Homework 1.2

Assignment 1.2 Date Assigned: **Wednesday September 7, 2022**
Date Due: **Mondy September 20, 2021**
Title: 2-D ***Method of Characteristics*** (M.O.C.) Grid Solver Development
Num. of Points: 10

- Code and Verify subroutines or scripts for
  - Initial Data Line along Expansion Section Wall
  - Internal Flow Unit Process
  - Centerline Intercept Unit Process (C- characteristic line)
  - Wall Intercept Unit Process (C+ characteristic line)
  - Minimum Length Nozzle Maximum Turning Angle
- Link and Sequence Unit Process Modules to Calculate M.O.C Grid

Homework 1.2 (2)

Solve Problem 11.1 in Anderson, page 429. (See Section 1.1 Notes for Example)

... *Minimum Length Nozzle with Maximum Turning Angle -- infinitesimal expansion section*

... $M_{exit} = 2.0$

... $D^* = 2.0$ cm

... *Assume* $\gamma = 1.4$

... *Repeat with* $\gamma = 1.2$

Solve Problem 11.2 in Anderson, Page 430 but with ...

... *Finite expansion section radius of Curvature equal to 1.5 x throat radius*

... $M_{exit} = 2.0$

... $D^* = 2.0$ cm

... *Assume* $\gamma = 1.4$

... *Repeat with* $\gamma = 1.2$

for all parts

Plot nozzle half-contours

Plot nozzle Mach number profile along upper wall and along centerline

Compare to Mach number profile calculated using A/A* equation

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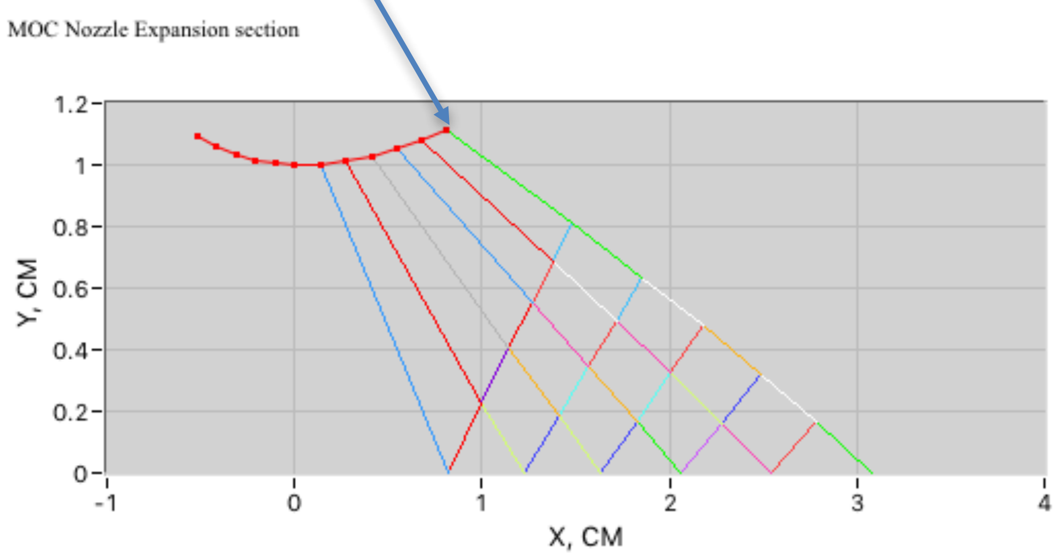
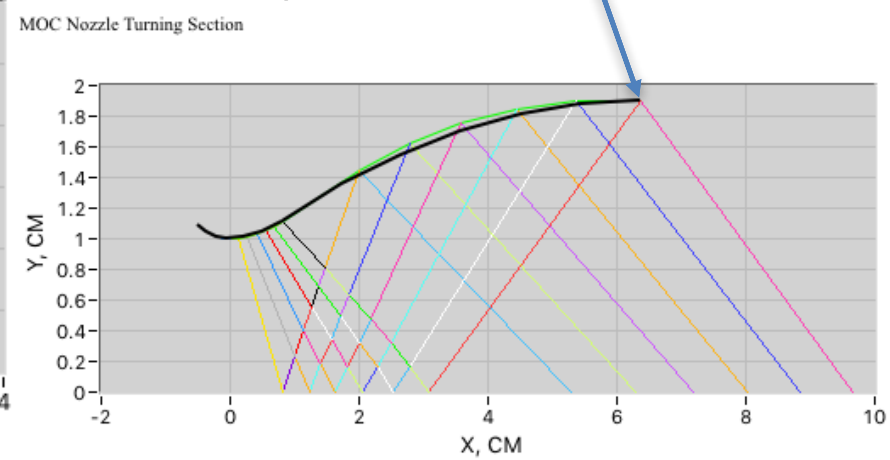
Homework 1.2 (3)

For each of the 4 Nozzle Contours above →

- Use the approximate "bell-curve" mapping technique to solve for P, Q, S, and T of the Parabolic Contour
- Plot Contours against the derived M.O.C contours, compare shapes
- Use M.O.C. values for θ_{max}, Nozzle length L_N, and Radius of Curvature R_c of the expansion section for these calculations
- Be sure to show Calculations for X_n, Y_n, P, Q, S, T, etc.
- Assume Nozzle exit angle is zero for each case
- Use "X" data from turning section to create Bell curve
- Assume X_n, Y_n are from final point on wall for *Expansion Section*



- Assume X_e, Y_e are from final point on Turning Section wall



Solution 1.2, part 1

- **Minimum Length Nozzle … <u>γ = 1.4</u>**

$$M_{exit} = 2.0 \rightarrow \nu(M_{exit}) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}}\left(2.0^2-1\right)\right\} - \tan^{-1}\sqrt{2.0^2-1} =$$

$$\frac{180}{\pi}\left(\left(\frac{1.4+1}{1.4-1}\right)^{0.5} \text{atan}\left(\left(\left(\frac{1.4-1}{1.4+1}\right)(2.0^2-1)\right)^{0.5}\right) - \text{atan}\left(\left((2.0^2-1)\right)^{0.5}\right)\right)$$

$$= 26.3798^{o}$$

$$\theta_{w_{Max}} = \frac{\nu_{exit}}{2} = 13.1899^{o}$$

$$M_{exit} = 2.0 \rightarrow \frac{A_{exit}}{A^*} = 1.6875$$

Solution 1.2, part 1 (4)

- **Minimum Length Nozzle … $\underline{\gamma = 1.2}$**
- **Let N=6**

$$M_{exit} = 2.0 \rightarrow \nu(M_{exit}) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\left\{\sqrt{\frac{\gamma-1}{\gamma+1}\left(2.0^2-1\right)}\right\} - \tan^{-1}\sqrt{2.0^2-1} =$$

$$= 31.46^{o}$$

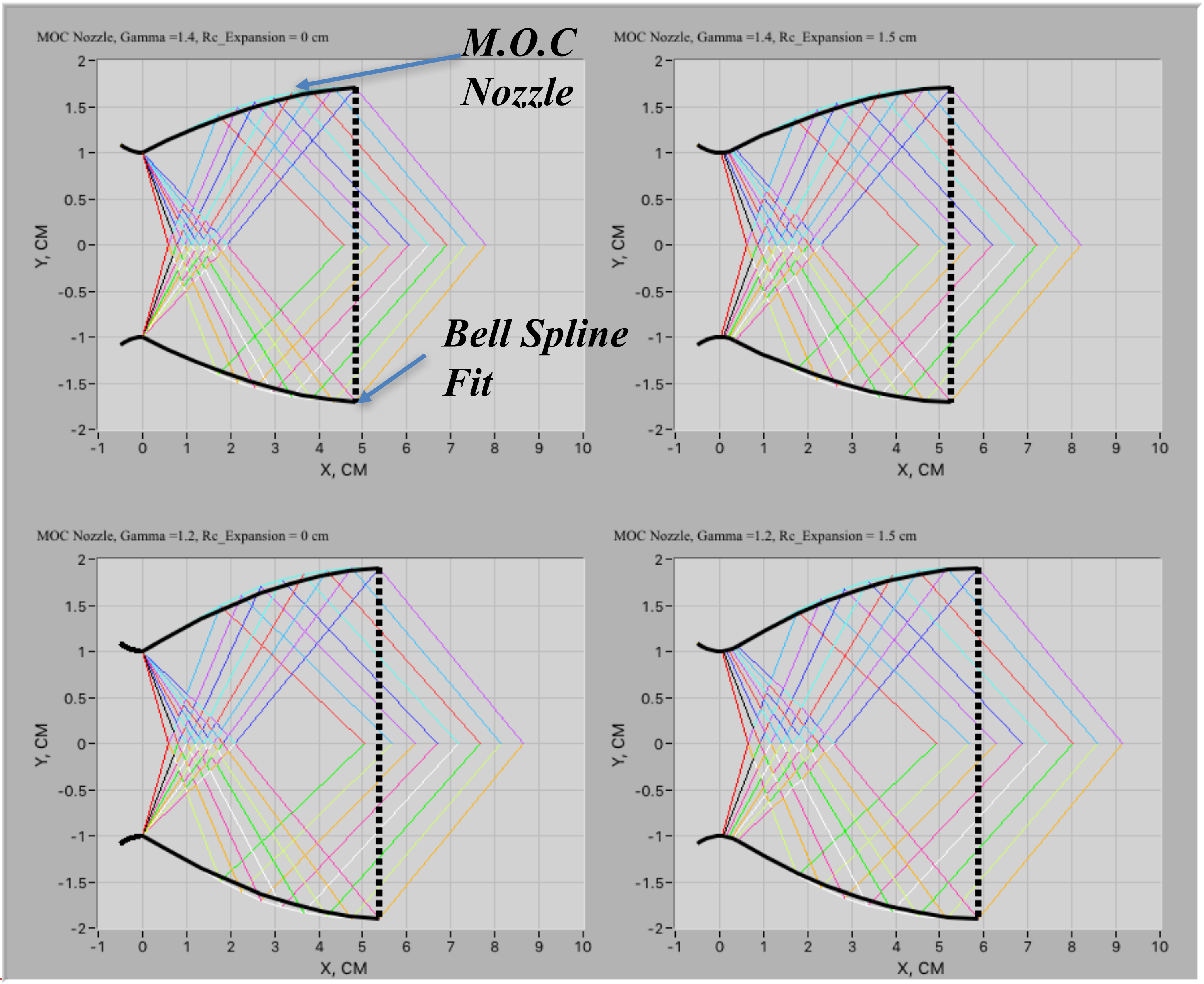
Maximium Turning Angle changes!

$$\theta_{w_{Max}} = \frac{\nu_{exit}}{2} = 15.731^{o}$$

$$M_{exit} = 2.0 \rightarrow \frac{A_{exit}}{A^*} = 1.884$$

Solution 1.2, Nozzle Profile Comparisons, N=8

M.O.C Nozzle Plot Clustzer



$\gamma = 1.4$

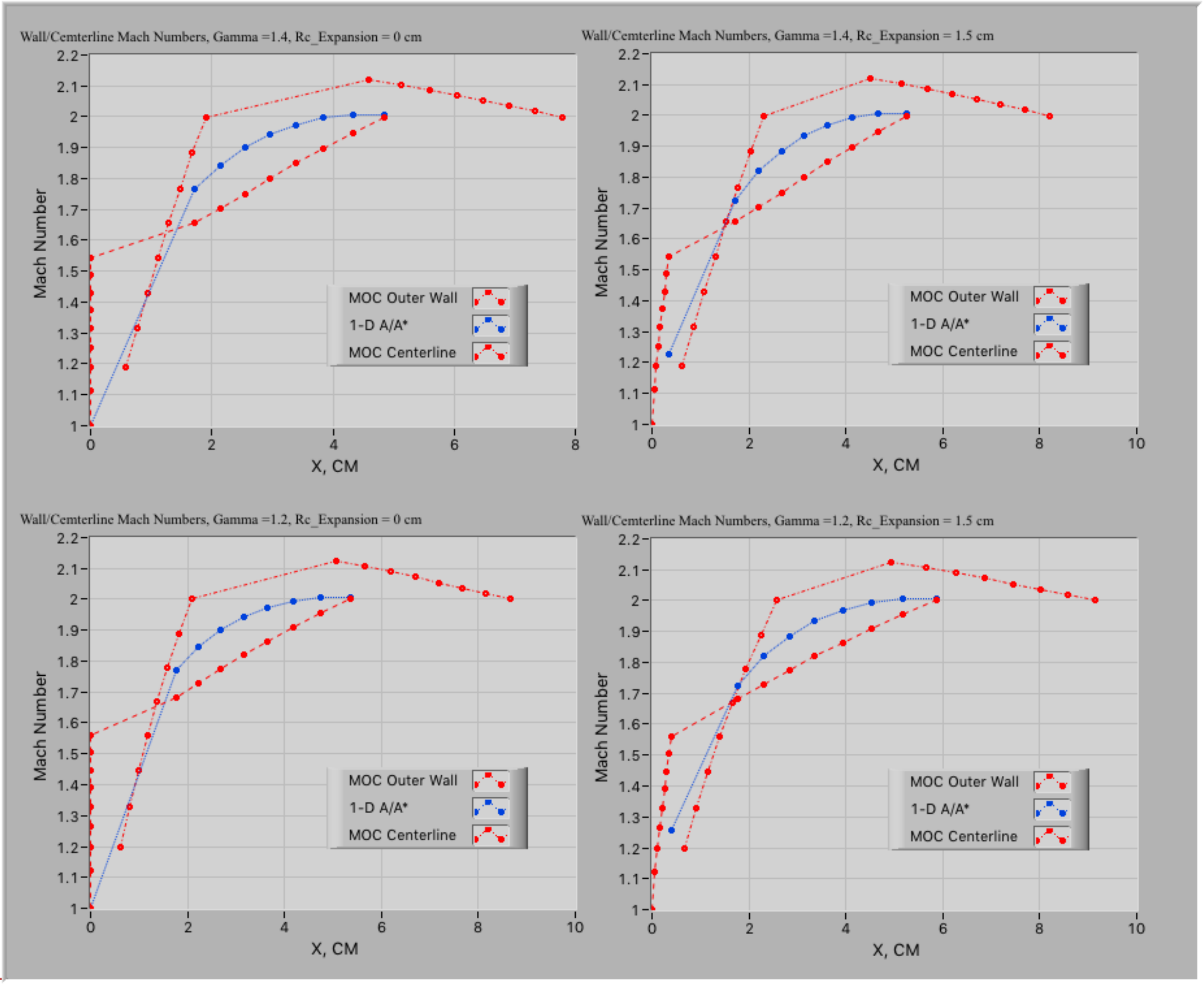
$\theta_{w_{Max}} = 13.1899^o$

$\gamma = 1.2$

$\theta_{w_{Max}} = 15.731^o$

Solution 1.2, Mach Number Comparisons, N=8

M.O.C. Mach Plot Cluster



$\gamma = 1.4$

$\theta_{w_{Max}} =$

$= 13.1899^o$

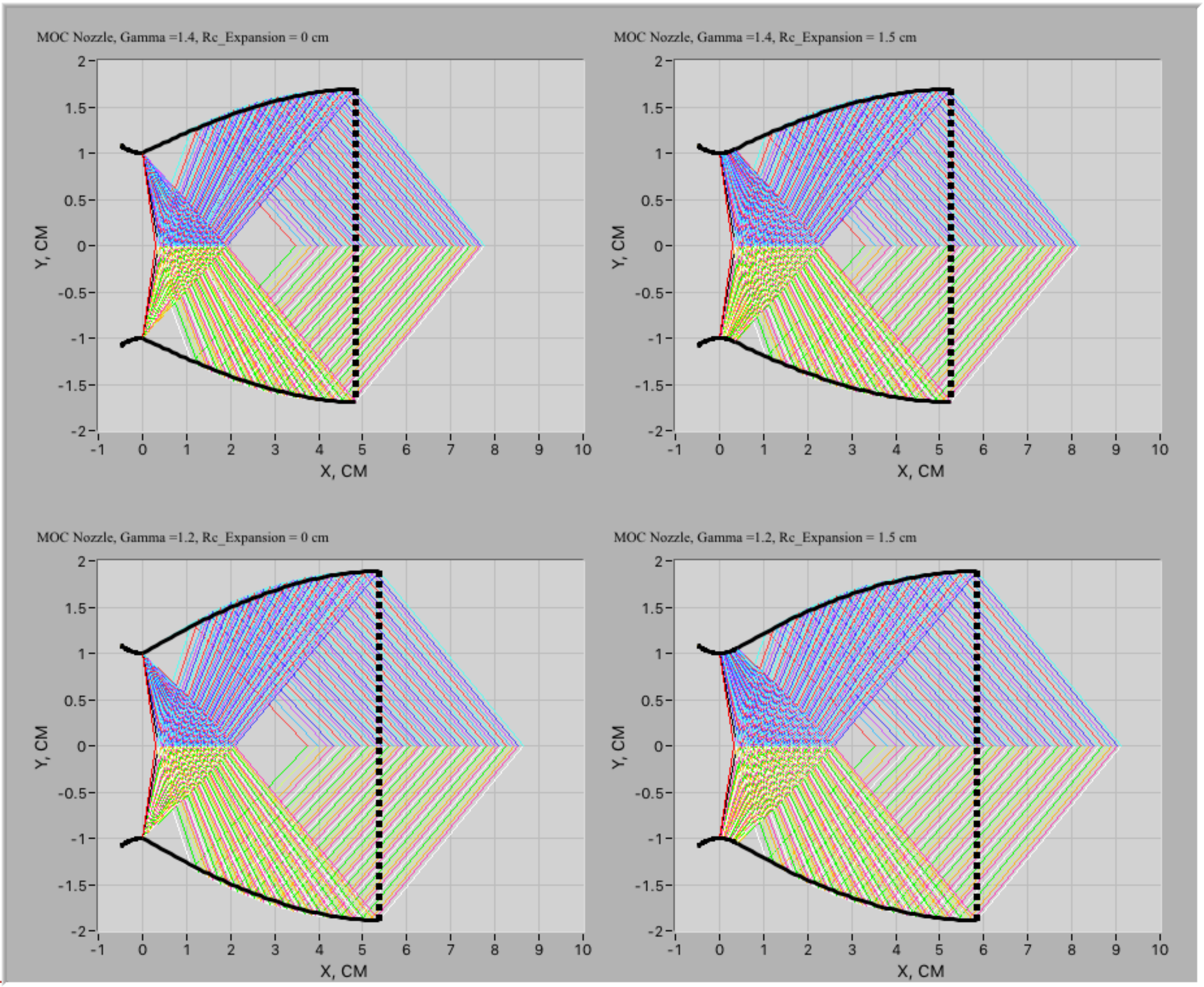
$\gamma = 1.2$

$\theta_{w_{Max}} =$

15.731^o

Solution 1.2, Nozzle Profile Comparisons, N=50

M.O.C Nozzle Plot Clustzer



$$\gamma = 1.4$$

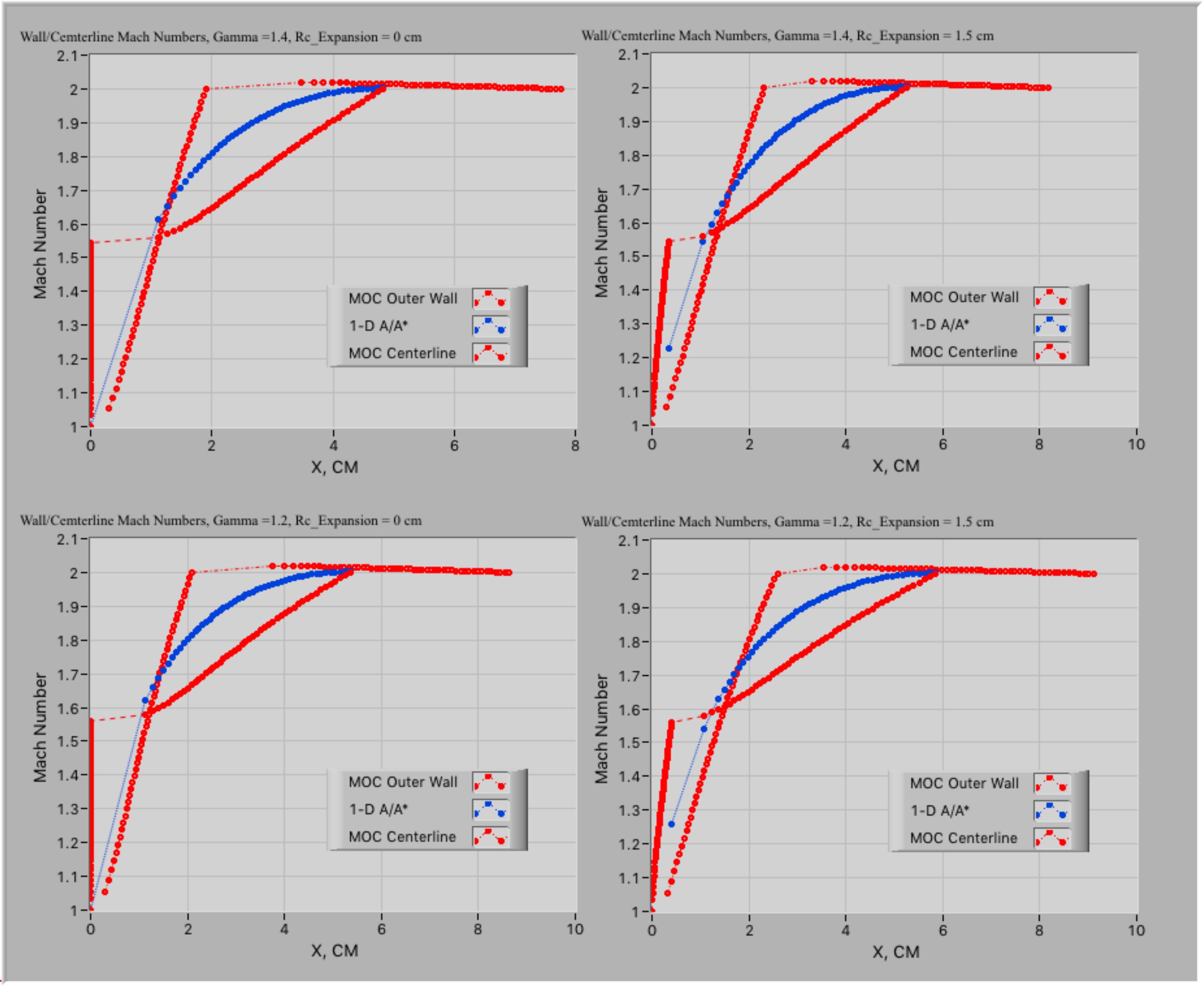
$$\theta_{w_{Max}} = 13.1899^{o}$$

$$\gamma = 1.2$$

$$\theta_{w_{Max}} = 15.731^{o}$$

Solution 1.2, Mach Number Comparisons, N=8

M.O.C. Mach Plot Cluster



$\gamma = 1.4$

$\theta_{w_{Max}} =$

$= 13.1899^{o}$

$\gamma = 1.2$

$\theta_{w_{Max}} =$

15.731^{o}

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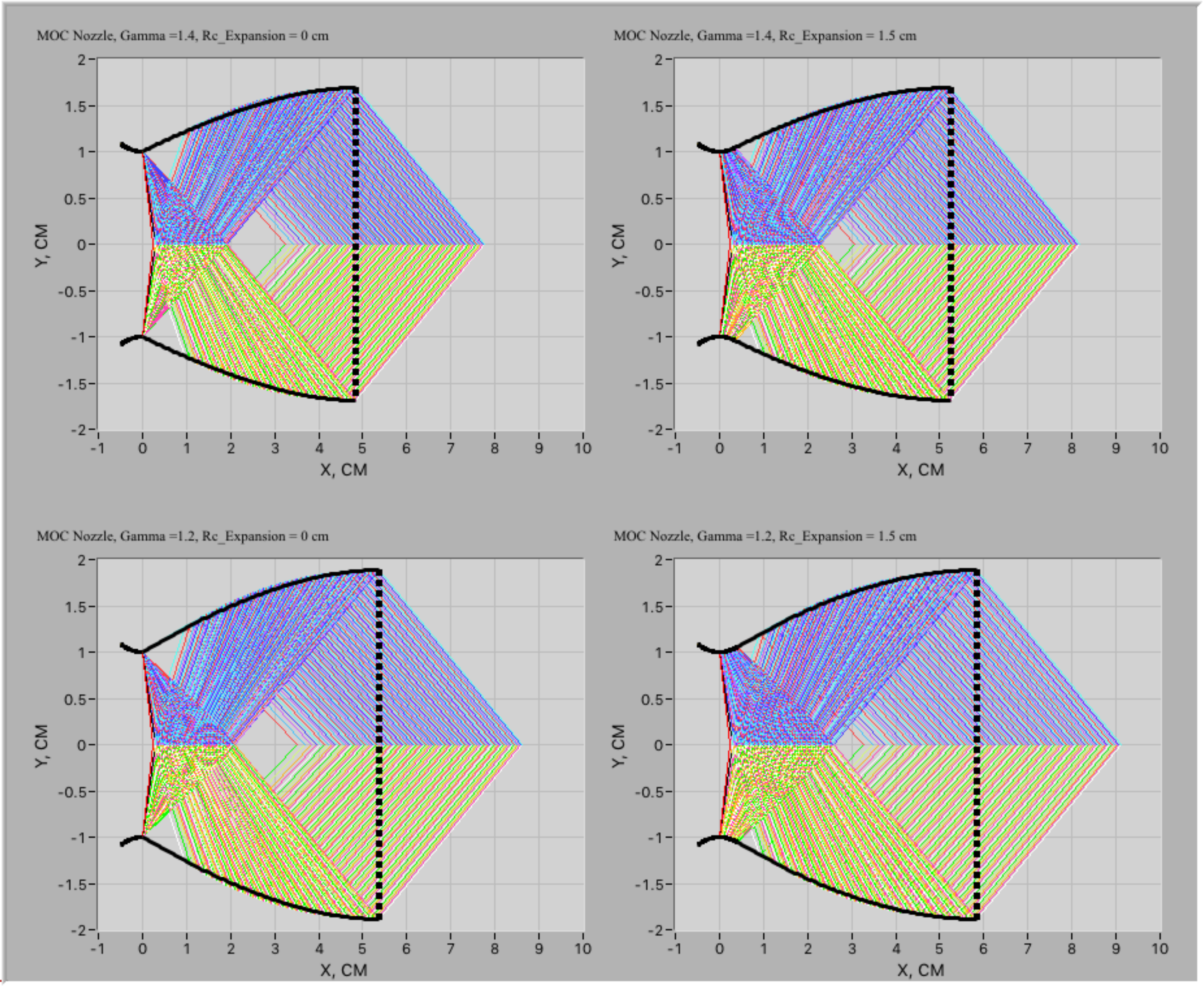
Solution 1.2, Nozzle Profile Comparisons, N=100

M.O.C Nozzle Plot Clustzer



$\gamma = 1.4$

$\theta_{w_{Max}} =$

$= 13.1899^o$

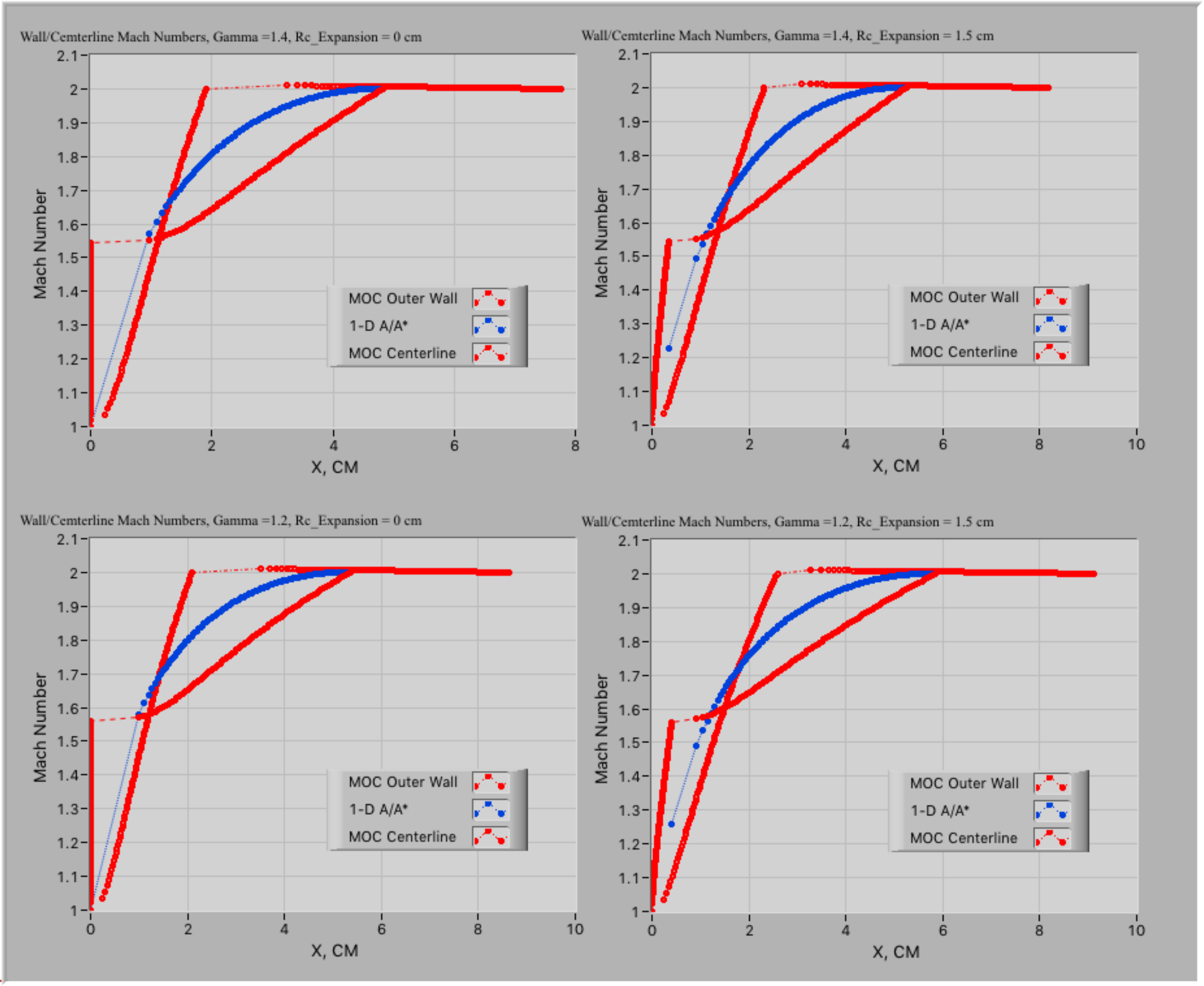
$\gamma = 1.2$

$\theta_{w_{Max}} =$

15.731^o

Solution 1.2, Mach Number Comparisons, N=100

M.O.C. Mach Plot Cluster



$\gamma = 1.4$

$\theta_{w_{Max}} =$

$= 13.1899^o$

$\gamma = 1.2$

$\theta_{w_{Max}} =$

15.731^o

Data Summary

Output Array, N=8

A/A*	Rc expansion, cm	Gamma	Theta Max	X_N	Y_N	M_N	X_{exit}	Y_{exit}	$M_{exit\ wall}$	$M_{exit\ CL}$	Mexit (A/A*)	P	S	Q	T
1.6875	0	1.4	13.1899	0	1	1.54353	4.84856	1.69773	1.99998	2.11321	2.00724	0.631005	-5.04437	-6.35882	40.4346
1.6875	1.5	1.4	13.1899	0.342269	1.03957	1.54353	5.25734	1.69796	1.99998	2.10017	2.0074	0.9359	-22.0851	-15.7405	247.763
1.884	0	1.2	15.7306	0	1	1.56078	5.37381	1.89541	2.00014	2.11493	2.00577	0.90992	-16.2131	-12.9034	166.498
1.884	1.5	1.2	15.7306	0.406682	1.05618	1.56078	5.86925	1.89616	2.00014	2.10071	2.00614	1.67437	-137.533	-49.3764	2438.03

Output Array, N=50

A/A*	Rc expansion, cm	Gamma	Theta Max	X_N	Y_N	M_N	X_{exit}	Y_{exit}	$M_{exit\ wall}$	$M_{exit\ CL}$	Mexit (A/A*)	P	S	Q	T
1.6875	0	1.4	13.1899	0	1	1.54353	4.83521	1.6901	1.99993	2.01469	2.00185	0.65474	-5.81499	-6.91639	47.8364
1.6875	1.5	1.4	13.1899	0.342269	1.03957	1.54353	5.2439	1.69012	1.99993	2.01303	2.00186	1.00083	-27.856	-18.1716	330.205
1.884	0	1.2	15.7306	0	1	1.56078	5.36333	1.88718	2.00005	2.01496	2.00172	0.947764	-18.8087	-14.1186	199.336
1.884	1.5	1.2	15.7306	0.406682	1.05618	1.56078	5.85614	1.88726	2.00005	2.01318	2.00176	1.84034	-187.336	-60.095	3611.41

Output Array, N=100

A/A*	Rc expansion, cm	Gamma	Theta Max	X_N	Y_N	M_N	X_{exit}	Y_{exit}	$M_{exit\ wall}$	$M_{exit\ CL}$	Mexit (A/A*)	P	S	Q	T
1.6875	0	1.4	13.1899	0	1	1.54353	4.833	1.68894	1.99988	2.00722	2.00103	0.658445	-5.94143	-7.00503	49.0705
1.6875	1.5	1.4	13.1899	0.342269	1.03957	1.54353	5.24173	1.68895	1.99988	2.00641	2.00103	1.01116	-28.8549	-18.573	344.956
1.884	0	1.2	15.7306	0	1	1.56078	5.36086	1.8858	1.99996	2.00744	2.00104	0.953745	-19.2388	-14.313	204.862
1.884	1.5	1.2	15.7306	0.406682	1.05618	1.56078	5.85347	1.88584	1.99996	2.00657	2.00105	1.86787	-196.58	-61.9655	3839.73