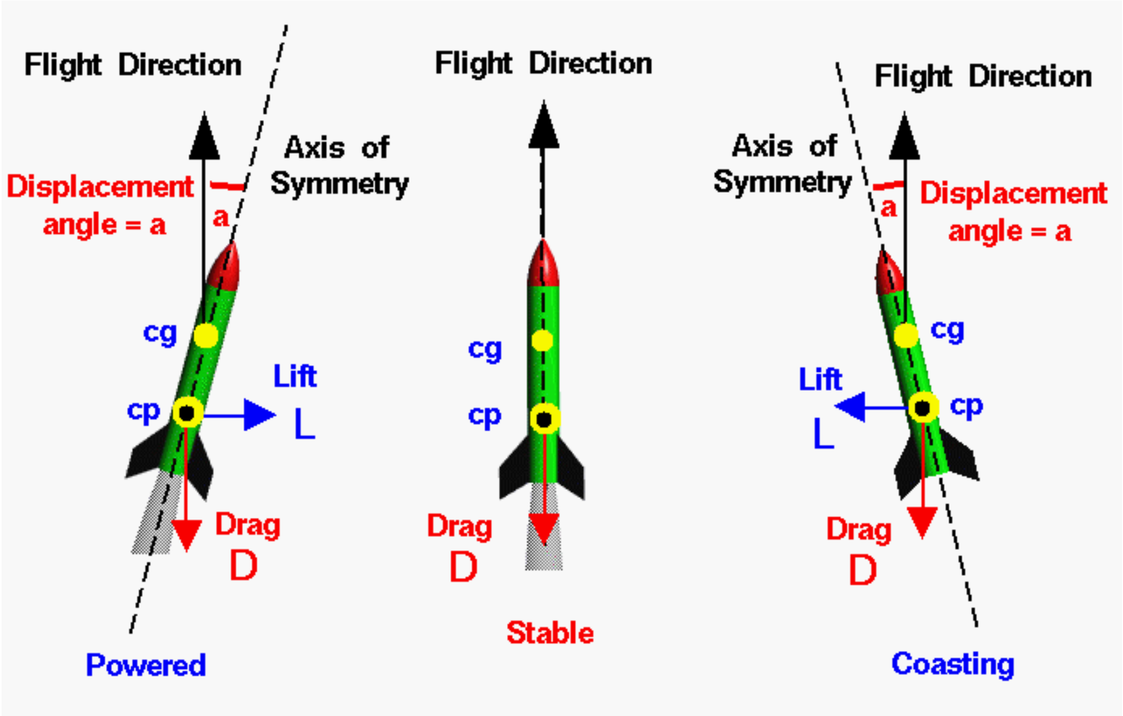
Section 7.3

Rocket Science Review 103: Estimating the Launch Vehicle Drag Coefficient

Newton's Laws as Applied to "Rocket Science"

... its not just a job ... its an adventure

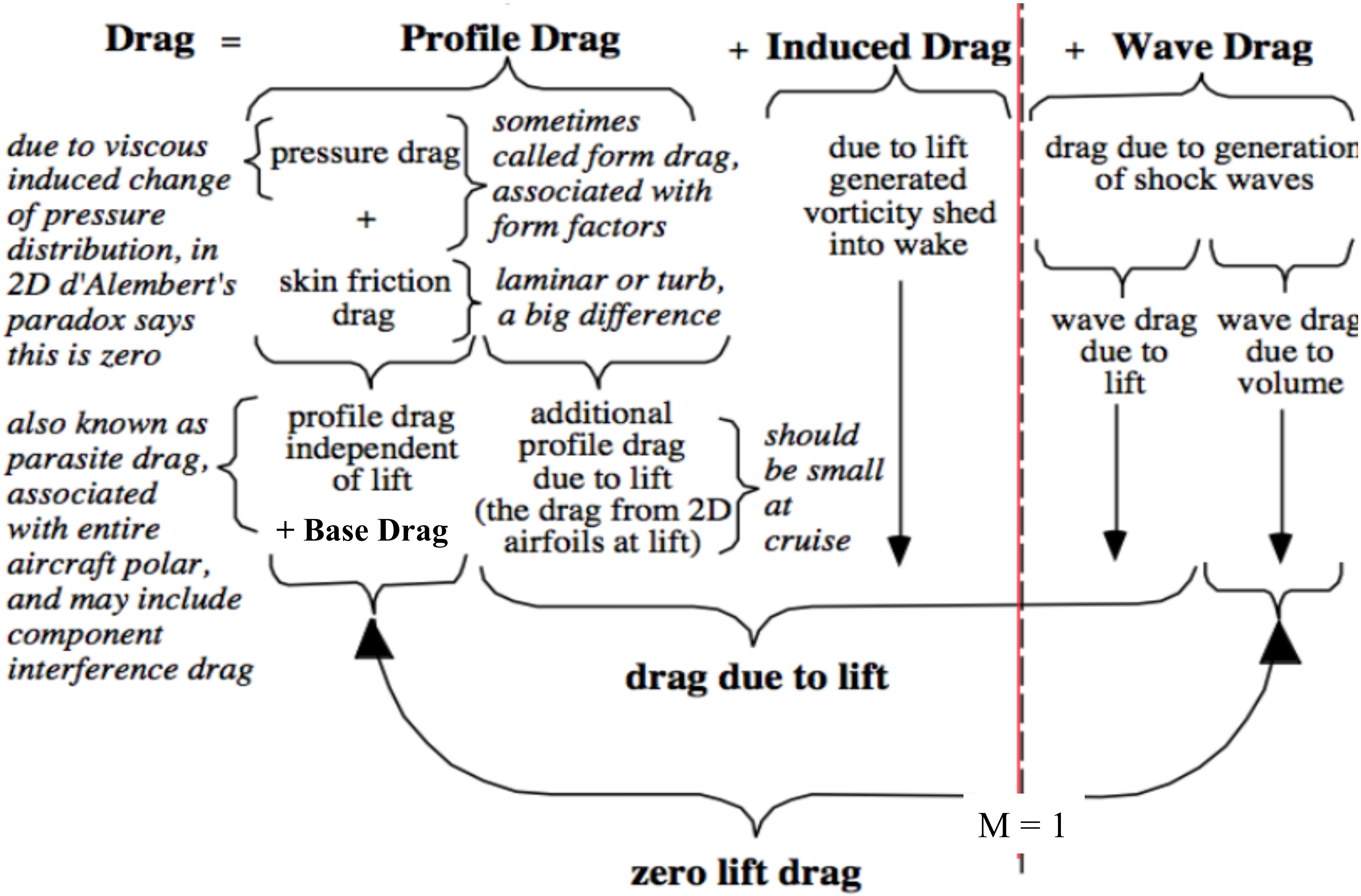
RS 101: Summary
External Forces Acting on Rocket



- **Lift – acts perpendicular to flight path *(non-conservative)***
- **Drag – acts along flight path *(non-conservative)***
- **Thrust – acts along longitudinal axis of rocket *(non-conservative)***
- **Gravity – acts downward *(conservative)***

Because lift acts perpendicular to flight path, drag is the primary dissipative force acting on airframe

Types of Aerodynamic Drag



Drag = **Profile Drag** + **Induced Drag** + **Wave Drag**

due to viscous induced change of pressure distribution, in 2D d'Alembert's paradox says this is zero

pressure drag

+

skin friction drag

sometimes called form drag, associated with form factors

laminar or turb, a big difference

due to lift generated vorticity shed into wake

drag due to generation of shock waves

wave drag due to lift

wave drag due to volume

also known as parasite drag, associated with entire aircraft polar, and may include component interference drag

profile drag independent of lift

+ Base Drag

additional profile drag due to lift (the drag from 2D airfoils at lift)

should be small at cruise

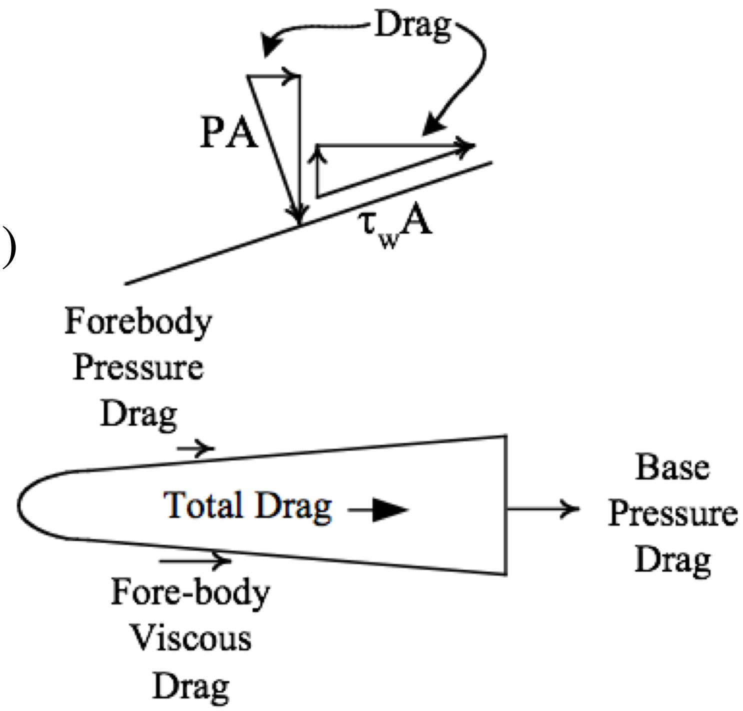
drag due to lift

M = 1

zero lift drag

Types of Aerodynamic Drag (2)

Drag Mechanisms Acting on Launch Vehicles (*LVs*)

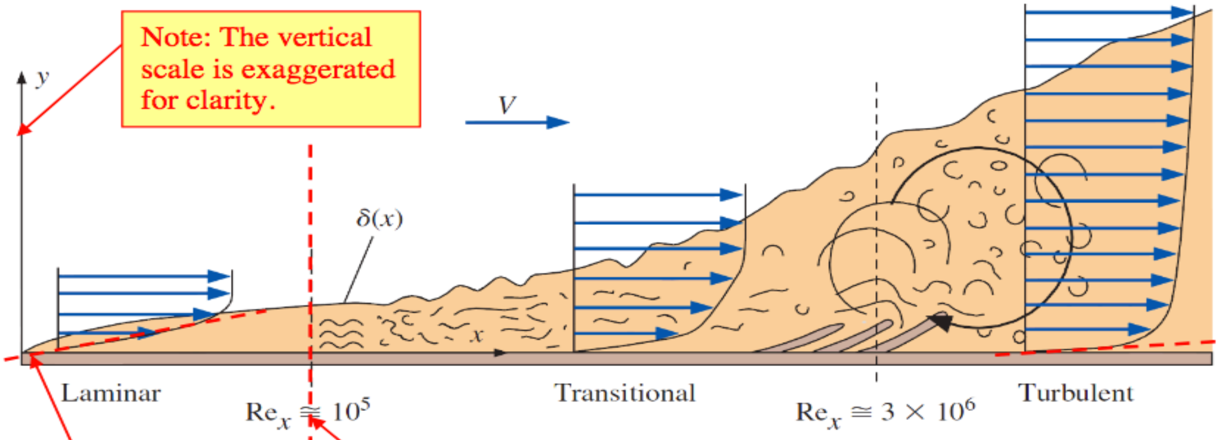


- Subsonic
  - Viscous drag
    - ***Wetted Area Skin Friction***
    - Flow Separation (*small for LVs*)
  - Pressure drag (form drag)
    - ***Forebody***
    - Interference (fin roots)
    - ***Base***
  - Induced (lift drag, *small for LVs*)
- Supersonic
  - Wave drag
  - Compressive ***drag due to lift***
- "Dither" Drag due to unsteady α modulation
- Total drag

Skin Friction Drag

$$C_F = \frac{\iint\limits_{surf} \tau_{wall} \cdot dA}{\bar{q} \cdot A_{wet}}$$

Flow Momentum lost due to surface viscosity



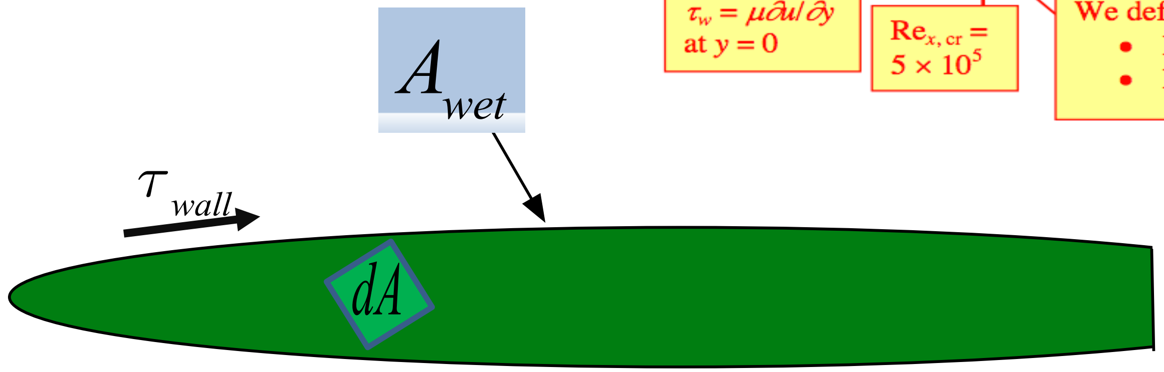
Note: The vertical scale is exaggerated for clarity.

$\tau_w = \mu \partial u / \partial y$ at $y = 0$

$Re_{x,\,cr} = 5 \times 10^5$

We define the engineering critical Reynolds number:

- $Re_x < Re_{x,\,cr}$ BL most likely laminar
- $Re_x > Re_{x,\,cr}$ BL most likely turbulent



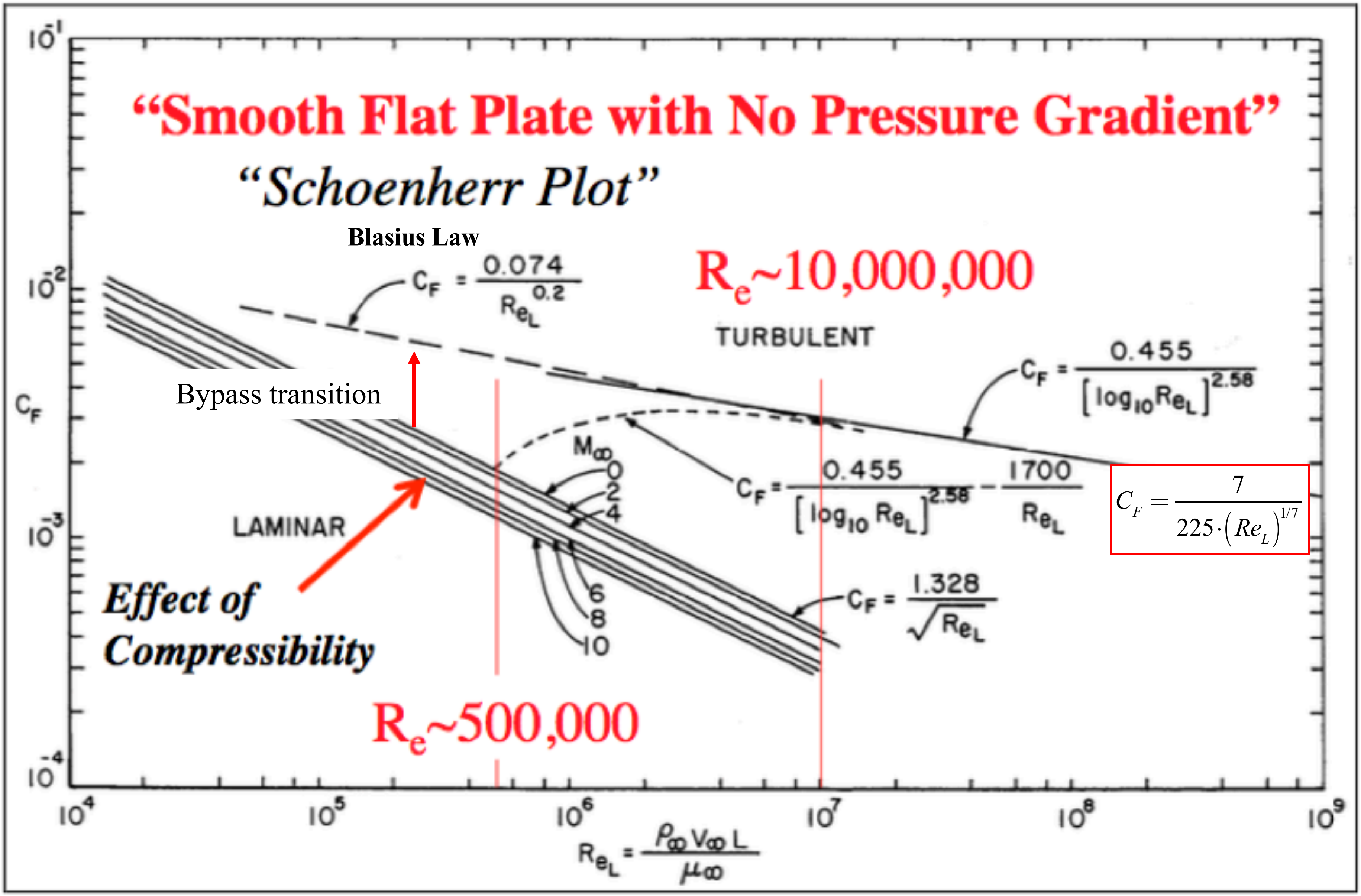
$$C_{D_F} = \frac{A_{wet}}{A_{ref}} \cdot C_F \rightarrow$$

C_F = skin friction coefficient

A_{wet} = forebody wetted surface area

A_{ref} = vehicle reference area, $\frac{\pi}{4} \cdot D_{max}^2$

2-D Skin Friction Drag



"Smooth Flat Plate with No Pressure Gradient"

"Schoenherr Plot"

Blasius Law

R_e~10,000,000

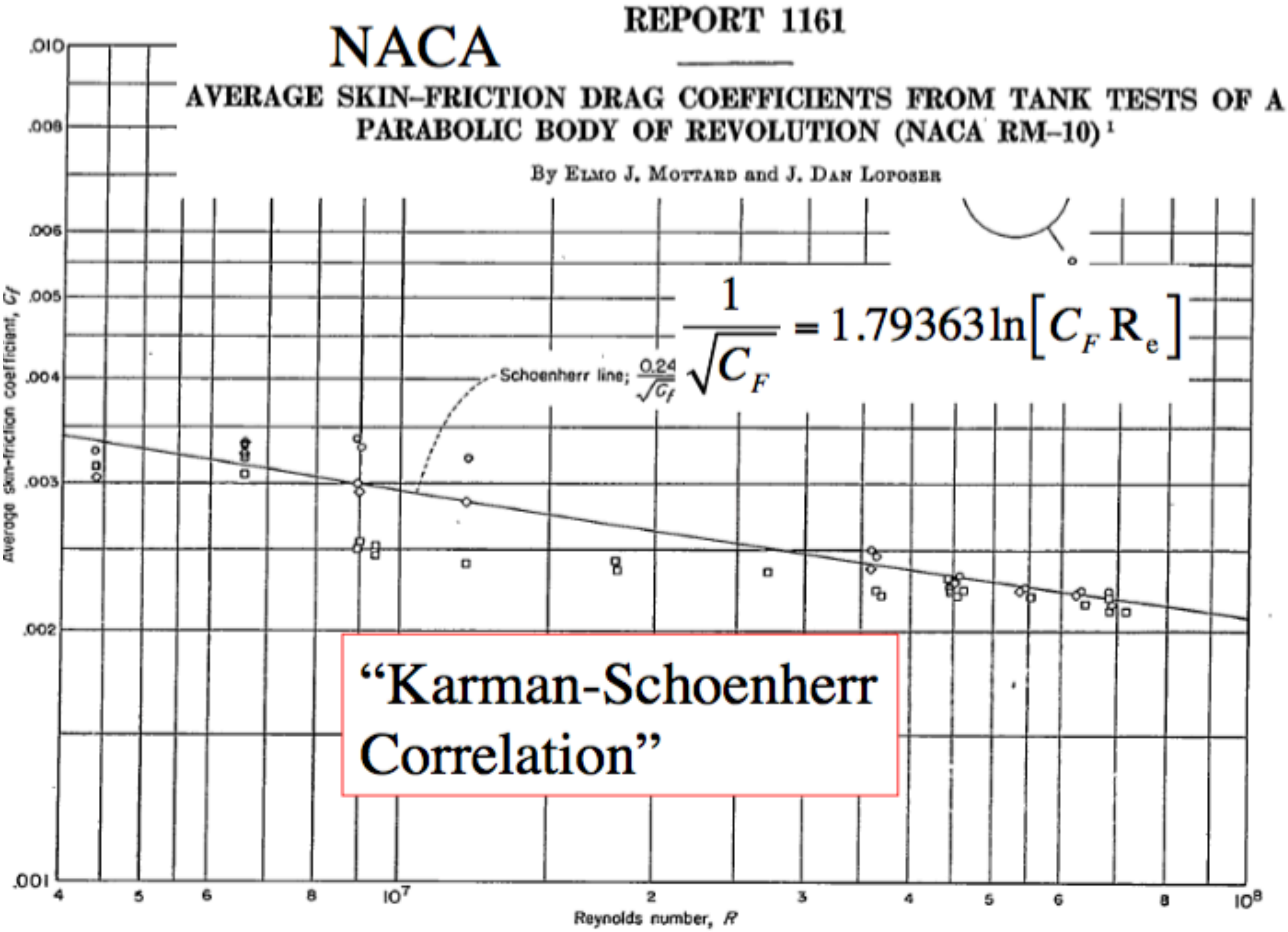
Bypass transition

$$C_F = \frac{7}{225 \cdot \left(Re_L\right)^{1/7}}$$

Effect of Compressibility

R_e~500,000

Axisymmetric 3-D Skin Friction Drag



NACA

REPORT 1161

AVERAGE SKIN-FRICTION DRAG COEFFICIENTS FROM TANK TESTS OF A PARABOLIC BODY OF REVOLUTION (NACA RM-10)[1]

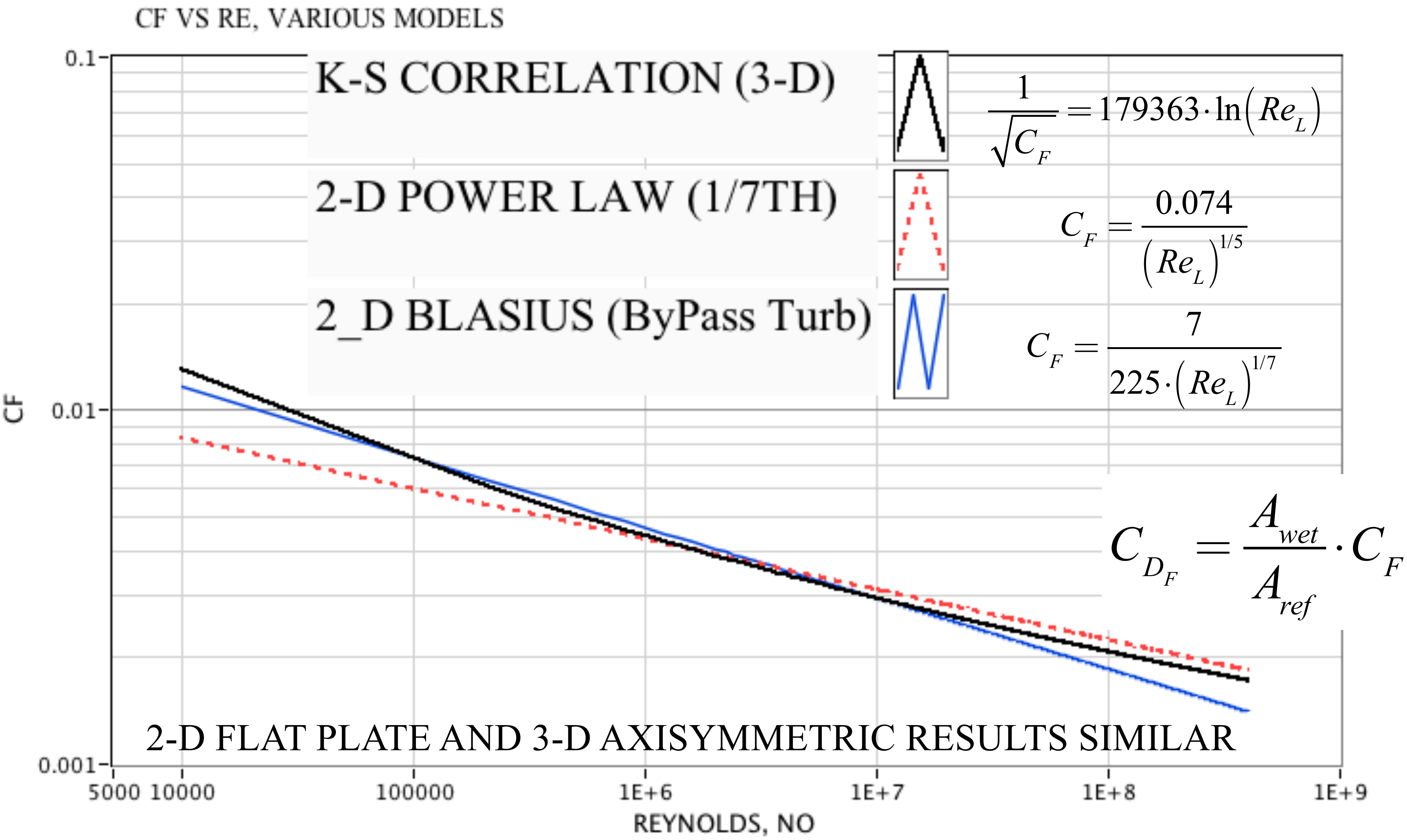
By Elmo J. Mottard and J. Dan Loposer

$$\frac{1}{\sqrt{C_F}} = 1.79363\ln\left[C_F \mathrm{R_e}\right]$$

"Karman-Schoenherr Correlation"

Figure 6.—Variation of average skin-friction coefficient, at the three measurement stations, with Reynolds number.

Skin Friction Model Comparisons



$$\frac{1}{\sqrt{C_F}} = 179363 \cdot \ln\left(Re_L\right)$$

$$C_F = \frac{0.074}{\left(Re_L\right)^{1/5}}$$

$$C_F = \frac{7}{225 \cdot \left(Re_L\right)^{1/7}}$$

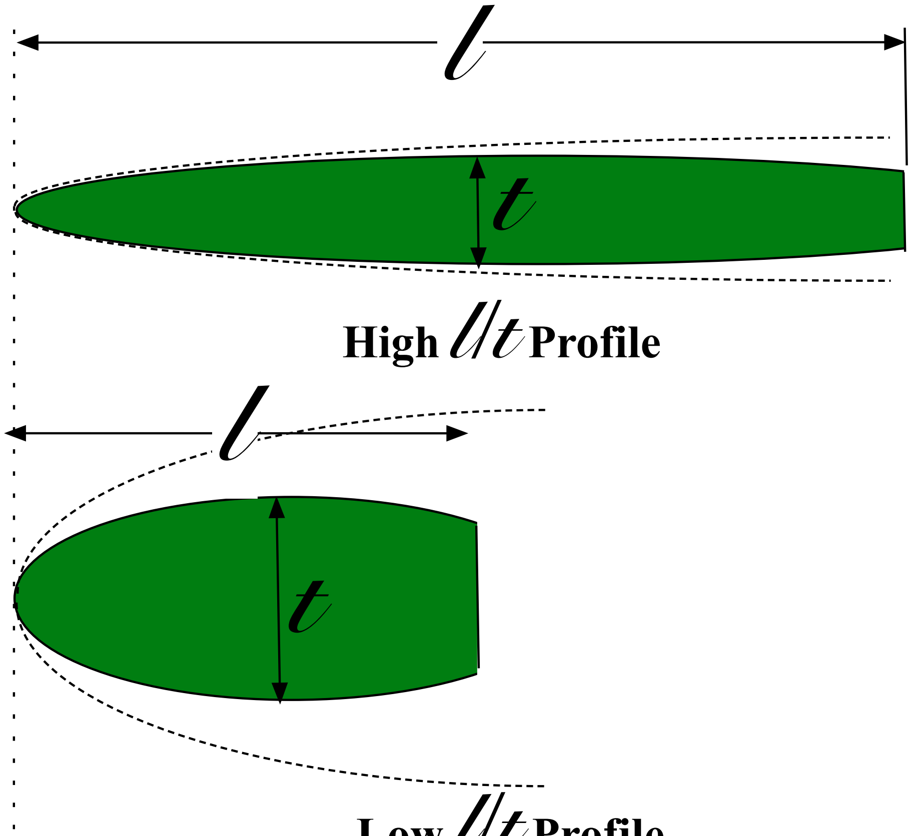
$$C_{D_F} = \frac{A_{wet}}{A_{ref}} \cdot C_F$$

2-D FLAT PLATE AND 3-D AXISYMMETRIC RESULTS SIMILAR

Forebody Form (Pressure) Drag

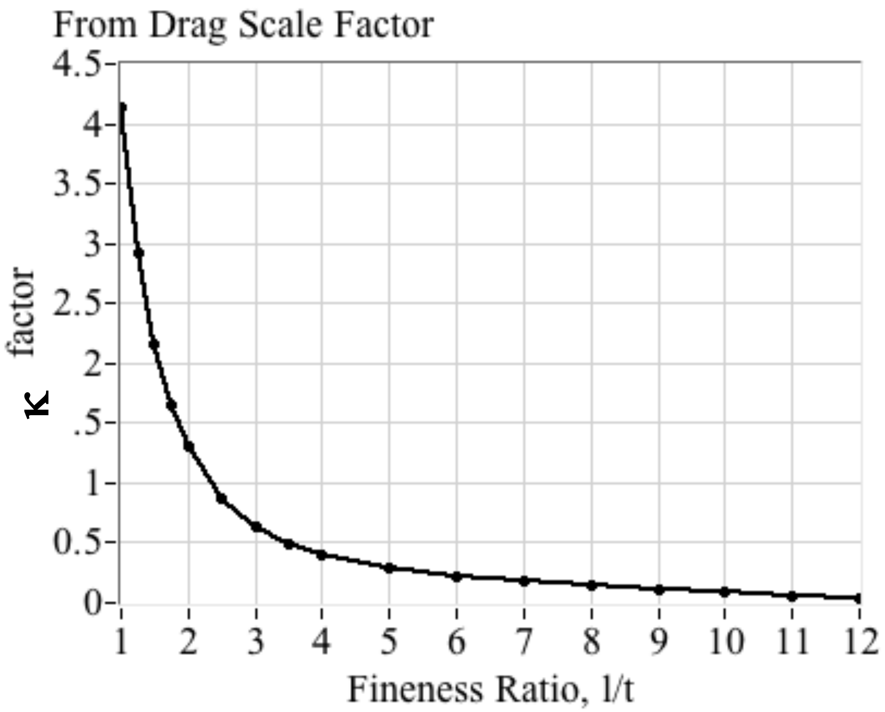
- Subsonic form drag directly related to boundary layer development along body.
- Knowing friction drag, subsonic form drag (C_{D_0}) correlated with skin friction coefficient (C_F) and body fineness ratio (L/D)

$$C_{D_0} = \kappa \cdot \left(\frac{A_{wet}}{A_{ref}} \cdot C_F \right)$$



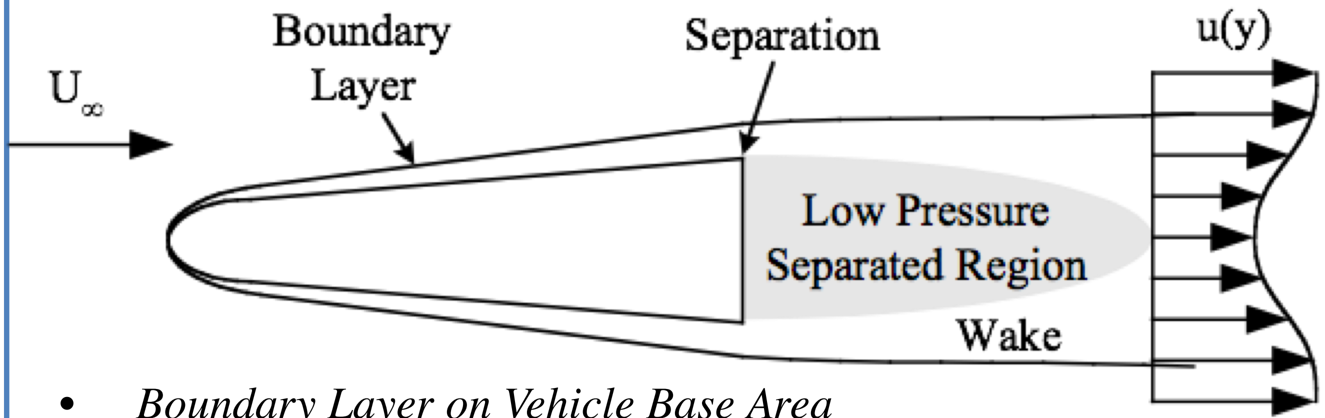
High l/t Profile

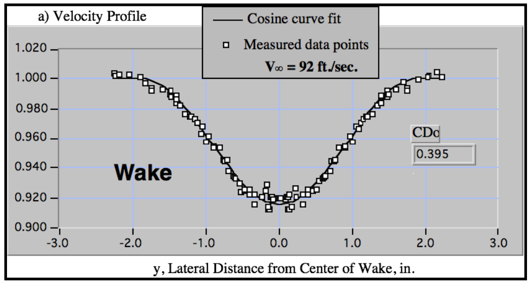
Low l/t Profile



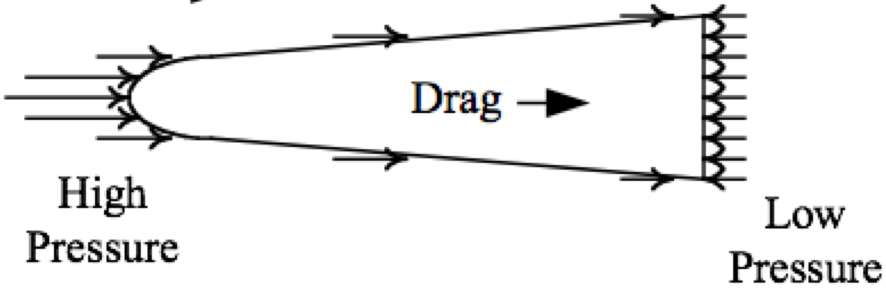
Drew and Jenn, "Pressure Drag Calculations on Axisymmetric Bodies of Arbitrary Mold Line," AIAA 90-0280, 28th Aerospace Sciences Meeting, 8-11 Jan., 1990, Reno NV.

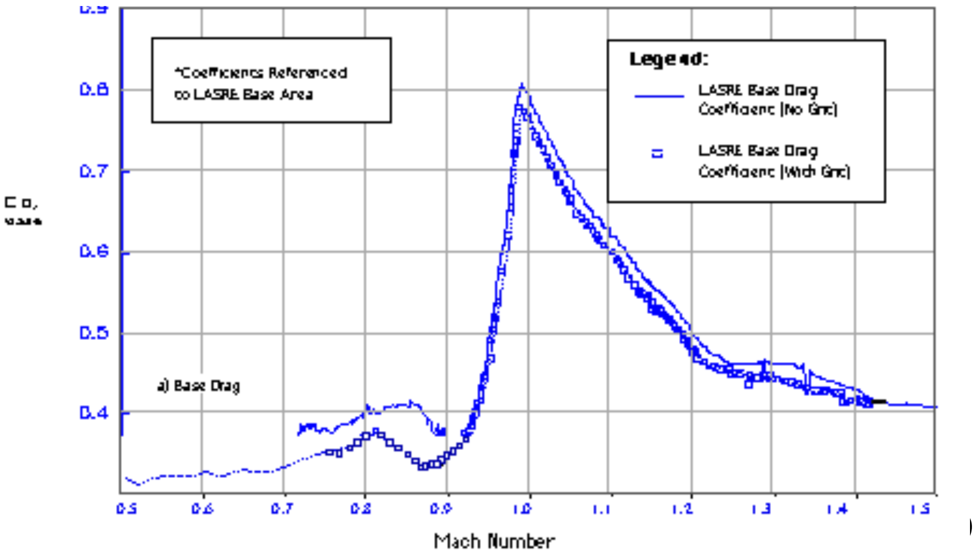
Base Drag



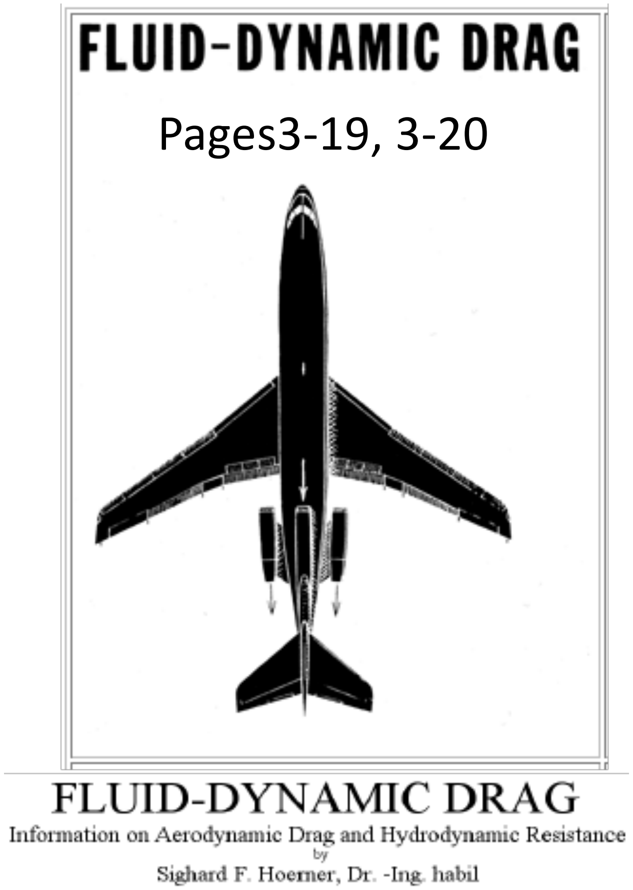


- *Boundary Layer on Vehicle Base Area Separates*
- *Low Pressure Separated Region Forms*
- *Low Pressure Causes a Large net Pressure Difference*
- ***Especially significant on Launch Vehicle after motor burnout***





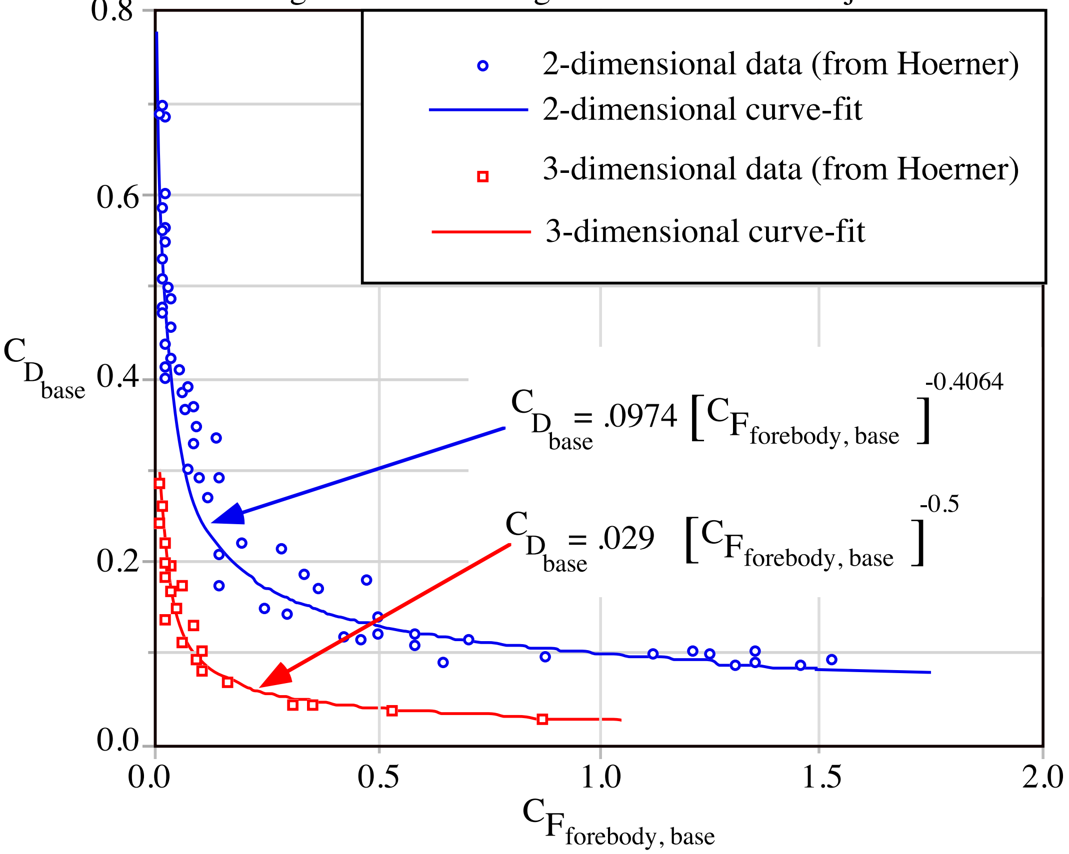
Base Drag (2)


FLUID-DYNAMIC DRAG

Pages3-19, 3-20

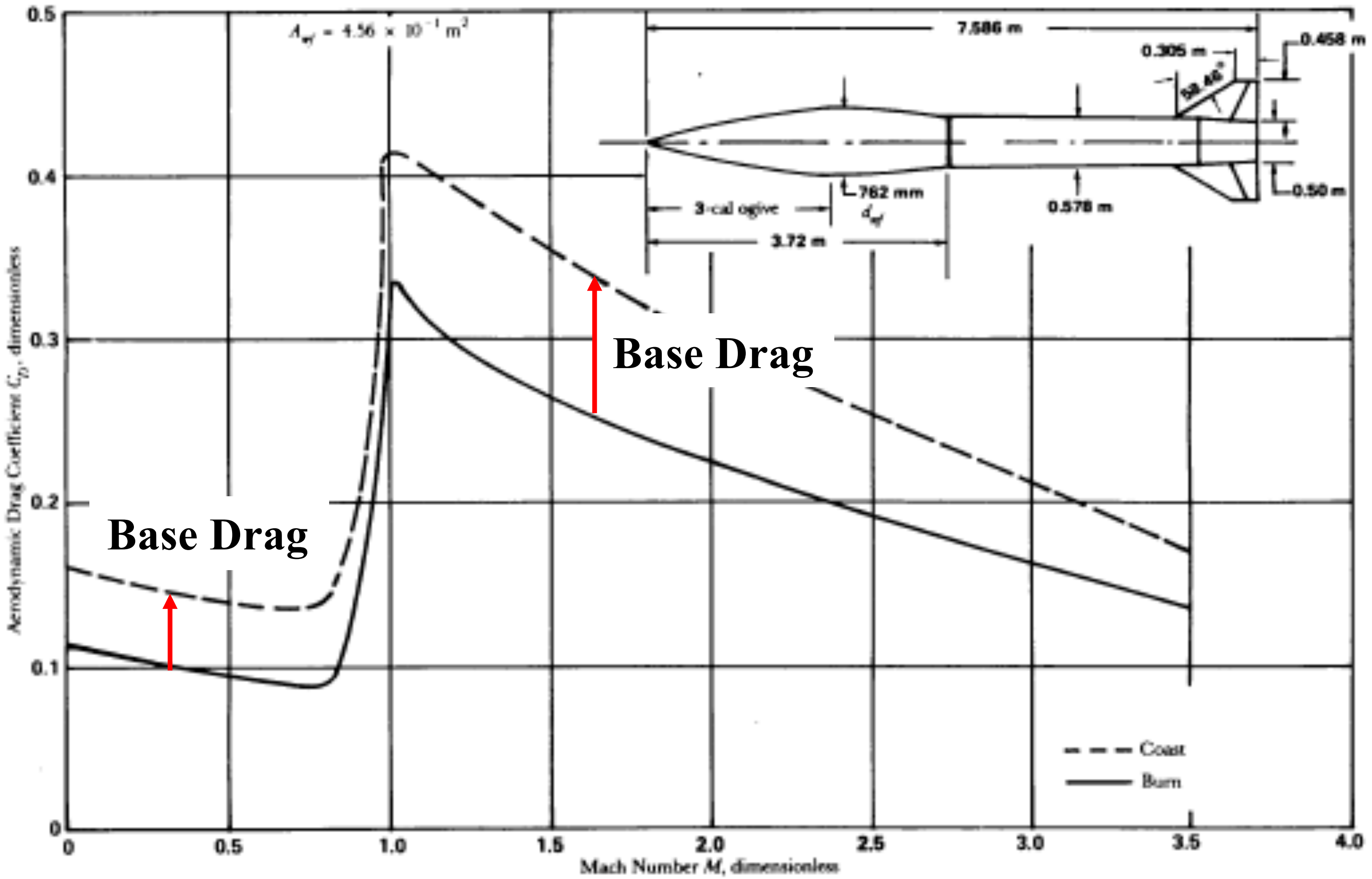
FLUID-DYNAMIC DRAG
Information on Aerodynamic Drag and Hydrodynamic Resistance
by
Sighard F. Hoerner, Dr. -Ing. habil

The Effect of the Viscous Forebody Drag on the Base Drag of a Blunt-Based Projectile



$$C_{D_{base}} = \frac{0.029}{\sqrt{\frac{A_{wet}}{A_{ref}} C_F}}$$

Base Drag (3)

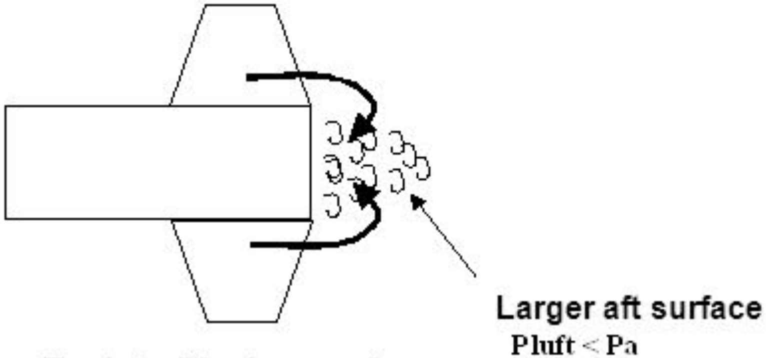


Drag Coefficient vs Mach Number—762-mm Rocket

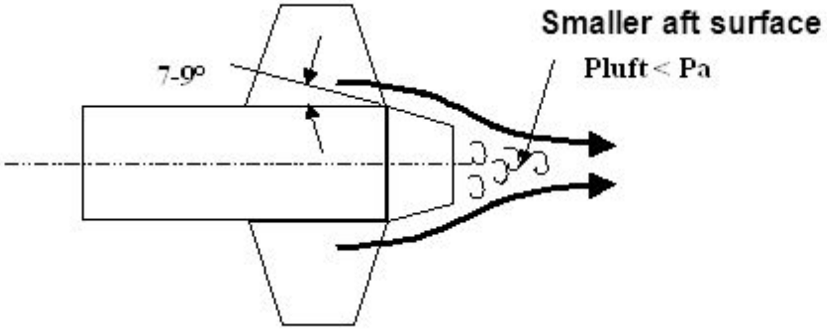
Burning Rocket Pressurizes Base Area, Eliminating Base Drag

Base Drag (4)

- Effect of "Boat Tailing" – Boat tail used on the rear end of rocket to reduce the base drag force by reducing area against which aft end pressure suction acts

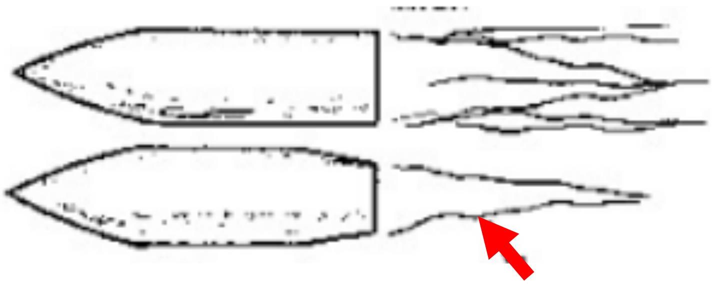


Rocket with sharp end

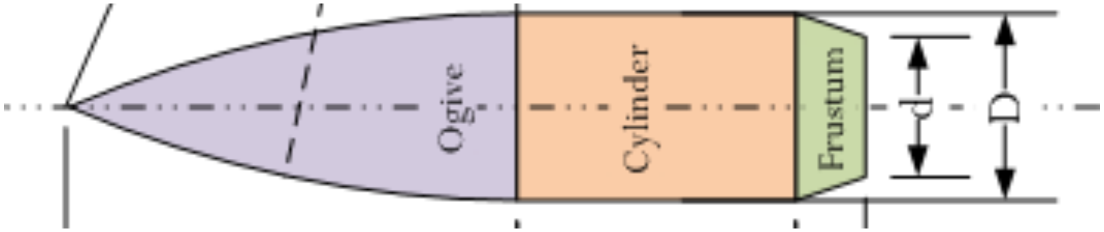


Rocket with conical end ("Boat-Tail")

- Effect of "Boat Tailing" – Boat tail Also serves to reduce the severity of the flow separation by reducing the exit turning angle



$$C_{D\,\substack{boat\\tail}} \approx C_{D\,base} \cdot \left(\frac{d}{D}\right)^2$$

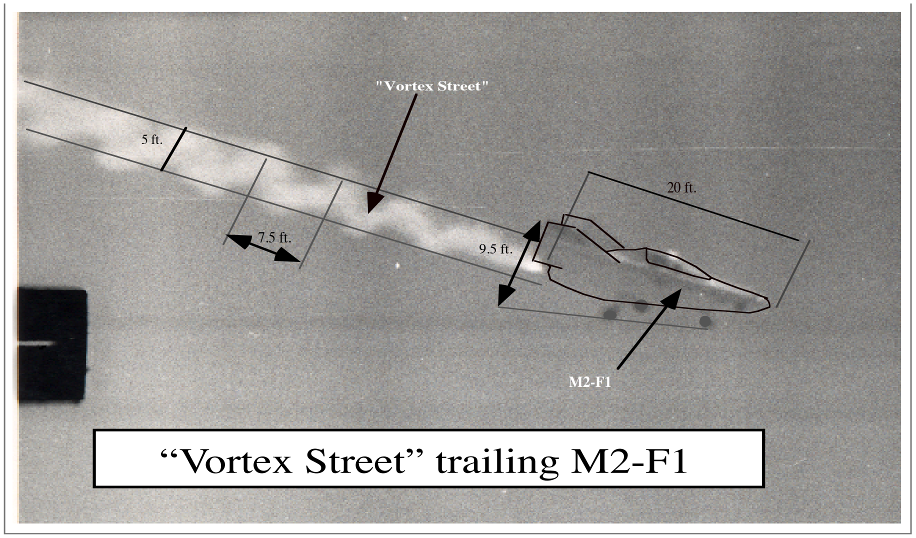


Fin “Dither” Drag

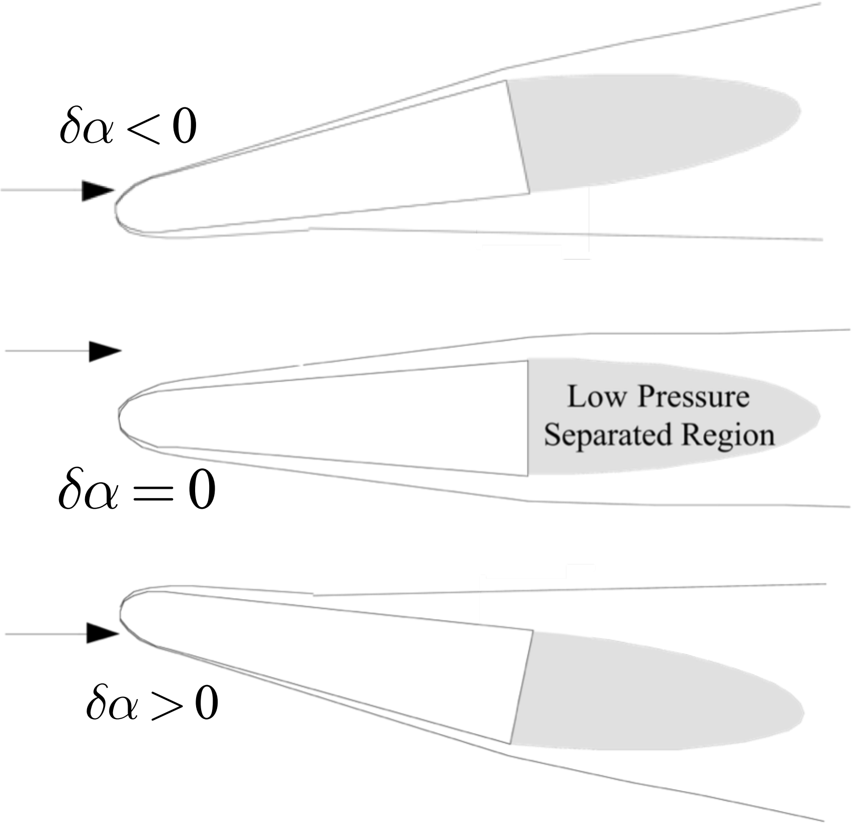
- Even along Ballistic Trajectory where nominal *α ~ 0*, fins can contribute induced drag to configuration due to unsteady “dither” or fin misalignment

- Shed Vortex from base is Unsteady and Contributes to Pitch Oscillations to Vehicle

- ”Dither” Drag due to Small angle of attack oscillations results in RMS drag contribution



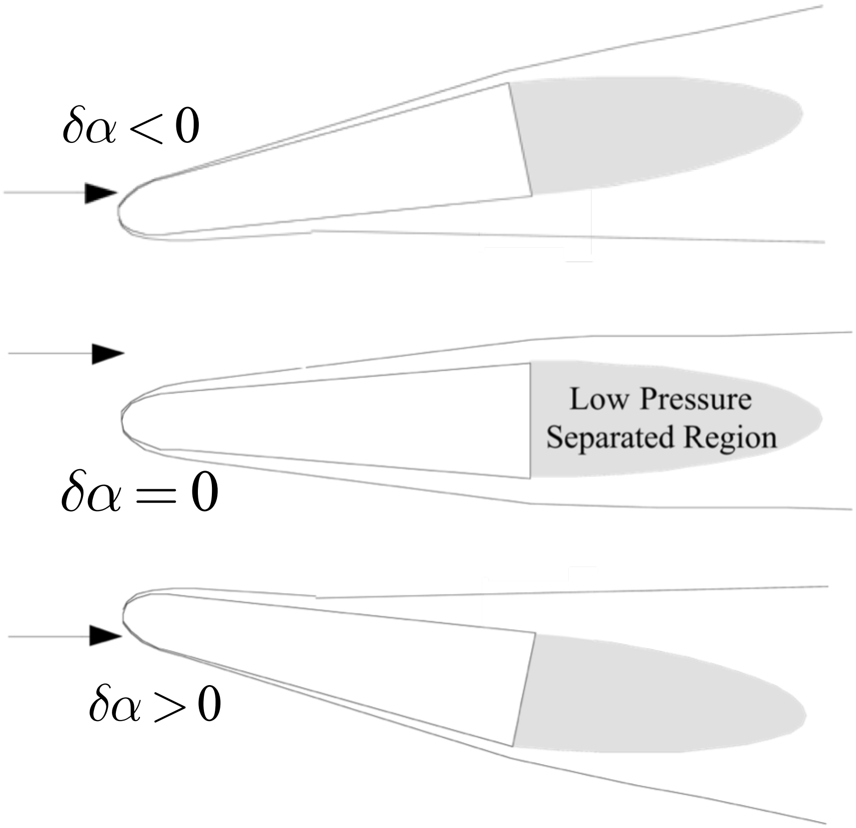
“Vortex Street” trailing M2-F1



Fin “Dither” Drag (2)

Dither” = Unsteady Induced Drag Component

$$C_{D_{"dither"}} = \left(\frac{A_{surf}}{A_{ref}}\right)\cdot\left(\frac{C_L^2}{\pi\cdot A_R}\right) = \left(\frac{A_{surf}}{A_{ref}}\right)\cdot\left(\frac{{C_{L_\alpha}}^2}{\pi\cdot A_R}\right)\cdot\delta\alpha^2_{dither}$$



$A_{surf} = Planform\ area$

$C_{L_\alpha} = Lift-curve\ slope$

$A_R = Aspect\ ratio$

$\delta\alpha_{dither} = RMS\ angle\ of\ attack\ dither$

Fin “Dither” Drag (3)

$$C_{D_{\text{"dither"}}} = \left(\frac{A_{surf}}{A_{ref}}\right)\cdot\left(\frac{C_{L_\alpha}{}^{2}}{\pi\cdot A_R}\right)\cdot\delta\alpha^{2} \rightarrow$$

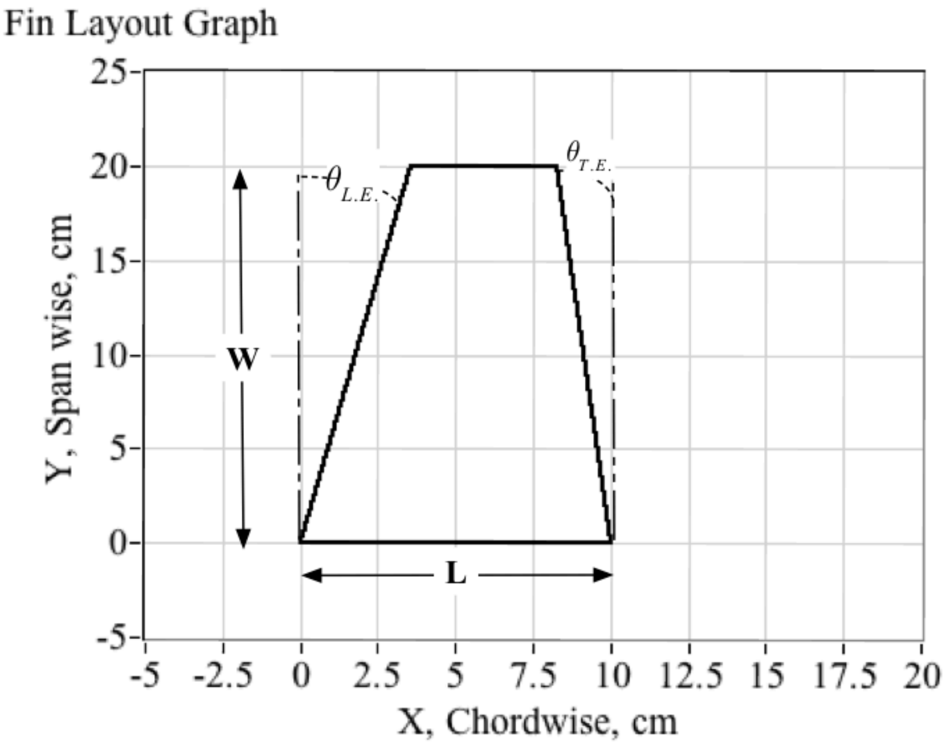
$$\boxed{\begin{aligned} A_{surf} &= L\cdot W - \frac{1}{2}\cdot W^{2}\cdot\left[\tan\left(\theta_{L.E.}\right)+\tan\left(\theta_{T.E.}\right)\right] \\ A_R &= \frac{W^{2}}{A_{surf}} \end{aligned}}$$

From Finite Airfoil Theory, Helmbold Equation:

$$C_{L_\alpha} = \frac{c_{L_\alpha}}{\sqrt{1+\left(\frac{c_{L_\alpha}}{\pi\cdot A_R}\right)^{2}} + \left(\frac{c_{L_\alpha}}{\pi\cdot A_R}\right)}$$

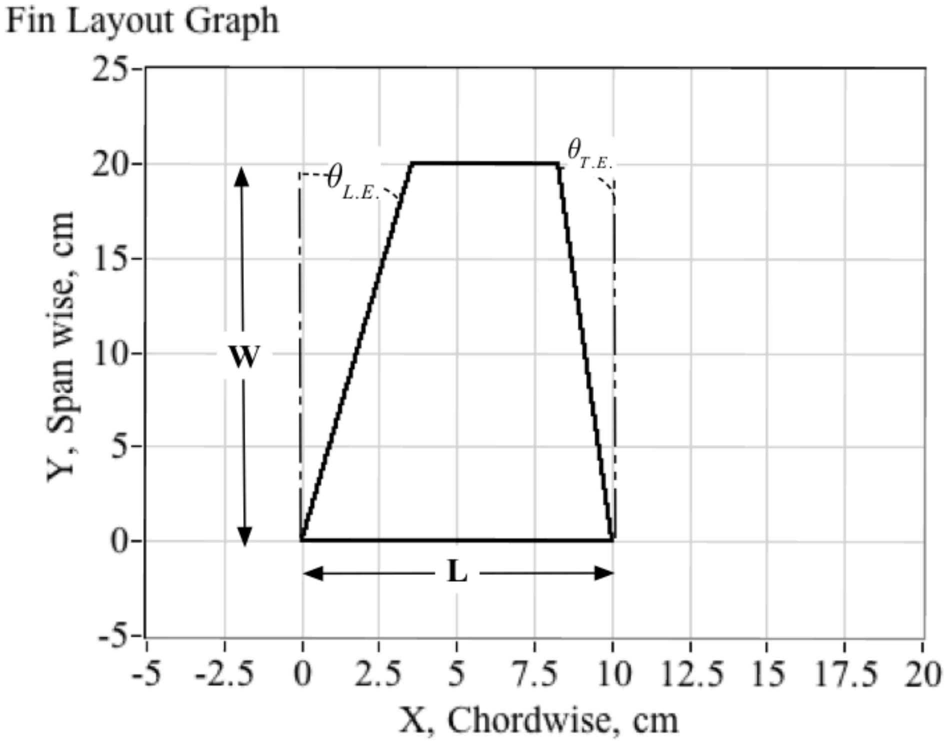
From Thin Airfoil Theory, Sectional Lift Curve Slope

$$c_{L_\alpha} = 2\pi \rightarrow C_{L_\alpha} = \frac{2\cdot\pi\cdot A_R}{2+\sqrt{4+A_R{}^{2}}}$$



Saltzman, E. J., Wang, C. K., and Iliff, K. W., “Aerodynamic Assessment of Flight-Determined Subsonic Lift and Drag Characteristics of Seven Lifting-Body and Wing-Body Reentry Vehicle Configurations,” NASA TP-2002-209032, November 2002.

Fin “Dither” Drag (4)


Fin Layout Graph

$$A_{surf} = L \cdot W - \frac{1}{2} \cdot W^2 \cdot \left[\tan\left(\theta_{L.E.}\right) + \tan\left(\theta_{T.E.}\right)\right]$$

$$A_R = \frac{W^2}{A_{surf}}$$

$$\left(C_{D_{\text{"dither"}}}\right)_{\substack{total\\fins}} = \sum_{i=1}^{N_{fins}} \left(\frac{A_{surf_i}}{A_{ref}}\right) \cdot \frac{\left(\dfrac{2 \cdot \pi \cdot A_{Ri}}{2 + \sqrt{4 + A_{R_i}{}^2}}\right)^2}{\pi \cdot A_{Ri}} \cdot \delta\alpha^2 = \sum_{i=1}^{N_{fins}} \left\{\left(\frac{A_{surf_i}}{A_{ref}}\right) \cdot \left(\frac{4 \cdot \pi \cdot A_{Ri}}{\left(2 + \sqrt{4 + A_{R_i}{}^2}\right)^2}\right)\right\} \cdot \delta\alpha^2$$

Fin Leading Edge Drag

- Stagnation Pressure Coefficient calculated based on Mach number Normal to leading edge of fins
- Scaled by leading edge area, *W•t*
- *Assumed fin thickness, t*

$$C_{p_{\max}} = \frac{q_c - p_\infty}{\bar{q}} = \frac{p_\infty \cdot \left(1+\frac{\gamma-1}{2}M_\perp^{\,2}\right)^{\frac{\gamma}{\gamma-1}} - p_\infty}{\frac{\gamma}{2}p_\infty M_\perp^{\,2}} = \frac{\left(1+\frac{\gamma-1}{2}\cdot\left(M_\infty\cdot\cos\theta_{L.E.}\right)^2\right)^{\frac{\gamma}{\gamma-1}}-1}{\frac{\gamma}{2}\cdot\left(M_\infty\cdot\cos\theta_{L.E.}\right)^2}$$

$$\left(C_{D_{L.E.}}\right)_{\substack{total\\fins}} = \sum_{i=1}^{N_{fins}}\left(\frac{W_i\cdot t_i}{A_{ref}}\right)\cdot\left\{\left(C_{P\max}\right)_{subsonic}\right\}_i = \sum_{i=1}^{N_{fins}}\left(\frac{W_i\cdot t_i}{A_{ref}}\right)\cdot\left\{\frac{\left(1+\frac{\gamma-1}{2}\cdot\left(M_\infty\cdot\cos\theta_{L.E.}\right)_i^{2}\right)^{\frac{\gamma}{\gamma-1}}-1}{\frac{\gamma}{2}\cdot\left(M_\infty\cdot\cos\theta_{L.E.}\right)_i^{2}}\right\}_i$$

Total Drag Correlation Model

$$\left(C_D\right)_{total} = C_{D_F} + C_{D_0} + C_{D_{base}} + C_{D_{Dither}} + C_{D_{L.E.}}$$

Skin Friction Drag

Nose/Body Profile Pressure Drag

Base Area Wake Drag

Unsteady Dither Drag, Fins

Fins, Leading Edge Pressure Drag

- **Medium Fidelity Engineering Model for “First Cut” Drag Coefficient Estimator.**

- **Calculates Subsonic Drag Coefficient in Incompressible Flight Regime $\underline{\sim}$ Mach 0.3**

- ***Rigorously, each term of the above equation should be scaled for compressibility at Higher Mach numbers***

- ***Operationally, Bulk scaling of*** $\left(C_D\right)_{total}$ ***is often used***

- ***Does not Model Wave Drag***

Correcting for Compressibility

$$D = \boxed{D_{friction} + D_{pressure}} + \boxed{D_{wave}}$$

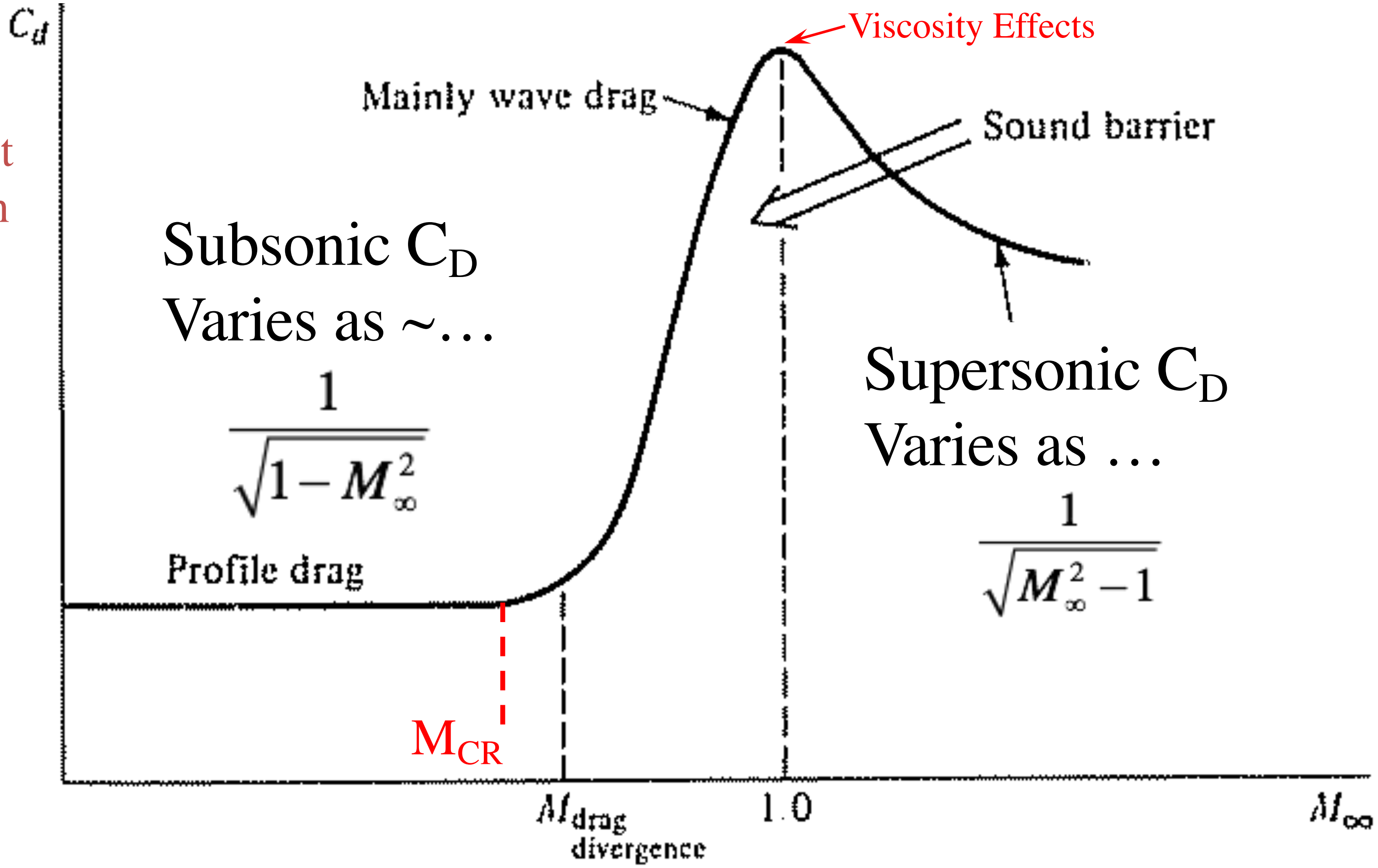
$$C_D = \boxed{C_{D,fric} + C_{D,press}} + \boxed{C_{D,wave}}$$

Only at transonic and supersonic speeds

$D_{wave} = 0$ for subsonic speeds below $M_{drag\text{-}divergence}$

Profile Drag

Profile Drag coefficient relatively constant with M_∞ at subsonic speeds



Viscosity Effects

Subsonic C_D Varies as ~…

$$\frac{1}{\sqrt{1-M_\infty^2}}$$

Supersonic C_D Varies as …

$$\frac{1}{\sqrt{M_\infty^2-1}}$$

M_{CR}

Credit: D. R. Kirk
FIT, 2011

Correcting for Compressibility (2)

- Several Simple Transformations exist that allows us to take compressible transonic flow and map back to an “equivalent” incompressible body

- Equivalently, compressibility corrections allow the pressure coefficient of an incompressible airfoil to be transformed into compressible flow on the same body. Since inviscid lift and drag are related directly to the pressure coefficient, similar corrections hold.

- Transformations are written as a function of Freestream Mach number.

$$\left\{C_L, C_D, C_p\right\}_{M_\infty} \equiv \left\{C_L, C_D, C_p\right\}_{M=0} \cdot f\left(M_\infty\right)$$

Correcting for Compressibility (3)

- ***Prandtl Glauert Rule: FIRST-ORDER CORRECTION***, the pressure coefficient, i.e. profile (pressure) drag at any point on a thin airfoil surface in a subsonic compressible flow is related to the pressure coefficient at the same point on the same airfoil in incompressible flow by

$$\left\{C_L, C_D, C_p\right\}_{M_\infty} \equiv \frac{\left\{C_L, C_D, C_p\right\}_{M=0}}{\sqrt{1 - M_\infty^2}}$$

- Correction valid from approximately M_{crit} to about $M=0.9$

- *Correction Not Valid in Supersonic Flow*

- *Applies to Wave and Profile Drag*

Correcting for Compressibility (4)

- ***Karman-Tsien Rule: FIRST-ORDER CORRECTION*,** the pressure coefficient, i.e. profile (pressure) drag at any point on a thin airfoil surface in a subsonic compressible flow is related to the pressure coefficient at the same point on the same airfoil in incompressible flow by

$$\left\{C_L, C_D, C_p\right\}_{M_\infty} \equiv \frac{\left\{C_L, C_D, C_p\right\}_{M=0}}{\sqrt{1-M_\infty^2} + \dfrac{M_\infty^2}{1+\sqrt{1-M_\infty^2}} \cdot \dfrac{\left\{C_L, C_D, C_p\right\}_{M=0}}{2}}$$

- Correction valid from approximately M_{crit} to about $M=0.98$

- *Correction Not Valid in Supersonic Flow*

- *Applies to Wave and Profile Drag*

Correcting for Compressibility (5)

- ***Laitone's Rule:*** Better Accounting for Isentropic Compressibility, and Heating of Local airflow

$$\left\{C_L, C_D, C_p\right\}_{M_\infty} \equiv \frac{\left\{C_L, C_D, C_p\right\}_{M=0}}{\sqrt{1-M_\infty^2} + \dfrac{M_\infty^2\left(1+\dfrac{\gamma-1}{2}M_\infty^2\right)}{1+\sqrt{1-M_\infty^2}} \cdot \dfrac{\left\{C_L, C_D, C_p\right\}_{M=0}}{2}}$$

- ***Ackeret Rule:*** the pressure coefficient, i.e. profile +wave drag at any point on a thin Airfoil surface in a supersonic flow at M_2 is related to the pressure coefficient at M_1 at the same point on the Airfoil by (*Applies to Wave/Profile Drag*)

$$\left\{C_L, C_D, C_P\right\}_{M_2} = \left\{C_L, C_D, C_P\right\}_{M_1} \cdot \frac{\sqrt{M_1^2-1}}{\sqrt{M_2^2-1}}$$

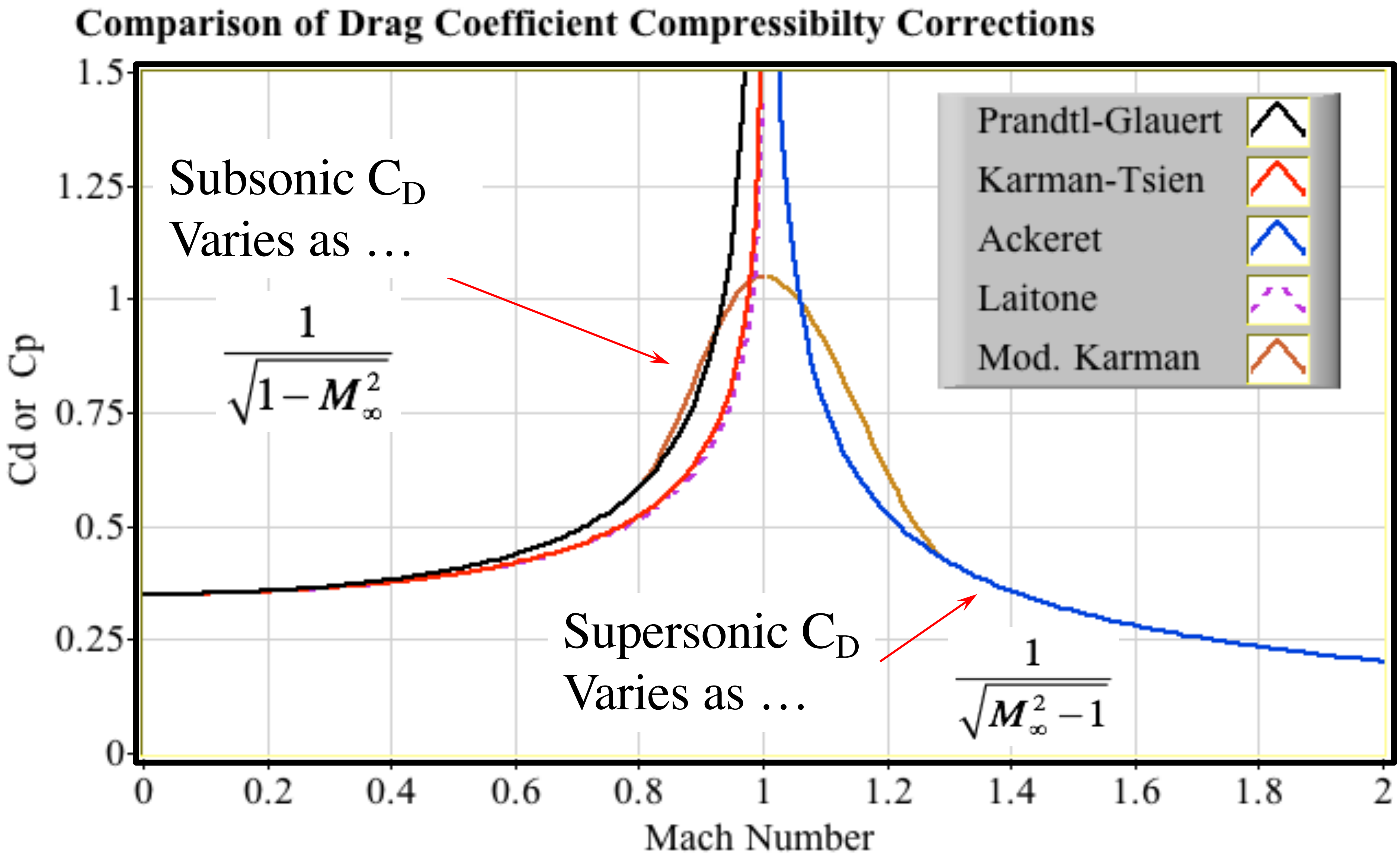
Correcting for Compressibility (6)

- ***Modified Karman Rule:*** Curve fit of Data to Match Known Drag Coefficient Profiles for Missile-Type Configurations

```
if(Mach < 0.80)
   CF_mkr=1.0/sqrt(1.0-Mach**2);
else
 {
      if (Mach < 1.0)
         CF_mkr= -240.740740741*(Mach**3) + 640.740740741*(Mach**2) -
                  559.259259260*Mach       +  162.259259259;
      else
      {
            if (Mach < 1.3)
                CF_mkr= 107.845968888*(Mach**3) - 375.848824765*(Mach**2) +
                      428.1597428660*Mach   - 157.156886989;
            else
                CF_mkr=1.0/sqrt( (Mach**2) -1.0);
      }
}
```

$$\left(C_D\right)_{M_\infty} = \left(C_{F_{MKR}}\right) \cdot \left(C_D\right)_{inc}$$

Correcting for Compressibility (7)



Subsonic C_D Varies as …

$$\frac{1}{\sqrt{1-M_\infty^2}}$$

Supersonic C_D Varies as …

$$\frac{1}{\sqrt{M_\infty^2-1}}$$

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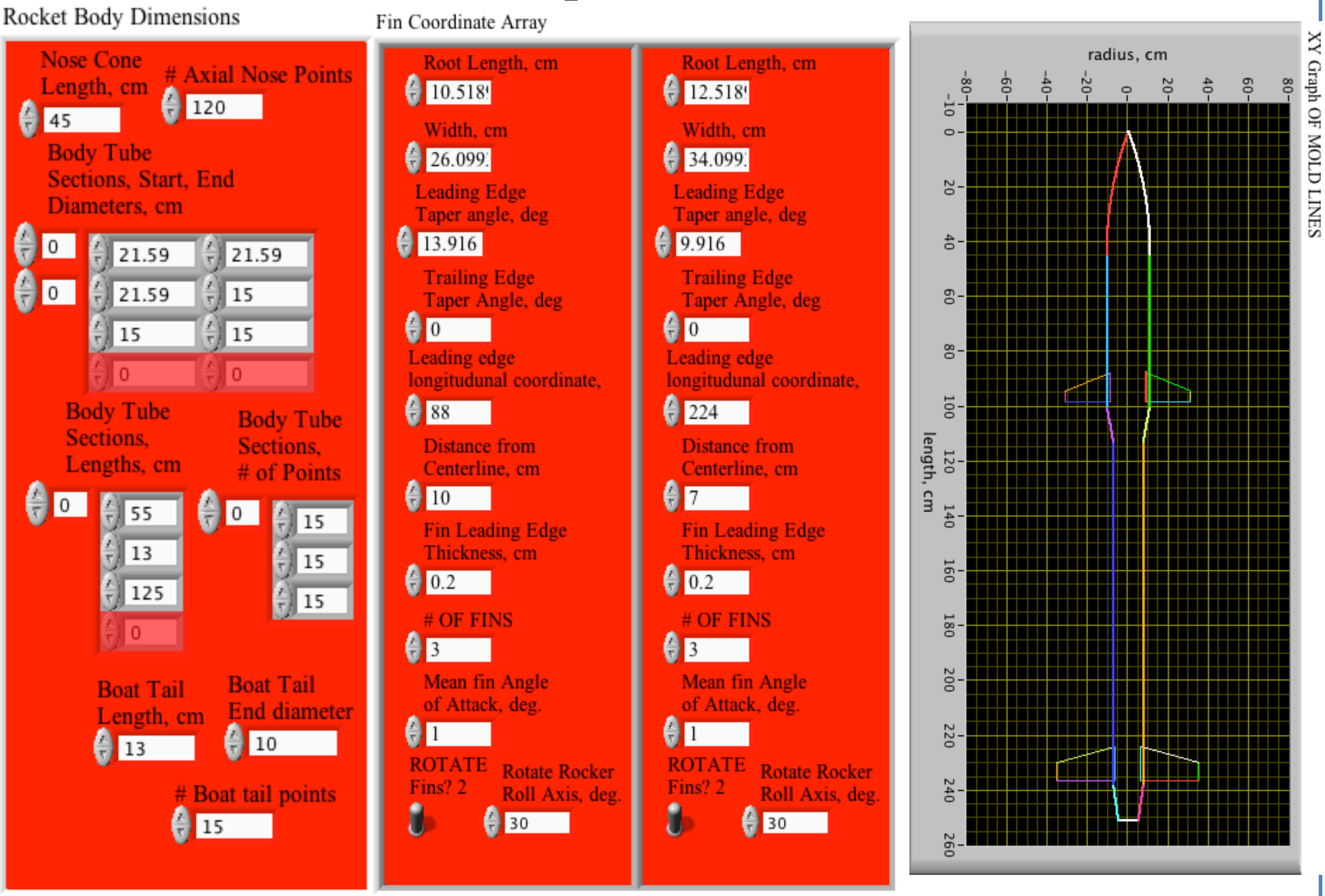
Mechanical & Aerospace
Engineering

Example Calculation



Rocket Body Dimensions

Nose Cone Length, cm: 45

Axial Nose Points: 120

Body Tube Sections, Start, End Diameters, cm

	Start	End
0	21.59	21.59
0	21.59	15
	15	15
	0	0

Body Tube Sections, Lengths, cm	Body Tube Sections, # of Points
0	0
55	15
13	15
125	15
0	

Boat Tail Length, cm: 13

Boat Tail End diameter: 10

Boat tail points: 15

Fin Coordinate Array

Root Length, cm	10.518	12.518
Width, cm	26.099	34.099
Leading Edge Taper angle, deg	13.916	9.916
Trailing Edge Taper Angle, deg	0	0
Leading edge longitudunal coordinate,	88	224
Distance from Centerline, cm	10	7
Fin Leading Edge Thickness, cm	0.2	0.2
# OF FINS	3	3
Mean fin Angle of Attack, deg.	1	1
ROTATE Fins? 2		
Rotate Rocker Roll Axis, deg.	30	30

Example Calculation (2)

Nose Cone Data

Fineness ratio, L/D
2.0843

X, cm
0
45

R, cm
40
9.6542

mean radius, cm
7.26315

Run Length, cm
47.082

Surface (Wetted) area, cm^
2053.61

Maximum Cross section area, cm^2
366.096

Boat tail data

Run Length, cm
13.2382

Wetted Area, cm^2
510.509

mean diameter, cm
12.5

Boat tail angle, deg
10.8855

Boat Tail Exit Area, cm^2
78.5398

Fin Output Data

surface area, cm^2	190.148	325.251
Aspect ratio	3.58233	3.57497
sectional Cl-alpha, 1/radian	6.28319	6.28319
CL-alpha, 1/radian	3.68821	3.68451
CL-alpha, 1/deg	0.0643713	0.0643068
CL	0.0643713	0.0643068
Induced drag, CDi	0.00019123	0.00032712
Leading Edge Pressure Coefficient	1.02138	1.02202
Leading Edge Pressure Drag Coeffi	0.014563	0.0190389
Total drag coefficient per fin	0.0147542	0.019366

Total Rocket Geometry Data

Maximum Cross Section area, cm^2
366.096

Run Length, METERS
2.5332

Total Wetted Area, cm^2
16024.7

Total Rocket Length, cm
251

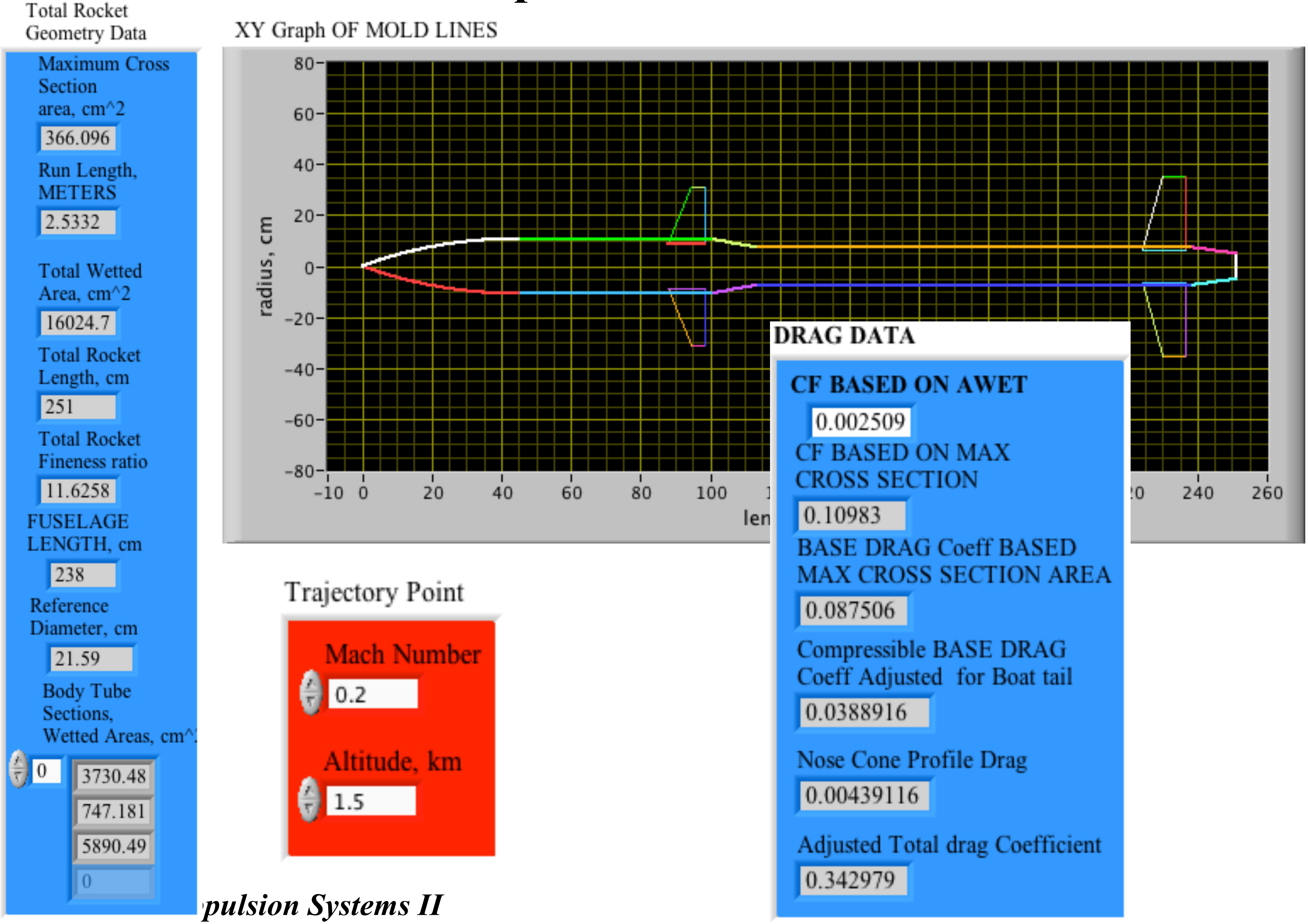
Total Rocket Fineness ratio
11.6258

FUSELAGE LENGTH, cm
238

Reference Diameter, cm
21.59

Body Tube Sections, Wetted Areas, cm^
0
3730.48
747.181
5890.49
0

Example Calculation (2)



Total Rocket Geometry Data

Maximum Cross Section area, cm^2: 366.096

Run Length, METERS: 2.5332

Total Wetted Area, cm^2: 16024.7

Total Rocket Length, cm: 251

Total Rocket Fineness ratio: 11.6258

FUSELAGE LENGTH, cm: 238

Reference Diameter, cm: 21.59

Body Tube Sections, Wetted Areas, cm^
0
3730.48
747.181
5890.49
0

XY Graph OF MOLD LINES

Trajectory Point

Mach Number: 0.2

Altitude, km: 1.5

DRAG DATA

CF BASED ON AWET: 0.002509

CF BASED ON MAX CROSS SECTION: 0.10983

BASE DRAG Coeff BASED MAX CROSS SECTION AREA: 0.087506

Compressible BASE DRAG Coeff Adjusted for Boat tail: 0.0388916

Nose Cone Profile Drag: 0.00439116

Adjusted Total drag Coefficient: 0.342979

Higher Fidelity Codes

- **Missile DATCOM** is a widely used semi-empirical datasheet component build-up method for the preliminary design and analysis of missile aerodynamics and performance. It has been in continual development for over twenty years, with the latest version released in March 2011

- DATCOM has traditionally been supplied free of charge by the United States Air Force to American defense contractors. The code is considered restricted under International Traffic in Arms Regulations (ITAR) and can not be distributed outside the United States.

- Use of latest release by NON-USA Nationals Requires Special Export Licensing Permissions.

- Missile DATCOM User's Manual, 2001 Release Version. http://www.dtic.mil/dtic/tr/fulltext/u2/a548461.pdf

Questions??