Section 7.3 Supplement
Rocket Science Review 103:
Estimating the Launch Vehicle Drag Coefficient

Newton's Laws as Applied to "Rocket Science"

... its not just a job ... its an adventure

Newtonian Flow Analysis

Fin Leading Edge Drag

- Stagnation Pressure Coefficient calculated based on Mach number Normal to leading edge of fins
- Scaled by leading edge area, *W•t*
- *Assumed fin thickness, t*

$$C_{p_{\max}} = \frac{q_c - p_\infty}{\bar{q}} = \frac{p_\infty \cdot \left(1 + \frac{\gamma - 1}{2} M_\perp{}^2\right)^{\frac{\gamma}{\gamma-1}} - p_\infty}{\frac{\gamma}{2} p_\infty M_\perp{}^2} = \frac{\left(1 + \frac{\gamma-1}{2} \cdot \left(M_\infty \cdot \cos\theta_{L.E.}\right)^2\right)^{\frac{\gamma}{\gamma-1}} - 1}{\frac{\gamma}{2} \cdot \left(M_\infty \cdot \cos\theta_{L.E.}\right)^2}$$

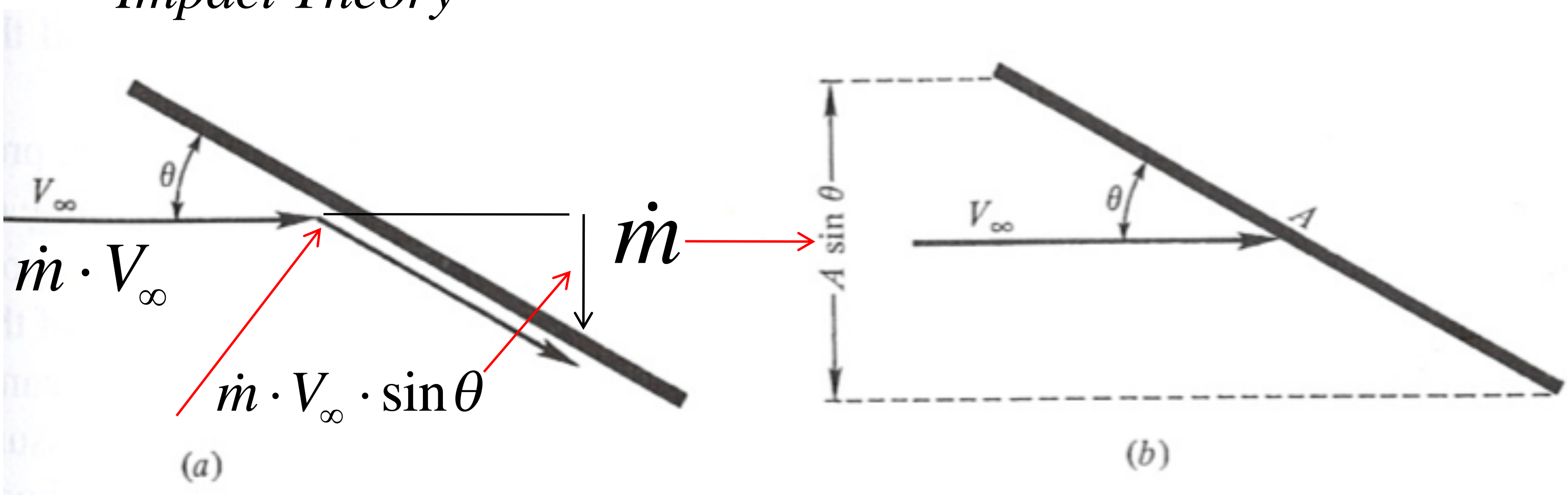
$$\left(C_{D_{L.E.}}\right)_{\substack{total\\fins}} = \sum_{i=1}^{N_{fins}} \left(\frac{W_i \cdot t_i}{A_{ref}}\right) \cdot \left\{\left(C_{P\max}\right)_{subsonic}\right\}_i = \sum_{i=1}^{N_{fins}} \left(\frac{W_i \cdot t_i}{A_{ref}}\right) \cdot \left\{\frac{\left(1 + \frac{\gamma-1}{2} \cdot \left(M_\infty \cdot \cos\theta_{L.E.}\right)_i^2\right)^{\frac{\gamma}{\gamma-1}} - 1}{\frac{\gamma}{2} \cdot \left(M_\infty \cdot \cos\theta_{L.E.}\right)_i^2}\right\}_i$$

Tends to Over-predict drag, Model can be refined using Newtonian Flow Theory

Newtonian Flow Analysis

- Newton had an Often unrecognized original contribution to Fluid Mechanics …. Propositions 34 and 35 in *"Principia…"*

- *"Impact Theory"*

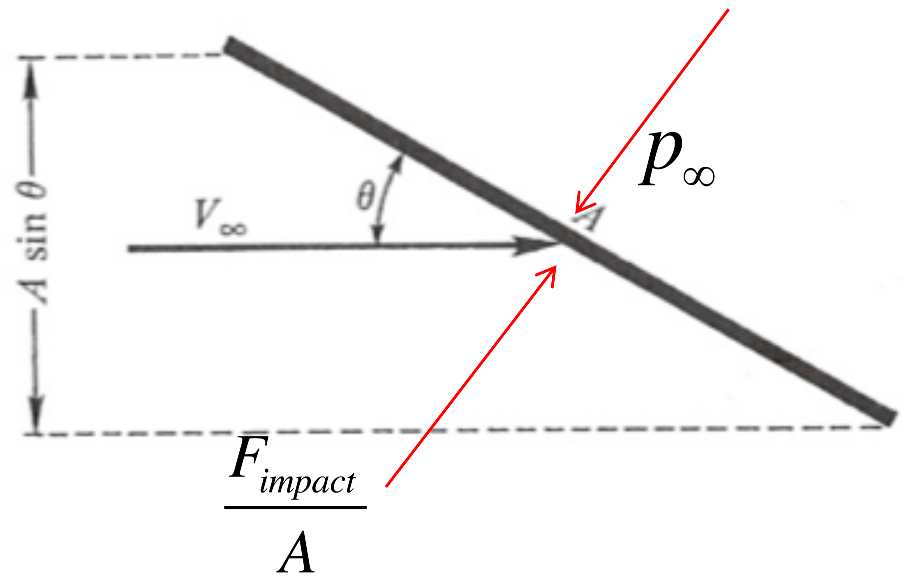


$$F_{impact} = change\ in\ momentum = \dot{m} \cdot V_\infty \cdot \sin\theta$$

$$\rightarrow F_{impact} = \rho_\infty \cdot V_\infty \cdot (A \sin\theta) \cdot V_\infty \cdot \sin\theta = \rho_\infty \cdot V_\infty^{\ 2} \cdot A \cdot \sin^2\theta$$

Newtonian Flow Analysis (2)

$$F_{impact} = \rho_\infty \cdot V_\infty^{\ 2} \cdot A \cdot \sin^2\theta \rightarrow \boxed{p(\theta) = \frac{F_{impact}}{A} + p_\infty = \rho_\infty \cdot V_\infty^{\ 2} \cdot \sin^2\theta + p_\infty}$$

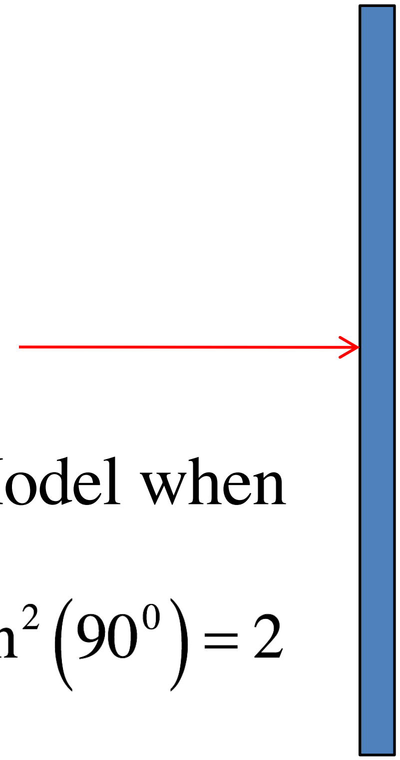


Newton Ignores the random motion of the molecules and Considers only the linear or translational motion

$$\boxed{C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} = \frac{\rho_\infty \cdot V_\infty^{\ 2} \cdot \sin^2\theta + p_\infty - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} = 2 \cdot \sin^2\theta}$$

Newtonian Flow Analysis (3)

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} = 2 \cdot \sin^2 \theta$$



What happens in Newtonian Model when

$$\theta = 90^0 \quad ? \; C_p(90^0) = 2 \cdot \sin^2(90^0) = 2$$

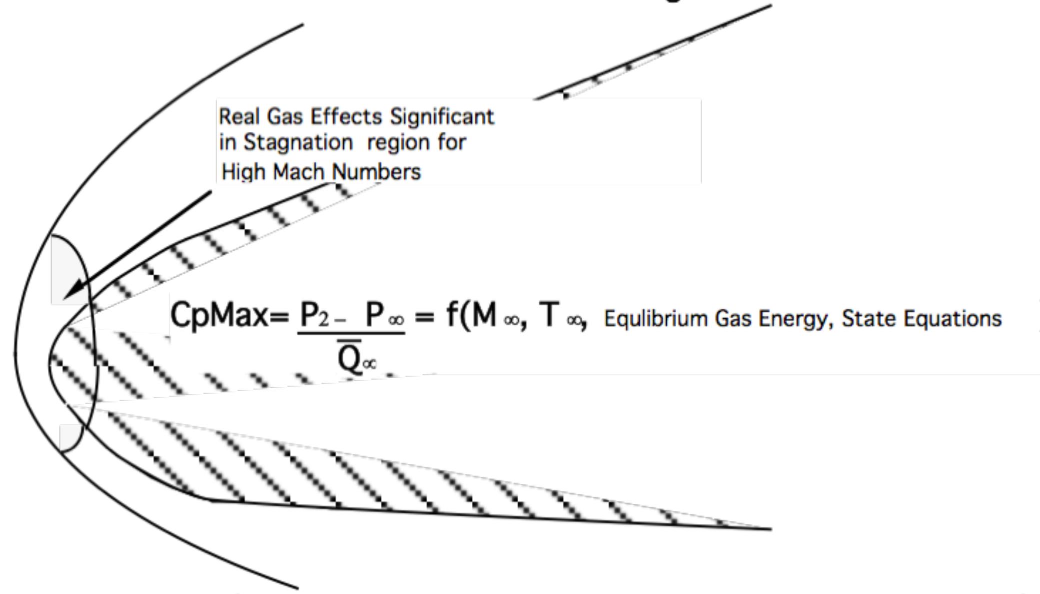
But for direct impact …

$$p(90^0) = P_0 \rightarrow C_p(90^0) = \frac{P_0 - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} \equiv C_{p_{\max}} \; !$$

Newtonian Flow Analysis (4)

$$C_p\left(90^0\right)=\frac{P_0-p_\infty}{\frac{1}{2}\rho_\infty\cdot V_\infty^{\ 2}}\equiv C_{p\ \max}$$

Schematic of the Bow Shock near the Stagnation Point



$$Cp_{\max}=\left[\frac{\left(P_{02}-p_\infty\right)}{\frac{1}{2}\rho_\infty V_\infty^{\ 2}}\right]=\frac{\left(P_{02}-p_\infty\right)}{\frac{\gamma}{2}p_\infty M_\infty^{\ 2}}=\frac{1}{\frac{\gamma}{2}M_\infty^{\ 2}}\left\{\frac{\left(\frac{\gamma+1}{2}M_\infty^{\ 2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)}M_\infty^{\ 2}-\frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}}-1\right\}$$

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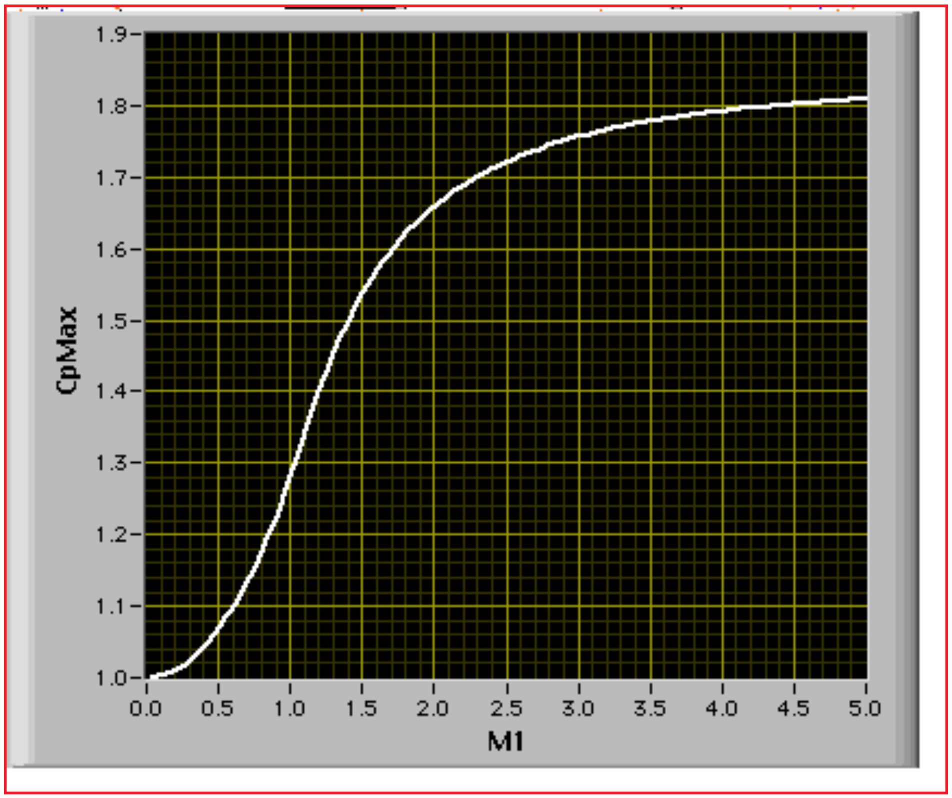
Newtonian Flow Analysis (5)

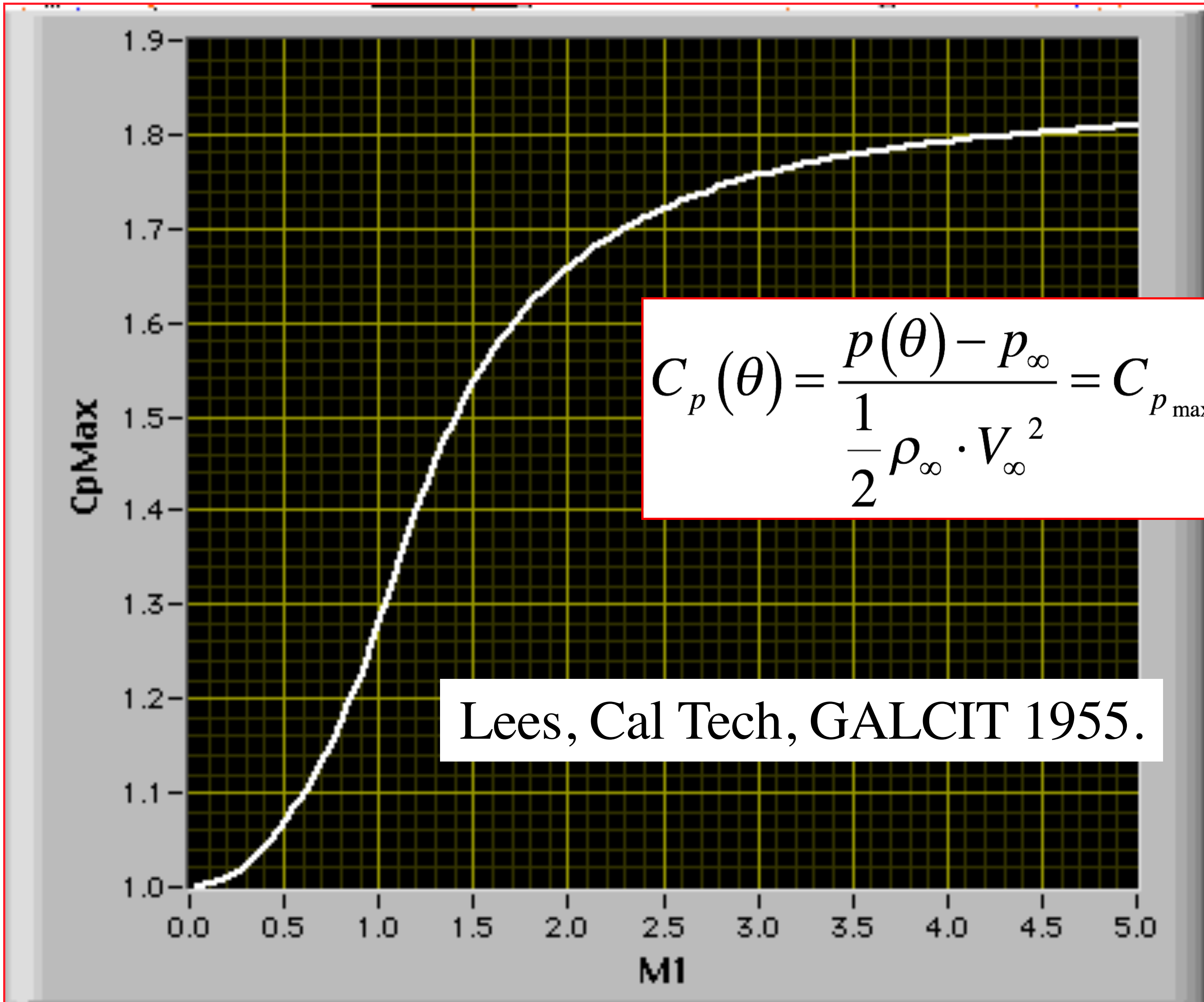
$$C_{p\,\max} = \frac{1}{\frac{\gamma}{2} M_\infty^{\,2}} \left\{ \frac{\left(\frac{\gamma+1}{2} M_\infty^{\,2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)} M_\infty^{\,2} - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}} - 1 \right\}$$



$$C_p\left(90^0\right) = \frac{P_0 - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\,2}} \equiv C_{p\,\max}$$

< 2 always for $\gamma = 1.4$

"Modified Newtonian Flow"



$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} = C_{p_{\max}} \cdot \sin^2\theta$$

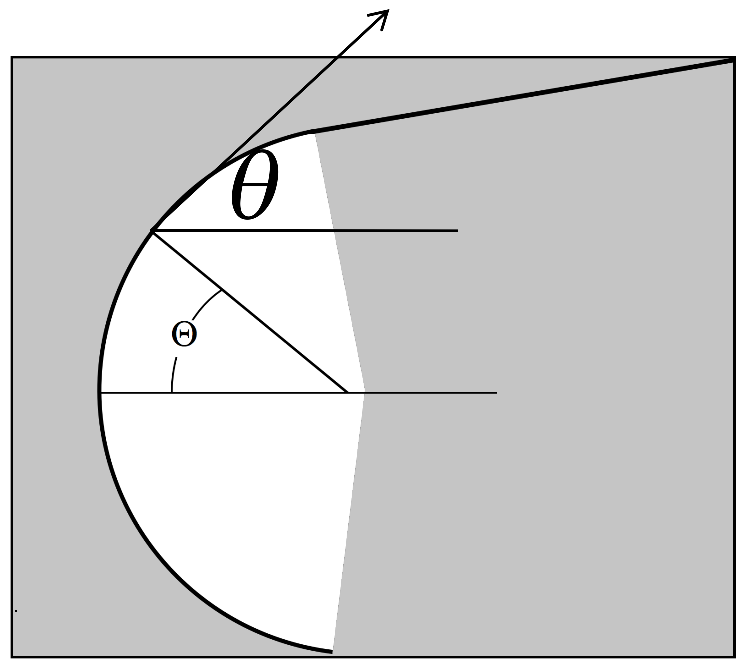
Lees, Cal Tech, GALCIT 1955.

Modified Newtonian Flow (2)

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} = C_{p_{\max}} \cdot \sin^2\theta$$

Often convenient to express model in terms of a Polar surface coordinate

$$\Theta = 90^o - \theta$$



$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} = C_{p_{\max}} \cdot \sin^2\left(90^0 - \Theta\right) =$$

$$C_{p_{\max}} \cdot \left[\sin\left(90^0\right) \cdot \cos(\Theta) - \sin(\Theta) \cdot \cos\left(90^0\right)\right]^2$$

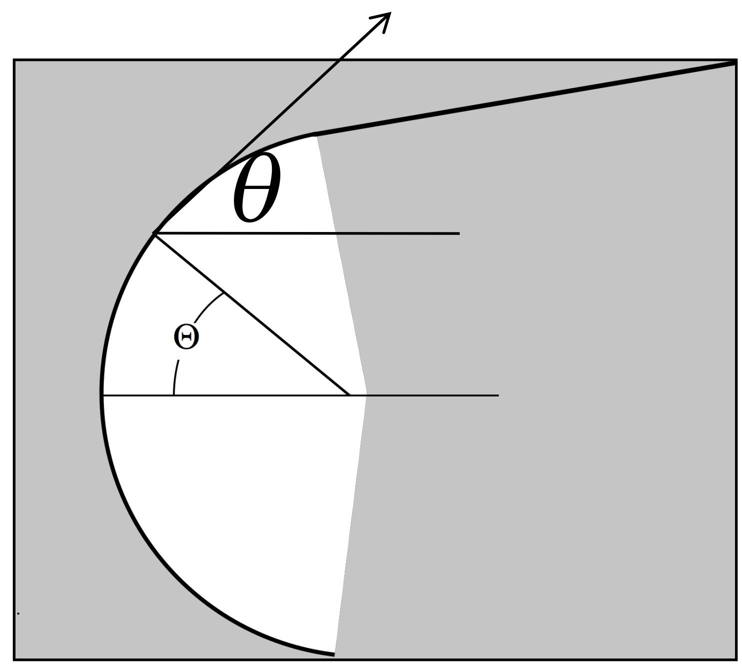
$$\Rightarrow C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} = C_{p_{\max}} \cdot \cos^2(\Theta)$$

Modified Newtonian Flow (3)

Expressed in terms of pressure ratio

$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^{\ 2}} = C_{p_{\max}} \cdot \cos^2(\Theta) \Rightarrow \frac{p(\Theta) - p_\infty}{\frac{\gamma}{2} p_\infty \cdot M_\infty^{\ 2}} = C_{p_{\max}} \cdot \cos^2(\Theta)$$

$$\boxed{\frac{p(\Theta)}{p_\infty} = 1 + \frac{\gamma}{2} \cdot M_\infty^{\ 2} \cdot C_{p_{\max}} \cdot \cos^2(\Theta)}$$



$$\Theta = 90^o - \theta$$

Modified Newtonian Flow (4)

$$\Rightarrow C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = C_{p_{\max}} \cdot \cos^2(\Theta)$$

For Newtonian model … Newton Ignores the random motion of the molecules and Considers only the linear or translational motion

"equivalent to infinite Mach number assumption"

- As derived in section 3

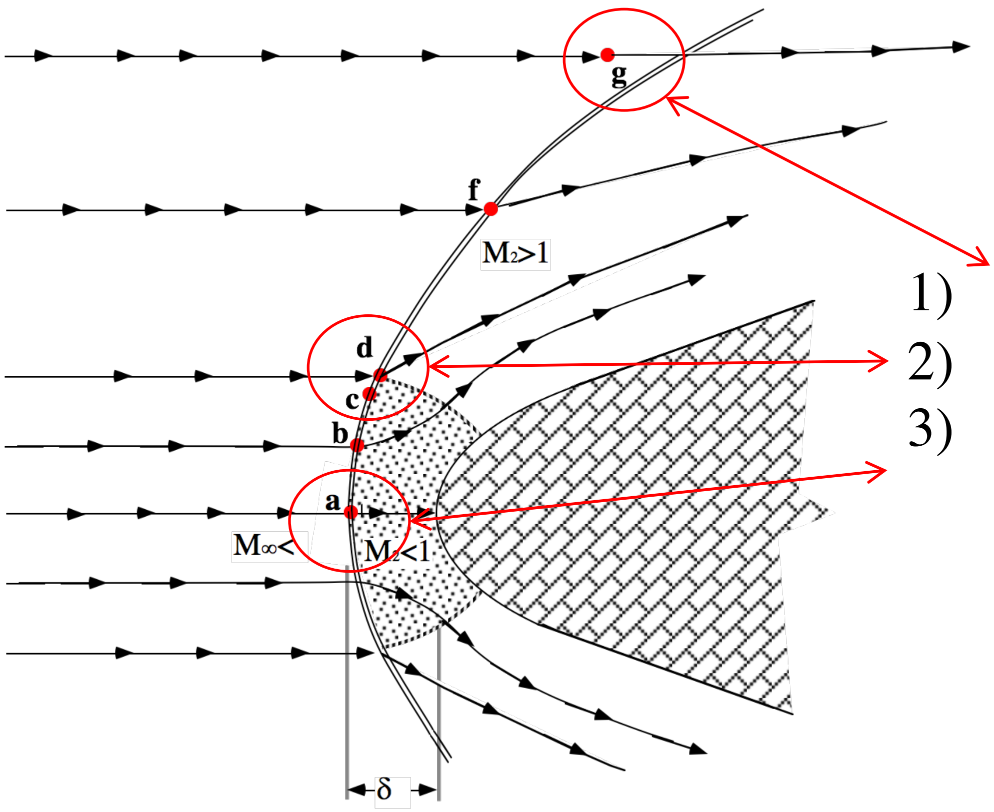
$$\frac{V^2/2}{e} = \frac{\gamma(\gamma-1)}{2} M^2$$

Mach number is a measure of the ratio of the fluid Kinetic energy to the fluid internal energy (direct motion To random thermal motion of gas molecules) -- Fundamental Parameter of Compressible Flow --

Example 3

- Example: Cylindrically Blunted 2-D wedge, M_∞=10.0, 1-cm Radius

- *Map conditions at three points along shock*

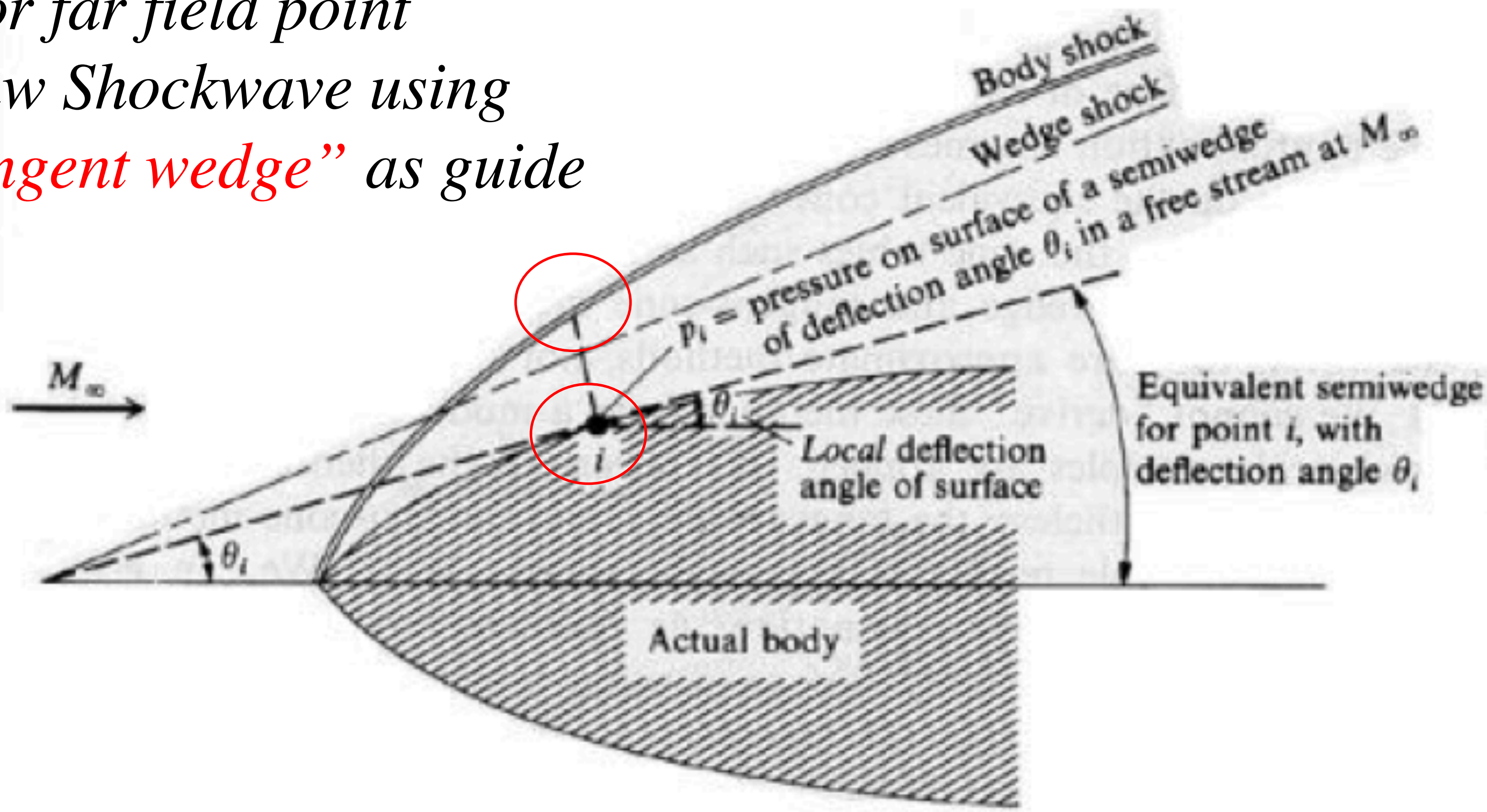


1) Far field (*g*)… "tangent wedge"
2) Sonic Point (*d*)
3) Stagnation Point (*a*)

Example 3 (2)

- Example Cylindrically Blunted 2-D wedge, M_∞=10.0 1-cm Radius

  - *For far field point Draw Shockwave using "tangent wedge" as guide*

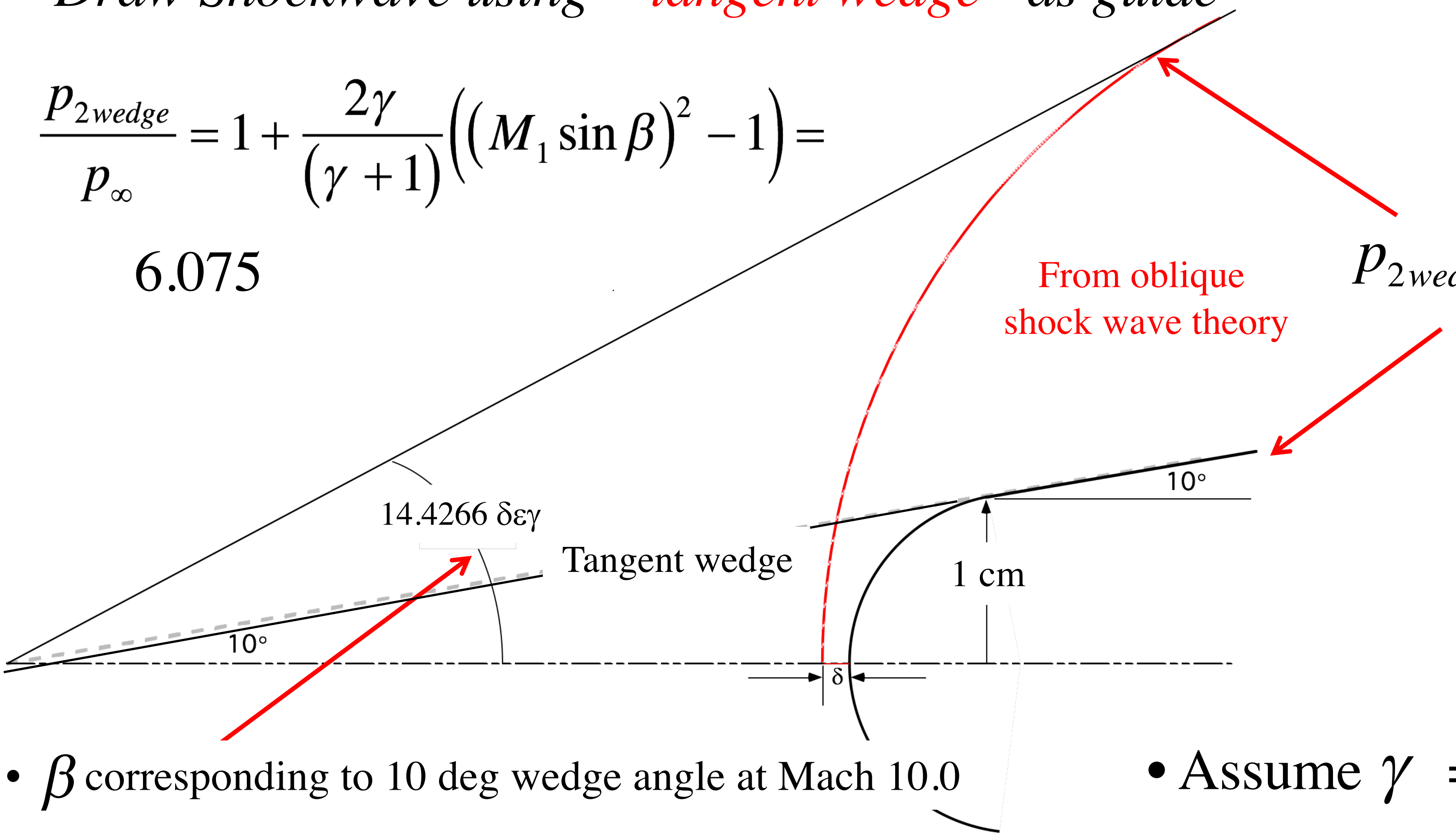


Example 3 (3)

- Example Cylindrically ' Blunted 2-D wedge, M_∞=10.0 1-cm Radius
- *Draw Shockwave using* *"tangent wedge"* *as guide*

$$\frac{p_{2wedge}}{p_\infty} = 1 + \frac{2\gamma}{(\gamma+1)}\left(\left(M_1 \sin\beta\right)^2 - 1\right) =$$

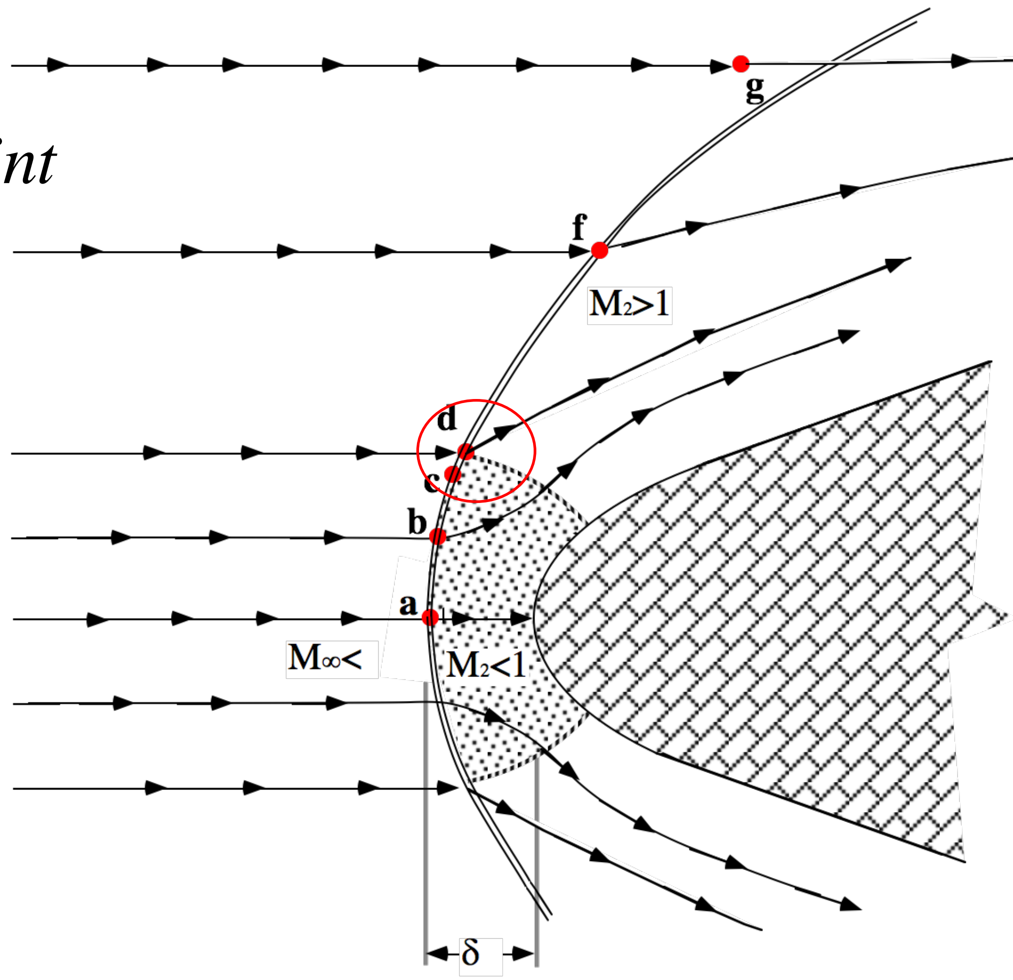
6.075



- β corresponding to 10 deg wedge angle at Mach 10.0
- Assume γ =1.4

Example 3: Sonic Point Surface Conditions behind detached shock wave: *(5)*

Sonic Point



Example 3 (6)

- Compute Standoff Distance

$$\frac{\delta}{D} = 0.193 \cdot e^{\frac{4.67}{M_\infty{}^2}} = \quad 2 \cdot 1 \cdot 0.193 \exp\left(\frac{4.67}{1.75^2}\right) = 0.4044 \text{ cm}$$

- Compute Sonic Point on Shock

$$\beta_{sonic} = \frac{180}{\pi}\sin^{-1}\sqrt{\frac{(\gamma-3)M_1{}^2+(\gamma+1)M_1{}^4+\sqrt{16\gamma M_1{}^4+\left[(\gamma-3)M_1{}^2+(\gamma+1)M_1{}^4\right]^2}}{4\gamma M_1{}^4}}$$

$$= 67.335 \text{ deg.}$$

- Compute Sonic Point on Surface

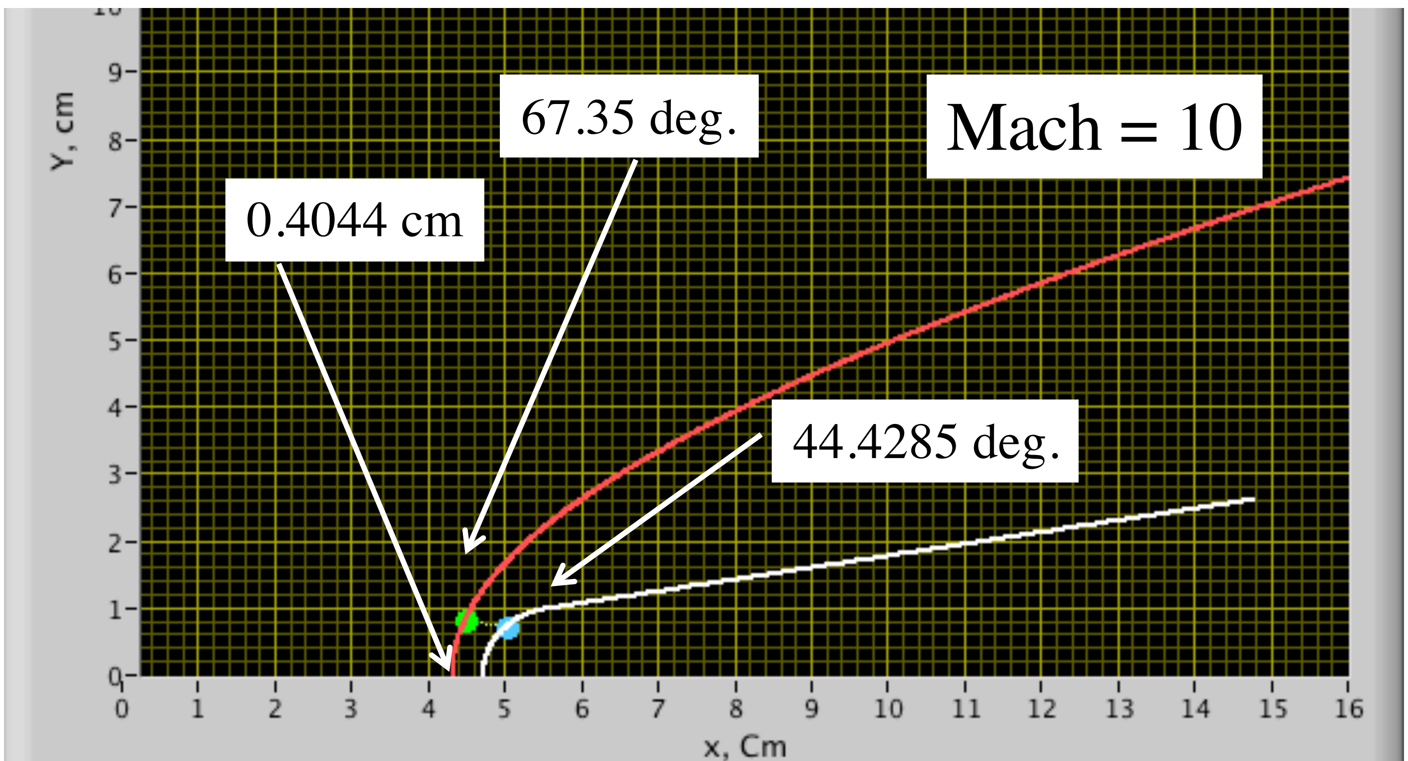
$$\tan(\theta_{sonic}) = \frac{2\left\{M_1{}^2\sin^2(\beta_{sonic})-1\right\}}{\tan(\beta_{sonic})\left[2+M_1{}^2\left[\gamma+\cos(2\beta_{sonic})\right]\right]} = \quad 44.428 \text{ deg.}$$

See for derivation: http://mae-nas.eng.usu.edu/MAE_5420_Web/section8/section8.3.pdf

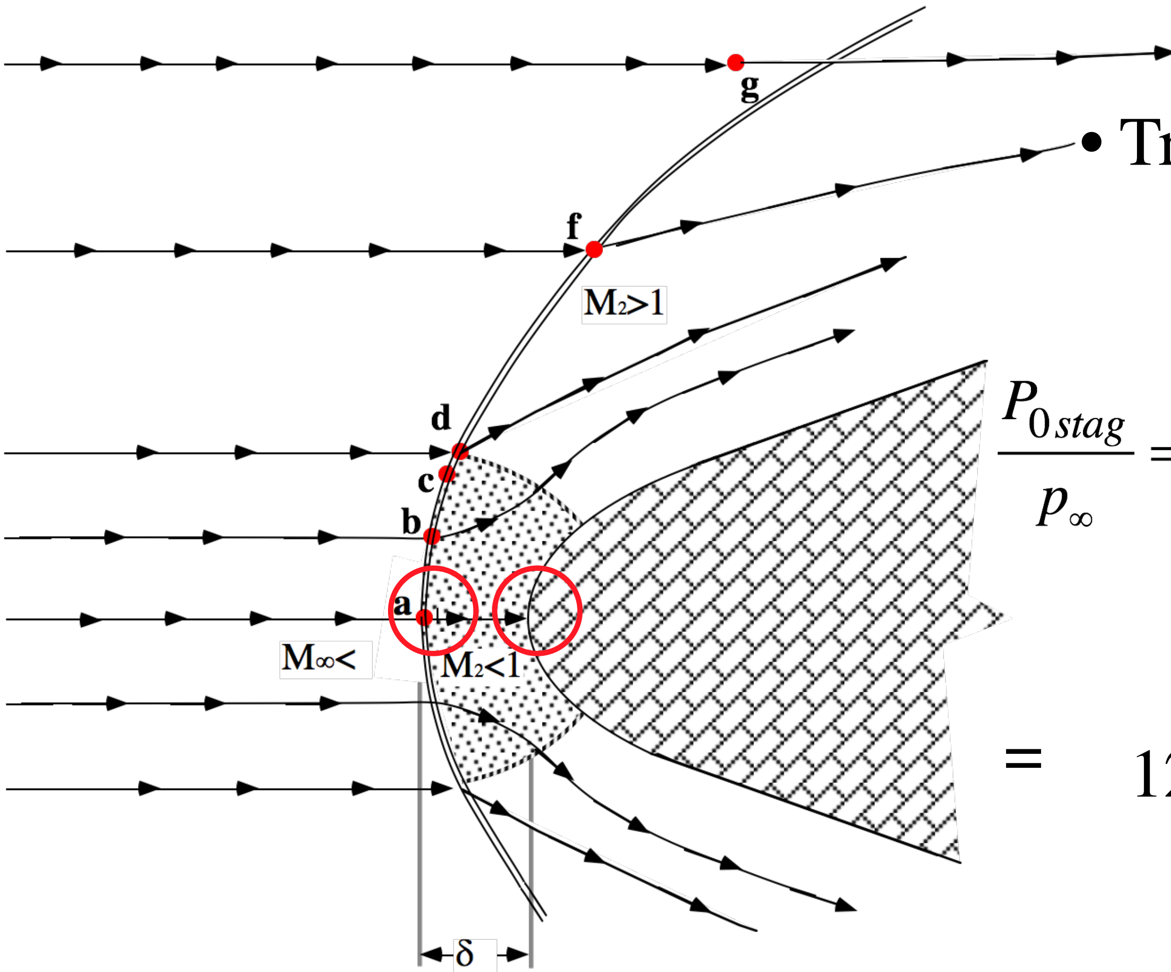
Example 3 (cont'd)

$$\frac{p_{sonic}}{p_\infty} = \frac{p_{sonic}}{P_{0sonic}}\frac{P_{0sonic}}{p_\infty} = \frac{1}{\left(1+\frac{\gamma+1}{2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}\left\{\frac{\left(\frac{\gamma+1}{2}\left[M_\infty \sin\left(\beta_{sonic}\right)\right]^2\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)}\left[M_\infty \sin\left(\beta_{sonic}\right)\right]^2 - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}}\right\} =$$

$$0.52828 \times 110.098512 = 64.16$$



Surface Conditions behind detached shock wave: *stagnation point* *(cont'd)*



- Treat as normal shockwave

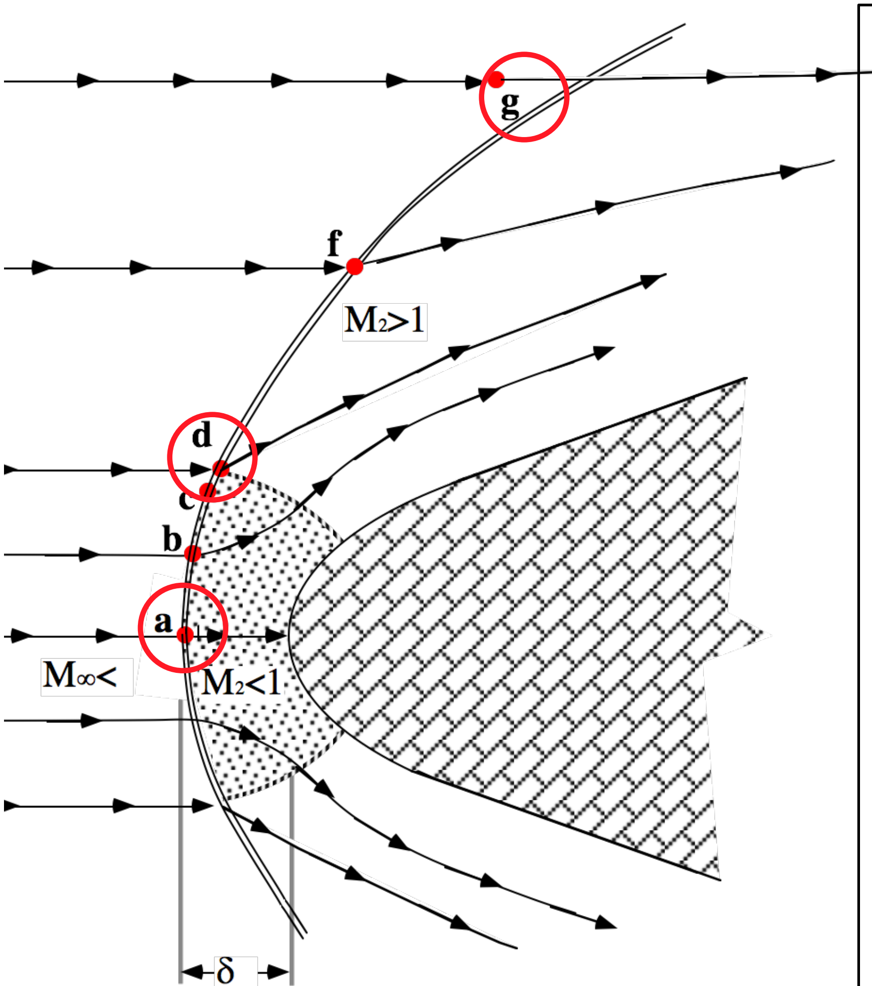
$$\frac{P_{0\,stag}}{p_\infty} = \left\{ \frac{\left(\frac{\gamma+1}{2} M_\infty^{\;2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}{\left(\frac{2\gamma}{(\gamma+1)} M_\infty^{\;2} - \frac{(\gamma-1)}{(\gamma+1)}\right)^{\left(\frac{1}{\gamma-1}\right)}} \right\}$$

$$= \quad 129.21$$

- $M_\infty = 10.0$

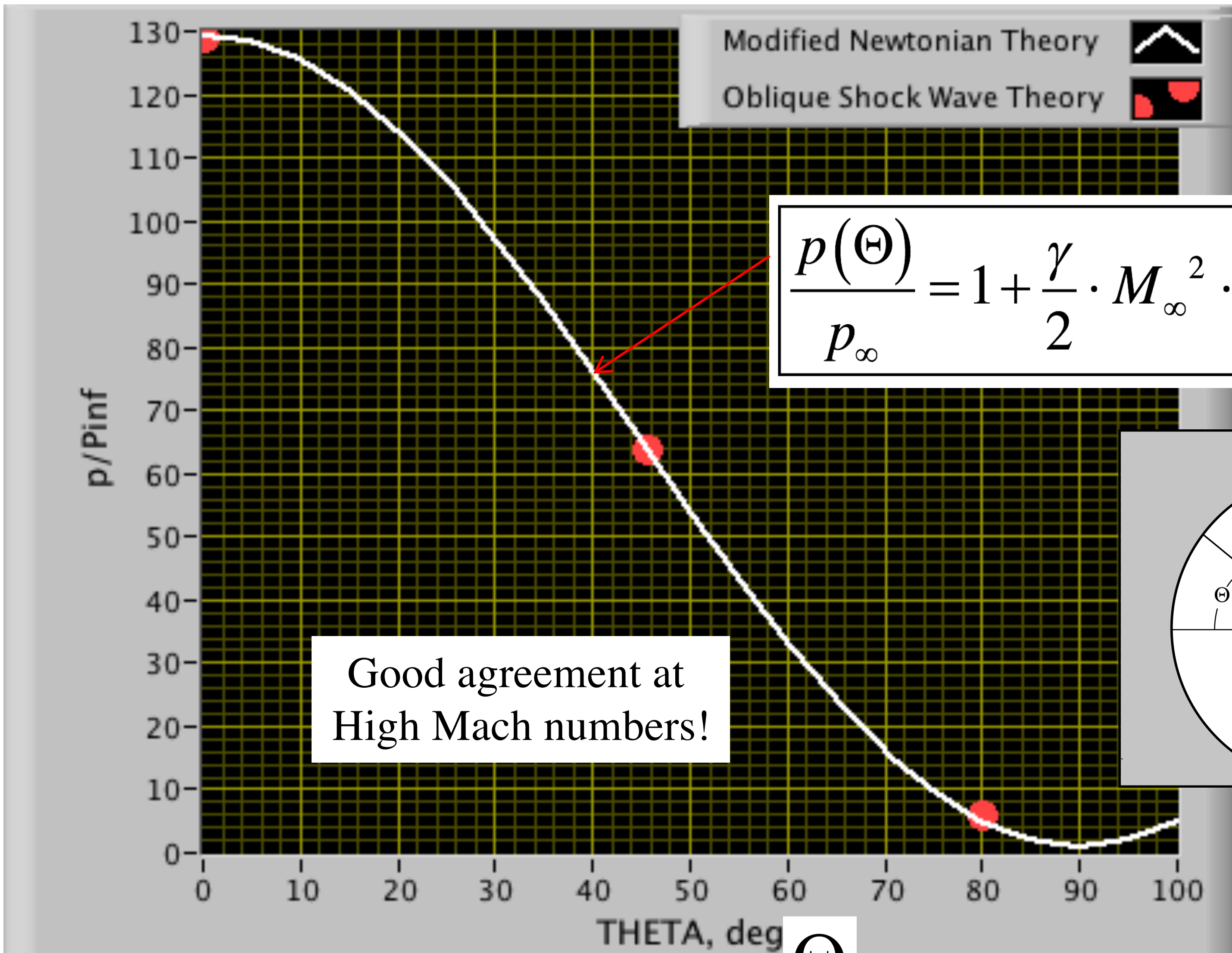
Plot Conditions along detached shock

summary $M_\infty = 10.0$



Point on Shock	beta, deg β	theta, deg θ	90°-theta Θ	$\frac{p_\theta}{p_\infty}$
a	90°	90°	0°	129.21
d	67.34°	44.43°	45.57°	64.16
g	14.43°	10°	80°	6.075

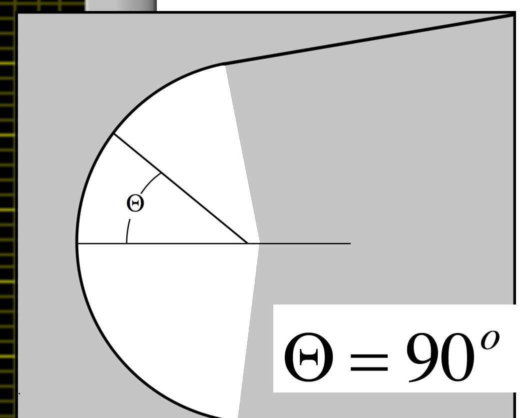
Surface Conditions behind detached shock wave: Mach 10.0



$$\frac{p(\Theta)}{p_\infty} = 1 + \frac{\gamma}{2} \cdot M_\infty{}^2 \cdot C_{p_{max}} \cdot \cos^2(\Theta)$$

Good agreement at High Mach numbers!

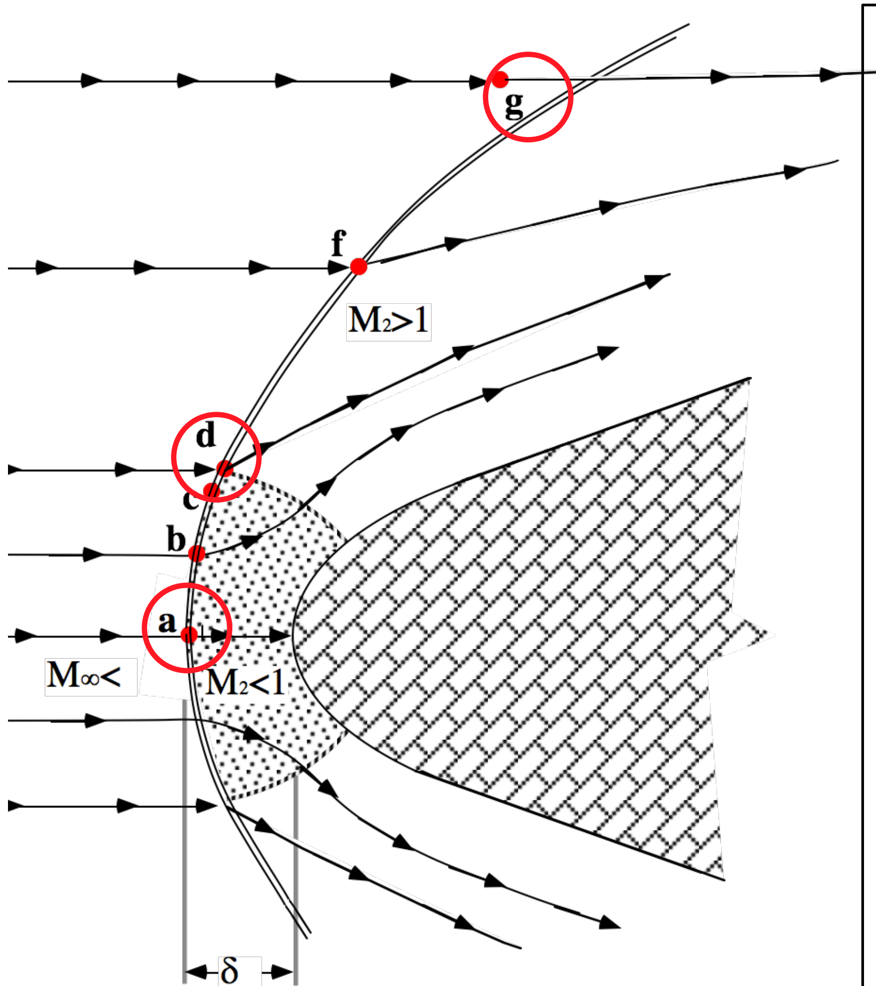
Θ



$$\Theta = 90^o - \theta$$

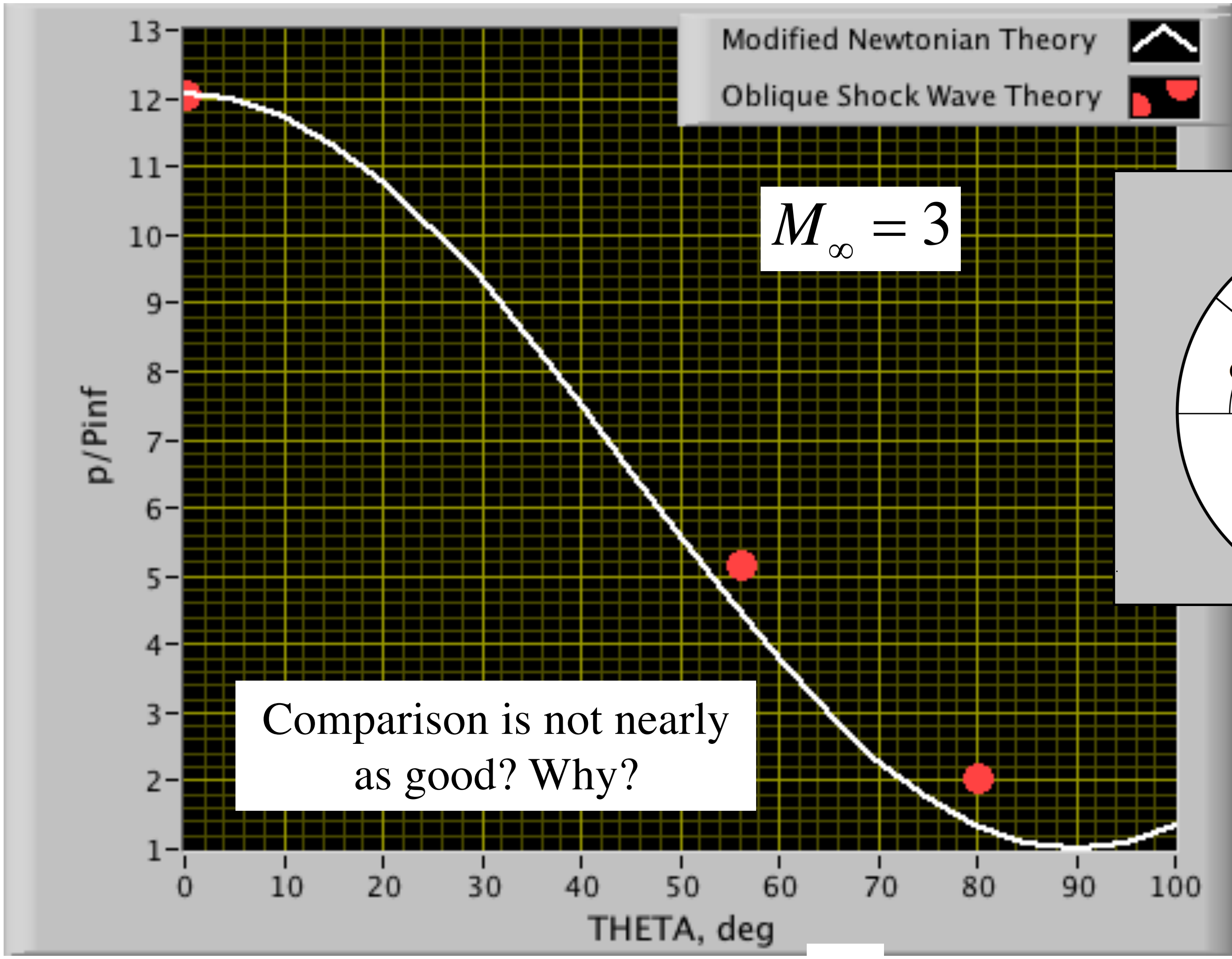
Repeat Oblique Shock Calculations at

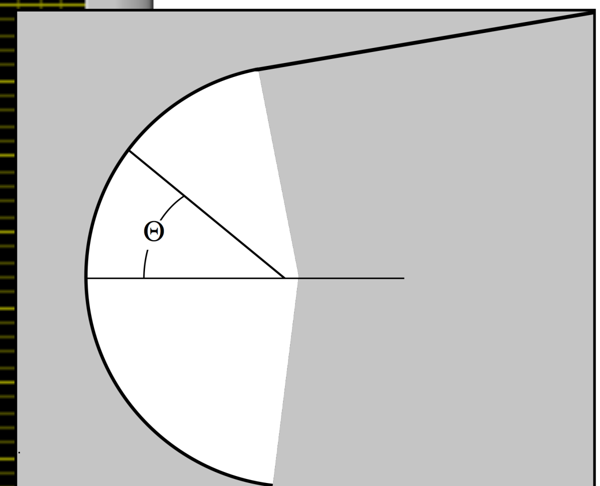
$$M_\infty = 3$$



Point on Shock	beta, deg β	theta, deg θ	90°- theta Θ	$\frac{p_\theta}{p_\infty}$
a	90°	90°	0°	12.06
d	63.77°	34.01°	55.99°	5.18
g	27.38°	10°	80°	2.05

Surface Conditions behind detached shock wave: *Mach 3.0*

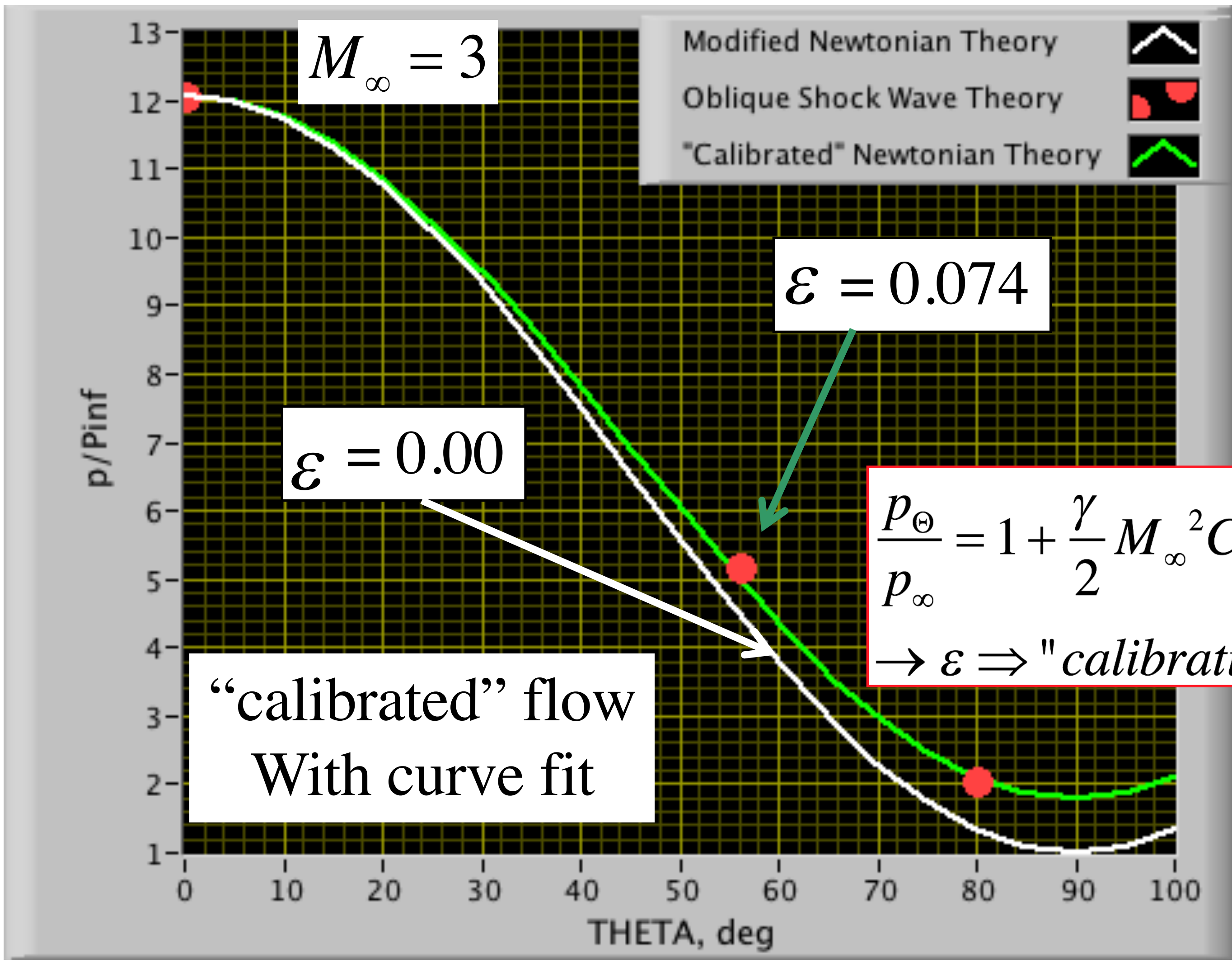


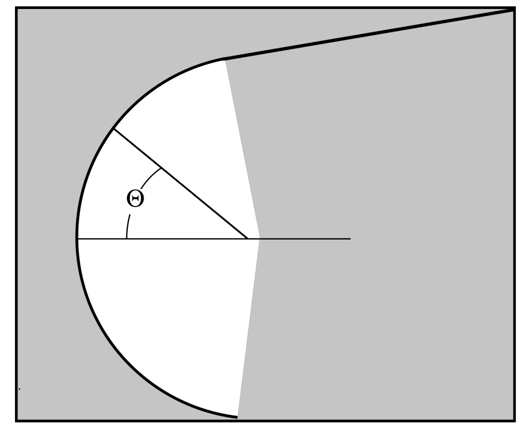


$$\Theta = 90^o - \theta$$

Θ

Surface Conditions behind detached shock wave: *Mach 3*





$$\frac{p_\Theta}{p_\infty} = 1 + \frac{\gamma}{2} M_\infty^{\;2} Cp_{Max}\left[\cos^2\Theta + \varepsilon \sin^2\Theta\right]$$

$$\rightarrow \varepsilon \Rightarrow \text{"}\mathit{calibration}\text{"} \approx 0.074$$

Surface Conditions behind detached shock wave: *Surface plot*

• In limit as M->infinity Modified Newtonian flow model is A good descriptor of the local surface pressures

$$\frac{p_\Theta}{p_\infty} = 1 + \frac{\gamma}{2} M_\infty{}^2 Cp_{Max} \cos^2 \Theta$$

• At lower Mach numbers we include a "calibration" term

$$\frac{p(\Theta)}{p_\infty} = 1 + \frac{\gamma}{2} \cdot M_\infty{}^2 \cdot C_{p_{\max}} \cdot \left[\cos^2(\Theta) + \varepsilon \cdot \sin^2(\Theta)\right]$$

Modified Newtonian Flow, Concluded

- Writing in terms of pressure coefficient

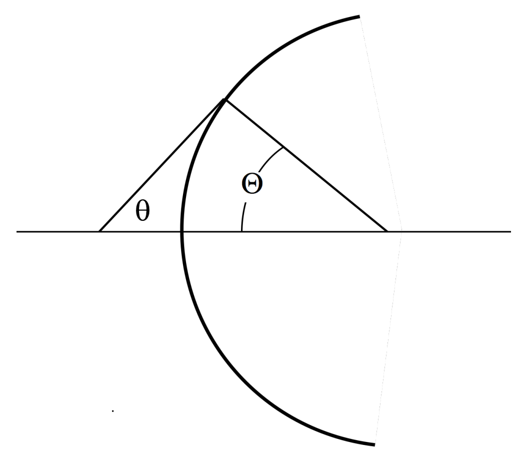
$$C_p(\Theta) = \frac{p(\Theta) - p_\infty}{\frac{\gamma}{2} \cdot p_\infty \cdot M_\infty^{\;2}} = C_{p\,\max} \cdot \left[\cos^2(\Theta) + \varepsilon \cdot \sin^2(\Theta)\right]$$

$$C_{p_{\max}} \cdot \left[\cos^2\Theta + \varepsilon \cdot \left(1 - \cos^2\Theta\right)\right] = C_{p_{\max}} \cdot \left[\left(1-\varepsilon\right) \cdot \cos^2\Theta + \varepsilon\right]$$

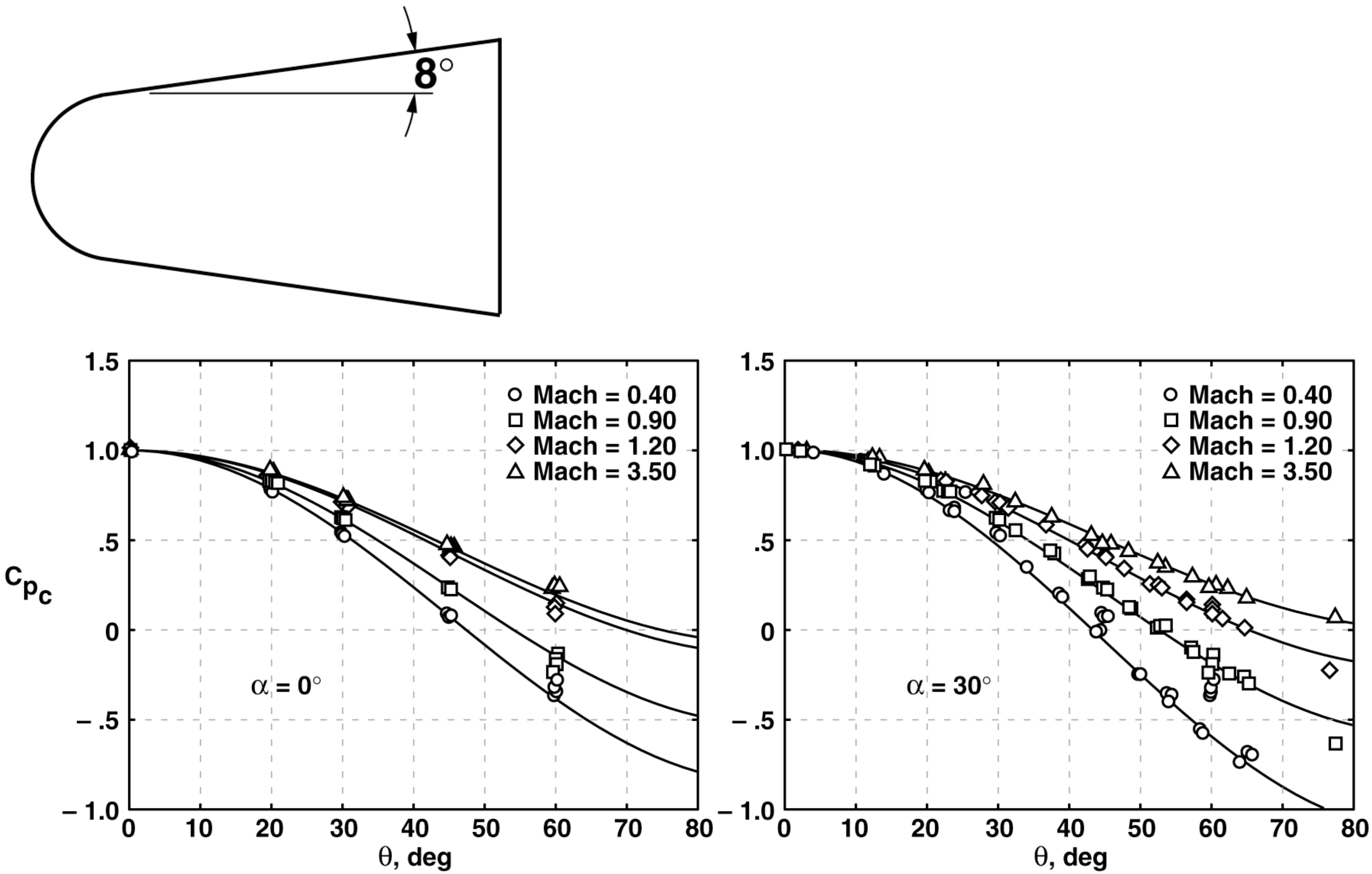
or...

$$C_p(\theta) = \frac{p(\theta) - p_\infty}{\frac{\gamma}{2} \cdot p_\infty \cdot M_\infty^{\;2}} = C_{p\,\max} \cdot \left[\sin^2(\theta) + \varepsilon \cdot \cos^2(\theta)\right] = C_{p_{\max}} \cdot \left[\left(1-\varepsilon\right) \cdot \sin^2(\theta) + \varepsilon\right]$$

- *"Calibrated Newtonian Flow"*
  Semi-empirical model
  Valid for very high speeds, 3-D
  And 2-D blunt bodies … accurate
  For both sonic and supersonic regions
  Behind detached shock wave



Calibration Examples: 8-deg Cone



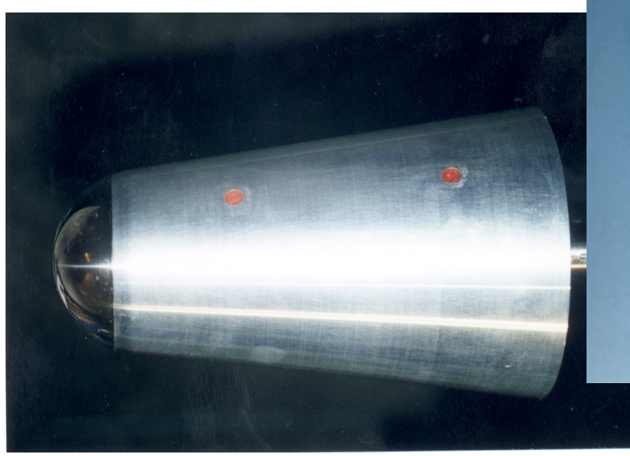
More Calibration Examples

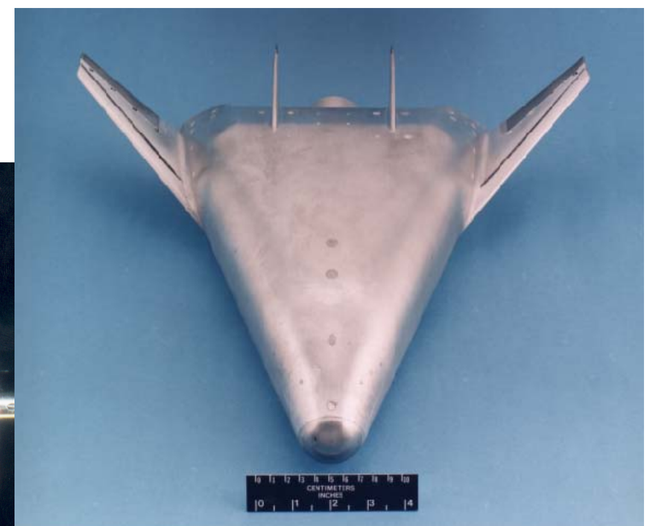
Analytical

- Potential solutions: sphere, cylinder, arbitrary ellipsoid

Wind Tunnel

- **8-deg sphere-cone, Rankine half-body, X-33 forebody (M=.3 - 4.75)**
- **9, 12.5, 18, and 45-deg sphere cones**
- **X-33**
- **X-34**
- **X-38**
- **Sphere**

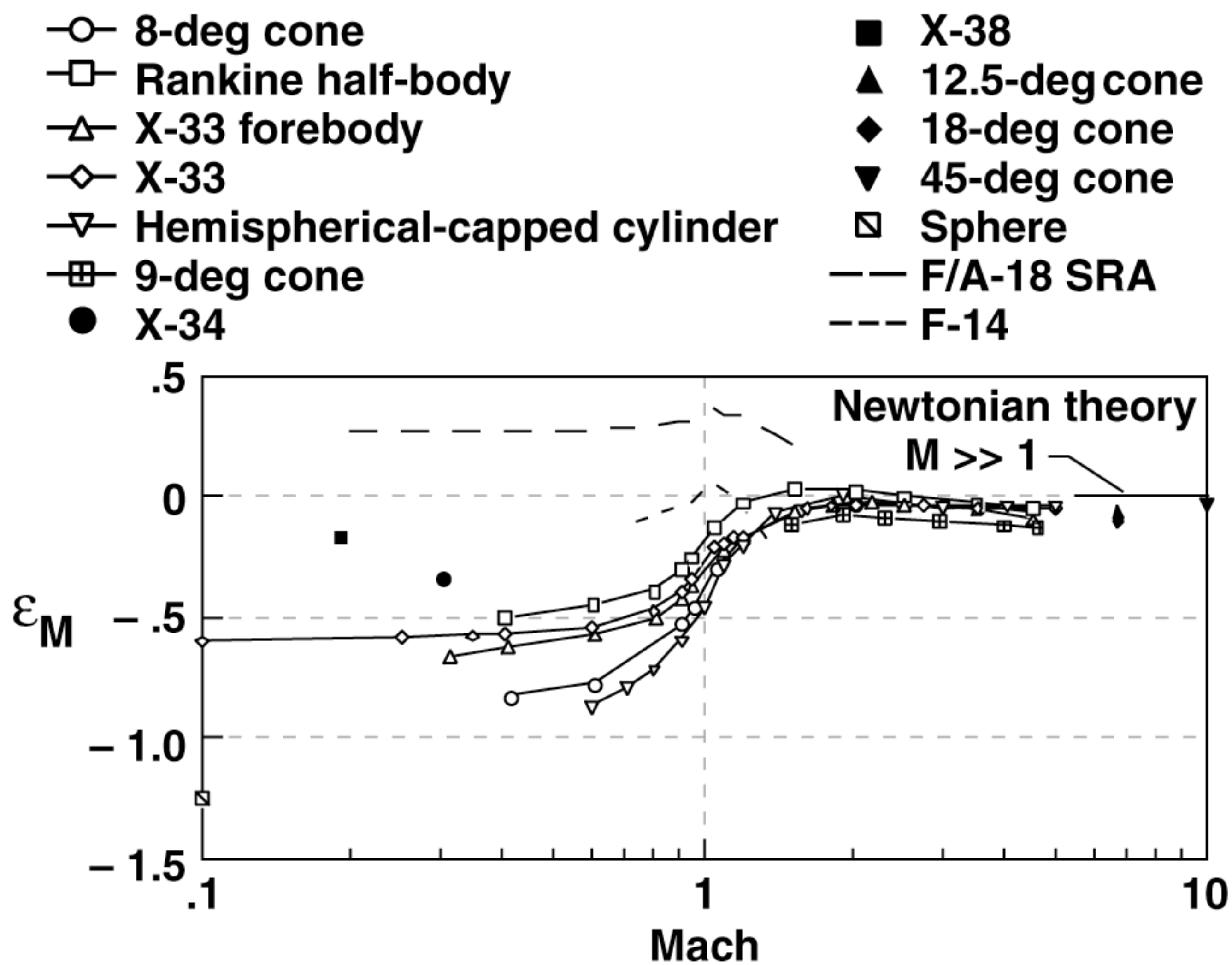




Flight Data

- F/A-18 SRA
- F-14

Mach Effect on Calibration (ε):



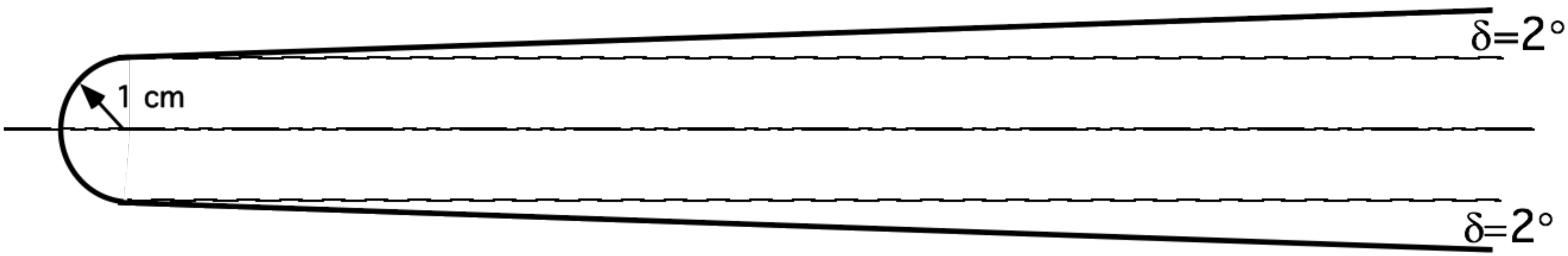
Bottom Line

- Even though flow across detached shock wave is extremely complex and is difficult to model theoretically, it turns out that simple engineering models do a good job with the pressure distribution

- Allow the drag due to detached shock wave to be estimated using very simple tools

Homework 7.1, Blunt Edge Drag

- Consider a 2° angle (δ) diamond-wedge wing, 2 meter chord (c), With A spherically blunted leading edge with a 1 cm radius (R)



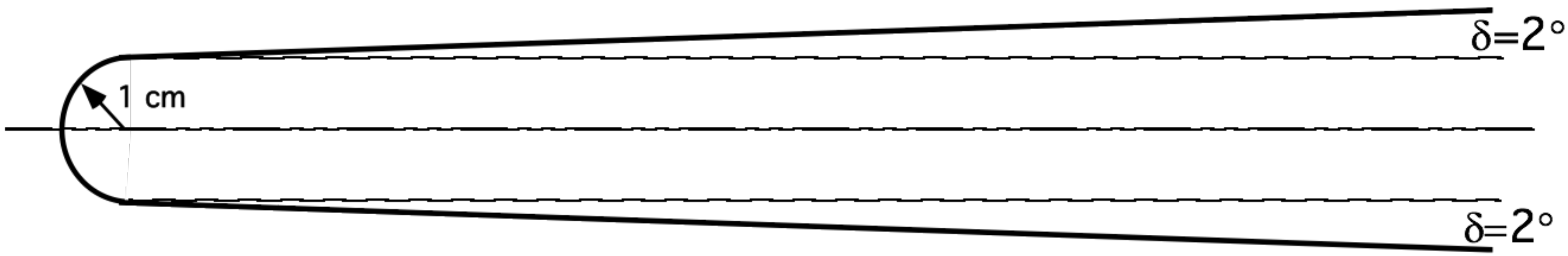
- Show that in general, the drag coefficient due to the the blunt leading edge is:

$$C_{D_{LE}} = \int_{-\left(\frac{\pi}{2}-\delta\right)}^{\left(\frac{\pi}{2}-\delta\right)} Cp(\Theta)\cos(\Theta)\frac{R}{c}\,d\Theta$$

Homework 7.1, Blunt Edge Drag (cont'd)

- Use the calibrated Newtonian Model

$$Cp(\Theta) = Cp_{max}\left[\cos^2(\Theta) + \varepsilon \sin^2(\Theta)\right] = C_{p_{max}} \cdot \left[(1-\varepsilon)\cdot\cos^2\Theta + \varepsilon\right]$$



… calculate the leading edge drag coefficient
at M_∞= 3.0 (ε$\underset{\sim}{}$0.074) AND M_∞= 10.0 (ε$\underset{\sim}{}$0.00)

- Compare to wave drag coefficient and skin drag coefficient (referenced to plan area) for the same conditions at 25 km altitude


Questions??