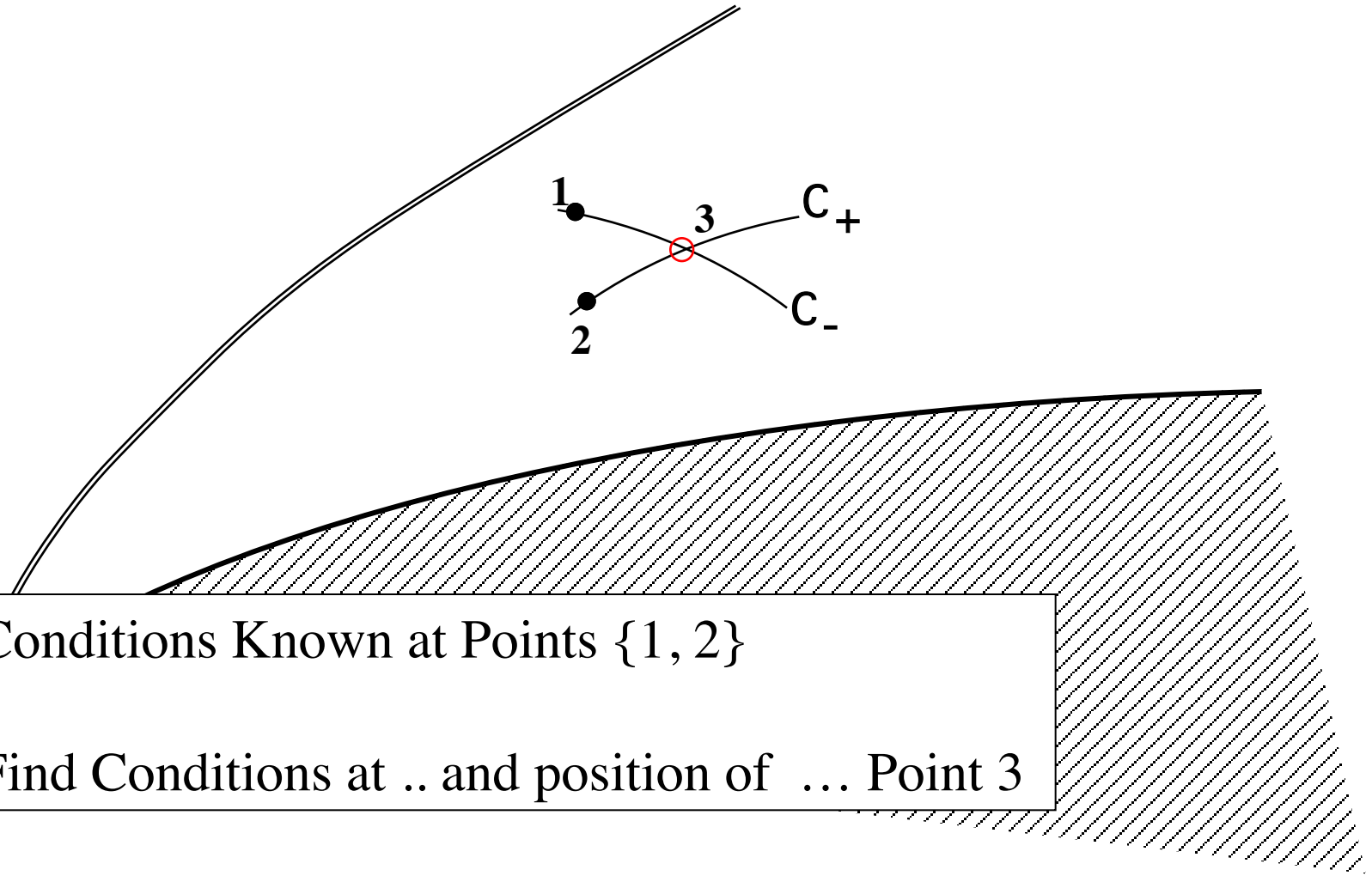
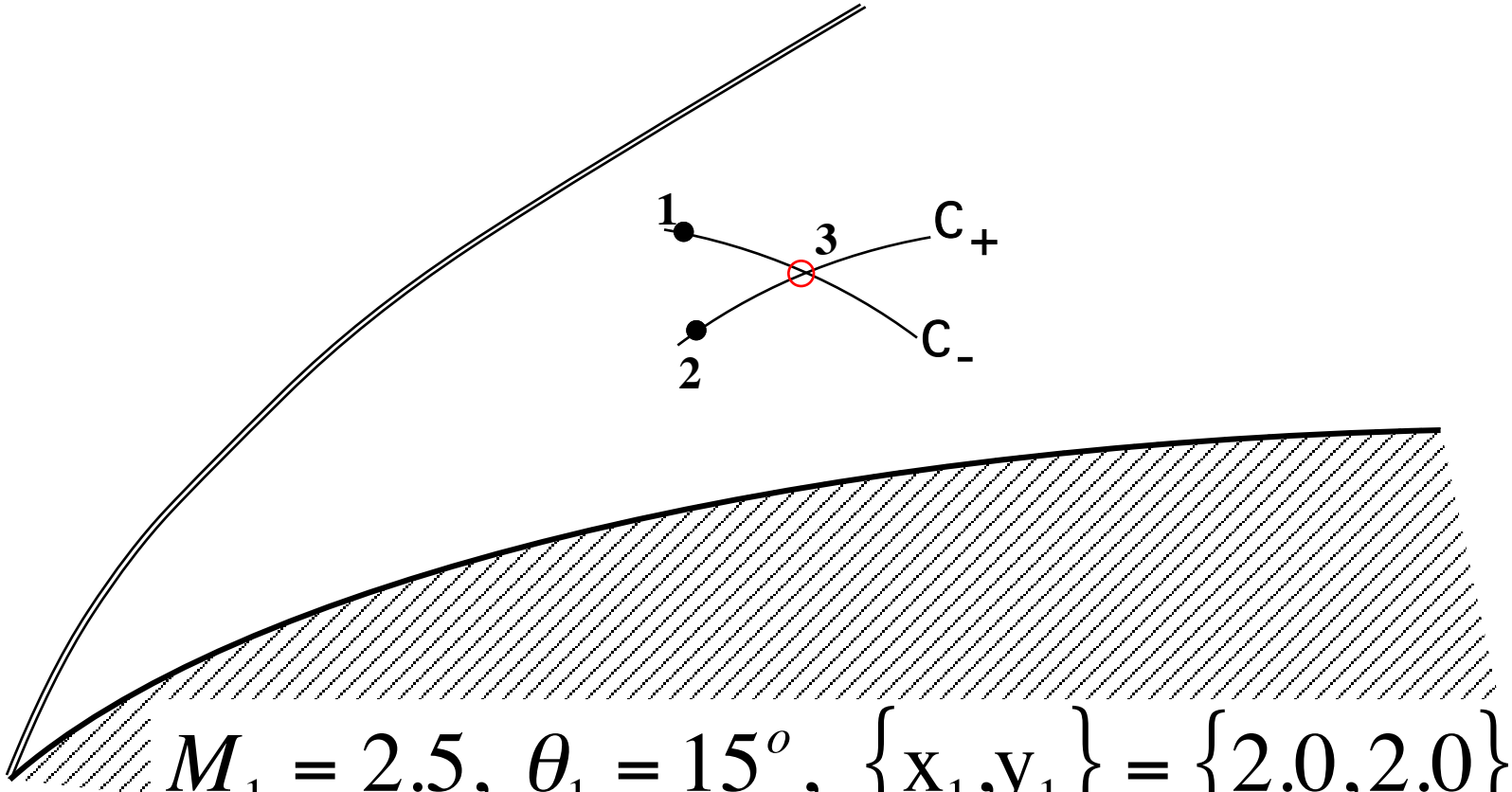
Homework 1 Solution



- Conditions Known at Points {1, 2}
- Find Conditions at .. and position of … Point 3

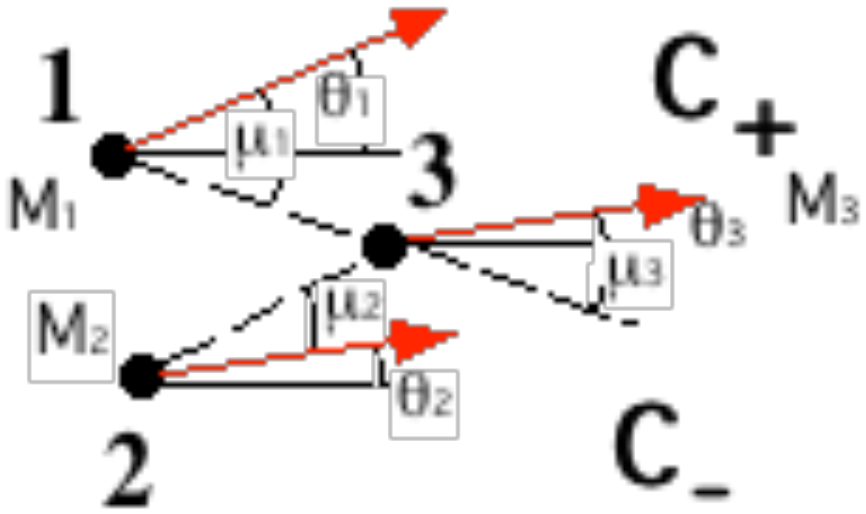
Homework 1 Solution (cont'd)



$$M_1 = 2.5,\ \theta_1 = 15^o,\ \{x_1, y_1\} = \{2.0, 2.0\}$$

$$M_2 = 2.00,\ \theta_2 = 5^o,\ \{x_2, y_2\} = \{2.5, 1.0\}$$

Homework 1 Solution: Internal Flow Example

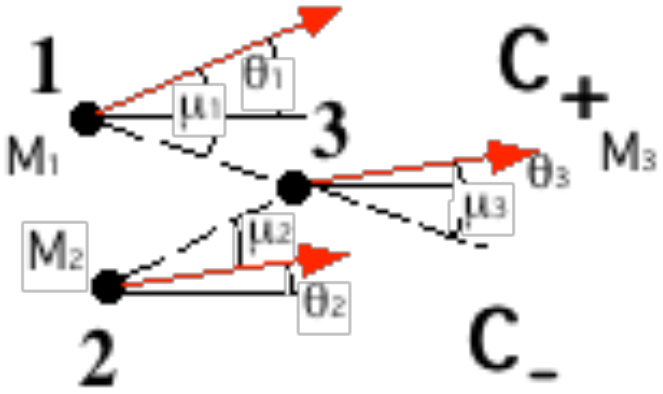


$$M_1 = 2.5,\ \theta_1 = 15^o,\ \{x_1, y_1\} = \{2.0, 2.0\}$$

$$M_2 = 2.00,\ \theta_2 = 5^o,\ \{x_2, y_2\} = \{2.5, 1.0\}$$

Homework 1 Solution: Internal Flow Example

(cont'd)



- Point 1, compute

$$\left\{\nu_1, \mu_1, \left(K_-\right)_1\right\}$$

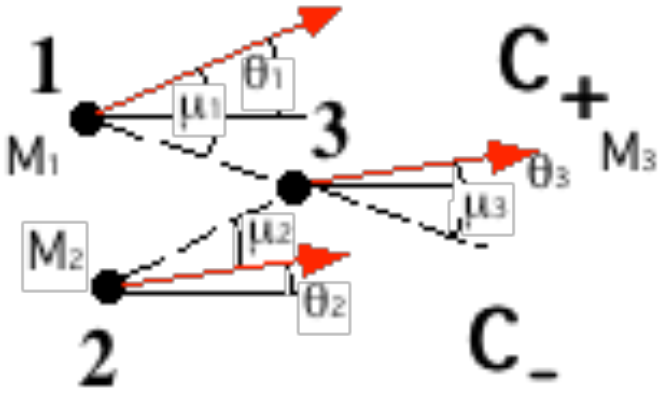
$$\nu_1 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(2.5^2-1\right)}\right\} - \tan^{-1}\sqrt{2.5^2-1} = 39.12^o$$

$$\mu_1 = \frac{180}{\pi}\sin^{-1}\left[\frac{1}{2.5}\right] = 23.58^o$$

$$\left(K_-\right)_1 = \theta_1 + \nu_1 = 15^o + 39.12^o = 54.12^o$$

Homework 1 Solution: Internal Flow Example

(cont'd)



- Point 2, compute

$$\left\{\nu_2, \mu_2, \left(K_+\right)_2\right\}$$

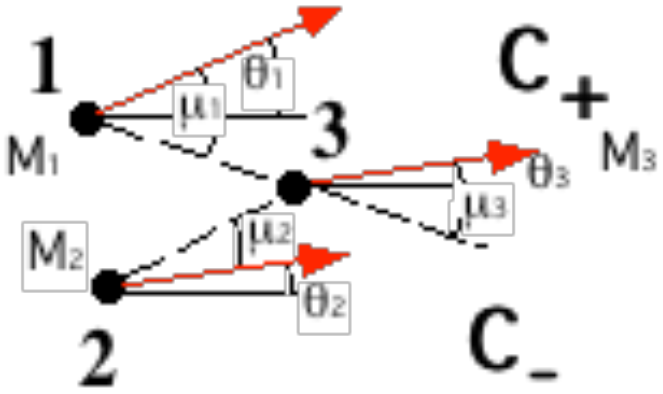
$$\nu_2 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(2^2-1\right)}\right\} - \tan^{-1}\sqrt{2^2-1} = 26.38^\circ$$

$$\mu_2 = \frac{180}{\pi}\sin^{-1}\left[\frac{1}{2}\right] = 30^o$$

$$\left(K_+\right)_2 = \theta_2 - \nu_2 = 5^o - 26.38^\circ = -21.38^\circ$$

Homework 1 Solution: Internal Flow Example

(cont'd)



- Point 3 Solve for

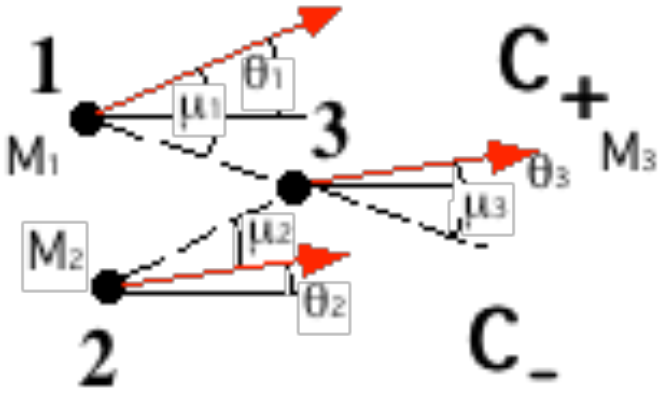
$$\{\theta_3, \nu_3\}$$

$$\theta_3 = \frac{(K_-)_1 + (K_+)_2}{2} = \frac{54.12^\circ + (-21.38^\circ)}{2} = 16.37^o$$

$$\nu_3 = \frac{(K_-)_1 - (K_+)_2}{2} = \frac{54.12^\circ - (-21.38^\circ)}{2} = 37.75^o$$

Homework 1 Solution: Internal Flow Example

(cont'd)



- Point 3 Solve for

$$\{M_3, \mu_3\}$$

$$M_3 = Solve\left[37.75^o \frac{\pi}{180} = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_3^{\,2}-1\right)}\right\} - \tan^{-1}\sqrt{M_3^{\,2}-1}\right]$$

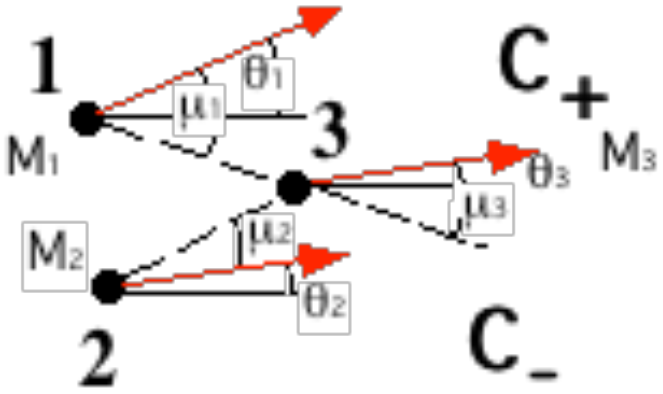
$$\sin(\mu) = \frac{1}{M}$$

$$M_3 = 2.442$$

$$\text{---> } \mu_3 = \quad 24.175^o$$

Homework 1 Solution: Internal Flow Example

(cont'd)



- Locate Point 3

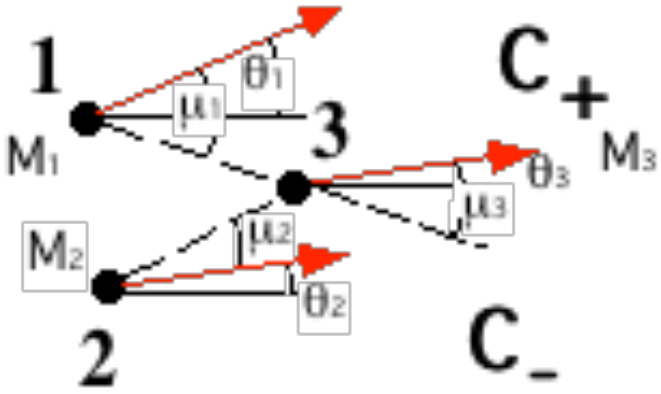
- Line Slope Angles

$$slope\{C_-\} = \frac{(\theta_1 - \mu_1) + (\theta_3 - \mu_3)}{2} = \frac{(15^o - 23.58^o) + (16.372^o - 24.175^o)}{2} = -8.192^o$$

$$slope\{C_+\} = \frac{(\theta_2 + \mu_2) + (\theta_3 + \mu_3)}{2} = \frac{(5^o + 30.00^o) + (16.372^o + 24.175^o)}{2} = 37.774^o$$

Homework 1 Solution: Internal Flow Example

(cont'd)

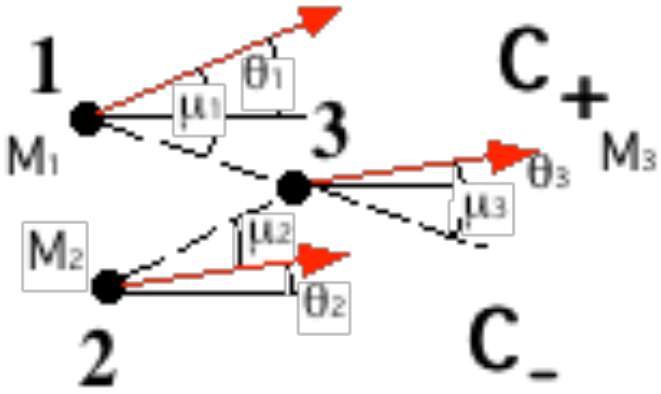


- Solve for $\{x_3, y_3\}$

$$\begin{bmatrix} \dfrac{y_3 - y_1}{x_3 - x_1} = \tan\left[slope\left\{C_-\right\}\right] \\ \dfrac{y_3 - y_2}{x_3 - x_2} = \tan\left[slope\left\{C_+\right\}\right] \end{bmatrix} \rightarrow \begin{bmatrix} y_3 = \left(x_3 - x_1\right)\tan\left[slope\left\{C_-\right\}\right] + y_1 \\ y_3 = \left(x_3 - x_2\right)\tan\left[slope\left\{C_+\right\}\right] + y_2 \end{bmatrix}$$

Homework 1 Solution: Internal Flow Example

(cont'd)

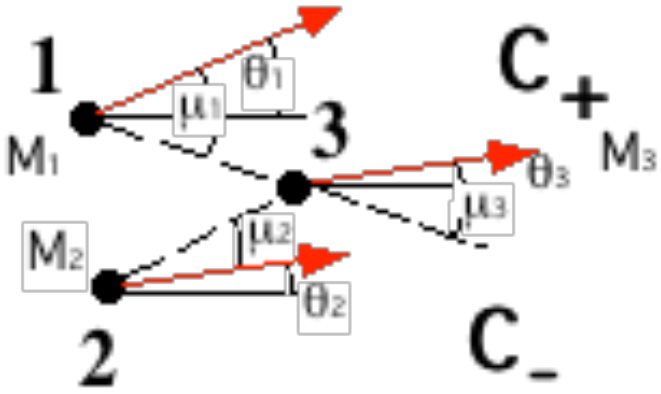


- Solve for $\{x_3, y_3\}$

$$\begin{bmatrix} x_3 = \dfrac{x_1 \tan\left[slope\{C_-\}\right] - x_2 \tan\left[slope\{C_+\}\right] + \boldsymbol{y2\text{-}y1}}{\tan\left[slope\{C_-\}\right] - \tan\left[slope\{C_+\}\right]} \\ y_3 = \dfrac{\tan\left[slope\{C_-\}\right]\tan\left[slope\{C_+\}\right](x_1 - x_2) - \tan\left[slope\{C_+\}\right]y_1 + \tan\left[slope\{C_-\}\right]y_2}{\tan\left[slope\{C_-\}\right] - \tan\left[slope\{C_+\}\right]} \end{bmatrix}$$

Homework 1 Solution: Internal Flow Example

(cont'd)



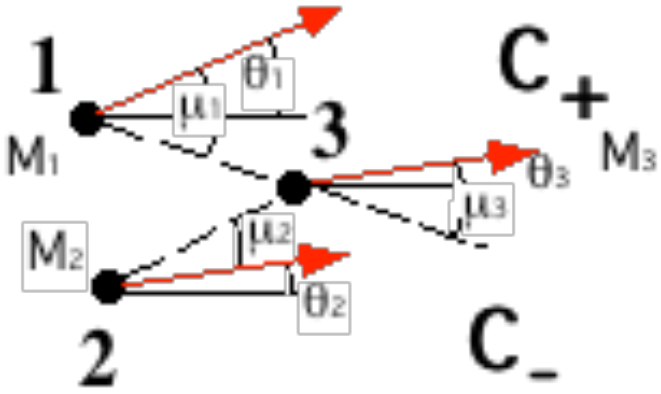
- Solve for $\{x_3, y_3\}$

$$x_3 = \frac{\left(\left(\left(\tan\left(\frac{\pi}{180}(-\mathbf{8.191})\right)\cdot 2\right) - \tan\left(\frac{\pi}{180}(37.774)\right)\right)\cdot 2.5\right) + \boldsymbol{1 - 2}\ \boldsymbol{)}}{\tan\left(\frac{\pi}{180}(-8.191)\right) - \tan\left(\frac{\pi}{180}(37.774)\right)}$$

$= \cancel{3.5095}$ *3.50996*

Homework 1 Solution: Internal Flow Example

(cont'd)



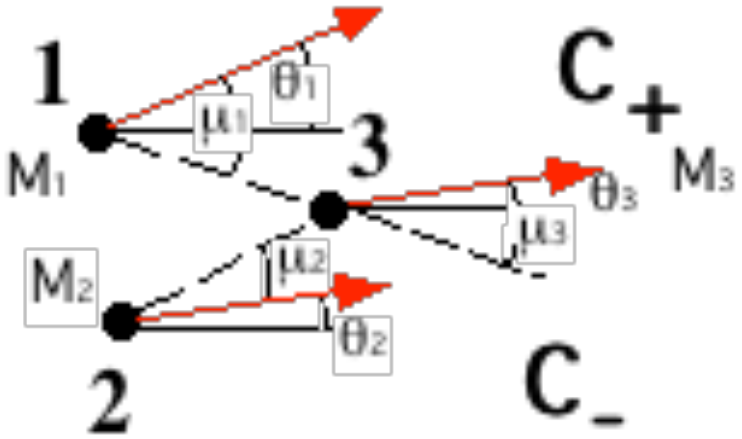
- Solve for $\{x_3, y_3\}$

$$y_3 = \frac{\tan\left(\frac{\pi}{180}(-8.191)\right)\cdot\tan\left(\frac{\pi}{180}(37.774)\right)\cdot(2-2.5) - \left(\tan\left(\frac{\pi}{180}(37.774)\right)\right)2 + \tan\left(\frac{\pi}{180}(-8.191)\right)\cdot 1}{\tan\left(\frac{\pi}{180}(-8.191)\right) - \tan\left(\frac{\pi}{180}(37.774)\right)}$$

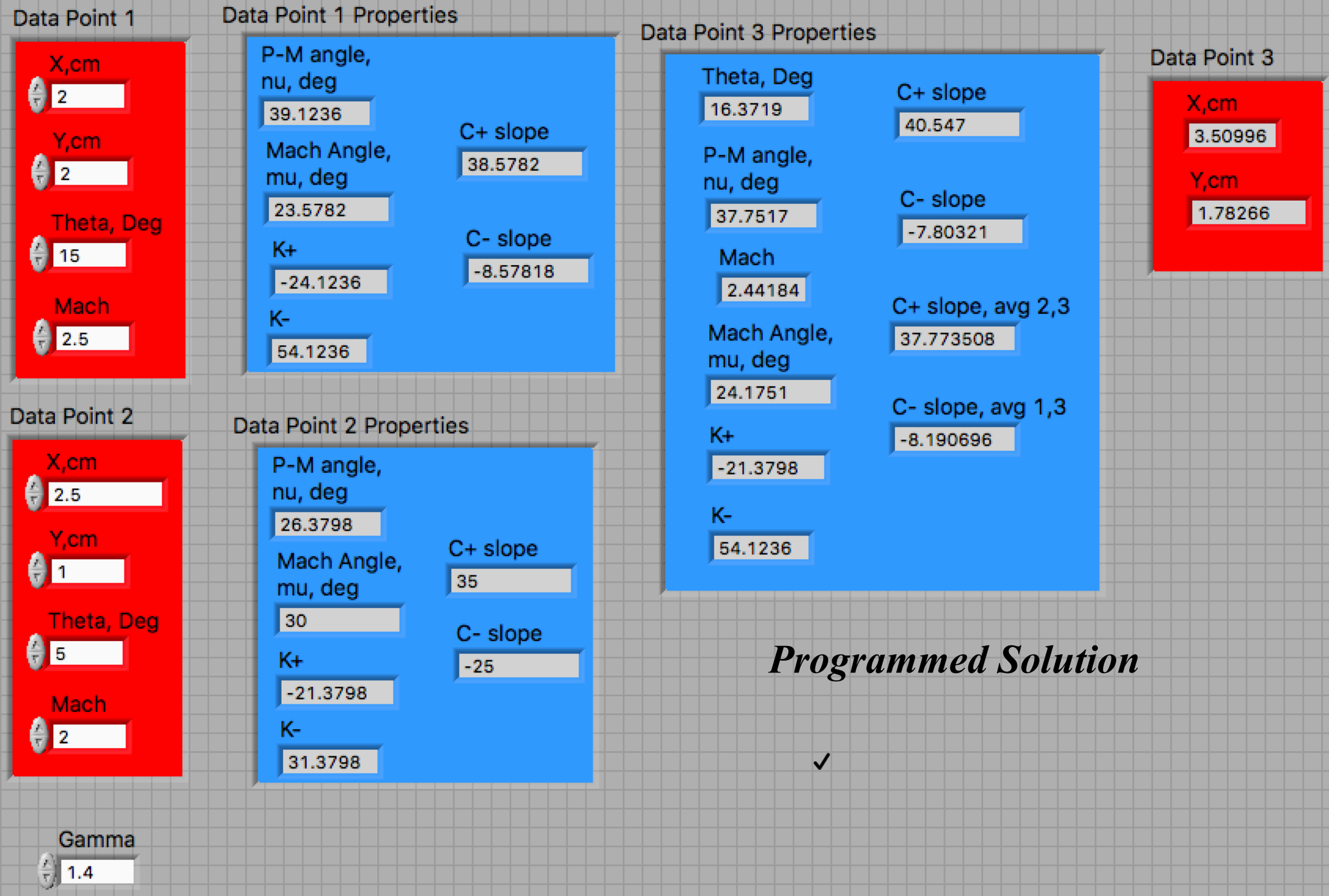
$$= 1.782$$

Homework 1 Solution: Internal Flow Example

(concluded)



$$\begin{bmatrix} M_1 \\ \theta_1 \\ x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 15^o \\ 2.0 \\ 2.0 \end{bmatrix} \rightarrow \begin{bmatrix} M_2 \\ \theta_2 \\ x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.00 \\ 5^o \\ 2.5 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} M_3 \\ \theta_3 \\ x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2.442 \\ 16.372^o \\ 3.510 \\ 1.782 \end{bmatrix}$$


Data Point 1
X,cm
2
Y,cm
2
Theta, Deg
15
Mach
2.5
Data Point 1 Properties
P-M angle, nu, deg
39.1236
Mach Angle, mu, deg
23.5782
K+
-24.1236
K-
54.1236
C+ slope
38.5782
C- slope
-8.57818
Data Point 2
X,cm
2.5
Y,cm
1
Theta, Deg
5
Mach
2
Data Point 2 Properties
P-M angle, nu, deg
26.3798
Mach Angle, mu, deg
30
K+
-21.3798
K-
31.3798
C+ slope
35
C- slope
-25
Gamma
1.4
Data Point 3 Properties
Theta, Deg
16.3719
P-M angle, nu, deg
37.7517
Mach
2.44184
Mach Angle, mu, deg
24.1751
K+
-21.3798
K-
54.1236
C+ slope
40.547
C- slope
-7.80321
C+ slope, avg 2,3
37.773508
C- slope, avg 1,3
-8.190696
Data Point 3
X,cm
3.50996
Y,cm
1.78266
Programmed Solution
✓