} \qquad = 7.22\%$$

~ 7.2 % error in delivered apogee Δ V due to aerodynamic drag

Drag Losses

$$\Delta E_{\substack{non- \\ conservative}} = \int_{path} \vec{F}_{\substack{non \\ -conservative}} \cdot d\vec{s} = \int_{path} \vec{F}_{\substack{non \\ -conservative}} \cdot \frac{d\vec{s}}{dt} \cdot dt$$

$$\int_{t} \vec{F}_{\substack{non \\ -conservative}} \cdot \vec{V} \cdot dt \approx \frac{\Delta V_{loss}^{2}}{2} \cdot M$$

non-conserv

• **Lift – acts perpendicular to flight path**
  .. Cannot effect energy level of rocket

• **Gravity – acts downward** *(conservative)*
  … cannot effect energy level of rocket, kinetic energy
  potential energy as rocket climbs

• *Apogee primarily influenced by burnout energy and atmospheric aerodynamc drag*
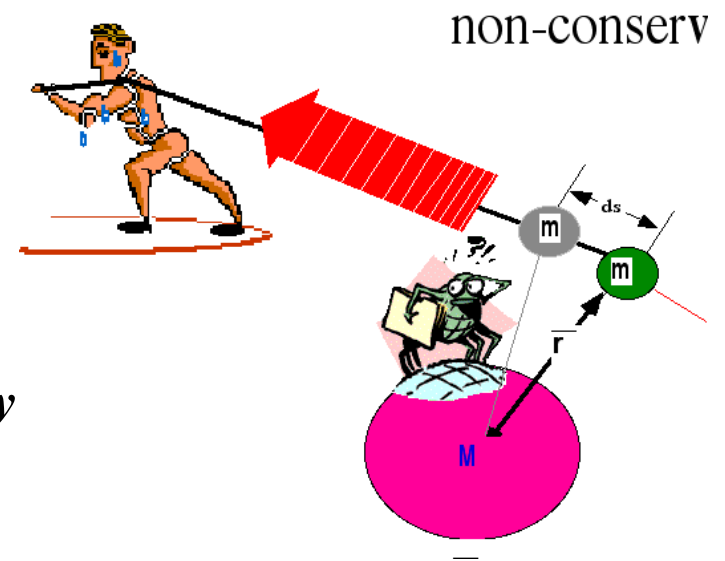
How Drag Losses Effect Peak Altitude

$$\Delta V_{drag} = \sqrt{2\int_0^t \frac{D_{rag}V}{M}dt} = \sqrt{2\int_0^t \frac{C_D A_{ref} \frac{1}{2}\rho V^2 \cdot V}{M}dt} = \sqrt{\int_0^t \frac{\rho \cdot V^3}{\beta}dt}$$

$$\Delta h_{drag} = \frac{\Delta V_{drag}^2}{2 \cdot g_0} = \frac{1}{2 \cdot g_0 \cdot \beta}\left[\int_0^t \rho \cdot V^3 \, dt\right]$$

Check units!

$$: \frac{1}{\frac{m}{\sec^2}\frac{kg}{m^2}}\frac{kg}{m^3}\left(\frac{m}{\sec}\right)^3 \sec = \frac{\sec^2 m^5}{m\ kg}\frac{kg}{m^3}\frac{1}{\sec^2} = m$$

How Drag Losses Effect Peak Altitude (2)

$$\Delta h_{drag} = \frac{1}{2 \cdot g_0 \cdot \beta} \left[\int_0^t \rho \cdot V^3 \, dt \right]$$

Correct peak altitude estimate

$$h_c = \frac{g_0}{4} \cdot \left(2 + P_{mf} \right) \cdot \left(I_{sp} \cdot \ln\left(1 + P_{mf} \right) \right)^2 - \Delta h_{drag} =$$

$$\frac{g_0}{4} \cdot \left(2 + P_{mf} \right) \cdot \left(I_{sp} \cdot \ln\left(1 + P_{mf} \right) \right)^2 - \frac{1}{2 \cdot g_0 \cdot \beta} \left[\int_0^t \rho \cdot V^3 \, dt \right]$$
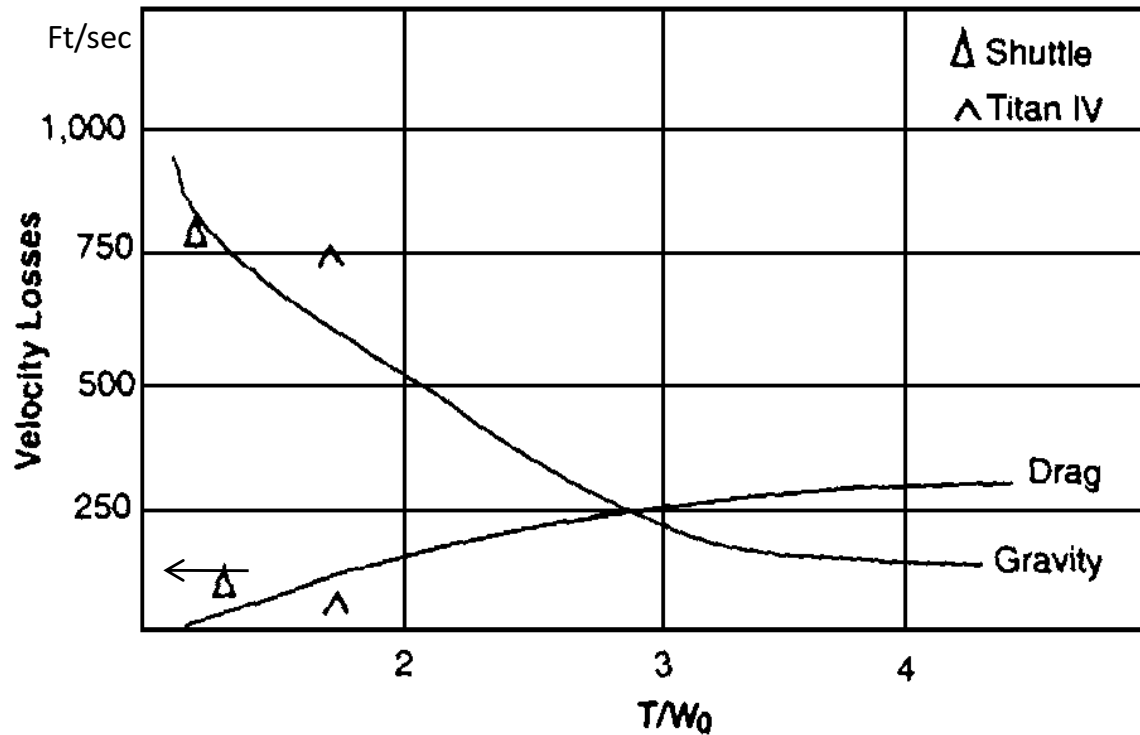
Path Independent   **Path Dependent**

"Rule of thumb" ~ drag loss is about 5-10% of delivered *Δ V* from motor

Drag Losses (3)

$$D_{rag} = C_D A_{ref} \frac{1}{2} \rho V^2 \rightarrow \Delta V_{drag} = \sqrt{A_{ref} \int_0^t \frac{C_D \rho V^3}{m} dt} = \sqrt{\int_0^t \frac{\rho V^3}{\beta} dt}$$

Depending
On thrust to-weight
Off of the pad
drag losses
can be significant
During motor burn

As much as 12-15% of
Potential altitude

... path dependent!

Must simulate trajectory

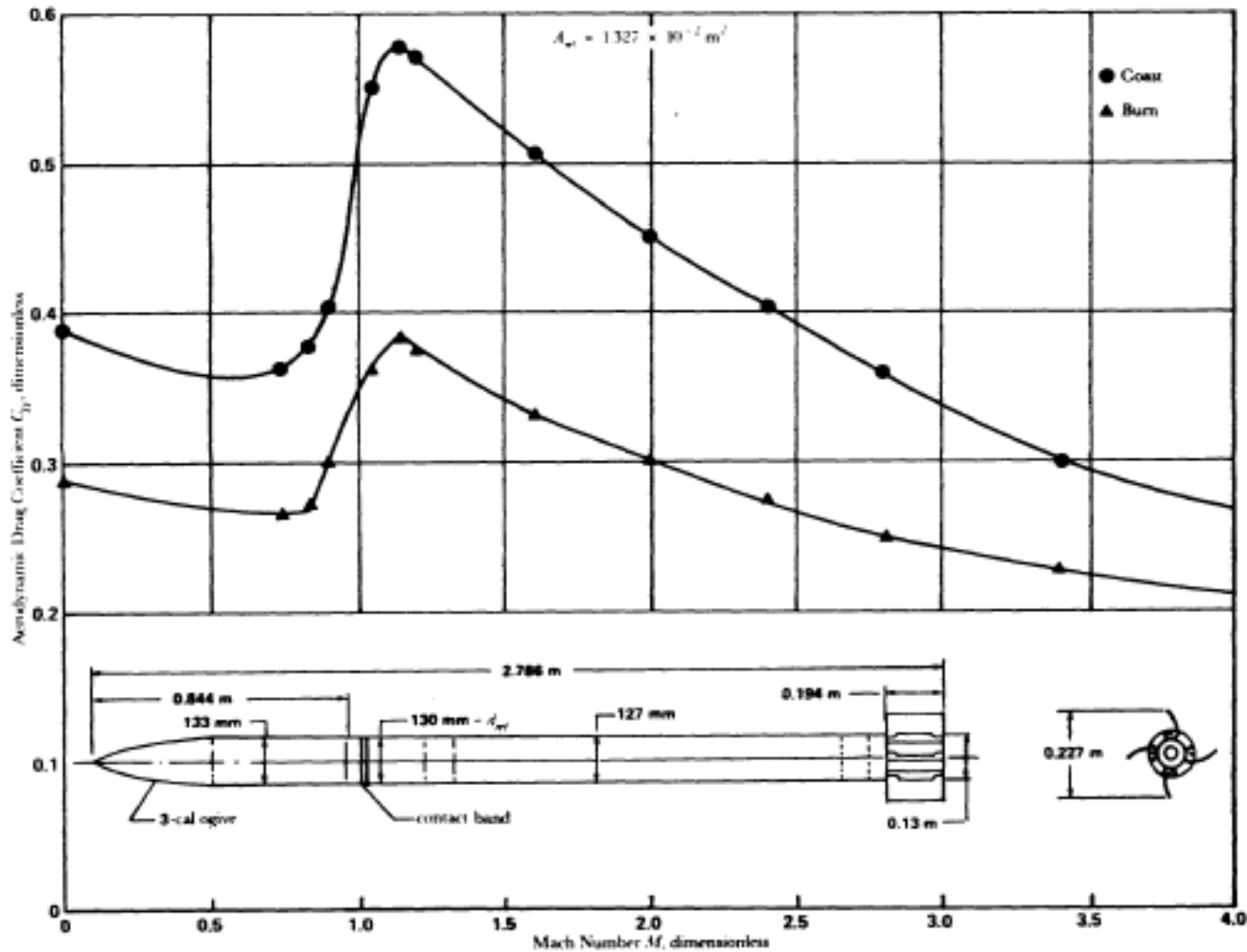# Drag Coefficient is Configuration Dependent

Figure 3-4. Drag Coefficient vs Mach Number-HO-mm Rocket

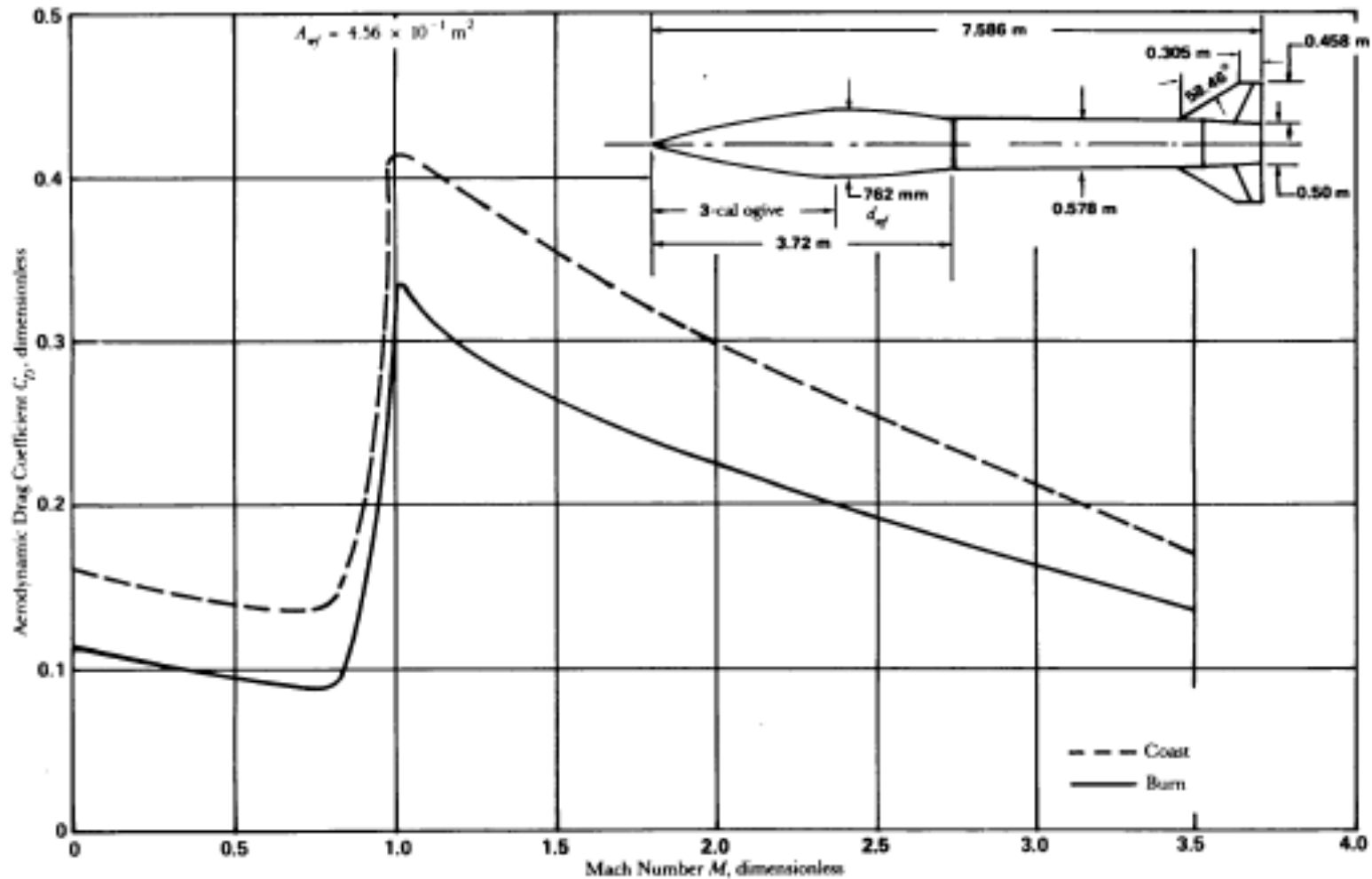# Drag Coefficient is Configuration Dependent (2)

Figure 3-5. Drag Coefficient vs Mach Number—762-mm Rocket

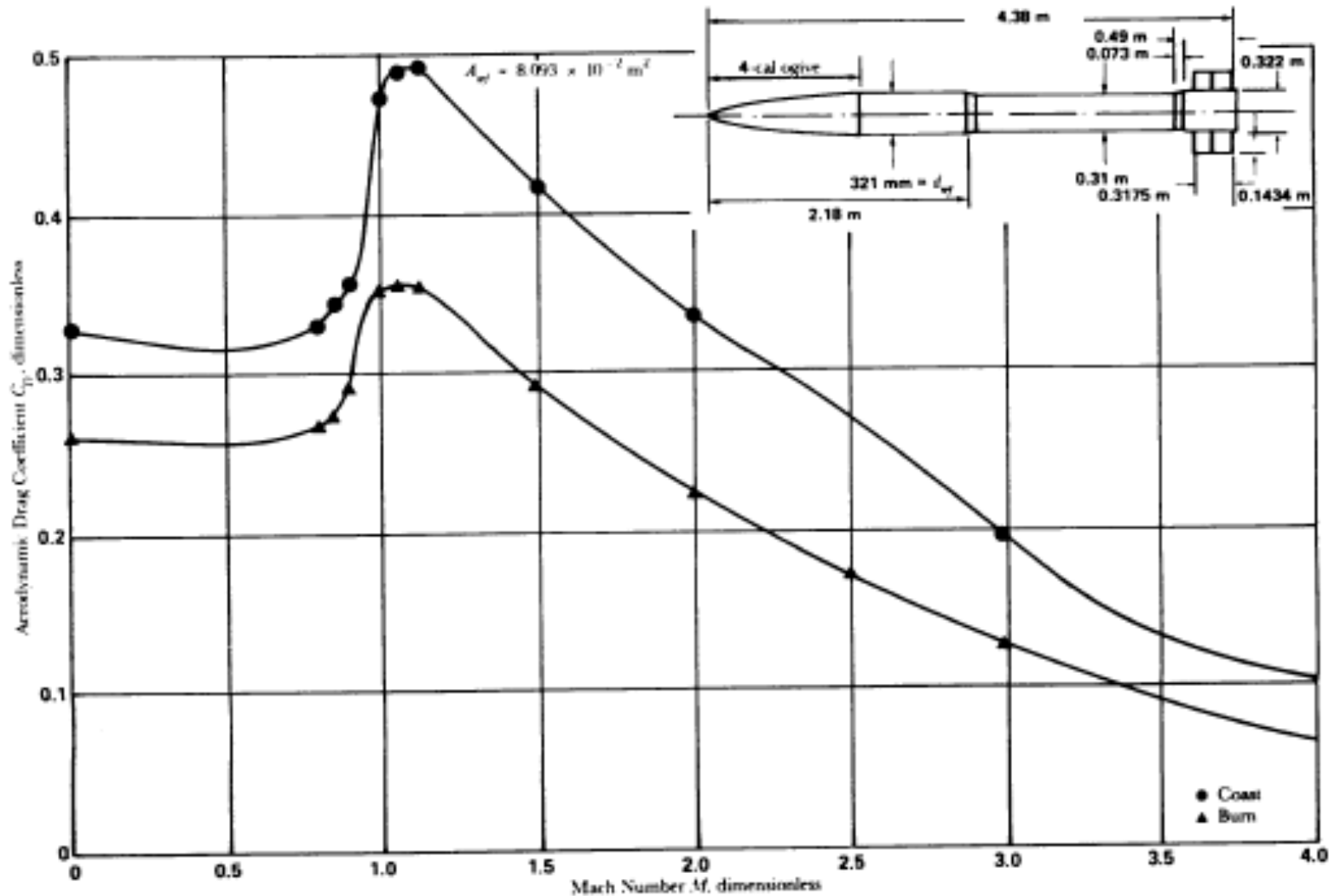# Drag Coefficient is Configuration Dependent (3)

Figure 3-6. Drag Coefficient vs Mach Numbr-321-mm Rocket

A Recipe for Energy Management

Conservation of Energy :

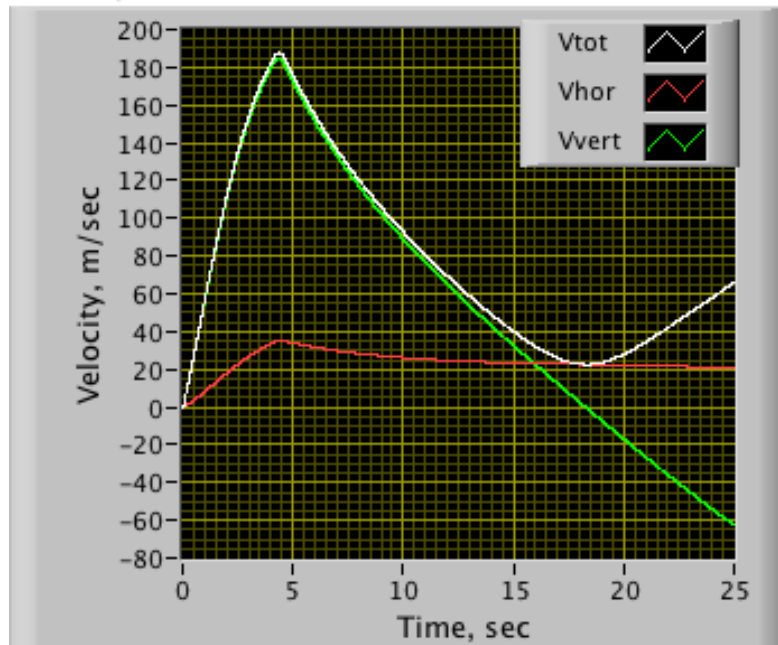*Potential* + *Kinetic Energy* = *Constant* − *Dissipated Energy*

$$g \cdot h_{apogee} + \frac{V^2_{apogee}}{2} = h_{burnout} + \frac{V^2_{burnout}}{2} - \int_{t_{burnout}}^{t_{apogee}} \frac{\rho \cdot V^3}{\beta} dt$$

$$\rightarrow h_{apogee} = h_{burnout} + \left(\frac{V^2_{burnout}}{2 \cdot g} - \frac{V^2_{apogee}}{2 \cdot g} \right) - \frac{1}{g} \int_{t_{burnout}}^{t_{apogee}} \frac{\rho \cdot V^3}{\beta} dt$$

• **Velocity and drag are Very high just after motor burnout .. But diminish near apogee**
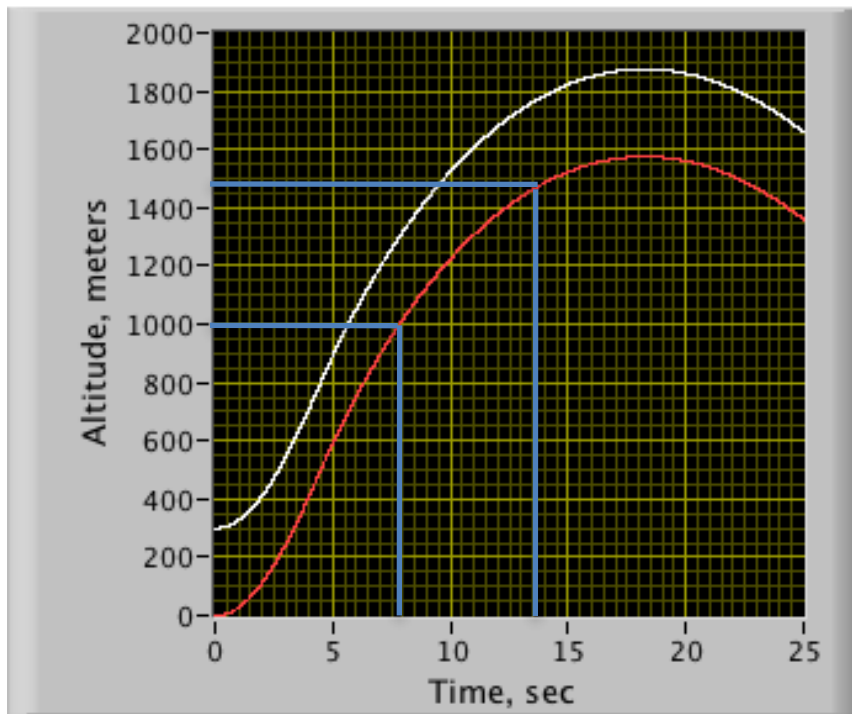
• **Specific Energy of Rocket becomes "more constant" with time**
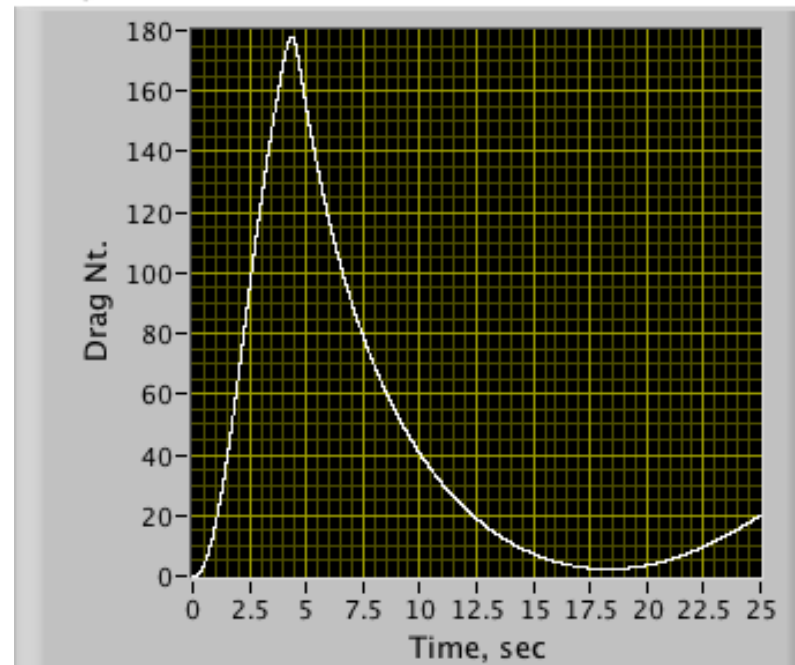
.. As drag
Diminishes
Near apogee

A Recipe for Energy Management (2)

• **Specific Energy of Rocket….**
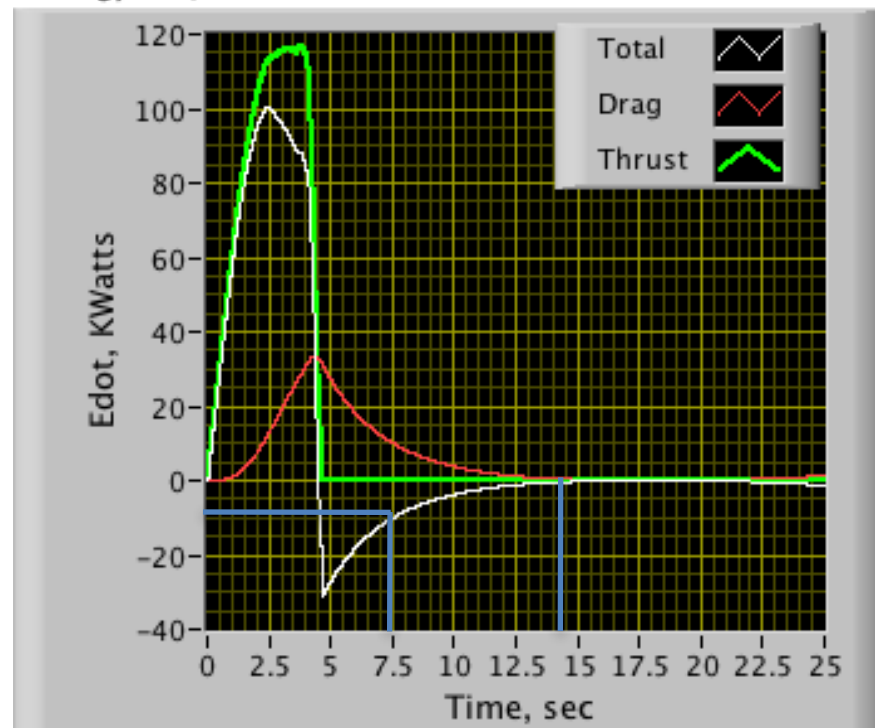
• At motor burn out Drag Energy Dissipation rate is ~3.5 times higher than at 1000 m AGL

• At 1500 m AGL Drag Energy Dissipation is essentially zero ..
Estimated energy level~ constant

A Recipe for Energy Management (3)

• **Potential Altitude as an Estimator of "Achievable Altitude" Becomes Increasingly More accurate as Apogee is Approached**

• **Use Augmentation Thrust to "Manage Energy" at waypoints of Increasing Altitude Along Probably trajectory**

Waypoint Array

0

Waypoint	h AGL max, m
1	1100

h AGL min, m	h AGL target, m
1000	1706.8

Waypoint	h AGL max, m
2	1609.3

h AGL min, m	h AGL target, m
1500	1609.32

Altitude

Thrust
Augmentation
Active

A Recipe for Energy Management (4)

Altitude
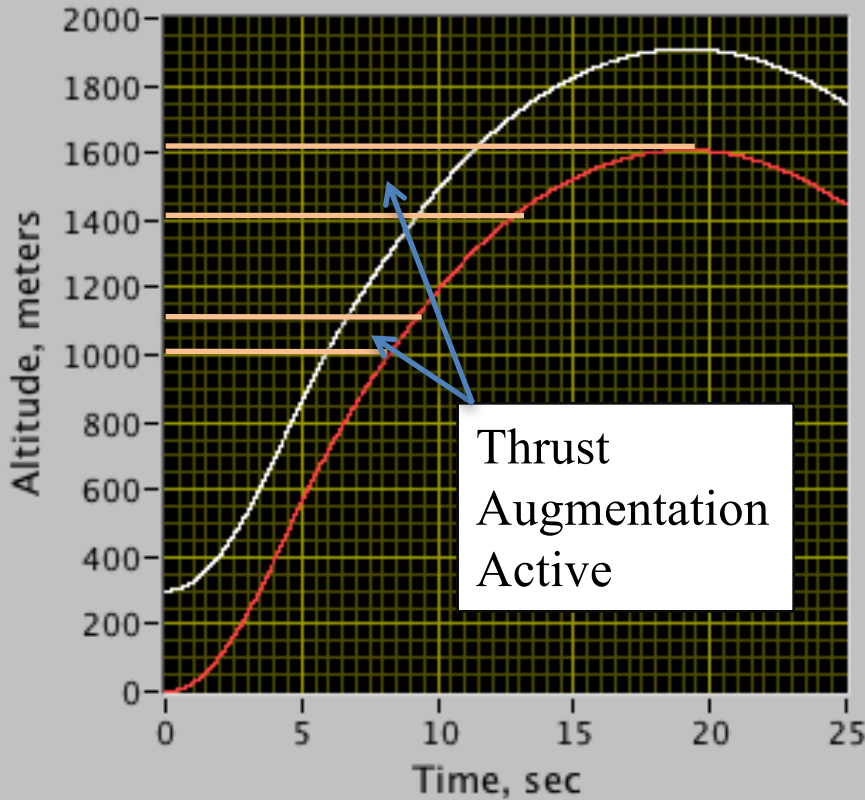
POTENTIAL ALTITUDE, agl
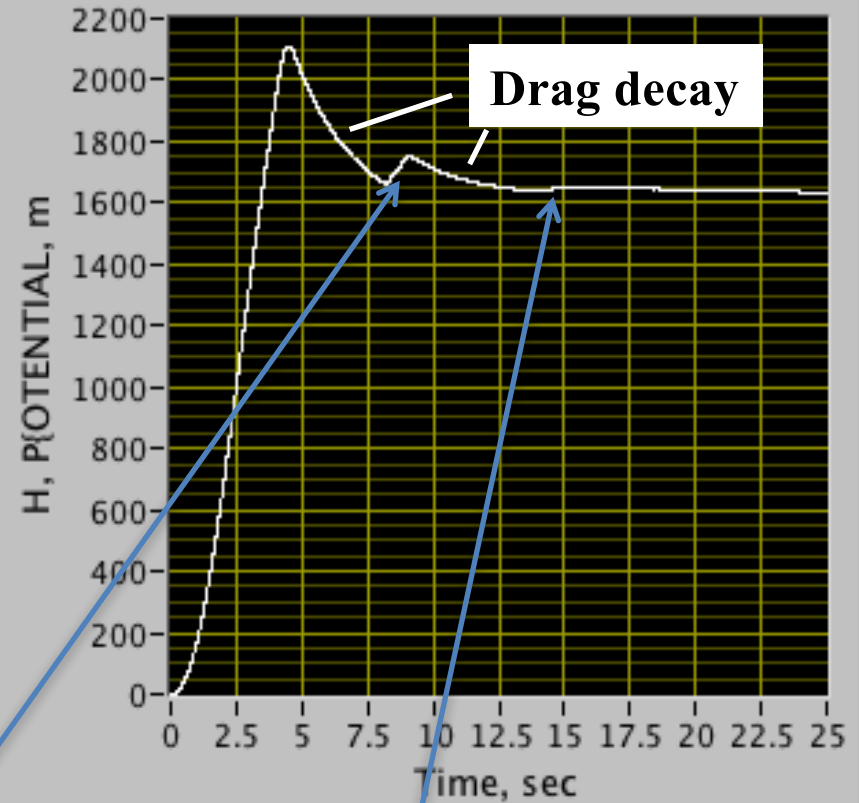
Thrust Augmentation Active

Drag decay

Waypoint Array

0

Waypoint	h AGL max, m
1	1100

h AGL min, m	h AGL target, m
1000	1706.8

Waypoint	h AGL max, m
2	1609.3:

h AGL min, m	h AGL target, m
1500	1609.32

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A Recipe for Energy Management (4)

POTENTIAL ALTITUDE, agl
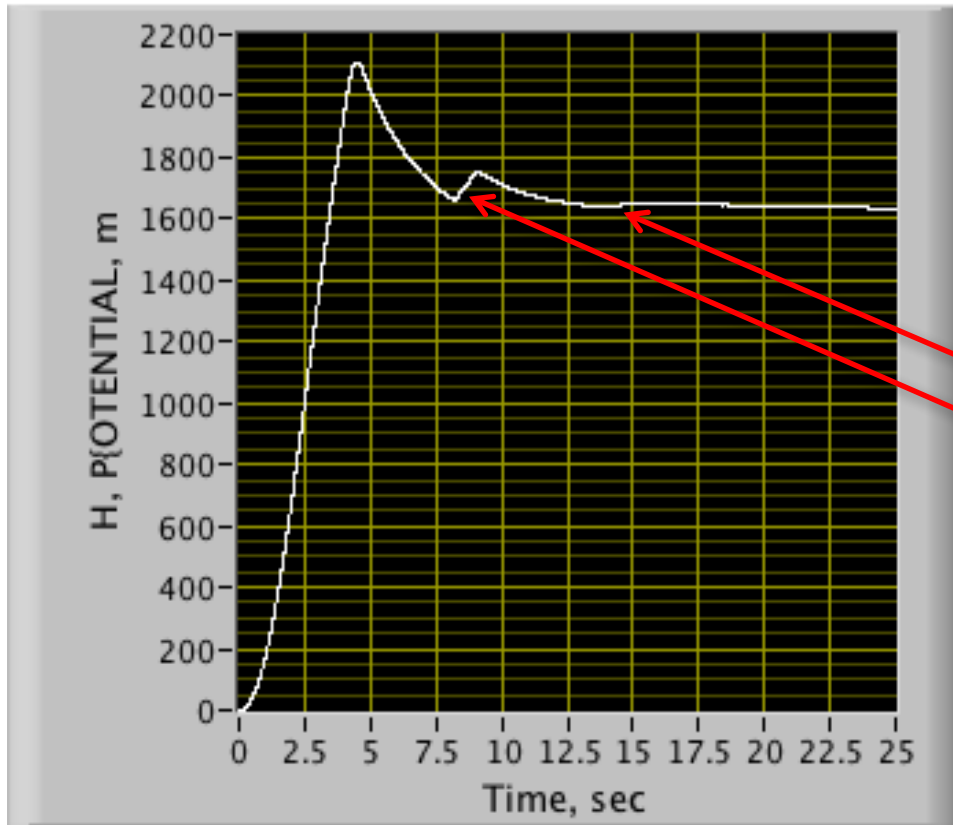
Thrust
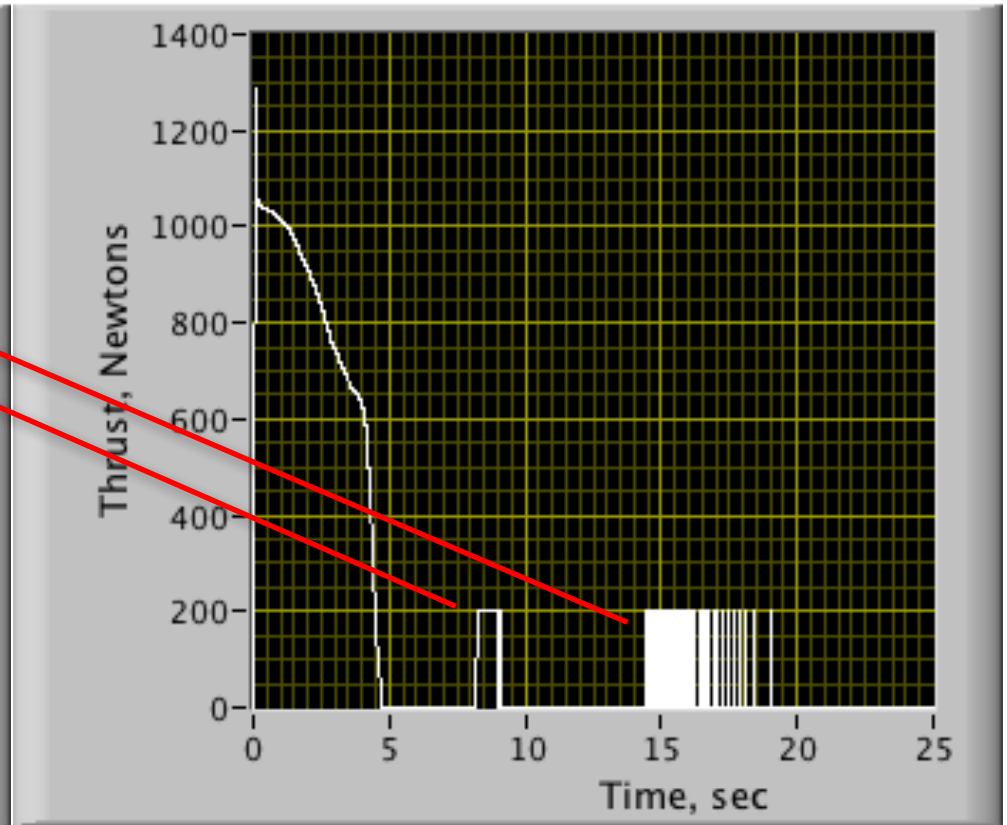
- First (mostly constant) Augmentation Impulse Boosts Energy to "achievable level"
Once we have calculated energy state (using IMU) … *1706.8 m = 5600 ft*

- Second Augmentation Impulse Boost and Maintains Energy level at Desired (Target Level)
Eneregy Level using Pulsed-modulation … *1609.32 m = 5280 ft*

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A Recipe for Energy Management (5)

Altitude

Peak ALTITUDE AGL, meters
1609.32

Peak ALTITUDE AGL, meters 2
1580.58

Thrust
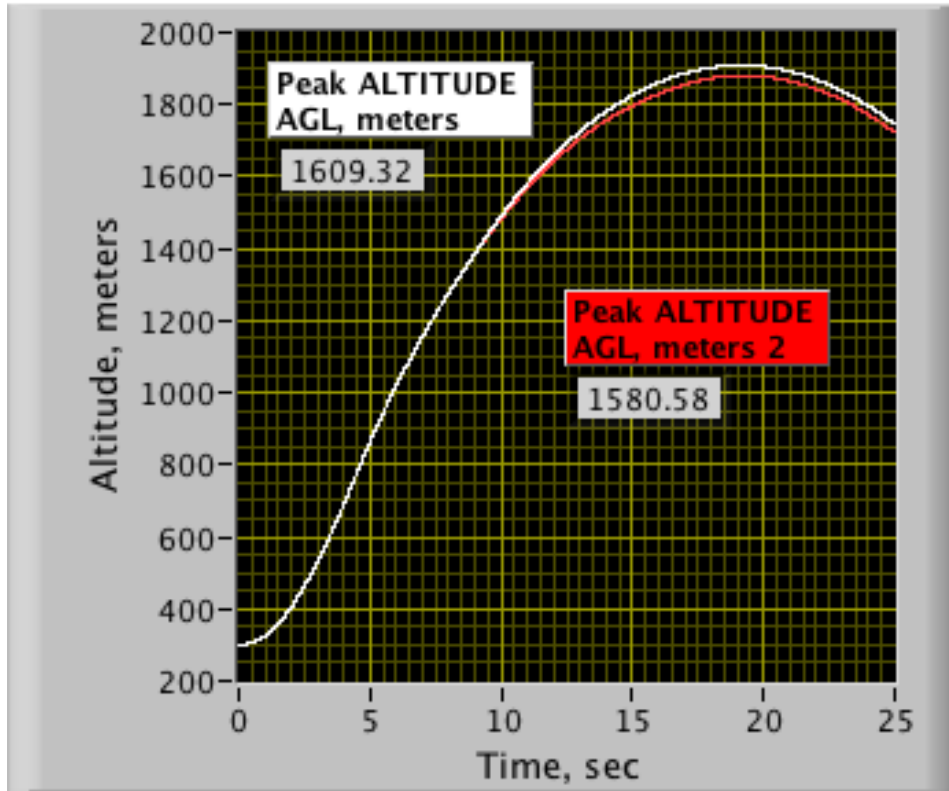
- 100 Nt Thrust
- 200 Nt Thrust
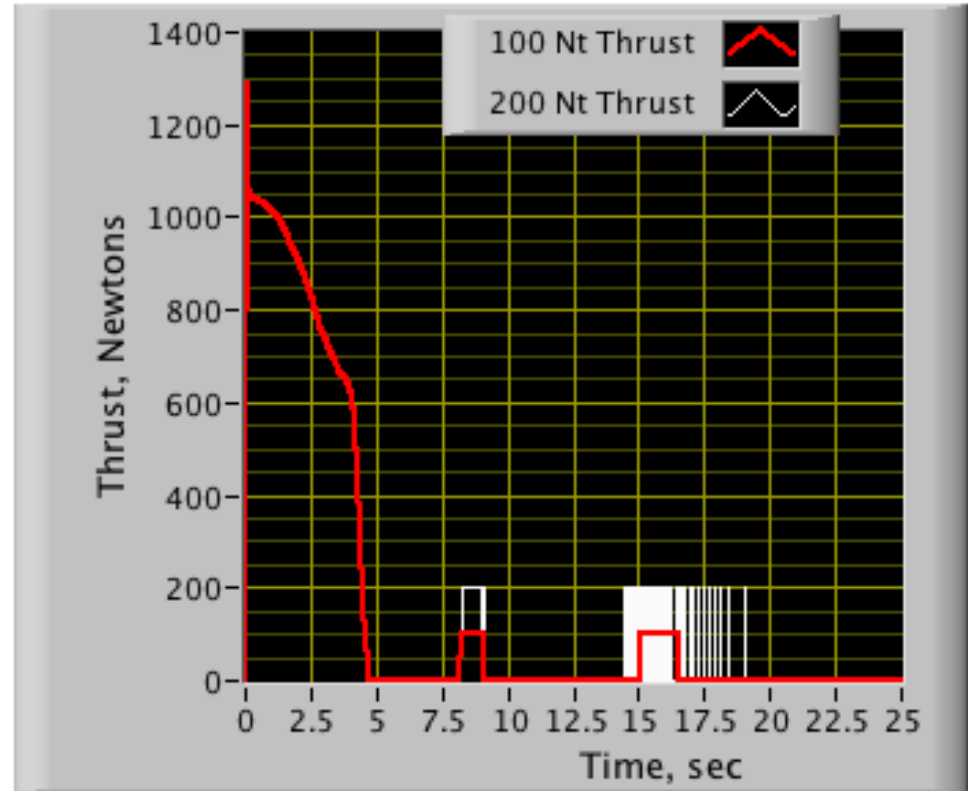
- Lower Augmentation Thrust levels allow for more precise modulation, but are less efficient and must "burn longer"

Waypoint Array

0

Waypoint	h AGL max, m
1	1100

h AGL min, m	h AGL target, m
1000	1706.8

Waypoint	h AGL max, m
2	1609.3:

h AGL min, m	h AGL target, m
1500	1609.32

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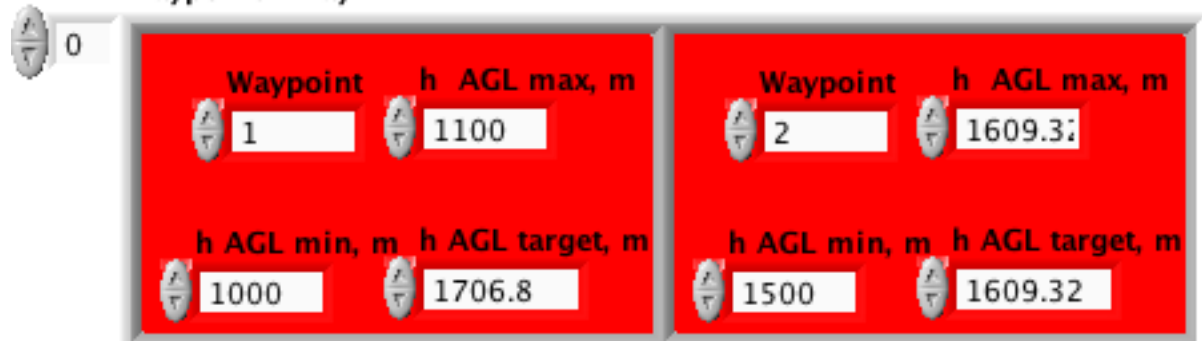
A Recipe for Energy Management (6)

Altitude

POTENTIAL ALTITUDE, agl

Thrust Augmentation Active
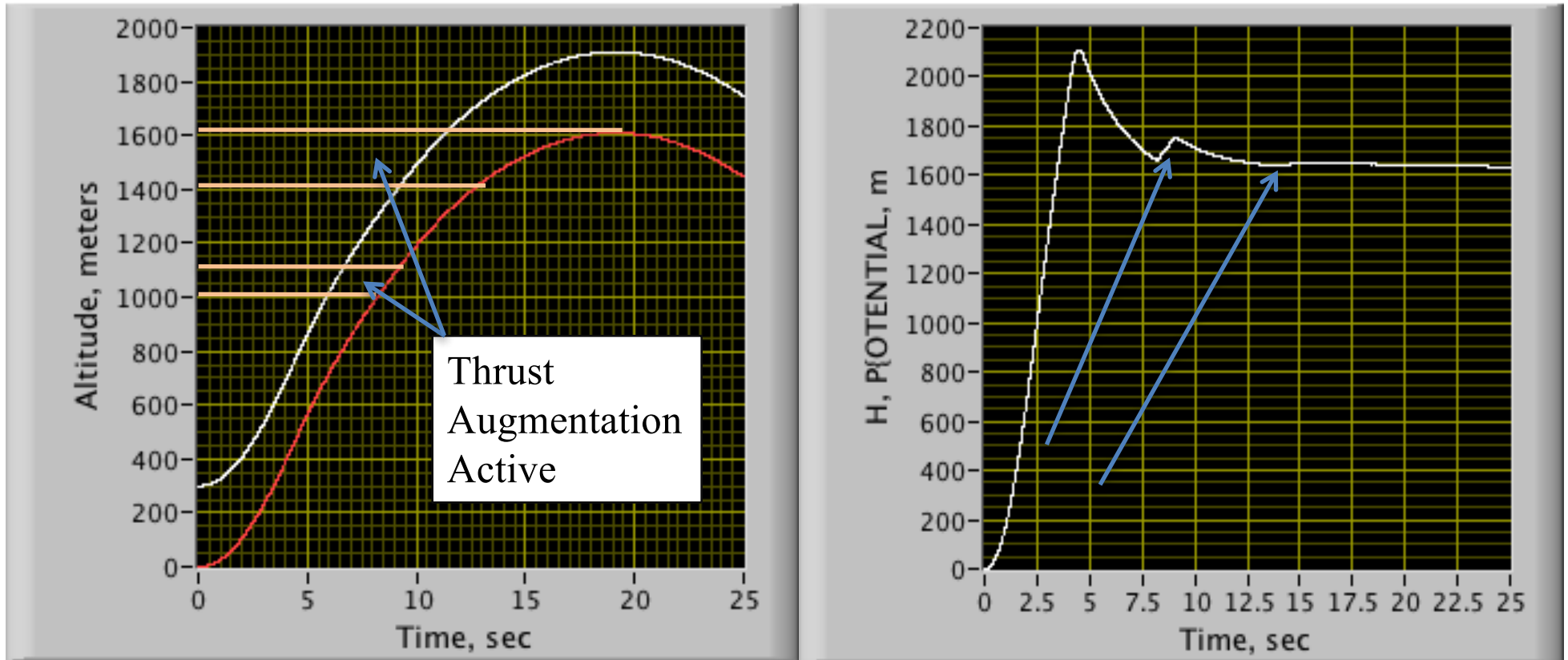
- Early Energy Management is More Effective, But less Precise

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A Recipe for Energy Management (7)

Altitude

Total Impulse, Nt-sec
3921.75

Peak ALTITUDE AGL, meters
1516.93
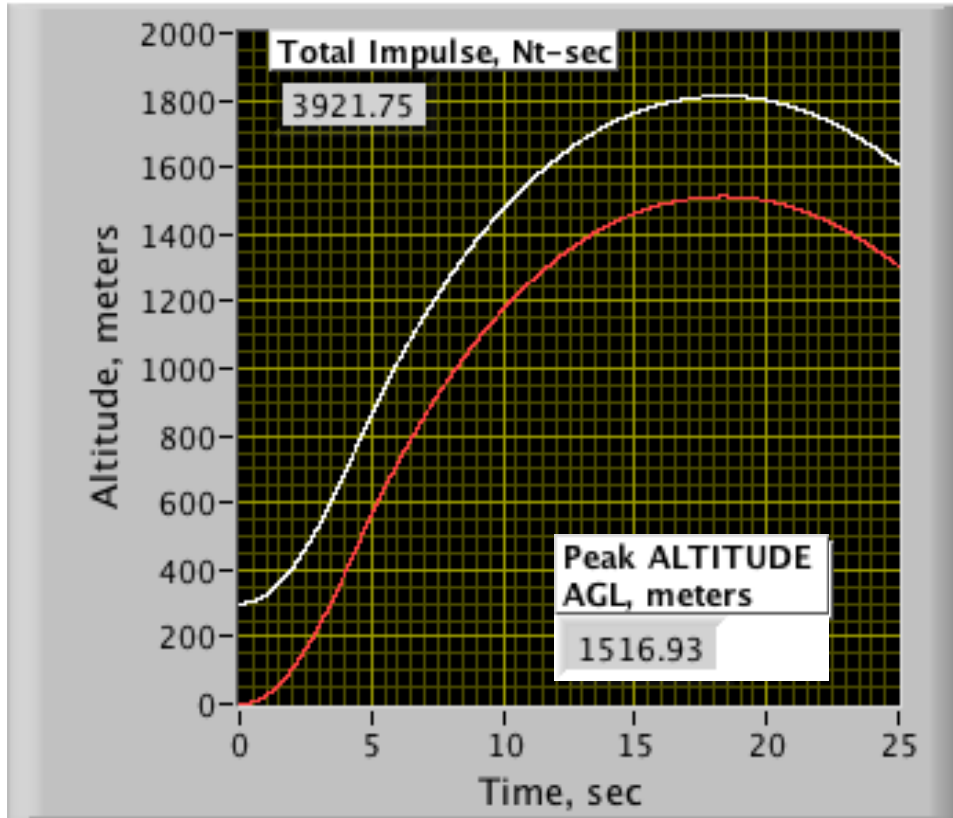
Thrust

Waypoint Array

0
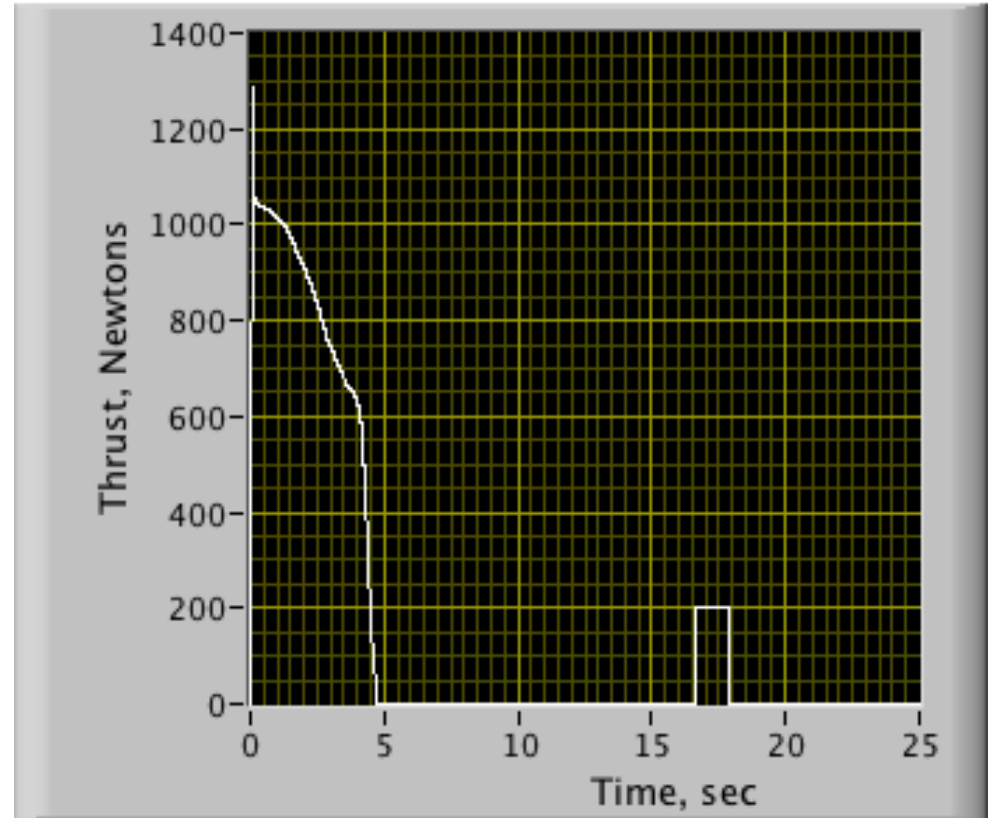
Waypoint	h AGL max, m
1	1609.32

h AGL min, m	h AGL target, m
1500	1609.32

- Single Waypoint Energy Management Near Apogee

- *Insufficient Propellant to hit target*

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A Recipe for Energy Management [8]

Altitude

Total Impulse, Nt-sec
3974.95

Peak ALTITUDE AGL, meters
1592.61
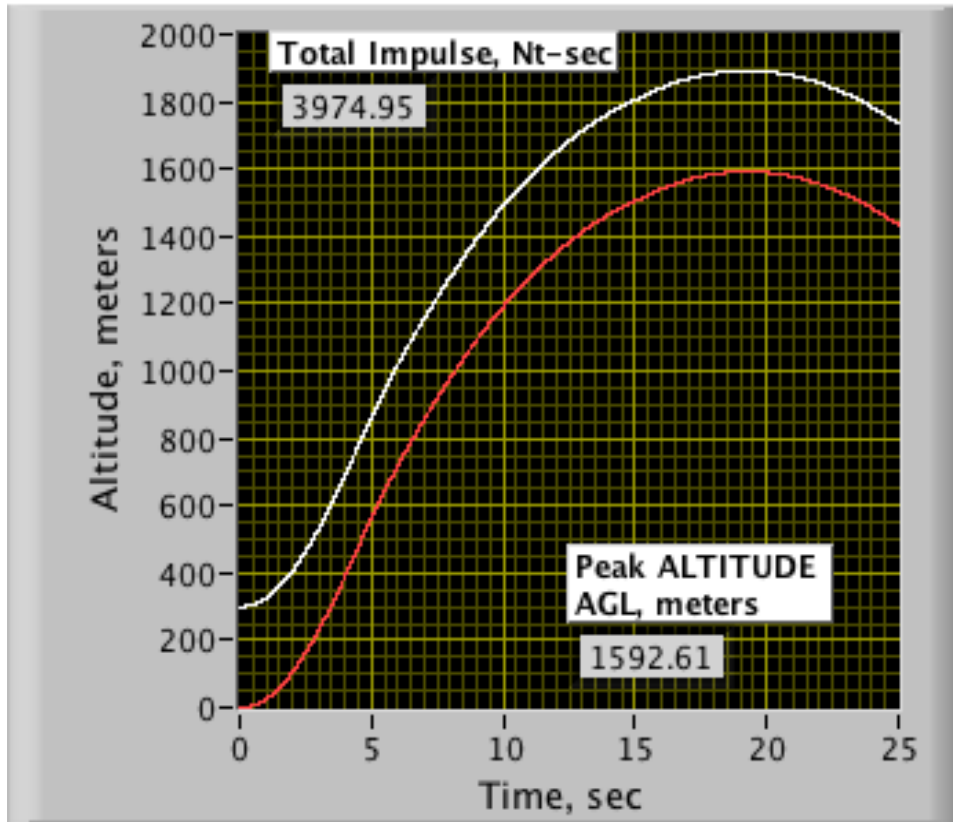
Thrust
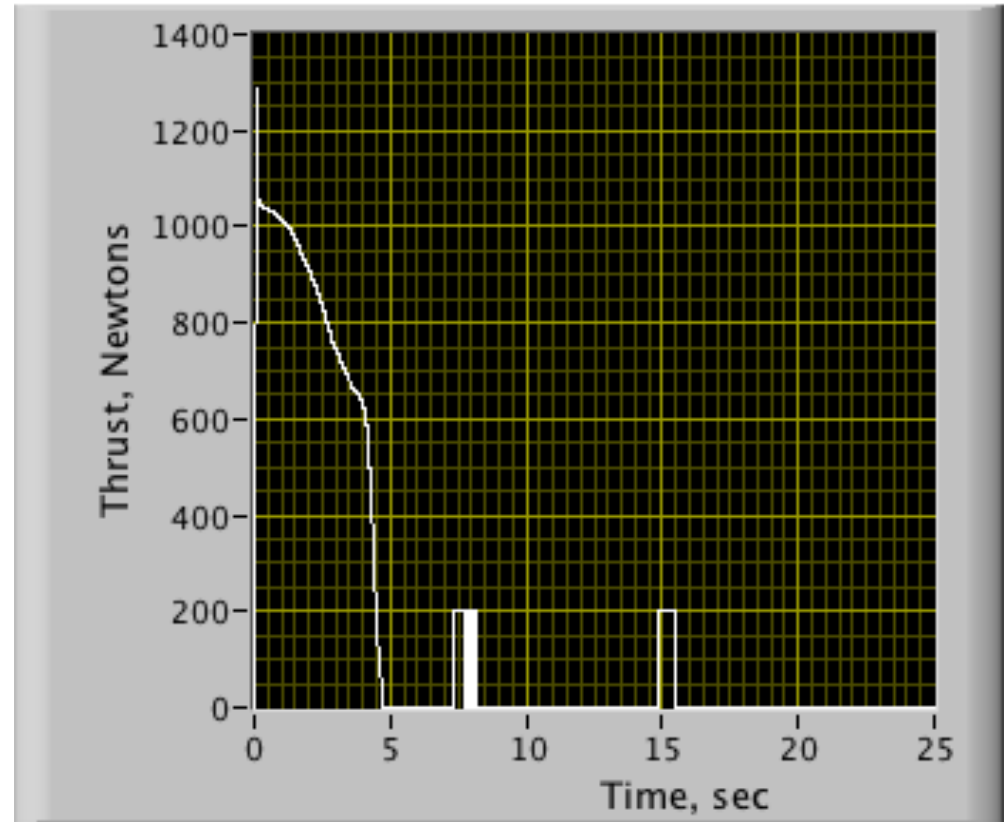
- Earlier Implementation of First Waypoint

- Insufficient Accuracy to Hit target

- *There will definitely be a Design "Sweet spot" .. here*

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Adjusting Potential Altitude Estimate for Effects of Horizontal Velocity
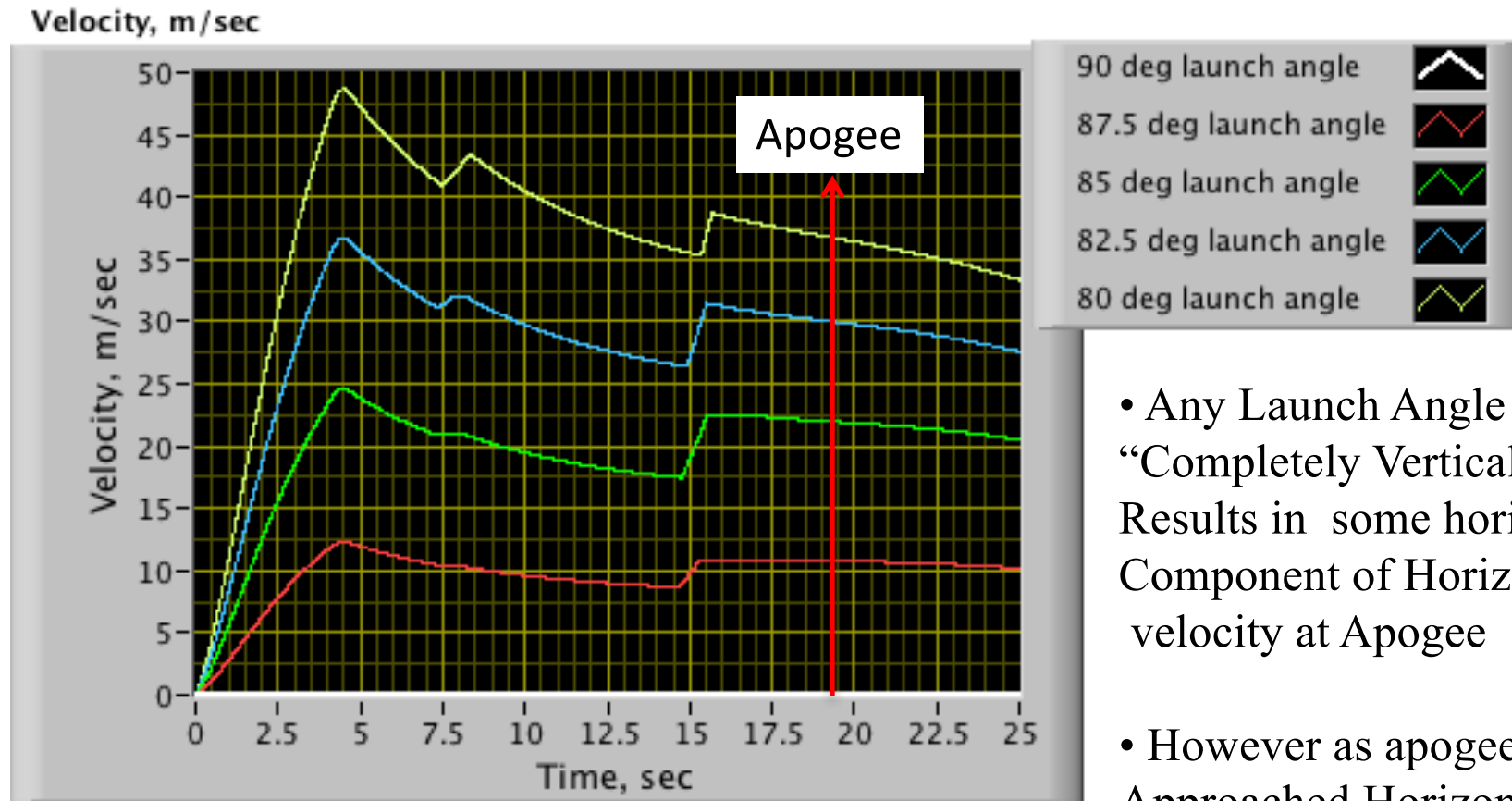
• Any Launch Angle not "Completely Vertical" Results in some horizontal Component of Horizontal velocity at Apogee

• However as apogee is Approached Horizontal Velocity Component becomes ~ constant

Adjusting Potential Altitude Estimate for Effects of Horizontal Velocity (2)

Velocity, m/sec
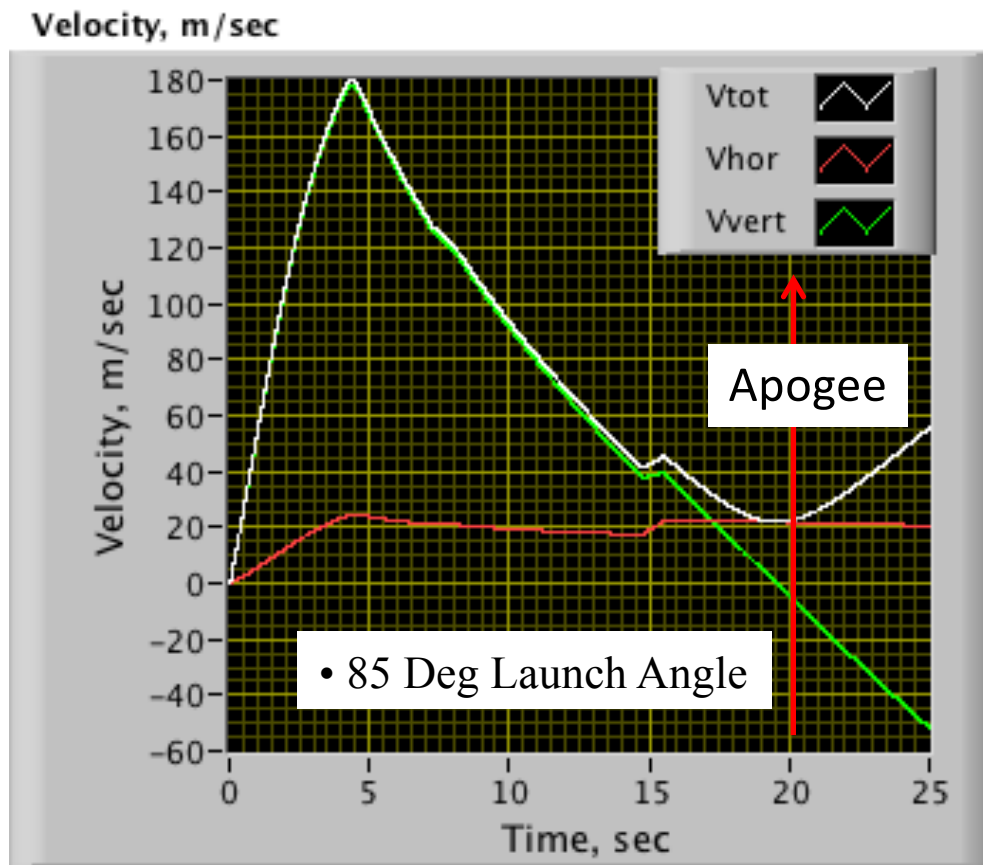
- Compared to total velocity of Vehicle Horizontal component ~ constant Very soon after motor burn out

- $V_{hor_{waypoint}} \sim V_{apogee}$

$$V_{hor\ waypoint} = V_{waypoint} \cdot \cos(\gamma) \approx V_{apogee}$$

$$\rightarrow \left| \begin{array}{l} \gamma = flight\ path\ angle \\[2ex] = \tan^{-1} \dfrac{\dot{h}}{V_{hor}} \end{array} \right.$$

Adjusting Potential Altitude Estimate for Effects of Horizontal Velocity (3)

$$h_{apogee} = h_{waypoint} + \left(\frac{V^2_{waypoint}}{2 \cdot g} - \frac{V^2_{apogee}}{2 \cdot g} \right) - \frac{1}{g} \int_{t_{waypoint}}^{t_{apogee}} \frac{\rho \cdot V^3}{\beta} dt =$$

$$= h_{waypoint} + \left(\left(\frac{V^2_{waypoint}}{2 \cdot g} \right)_{vertical} + \left(\frac{V^2_{waypoint}}{2 \cdot g} \right)_{horizontal} - \frac{V^2_{apogee}}{2 \cdot g} \right) - \frac{1}{g} \int_{t_{waypoint}}^{t_{apogee}} \frac{\rho \cdot V^3}{\beta} dt$$

$$\boxed{ \rightarrow \left(\frac{V^2_{waypoint}}{2 \cdot g} \right)_{horizontal} \approx \frac{V^2_{apogee}}{2 \cdot g} }$$

$$\rightarrow h_{apogee} = h_{waypoint} + \left(\frac{V^2_{waypoint}}{2 \cdot g} - \frac{V^2_{apogee}}{2 \cdot g} \right) - \frac{1}{g} \int_{t_{waypoint}}^{t_{apogee}} \frac{\rho \cdot V^3}{\beta} dt =$$

$$\rightarrow h_{apogee} = h_{waypoint} + \left(\frac{V^2_{waypoint}}{2 \cdot g} \right)_{vertical} - \frac{1}{g} \int_{t_{waypoint}}^{t_{apogee}} \frac{\rho \cdot V^3}{\beta} dt$$

"0" Near Apogee

Adjusting Potential Altitude Estimate for Effects of Horizontal Velocity (4)

$$\rightarrow h_{apogee} = h_{waypoint} + \left(\frac{V^2_{waypoint}}{2 \cdot g} \right)_{vertical} - \frac{1}{g} \int_{t_{waypoint}}^{t_{apogee}} \frac{\rho \cdot V^3}{\beta} dt$$

$$\rightarrow \boxed{\widehat{h}_{potential} = h_{waypoint} + \frac{V^2_{waypoint} \cdot \sin^2(\gamma)}{2 \cdot g}}$$

• Non-optimal strategy
.. But it works pretty well

• Some potential that
non-linear "bang-bang" or
Dead-band controller may
Be more propellant efficient

• But $\widehat{h}_{potential}$ is a critical
feedback parameter

continuously estimate ...

$$\left(\widehat{h}_{potential} \right)_t = h_{(t)} + \frac{V^2_{(t)} \cdot sin^2(\gamma_{(t)})}{2 \cdot g_{(t)}}$$

at waypoint ...we have a very simple control strategy....

$$...if \left[(h_{min} \le h \le h_{max}) \,\&\&\, \left(h < \widehat{h}_{potential} \right) \right]$$

$$\quad "thrust\ on"$$

$$...else$$

$$\quad "thrust\ off"$$

Accounting for Drag Losses In Potential Altitude

Ignoring drag …. At any point along the trajectory …

$$h_{potential} = h(t) + \frac{V(t) \cdot \sin(\gamma)}{2 \cdot g}$$

$$since \rightarrow V_{hor} = V(t) \cdot \cos(\gamma) \approx constant$$

But because of drag …. The true apogee will be…

Drag Loss

$$\Delta h_{drag}$$

$$h_{apogee} = h_{potential} - \left[\int_{t}^{t_{apogee}} \left(\frac{1}{2} \cdot \rho \cdot V^2 \right) \cdot \left(\frac{C_D \cdot A_{ref}}{m \cdot g} \right) \cdot V \cdot d\tau \right]$$

Accounting for Drag Losses
In Potential Altitude $_2$

True, Potential Altitude and Drag Loss
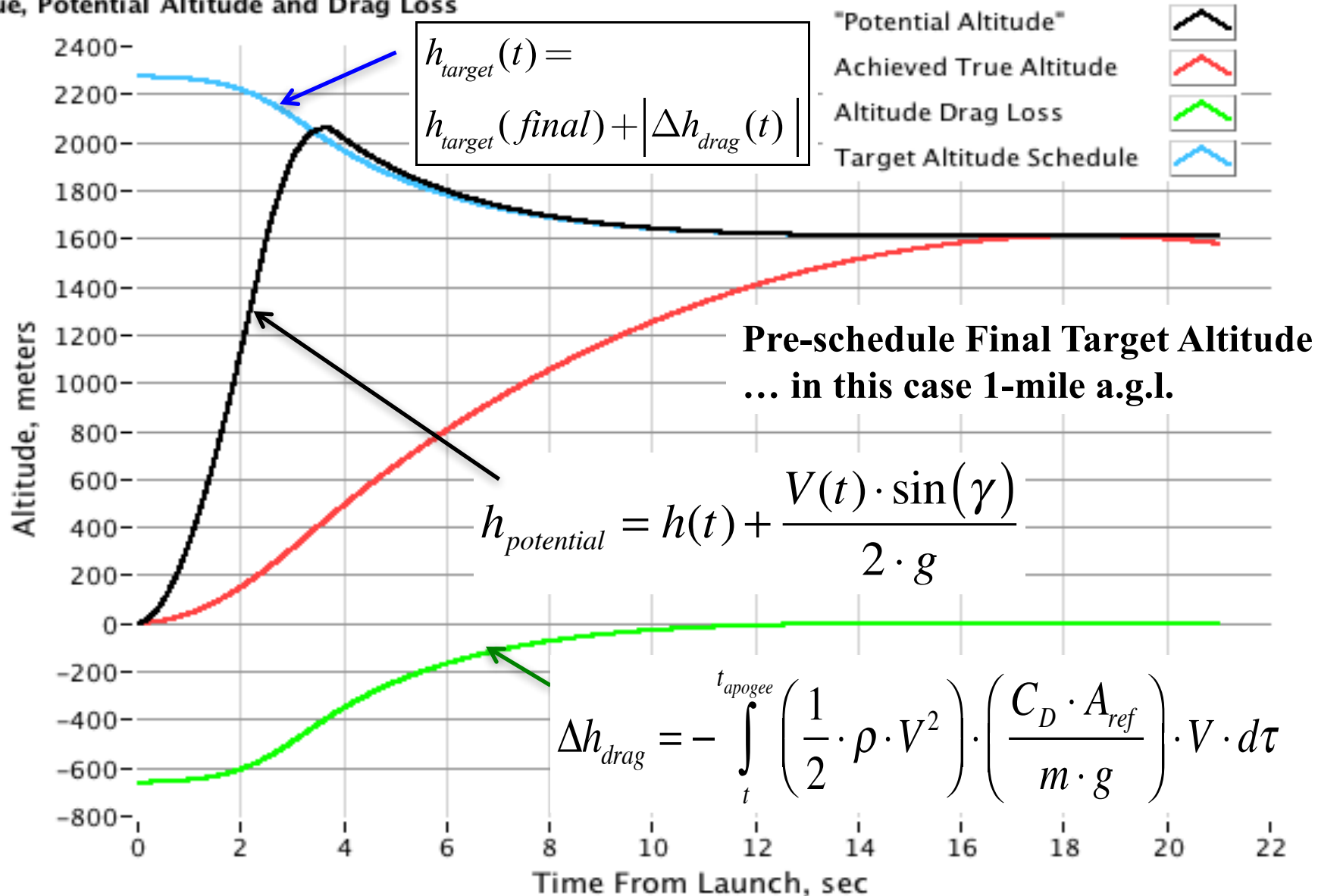
$$h_{target}(t) =$$
$$h_{target}(final) + \left| \Delta h_{drag}(t) \right|$$

"Potential Altitude"
Achieved True Altitude
Altitude Drag Loss
Target Altitude Schedule

Pre-schedule Final Target Altitude … in this case 1-mile a.g.l.

$$h_{potential} = h(t) + \frac{V(t) \cdot \sin(\gamma)}{2 \cdot g}$$

$$\Delta h_{drag} = - \int_{t}^{t_{apogee}} \left(\frac{1}{2} \cdot \rho \cdot V^2 \right) \cdot \left(\frac{C_D \cdot A_{ref}}{m \cdot g} \right) \cdot V \cdot d\tau$$

Time From Launch, sec

Altitude, meters

True, Potential Altitude and Drag Loss

Legend:
- "Potential Altitude"
- Achieved True Altitude
- Altitude Drag Loss
- Target Altitude Schedule

$$h_{target}(t) =$$
$$h_{target}(final) + \left| \Delta h_{drag}(t) \right|$$

Pre-schedule target
Altitude … at 1609.35 m

Low Energy Launch
W/ Thrust
Augmentation

$$h_{potential} = h(t) + \frac{V(t) \cdot \sin(\gamma)}{2 \cdot g}$$

$$\Delta h_{drag} = - \int_{t}^{t_{apogee}} \left(\frac{1}{2} \cdot \rho \cdot V^2 \right) \cdot \left(\frac{C_D \cdot A_{ref}}{m \cdot g} \right) \cdot V \cdot d\tau$$

38
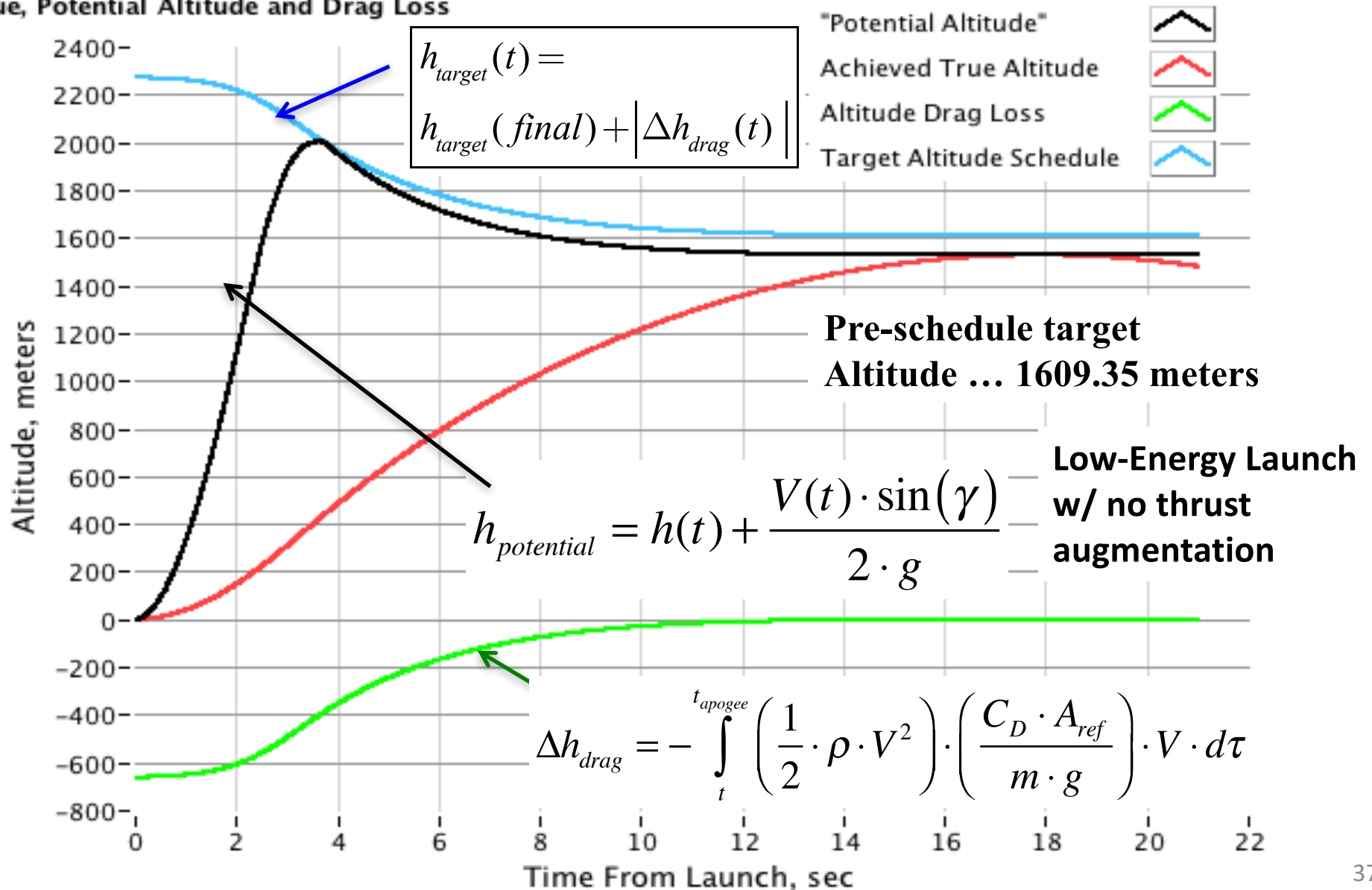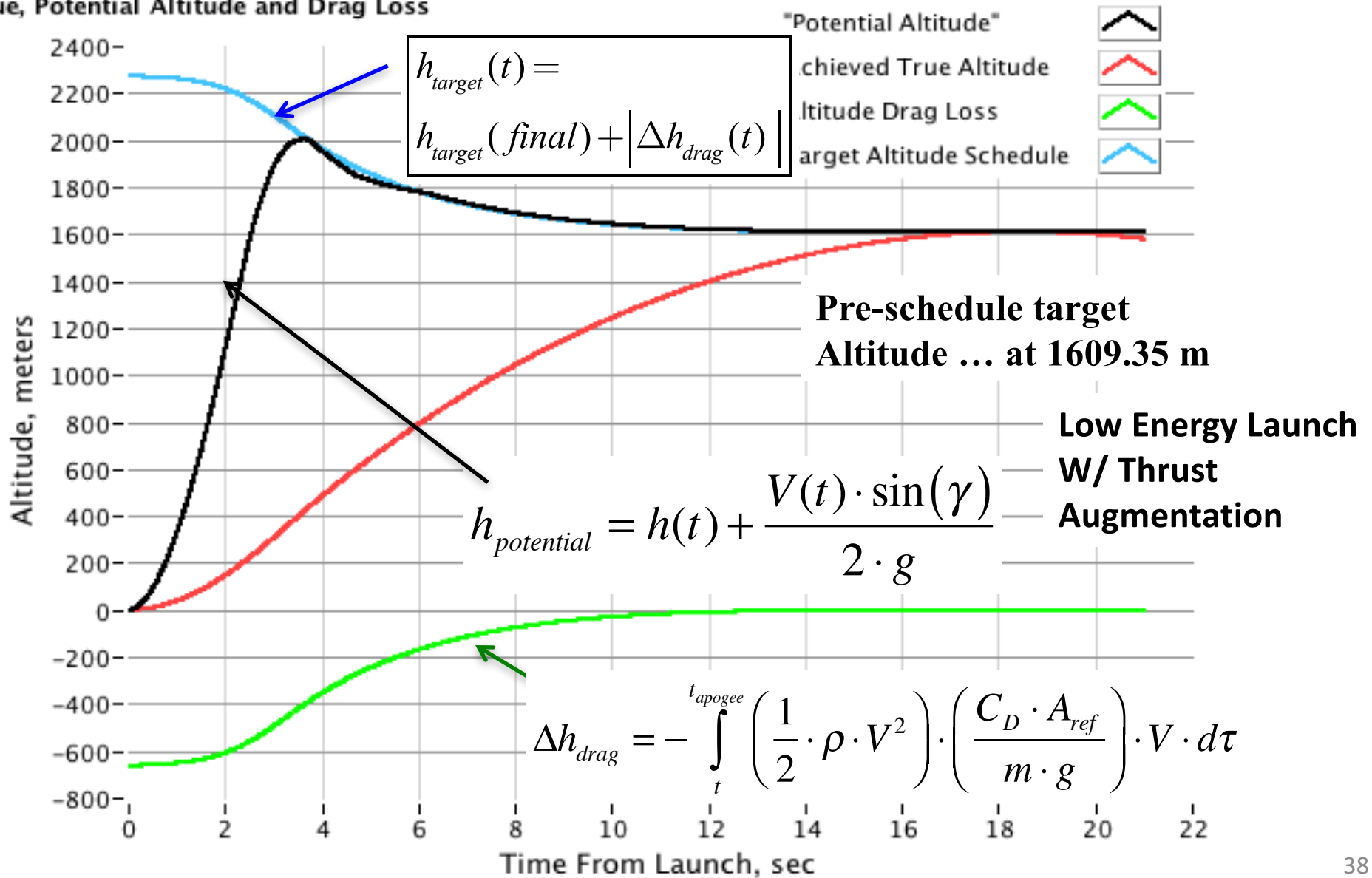
Accounting for Drag Losses
In Potential Altitude [6]

Thrust

Augmentation thrust
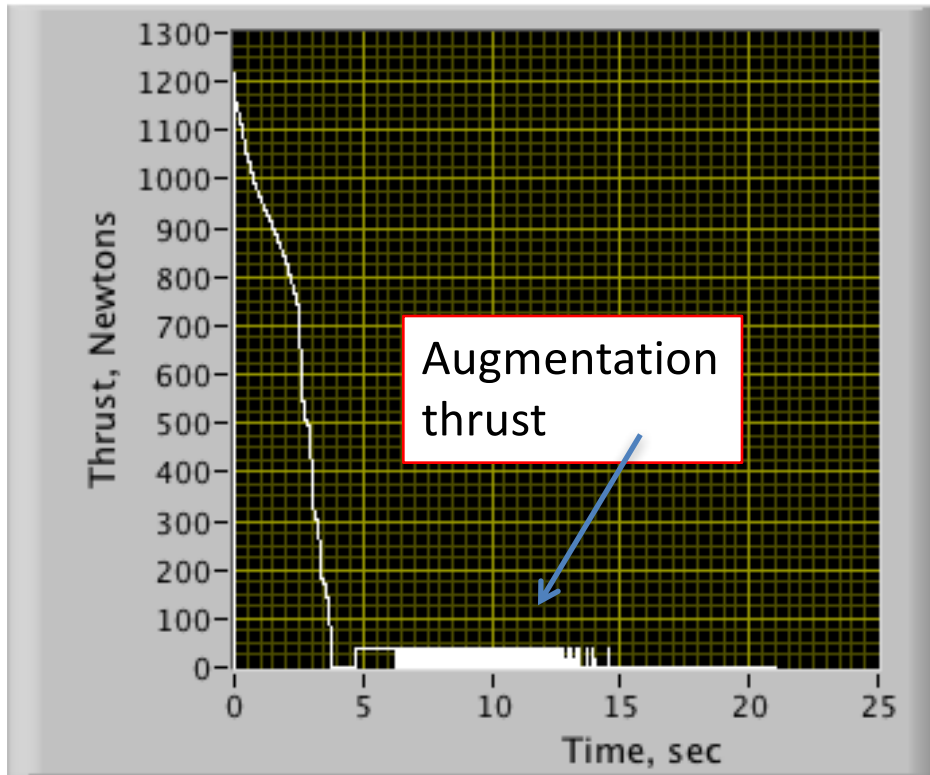
Acceleration
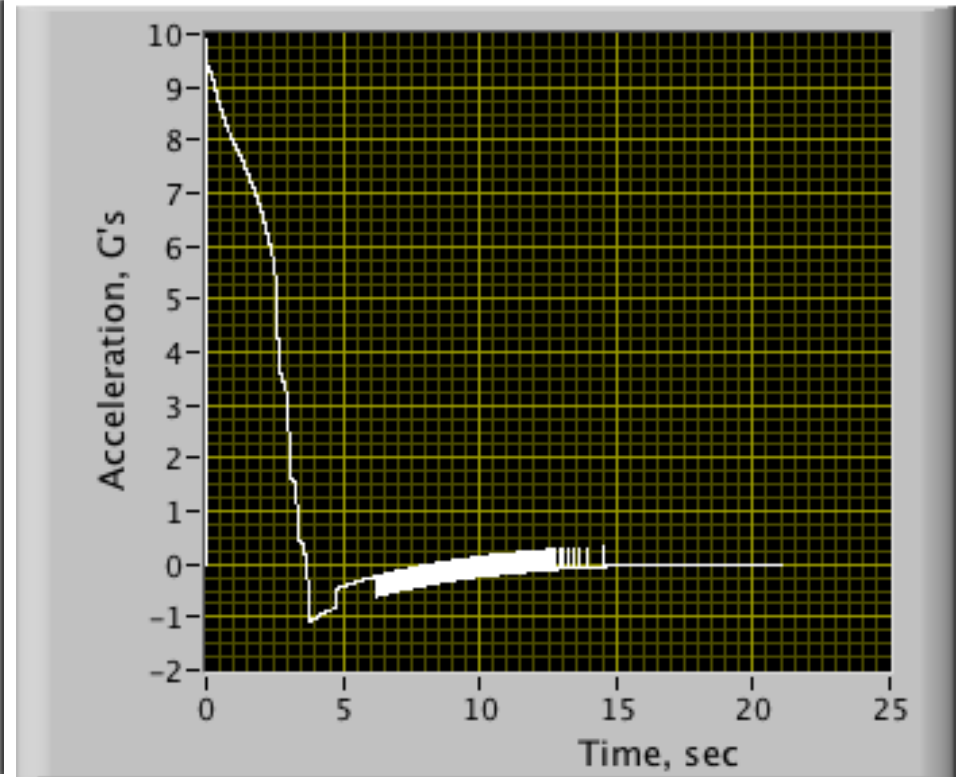
Energy Management w/ Target Envelope (1)
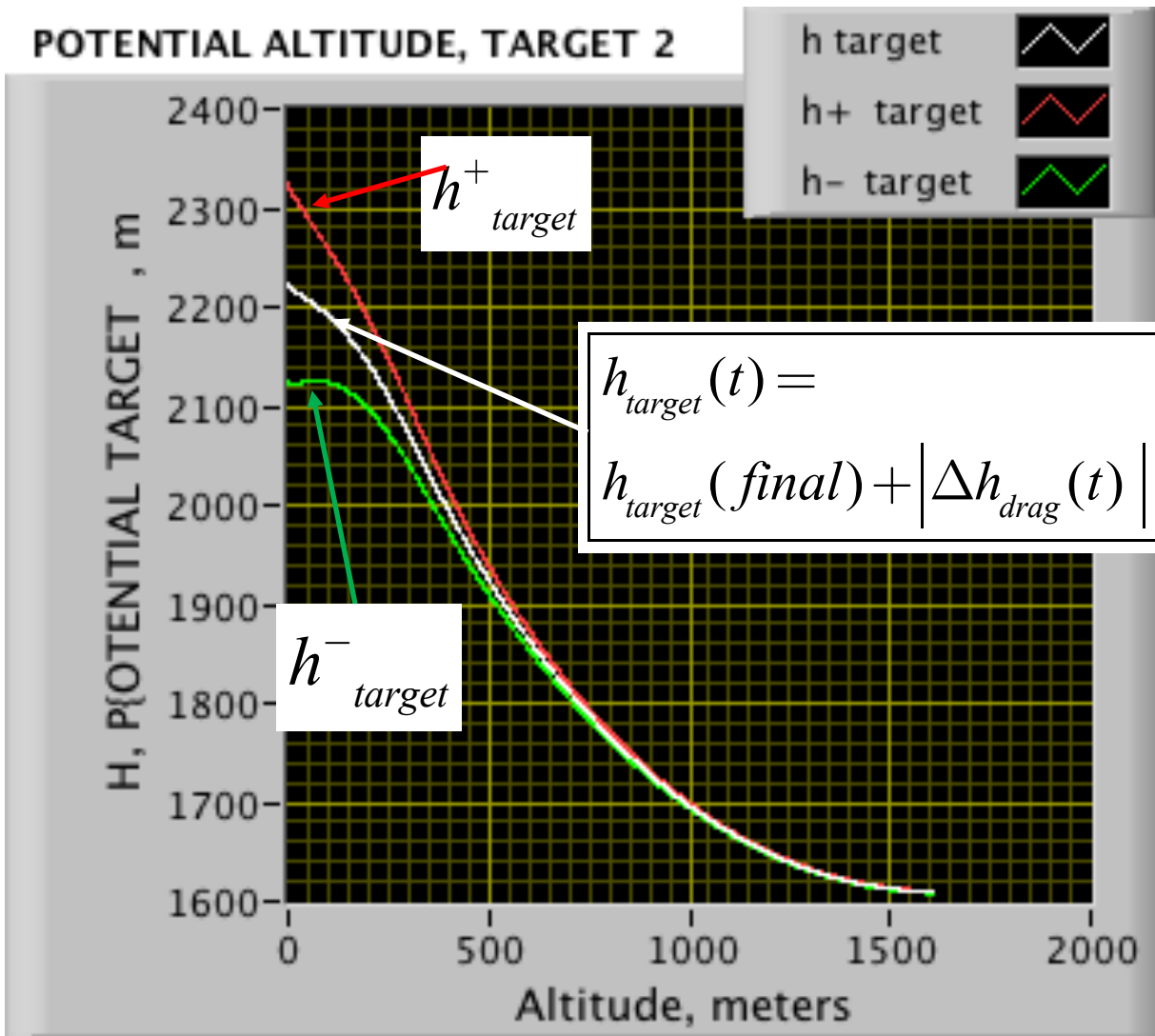
POTENTIAL ALTITUDE, TARGET 2

h target		⟋⟍
h+ target		⟋⟍
h− target		⟋⟍

h^+_{target}

h^-_{target}

$$h_{target}(t) = h_{target}(final) + \left| \Delta h_{drag}(t) \right|$$

• **Enveloping allows for final position error tolerance while significantly reducing required control activity**

$$h^+_{target}(t) = h_{target}(t) + \delta h_h$$

$$h^-_{target}(t) = h_{target}(t) - \delta h_h$$

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Energy Management w/ Target Envelope (2)

- Use first order decay as a function of altitude for envelope…

$$h^{\pm}_{target} = h_{\text{target}} \pm \delta h_t \rightarrow \textit{first order decay} \ .... \ \dot{\delta h_t} + \frac{1}{\tau_h} \cdot \delta h_t = \frac{1}{\tau} \delta h_{\text{max}}$$

$$\textit{multiply by} \rightarrow e^{\frac{h}{\tau_h}}$$

$$\rightarrow e^{\frac{h}{\tau_h}} \cdot \left(\dot{\delta h_t} + \frac{1}{\tau_h} \cdot \delta h \right) = \frac{e^{\frac{h}{\tau_h}}}{\tau_h} \cdot \delta h_{\text{max}} \rightarrow \frac{d}{dt} \left(e^{\frac{h}{\tau_h}} \cdot \delta h \right) = \frac{e^{\frac{h}{\tau_h}}}{\tau_h} \cdot \delta h_{\text{max}}$$

- Integrate from launch to $h(t)$ …

$$\int_{t(0)}^{t} \frac{d}{ds} \left(e^{\frac{h}{\tau_h}} \cdot \delta h \right) \cdot ds = \int_{0}^{h(t)} \frac{\delta h_{\text{max}}}{\tau_h} \cdot e^{\frac{h}{\tau_h}} dh \rightarrow \boxed{e^{\frac{h(t)}{\tau_h}} \cdot \delta h(t) - e^{\frac{h(0)}{\tau_h}} \cdot \delta h(0) = \frac{\delta h_{\text{max}}}{\tau_h} \cdot \left(e^{\frac{h(t)}{\tau_h}} \cdot \tau_h - e^{\frac{h(0)}{\tau_h}} \cdot \tau_h \right)}$$

Energy Management Target Envelope (3)

- Assuming $h(0) = 0$,

$$e^{\frac{h(t)}{\tau_h}} \cdot \delta h(t) - \delta h(0) = \delta h_{max} \cdot \left(e^{\frac{h(t)}{\tau_h}} - 1 \right)$$

and Multiply thru by $e^{-\frac{h(t)}{\tau_h}}$

$\rightarrow$

$$\delta h(t) = e^{-\frac{h(t)}{\tau_h}} \cdot \delta h(0) + \delta h_{max} \cdot \left(1 - e^{-\frac{h(t)}{\tau_h}} \right)$$

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Collected Target Altitude Envelope

$$\delta h_{(h)} = \delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}} \right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0$$

$$h_{target}^+ = h_{target} + \left(\delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}} \right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0 \right)$$

$$h_{target}^+ = h_{target} - \left(\delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}} \right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0 \right)$$

$$\rightarrow \begin{array}{l} \delta h_{\max} \rightarrow \textit{sets final envelope error} \\ \delta h_0 \rightarrow \textit{sets initial envelope error} \\ \tau_h \rightarrow \textit{sets envelope decay rate} \end{array}$$

Example Envelopes (1)

POTENTIAL ALTITUDE, TARGET 2

Legend:
- h target
- h+ target
- h− target

$$\delta h_{\max} = 1_{meter}$$

$$\delta h_0 = 100_{meters}$$

$$\tau_h = 250_{meters}$$

$$\delta h_{(h)} = \delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}}\right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0$$

$$h_{target}^{+} = h_{target} + \left(\delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}}\right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0\right)$$

$$h_{target}^{+} = h_{target} - \left(\delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}}\right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0\right)$$

$$\rightarrow \begin{vmatrix} \delta h_{\max} \rightarrow sets\ final\ envelope\ error \\ \delta h_0 \rightarrow sets\ initial\ envelope\ error \\ \tau_h \rightarrow sets\ envelope\ decay\ rate \end{vmatrix}$$

Envelope parameters

Delta Altitude decay, meters
250

delta h, max, meters
1

delta h, 0, meters
100

Final Envelope "Error"
3.61342

HPA Propellant Margin, %
67.5646

Example Envelopes (2)

POTENTIAL ALTITUDE, TARGET 2

Legend	
h target	(black)
h+ target	(red)
h- target	(green)

Box on plot:
$$\delta h_{\max} = 0\ _{meter}$$
$$\delta h_0 = 200\ _{meters}$$
$$\tau_h = 400\ _{meters}$$

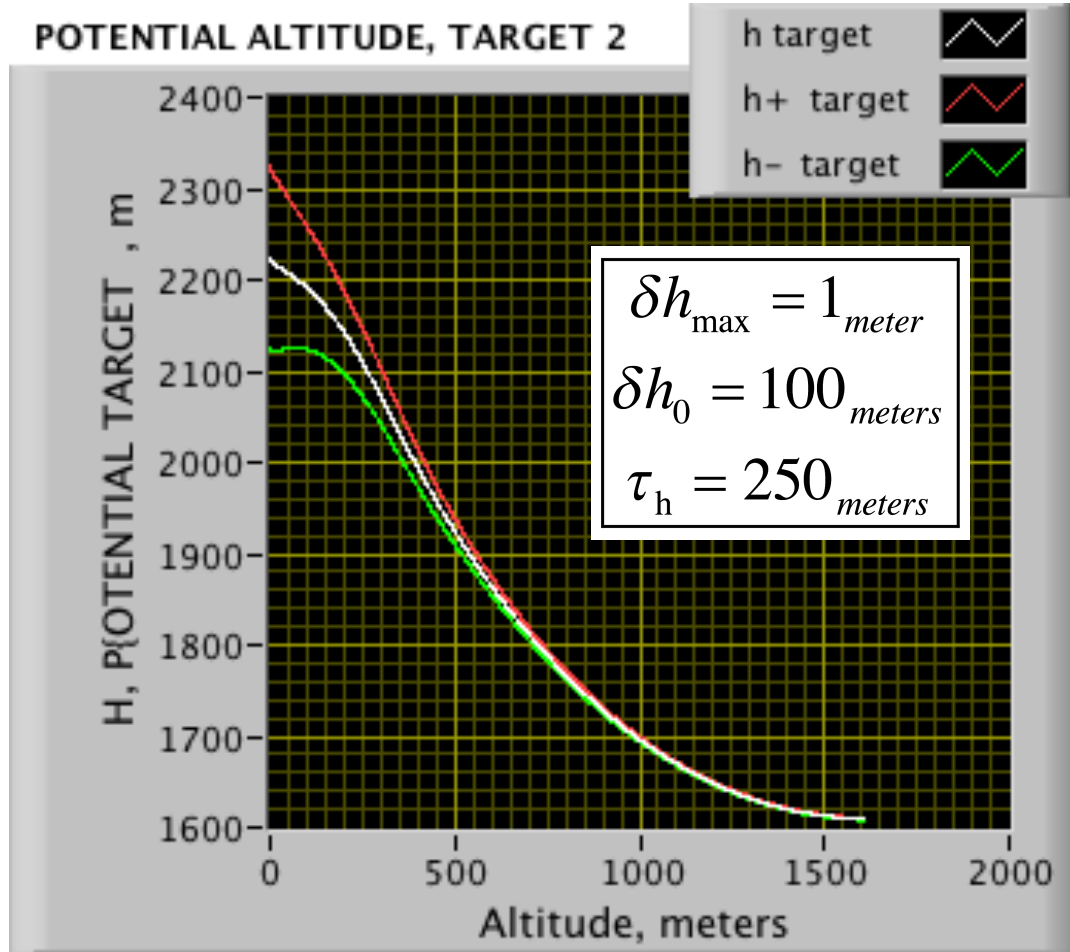Plot axes: H, POTENTIAL TARGET , m (y-axis from 1600 to 2500); Altitude, meters (x-axis 0 to 2000)
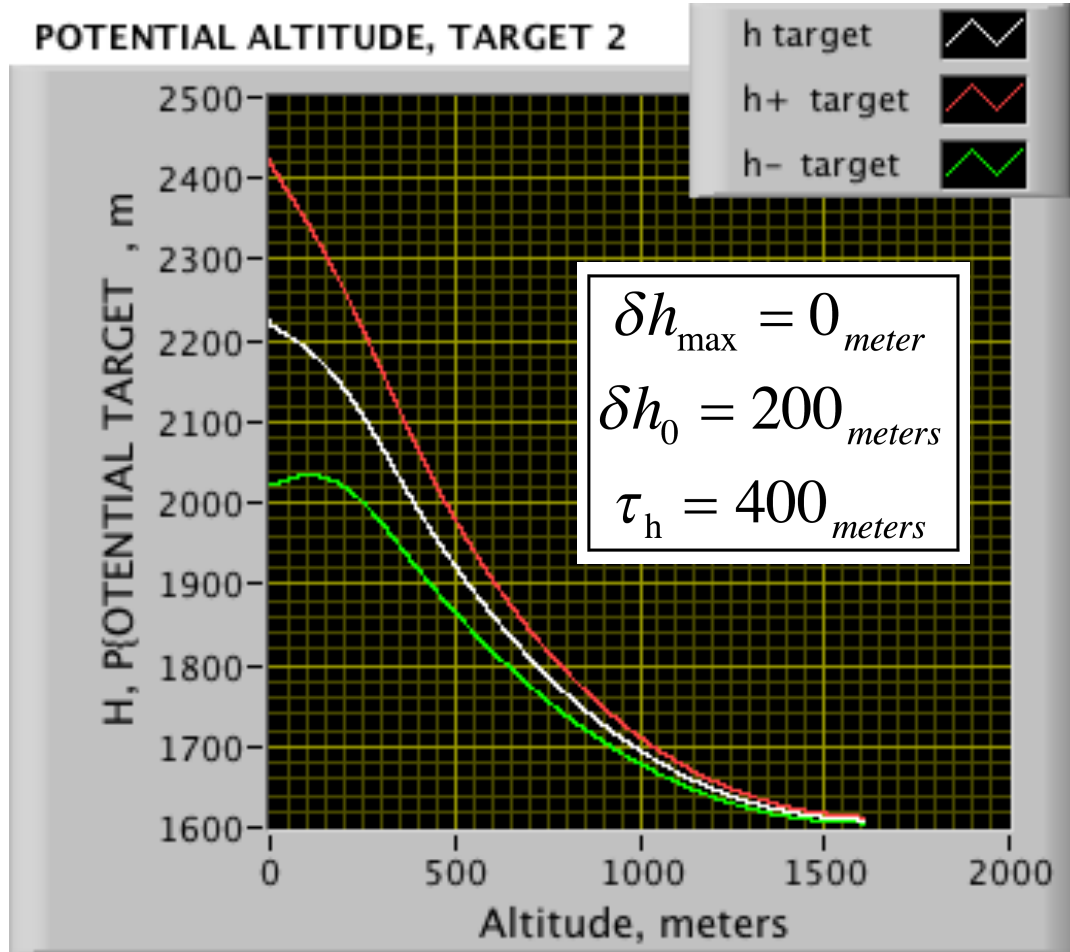
$$\delta h_{(h)} = \delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}} \right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0$$

$$h_{target}^+ = h_{target} + \left(\delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}} \right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0 \right)$$

$$h_{target}^+ = h_{target} - \left(\delta h_{\max} \cdot \left(1 - e^{-\frac{h}{\tau_h}} \right) + e^{-\frac{h}{\tau_h}} \cdot \delta h_0 \right)$$

$\rightarrow$
$\delta h_{\max} \rightarrow$	*sets final envelope error*
$\delta h_0 \rightarrow$	*sets initial envelope error*
$\tau_h \rightarrow$	*sets envelope decay rate*

• Enveloping allows for final position error tolerance while significantly reducing required control activity

Envelope parameters

Delta Altitude decay, meters
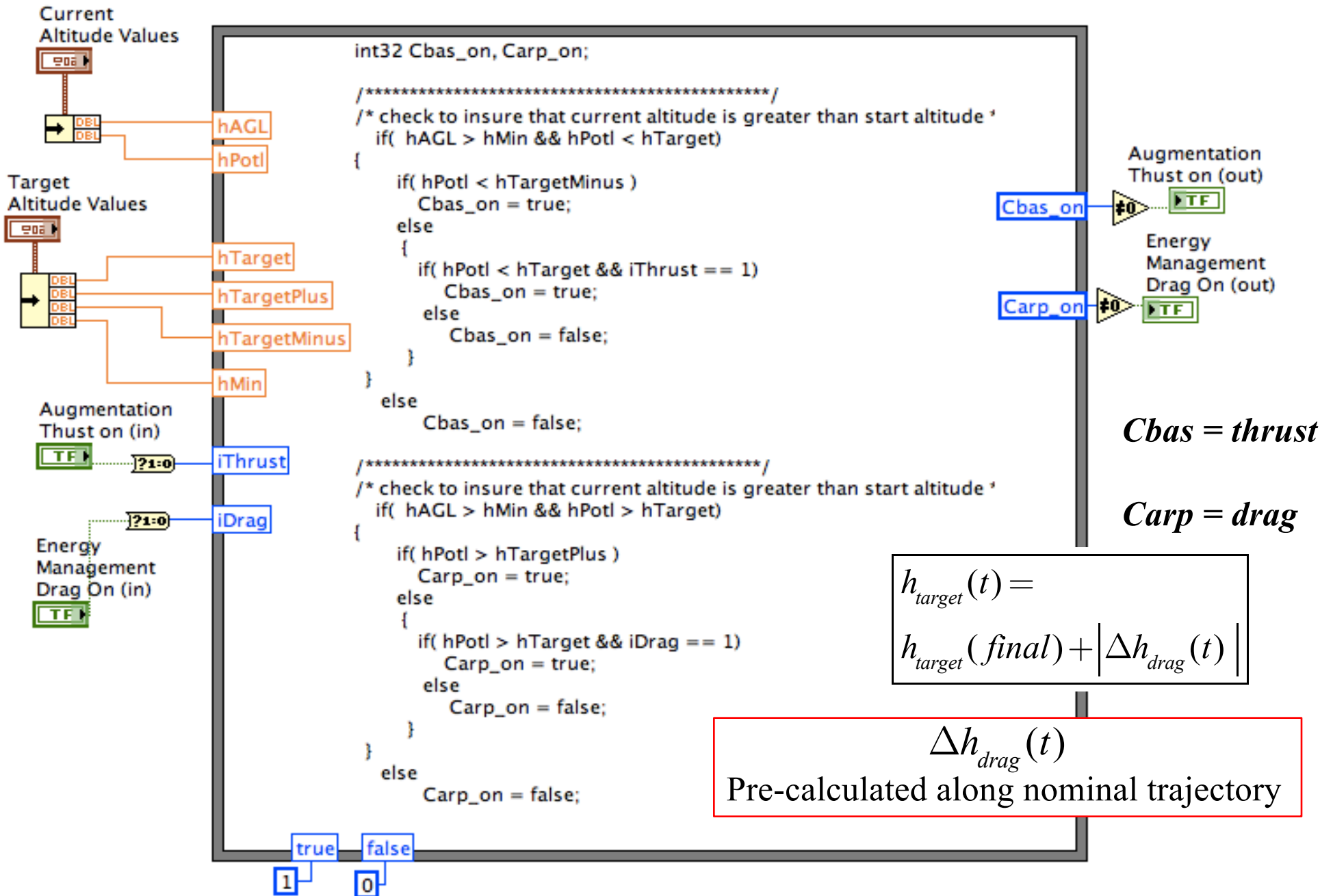400

delta h, max, meters
0

delta h, 0, meters
200

Final Envelope "Error"
3.61342

HPA Propellant Margin, %
72.0604

Energy Management Control Logic

```
int32 Cbas_on, Carp_on;

/***********************************************/
/* check to insure that current altitude is greater than start altitude *
  if( hAGL > hMin && hPotl < hTarget)
  {
      if( hPotl < hTargetMinus )
          Cbas_on = true;
      else
      {
        if( hPotl < hTarget && iThrust == 1)
            Cbas_on = true;
        else
            Cbas_on = false;
      }
  }
    else
        Cbas_on = false;

/***********************************************/
/* check to insure that current altitude is greater than start altitude *
  if( hAGL > hMin && hPotl > hTarget)
  {
      if( hPotl > hTargetPlus )
          Carp_on = true;
      else
      {
        if( hPotl > hTarget && iDrag == 1)
            Carp_on = true;
        else
            Carp_on = false;
      }
  }
    else
        Carp_on = false;
```

Current Altitude Values

Target Altitude Values

Augmentation Thust on (in)

Energy Management Drag On (in)

hAGL

hPotl

hTarget

hTargetPlus

hTargetMinus

hMin

iThrust

iDrag

Augmentation Thust on (out)

Energy Management Drag On (out)

Cbas_on

Carp_on

true   false

1   0

Cbas = thrust

Carp = drag

$$h_{target}(t) =$$
$$h_{target}(final) + \left| \Delta h_{drag}(t) \right|$$

$$\Delta h_{drag}(t)$$
Pre-calculated along nominal trajectory

Questions??