Homework 3, Part2

a) Re-derive the Conical (3-D) Aerospike Contour Design Rules (*Slide 31)* for a two dimensional (Linear) Nozzle

b) For Aerospike Nozzle use Sonic Throat section, assume axi-symmetric design, full spike length .. For Aerospike Nozzle use Sonic Throat section, assume axi-symmetric …. design, full spike length .. Design a Conical *aerospike nozzle replacement* for the RS-27A Nozzle (Delta II Stage 1)

… i) First Compare RS-27A Nozzle Length with minimum length nozzle of same expansion ratio (assume conical nozzle with 15.25:1 expansion ratio but actual RS-27 A θ_{exit} = 30.5 deg.) …. *Plot both minimum Length and RS-27A Contours*

… ii) Calculate and plot design Aerospike spike contour of same expansion ratio as RS-27A NOZZLE

… iii) Calculate design altitude for this expansion ratio and plot design Mach number and pressure profile along spike LENGTH

… iv) Plot delivered Thrust and I_{sp} as a function of altitude for 1) RS-27A ACTUAL, 2) RS-27A MINIMUM LENGTH (CONICAL), and 3) **Truncated Aerospike** *NOZZLE*

assume chamber properties identical to RS-27A

$$\theta_{MAX_{RS-27A}} = \frac{\nu_{EXIT}}{2}$$

Homework 3, Part 3

(i) For Aerospike Nozzle from part 2, Truncate the spike at the point on the ramp where the pressure exactly equals sea level ambient pressure (101.325 kPa)

- This truncation will ensure no shockwaves on nozzle at Launch condition, which are the most over-expanded

(ii) Calculate the base pressure using the "Rocketdyne Model" from Onofri, pg. 16

$$p_{base} = 0.58 \cdot P_0 \cdot \left(\frac{C_{F,\max,d} - C_{F,core}}{\varepsilon_{base}} \right) = 0.58 \cdot \left(\frac{F_{,\max,d} - F_{,core}}{A_{base}} \right)$$

$F_{,\max,d} \rightarrow$ *Full Ramp Thrust, Design Condition*

$F_{,core} \rightarrow$ *Accumulated Thrust at Truncation Point, Design Condition*

Plot delivered Thrust and Isp as a function of altitude for

(iii) Stage 1a: Aerospike RS-27a Stage + 3 x Gem40 ... Assume conventional Nozzles for Gem-40 boosters, truncated aerospike for RS-27A! 0 to 16.31 km altitude

(iv) stage 1b: 16.31 km altitude 105.52 km altitude (RS-27A. Aerospike Only)

Use conventional nozzles for Gem-40's

Homework 3, Part 3 (continued)

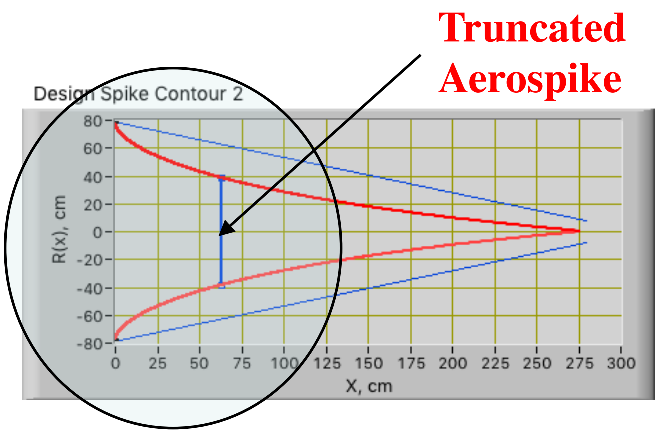
(v) ... Calculate mean Isp over the operating range of the First stage (use above generated data) Use "2/3rds" rule (RS-27A, truncated Aerospike)

Stage "1a"

$$Use \rightarrow \left(I_{sp}\right)_{eff} = \frac{2}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\3x\ Gem40}}\right]_{launch} + \frac{1}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\3x\ Gem40}}\right]_{\substack{Gem40\\Burnout}}$$

Stage "1b"

$$Use \rightarrow \left(I_{sp}\right)_{eff} = \frac{2}{3}\left(I_{sp}\right)_{\substack{R2-27A\\initial\\(16.31km)}} + \frac{1}{3}\left(I_{sp}\right)_{\substack{R2-27A\\final\\(105.52\,km)}}$$



... (vi) Re-work delta II payload analysis using new mean I_{sp}'s for stage "1a" and "1b" with the RS-27A aerospike nozzle

... (vii) compare to earlier results using standard conical nozzle for stage 1.. assume conventional nozzles for Both Gem-40 and AJ10-118 Second Stage Engine

Stage 1 Properties

- Boeing Delta II Rocket…Stage 1
  - Sea Level Thrust: 890kN
  - Vacuum Thrust: 1085.8 kN
  - **Nozzle Expansion Ratio: 15.2503:1**
  - **Conical Nozzle, 30.5 deg exit angle**

- Combustion Properties:
  (RS-27A Rocketdyne Engine)
  - Lox/Kerosene, Mixture Ratio: 2.24:1
  - **Chamber Pressure (P_0): 5161.463 kPa**
  - Combustion temperature (T_0): 3455 K
  - γ = 1.2220
  - M_W = 21.28 *kg/kg-mol*

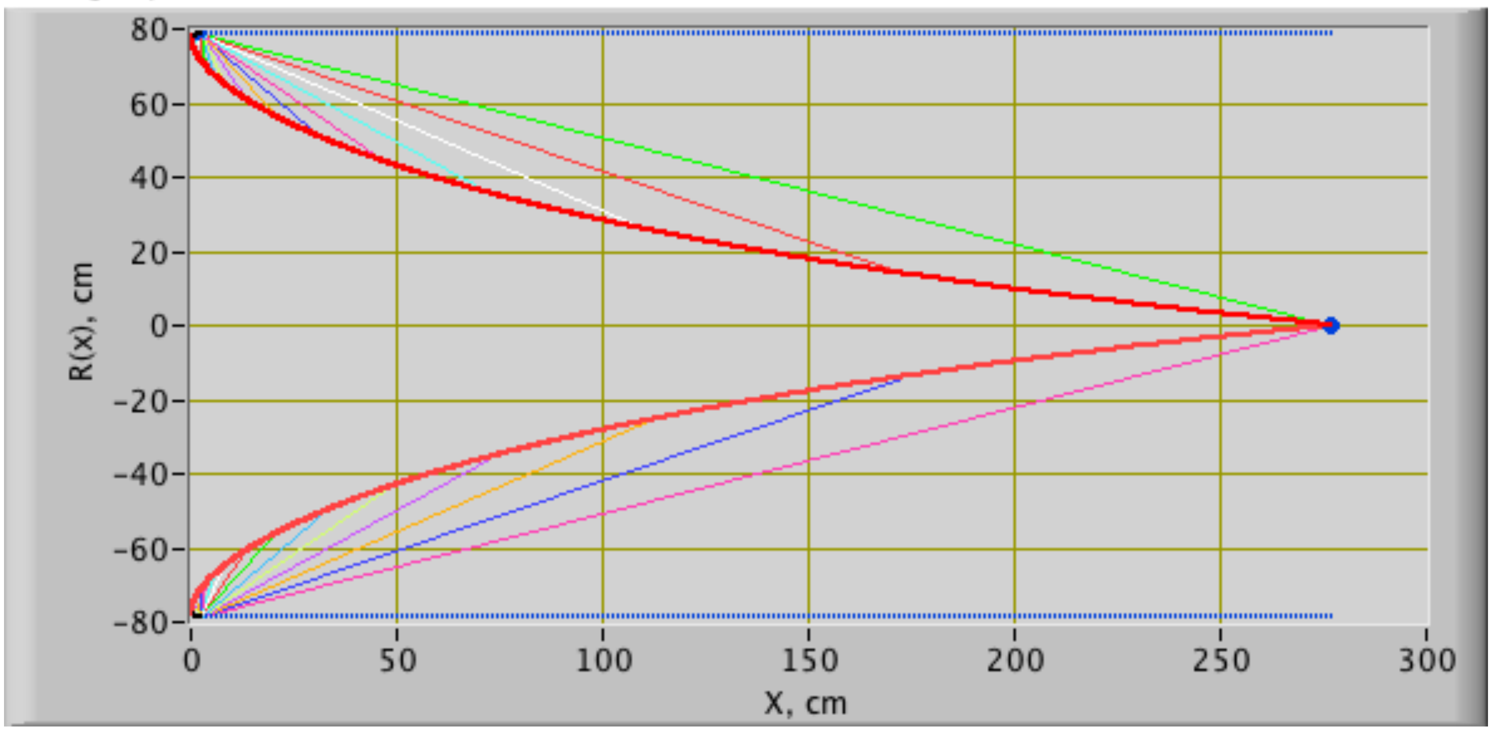
- Propellant Mass: 97.08 Metric Tons
- Stage 1 Launch Mass: 101.8 Metric Tons

Spike Design Characteristics

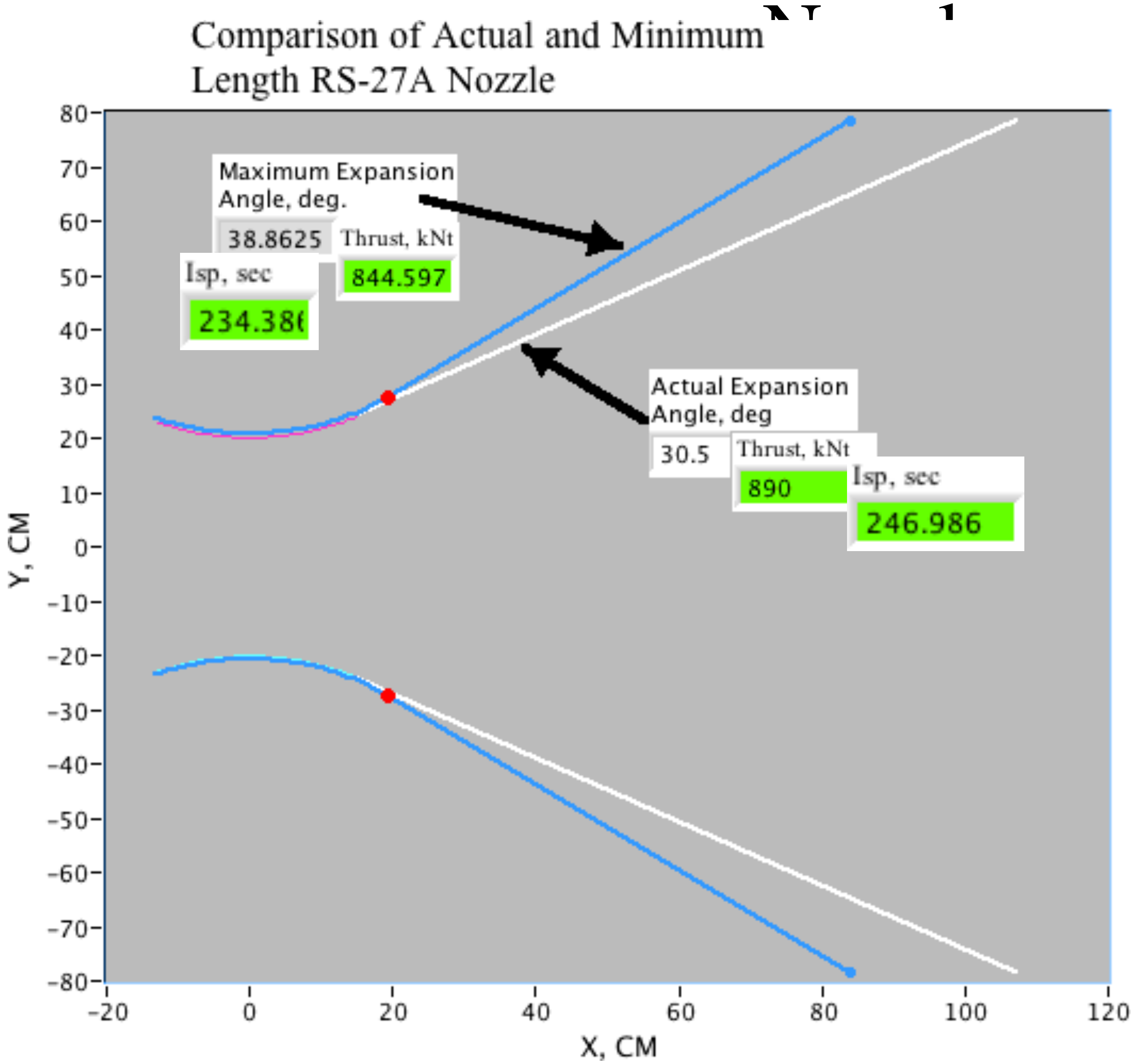
... iii) Calculate design altitude for this expansion ratio and plot design mach number and pressure profile along spike, assume 15.2503:1 expansion ratio and chamber properties identical to RS-27A

Design Altitude, km 8.16812 → Pa, kPa 34.74

Design Spike Contour



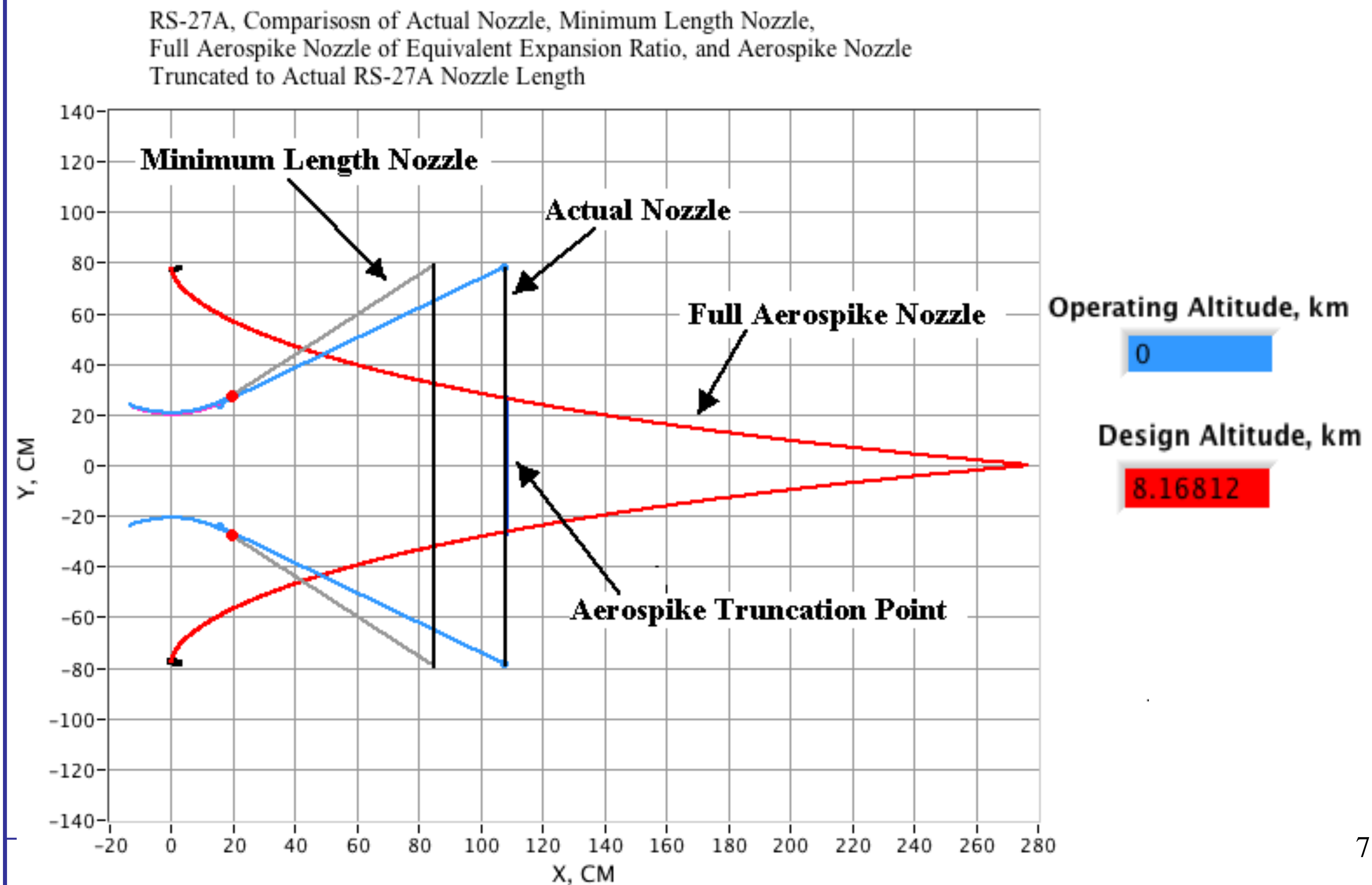
Compare RS-27A Nozzle to Minimum Length Nozzle



- Slight Thrust For Minimum Length Nozzle Due to Higher Exit Angle

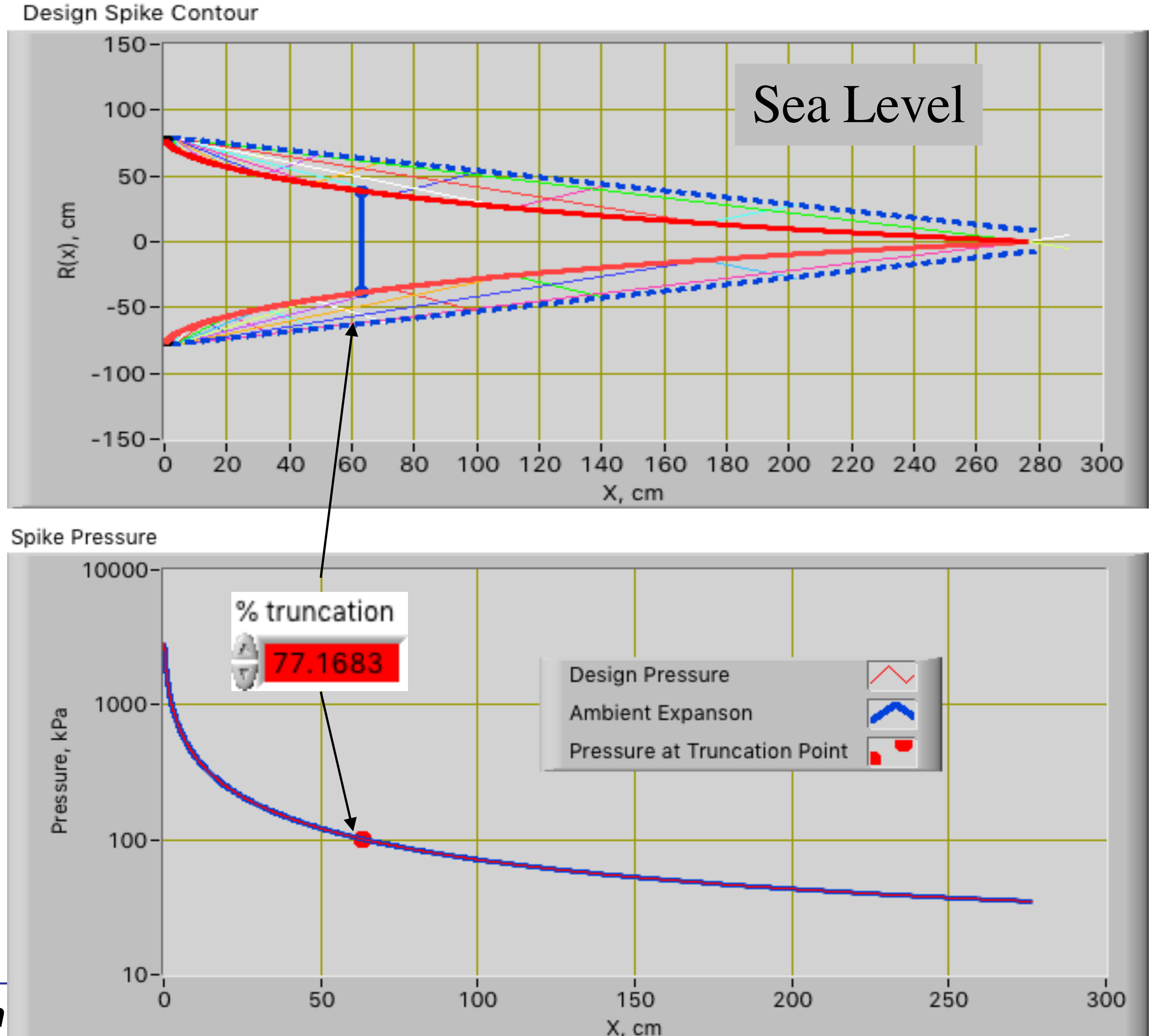
- Launch Thrust, I_{sp} Levels

Compare RS-27A Nozzle to Aerospike Nozzle



Spike Design Characteristics (1)

Truncate Spike Such that pressure at truncation Point equals $= P_{sl}$



Calculate base pressure using "Rocketdyne Model" from Onofri, pg.

$$p_{base} = 0.58 \cdot P_0 \cdot \left(\frac{C_{F,\max,d} - C_{F,core}}{\varepsilon_{base}} \right) = 0.58 \cdot \left(\frac{F_{,\max,d} - F_{,core}}{A_{base}} \right)$$

$F_{,\max,d} \rightarrow$ *Full Ramp Thrust, Design Condition*

$F_{,core} \rightarrow$ *Accumulated Thrust at Truncation Point, Design Condition*

Truncated Thrust terms

Base Pressure, kPa	19.9261
Base Drag, kNt	-37.9658
RampThrust, kNt	808.459
TotalThrust, kNt	942.649
Isp, sec	261.595

Thrusts

Fmax,d, kNt	1094.38
Fcore, kNt 2	1078.35

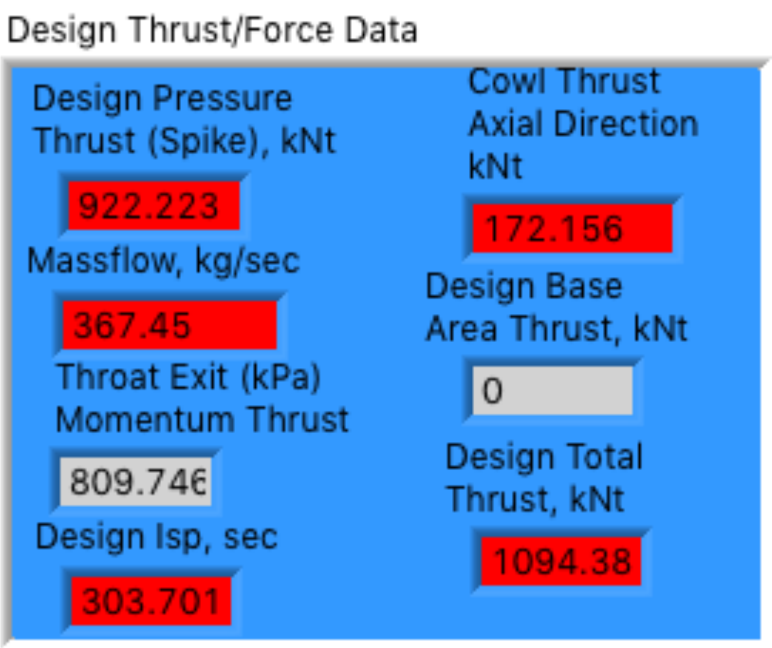
$$P_{base} = 0.58 \frac{(1094.38 - 1078.35)}{4664.17 \cdot 0.0001} = 19.9337 \text{ kPa}$$

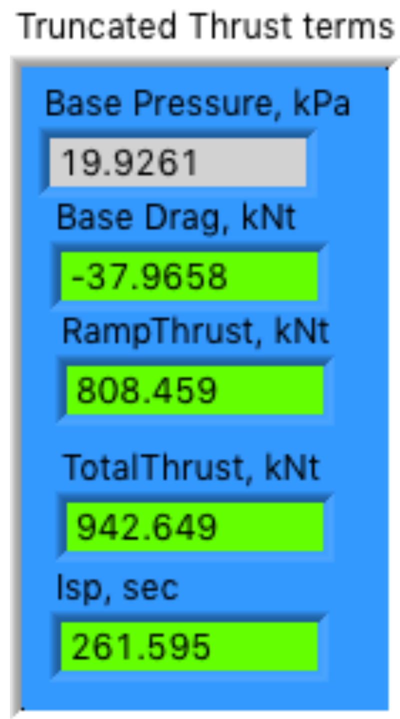
Data at Truncation

R value at truncation, cm	38.5312
Theta at truncation, deg	14.2576
Mach Number at Truncation	3.06432
Spike Surface Pressure at Truncation, kPa	101.325
Spike Surface Temperature at Truncation, deg. K	1691.72
Spike Surface Velocity at Truncation, deg. K	2754
Spike Truncated base area cm^2	4664.17
Spike Truncated Length cm	63.16

% truncation: 77.1683

Calculate Launch Thrust





$$T_{spike} = T_{throat} + T_{ramp} + T_{base} \quad =$$

$$172.156 + 808.459 - 37.9658 = 942.649 \quad kNt$$

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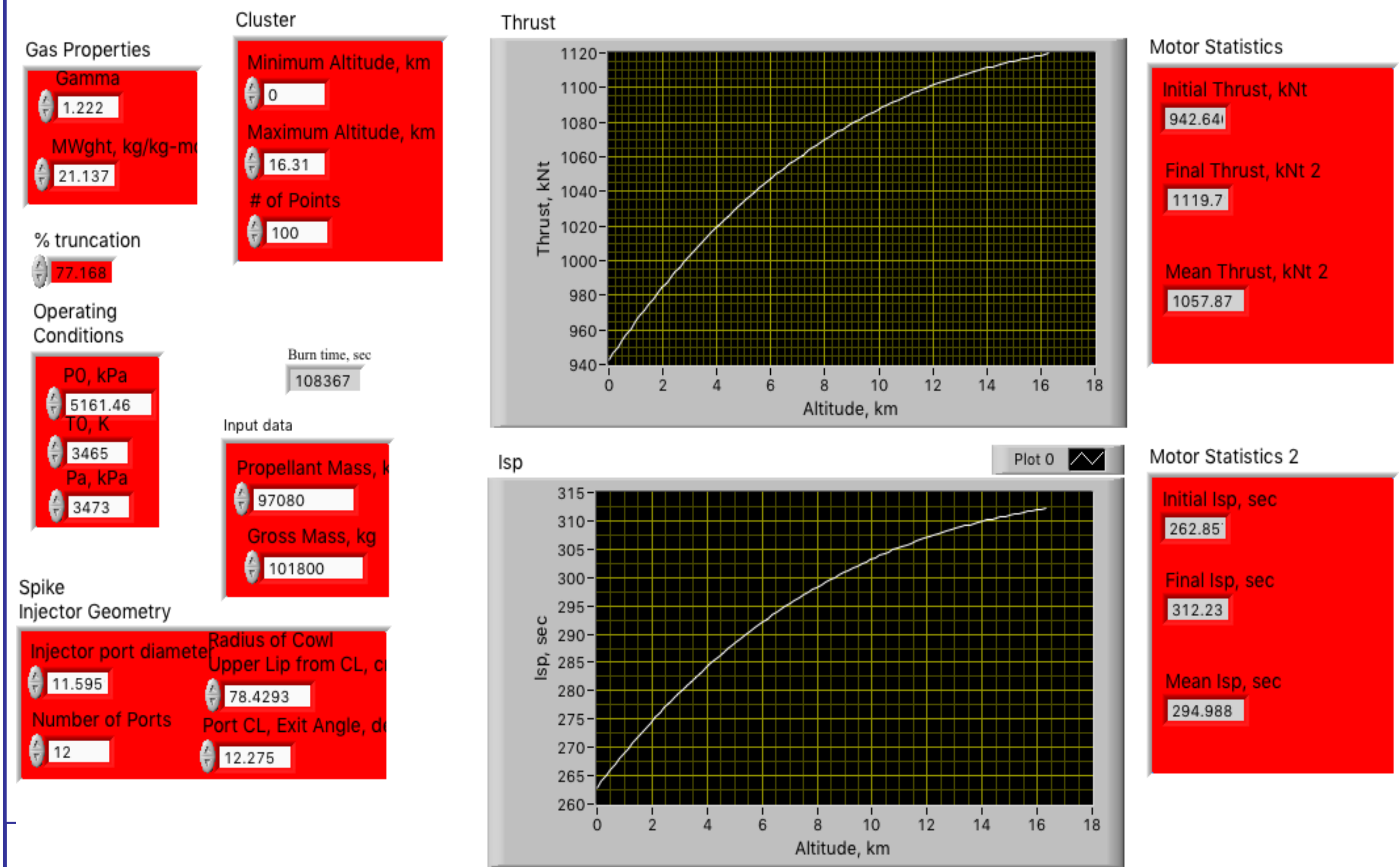
Spike Design Characteristics (2)

Aerospike Versus Altitude



Spike Design Characteristics (3)

Gem 40 vs Altitude

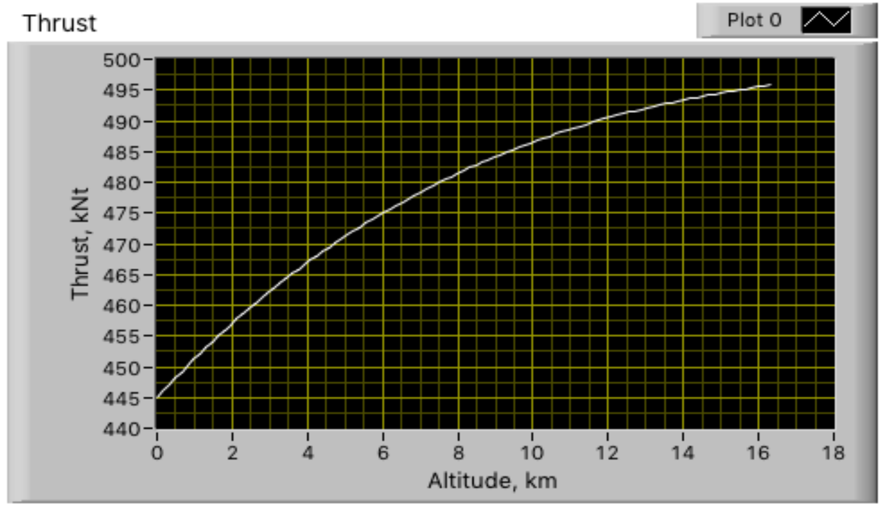
Input data

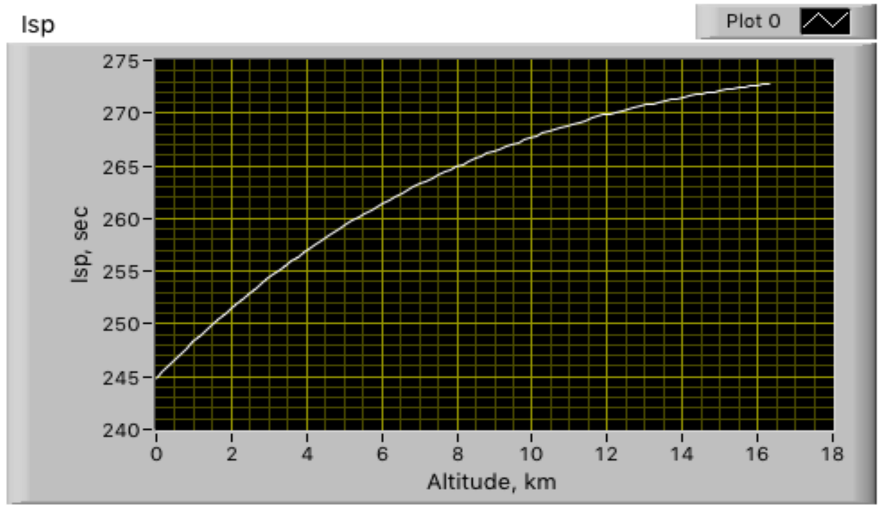
Starting Mach: 3.600000
gamma: 1.2000
A/A*: 10.65000
iterations: 21
% error in (A/A*): 0.00000
P01, kPa: 5652.66
T01, deg. K: 3600
A*, M^2: 0.052125
MW: 28.15
Actual Expansion Angle, deg: 20
Propellant Mass, k: 11765
Gross Mass, kg: 13080

Cluster

Minimum Altitude, km: 0
Maximum Altitude, km: 16.31
# of Points: 100

Mean Thrust, kN: 478.025
Mean Isp, sec: 263.042
Burn time, sec: 63.4878



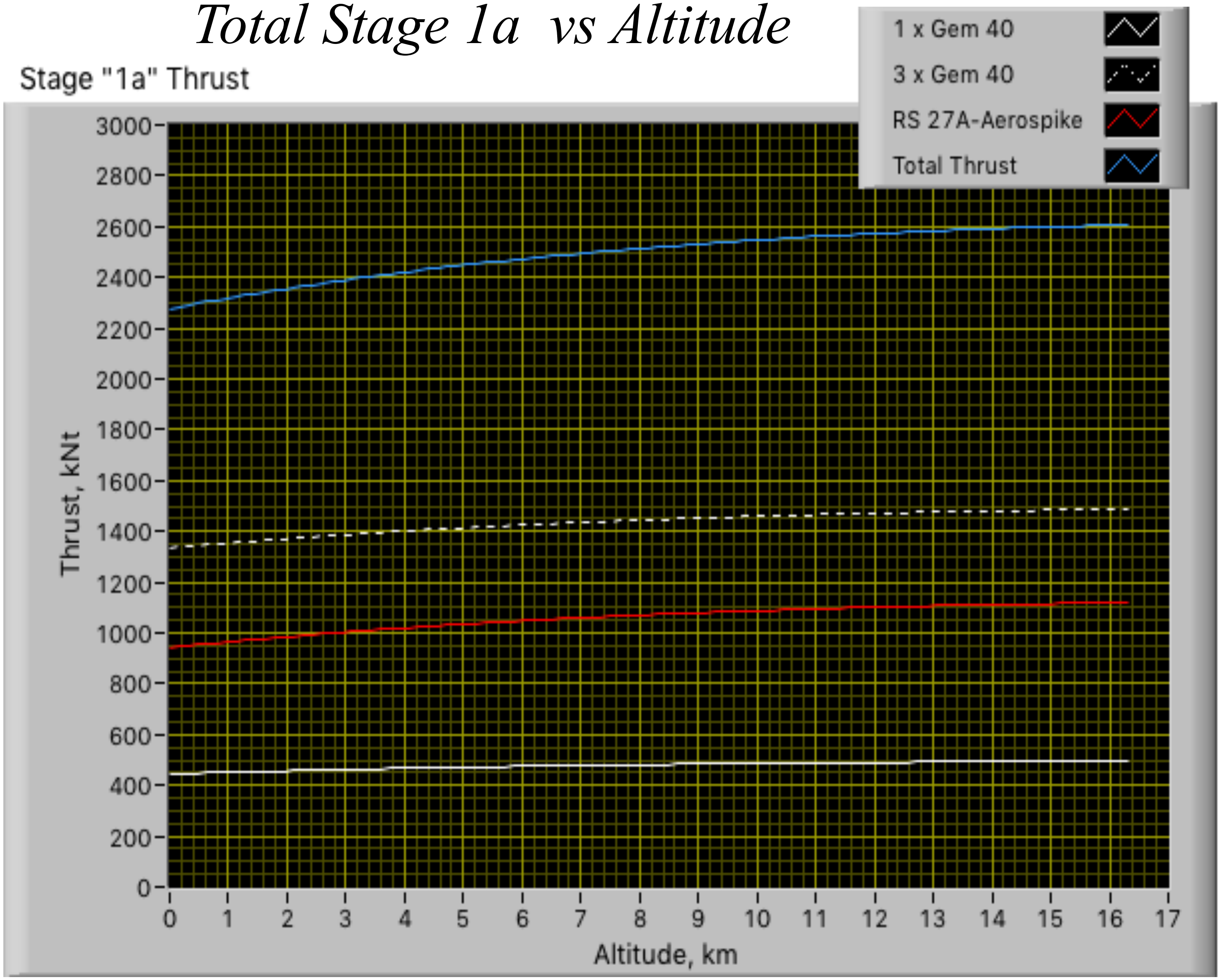


Isentropic Output parameters

Exit mach Number: 3.322236
Pexit, kPa: 65.210
Texit, deg. K: 1711.2
Vexit, m/sec: 2587.35
Mdot, kg/sec: 185.311
Thrust, kNt: 495.769
Isp, sec: 272.80
Exit Area, M^2: 0.55513
Cstar, m/sec: 1590
Max Isp, sec: 364.247
Max Thrust, Kn: 661.943
Ce, m/.sec: 2.67533
Volumetric flow rate, m^3/sec: 3.32223
Chamber density kg/m^2: 55.7792
Rg, J/kg-deg-K: 295.36102
Lambda, divergence: 0.9698463
Burn time, sec: 63.487757

Spike Design Characteristics (4)

Total Stage 1a vs Altitude



Motor Statistics

Total Take Off Thrust, kNt

2277.5

Mean GEM 40 Thrust, kNt

478.02

Mean RS-27A Thrust, kNt

1057.87

Total Mean Thrust, kNt

2491.95

% truncation

77.168

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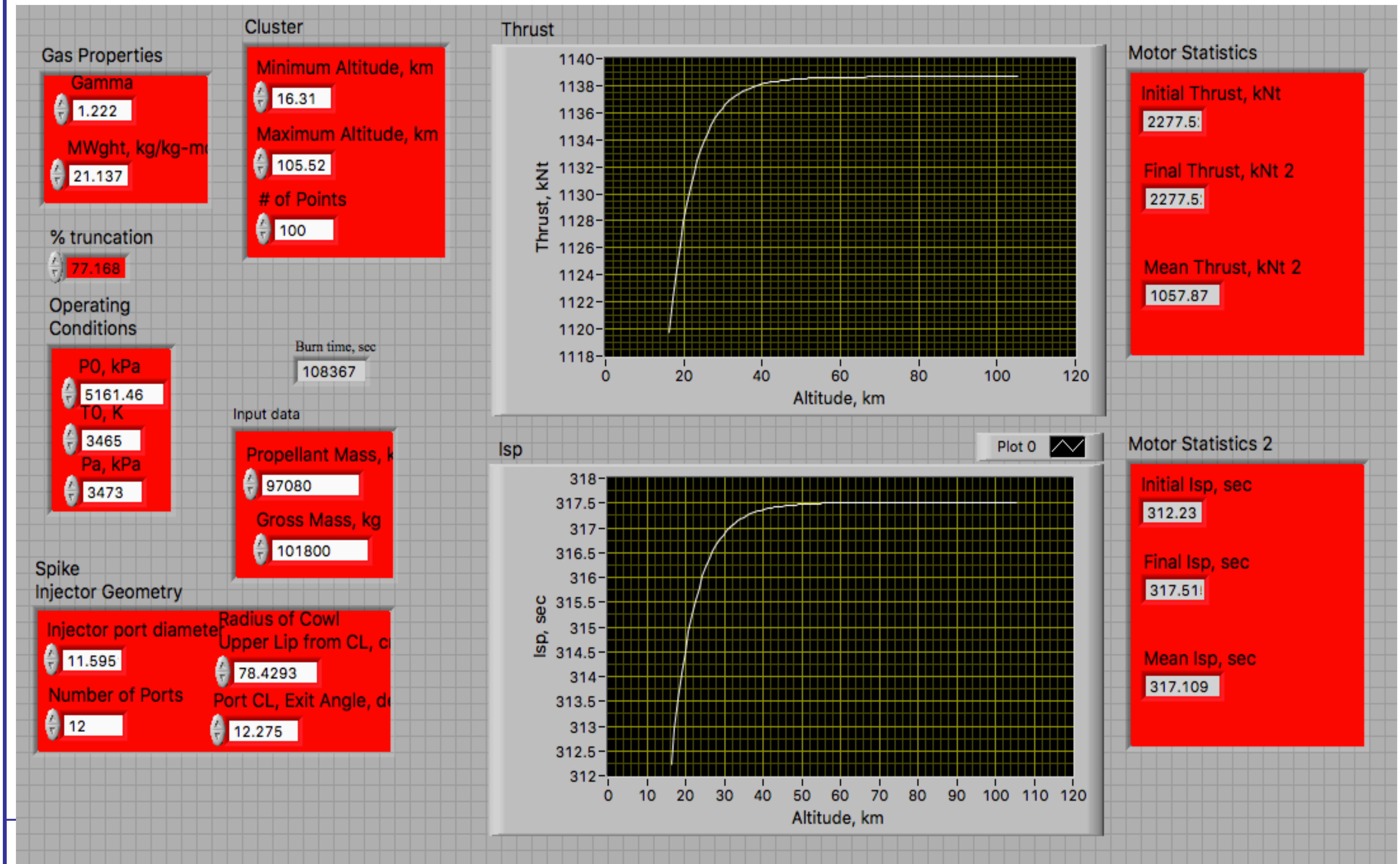
Spike Design Characteristics (5)

Total Stage 1b vs Altitude



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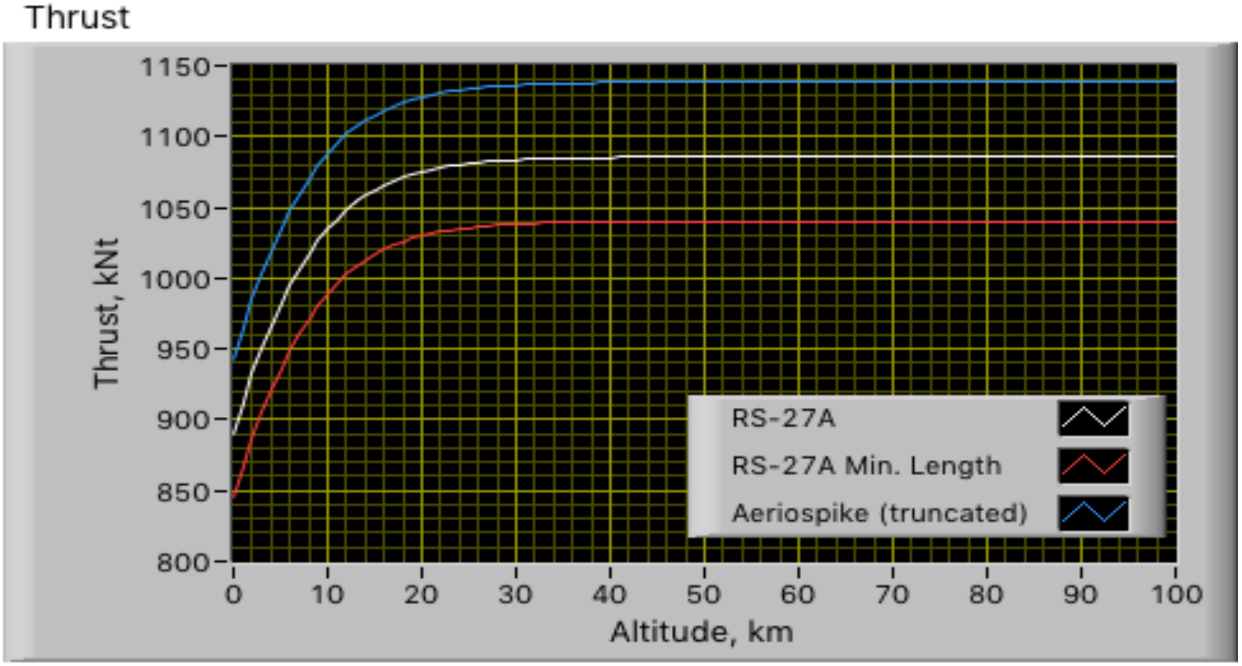
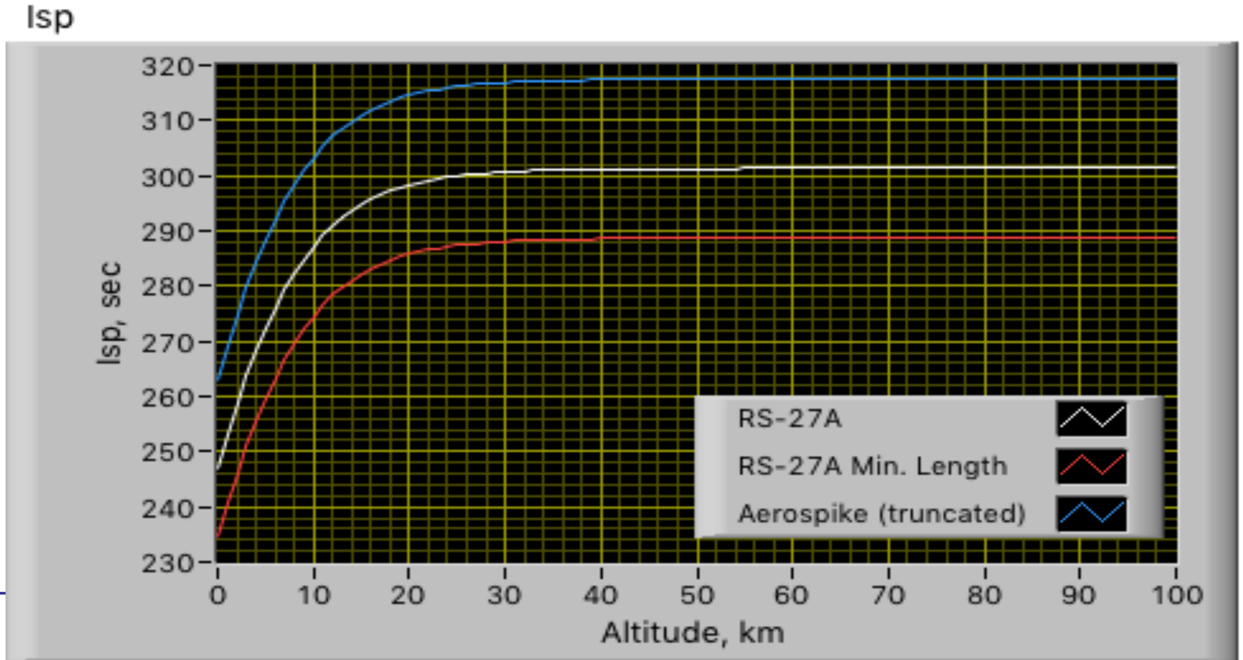
Nozzle Comparison Summary

	Launch Thrust, kNt	Vacuum Thrust, kN	Design Thrust, kNt	Launch I_{sp}, sec	Vacuum I_{sp}, sec	Design Isp, sec	Length, cm
RS-27A Normal Nozzle	**890.0**	**1085.8**	**1018.7**	**247.0**	**301.3**	**282.7**	**107.3**
RS-27A Minimum Length Nozzle	**844.6**	**1040.4**	**973.3**	**234.4**	**288.7**	**270.3**	**83.8**
RS-27A Truncated Aerospike Nozzle, 77.168% **(Ambient Length)**	**942.6**	**1138.7**	**1071.5**	**262.9**	**317.5**	**298.8**	**63.2**

Spike Wins all Around!

Spike Performance Comparisons (1)





Delta II Launch Analysis

... iv) Plot delivered Thrust and I_{sp} as a function of altitude for RS-27a Stage
stage 1a: 0 to 16.31 km altitude (RS-27A + 3 x Gem40)
stage 1b: 16.31 km altitude 105.52 km altitude (RS-27A)
... Assume conventional Nozzles for Gem-40 boosters, truncated aerospike for RS-27A

Stage "1a"

- Aerospike Nozzle Thrust, I_{sp} at Sea Level:

942.6 kNt, 262.9 sec

- Aerospike Nozzle Thrust, I_{sp} at 16.31 km ($P_{amb} = 9.797\ kPa$):

1119.7 kNt, 312.2 sec

- Nozzle Mass Flow: *367.445 kg/sec*

… Stage "1a" (4)

vi) Effective Specific Impulse

(3 x Gem 40 + RS-27A/w aerospike over operating altitude range

$$\left(I_{sp}\right)_{launch} = \frac{\left(F_{RS-27A} + 3 \cdot F_{Gem40}\right)_{launch}}{g_0 \cdot \left(\dot{m}_{RS-27A} + 3 \cdot \dot{m}_{Gem40}\right)} = \left(\frac{942.646 + 3 \cdot 442.95}{367.447 + 3 \cdot 185.315}\right)\frac{1000}{9.8067}$$

$= 250.84$ sec

$$\left(I_{sp}\right)_{\substack{Gem40\\burnout}} = \frac{\left(F_{RS-27A} + 3 \cdot F_{Gem40}\right)_{\substack{Gem40\\burnout}}}{g_0 \cdot \left(\dot{m}_{RS-27A} + 3 \cdot \dot{m}_{Gem40}\right)} = \left(\frac{1119.7 + 3 \cdot 493.765}{367.447 + 3 \cdot 185.315}\right)\frac{1000}{9.8067}$$

$= 287.23$sec

$$\left(I_{sp}\right)_{eff} = \frac{2}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\3x\ Gem40}}\right]_{Launch} + \frac{1}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\3x\ Gem40}}\right]_{\substack{Gem40\\Burnou}} = \frac{2}{3}(250.84) + \frac{1}{3}(287.23)$$

$= 262.97$sec

Delta II Launch Analysis

... iv) Plot delivered Thrust and I_{sp} as a function of altitude for RS-27a Stage
stage 1a: 0 to 16.31 km altitude (RS-27A + 3 x Gem40)
stage 1b: 16.31 km altitude 105.52 km altitude (RS-27A)
... Assume conventional Nozzles for Gem-40 boosters, full aerospike for RS-27A

Stage "1b"

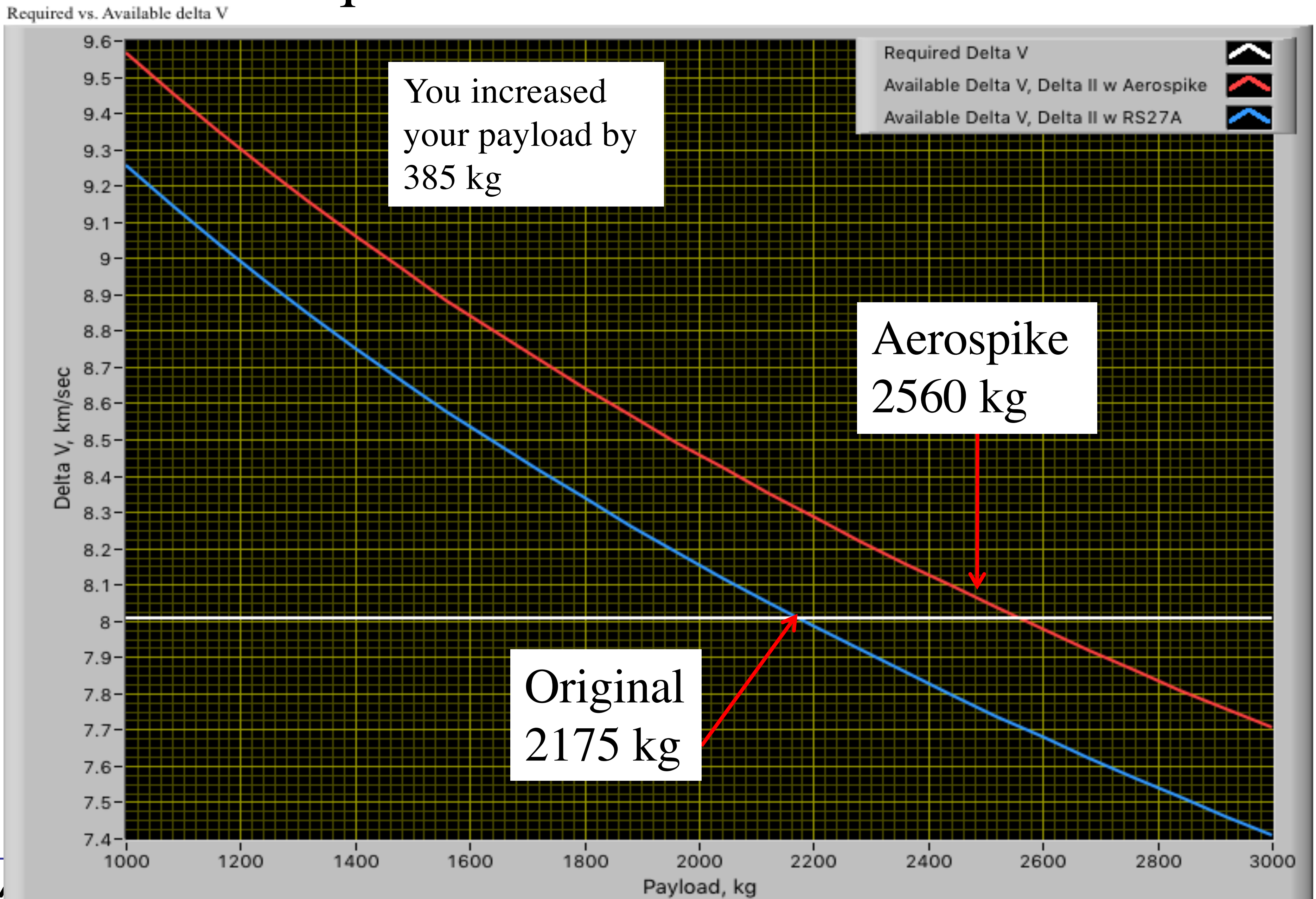
- Aerospike Nozzle (MECO) Burnout Thrust, I_{sp} :

1138.66 kNt, 317.52 sec

$$\left(I_{sp}\right)_{eff} = \frac{2}{3}\left[\left(I_{sp}\right)_{R2-27A}\right]_{\substack{Gem40\\Burnou}} + \frac{1}{3}\left[\left(I_{sp}\right)_{R2-27A}\right]_{\substack{R2-27A\\MECO}} =$$

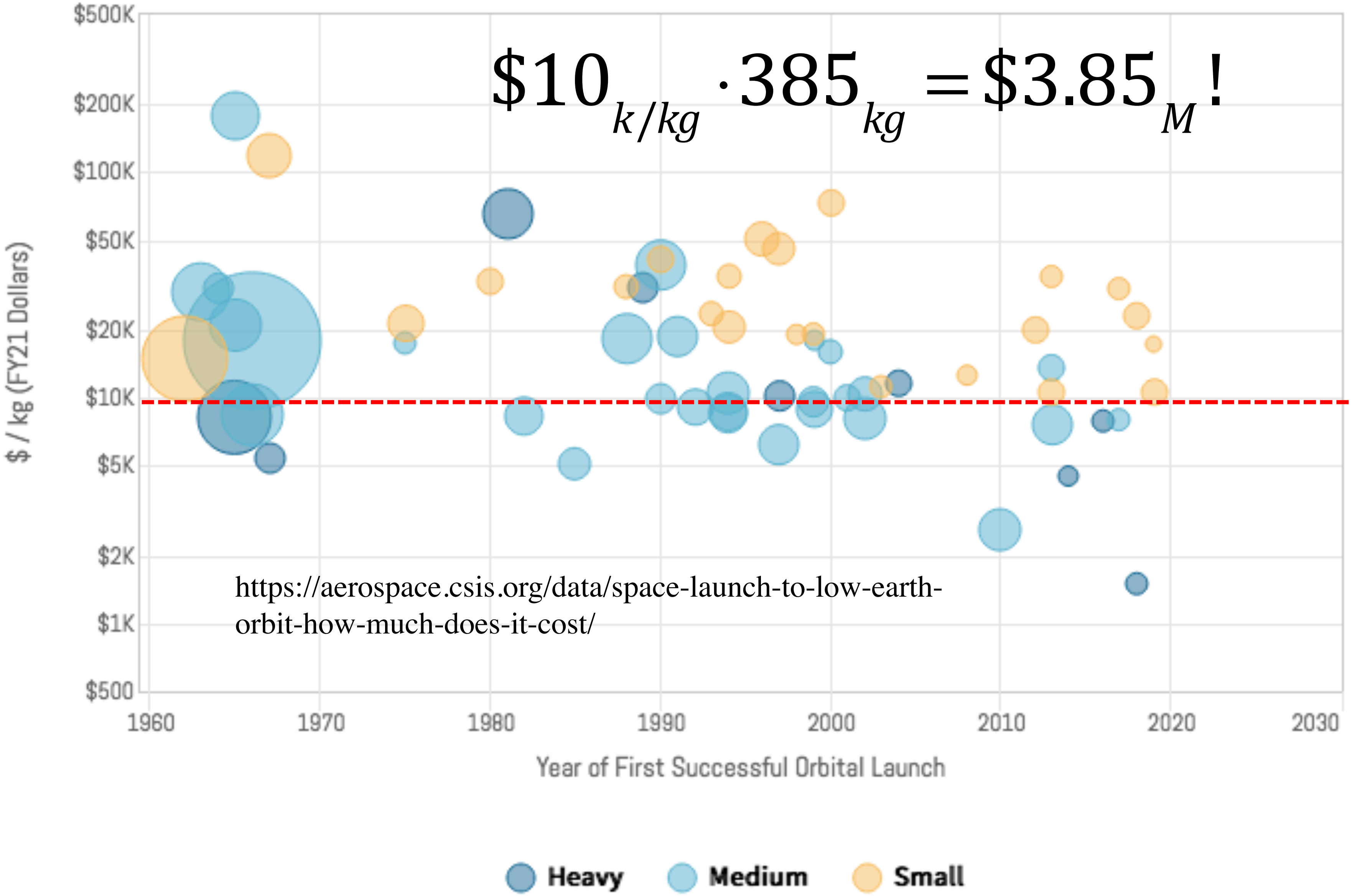
$$\frac{2}{3}(312.2) + \frac{1}{3}(317.52) = 313.97 \text{ sec}$$

Required versus Available Delta V



Payload Cost Estimates?

$$\$10_{k/kg} \cdot 385_{kg} = \$3.85_{M}!$$



https://aerospace.csis.org/data/space-launch-to-low-earth-orbit-how-much-does-it-cost/

Live Metal Spot Price (24hrs) Oct 11, 2021 at 15:48 EST		
Silver Spot Prices	**Today**	**Change**
Silver Price Per Ounce	$22.7	-0.06
Silver Price Per Gram	$0.73	0
Silver Price Per Kilo	$729.82	-1.93

$$\frac{3.86 \cdot 10^{6}}{729.82 \cdot 2175} = 2.43171$$

You saved MORE 2.4 times the cost of your original payload .. IF IT WAS MADE OUT OF 100% STERLING!