Homework 2, part 2

1) For Aerospike Nozzle use Sonic Throat section, assume axi-symmetric design, full spike length. For Aerospike Nozzle use Sonic Throat section, assume axi-symmetric design, full spike length. Design a Conical aerospike nozzle replacement for the RS-27A Nozzle

... i) First Compare RS-27A Nozzle Length with minimum length nozzle of same expansion ratio (assume conical nozzle with 15.2503:1 expansion ratio, \( \theta_{exit} = 30.5 \) deg.) .... Plot both Contours

... ii) Calculate and plot design spike contour,

... iii) Calculate design altitude for this expansion ratio and plot design mach number and pressure profile along spike, assume 15.2503:1 expansion ratio and chamber properties identical to RS-27A

... iv) Plot delivered Thrust and \( I_{sp} \) as a function of altitude for RS-27a Stage
  
  stage 1a: 0 to 16.31 km altitude (RS-27A + 3 x Gem40)
  stage 1b: 16.31 km altitude 105.52 km altitude (RS-27A)

... Assume conventional Nozzles for Gem-40 boosters
Homework 2 (2)

... iv) Calculate mean $I_{sp}$ over the operating range of the First stage (use above generated data) Use “2/3rds” rule

Stage “1a”

\[ \text{Use} \rightarrow \left( I_{sp} \right)_{\text{eff}} = \frac{2}{3} \left[ \left( I_{sp} \right)_{R2-27A+3x\ Gem40} \right]_{\text{launch}} + \frac{1}{3} \left[ \left( I_{sp} \right)_{R2-27A+3x\ Gem40} \right]_{\text{Gem40 Burnout}} \]

Stage “1b”

\[ \text{Use} \rightarrow \left( I_{sp} \right)_{\text{eff}} = \frac{2}{3} \left( I_{sp} \right)_{R2-27A \text{ initial (16.31 km)}} + \frac{1}{3} \left( I_{sp} \right)_{R2-27A \text{ final (105.52 km)}} \]

... v) Re-work delta II Homework problem (HW 2) using new mean $I_{sp}$’s for stage “1a” and “1b” with the RS-27A aerospike nozzle... compare to earlier results using standard conical nozzle for stage 1

assume conventional nozzles for Both Gem-40 and AJ10-118 Second Stage Engine
3-D Nozzle Algorithm
Aerospike Nozzle

\[
\tan \phi_x = \frac{R_e - R_x}{X_x} \rightarrow \phi_x = v_e - v_x + \mu_x \\
X_x = \frac{R_{exit} - R_x}{\tan(v_e - v_x + \mu_x)} \\
R_x = R_{exit} \sqrt{1 - \frac{\sin(v_e - v_x + \mu_x)}{\varepsilon}} \left[ \left( \frac{2 + \gamma - 1}{\gamma + 1} \right) \left( 1 + \frac{\gamma - 1}{2} M_x^2 \right) \right]^{\gamma + 1 \over 2(\gamma - 1)} \\
\sin(\mu_x) = \frac{1}{M_x}
\]
Stage 1 Properties

• Boeing Delta II Rocket…Stage 1
  - Sea Level Thrust: 890 kN
  - Vacuum Thrust: 1085.8 kN
  - Nozzle Expansion Ratio: 15.2503:1
  - Conical Nozzle, 30.5 deg exit angle

• Combustion Properties:
  (RS-27A Rocketdyne Engine)
  - Lox/Kerosene, Mixture Ratio: 2.24:1
  - Chamber Pressure \( (P_0) \): 5161.463 kPa
  - Combustion temperature \( (T_0) \): 3455 K
  - \( \gamma = 1.2220 \)
  - \( M_W = 21.28 \text{ kg/kg-mol} \)

- Propellant Mass: 97.08 Metric Tons
- Stage 1 Launch Mass: 101.8 Metric Tons
Include 2-D Spike Contour on Plot

RS-27A, Comparisons of Actual Nozzle, Minimum Length Nozzle, Full 3-D Aerospike Nozzle of Equivalent Expansion Ratio, and Aerospike Nozzle Truncated to Actual RS-27A Nozzle Length, 2-D aerospike

- 3-D “flow relief” effect
- Allows conical Aerospike to “turn corner” faster than 2-D spike and still preserve isentropic flow

Minimum Length Nozzle
Actual RS-27A Nozzle
2-D (Ramp) Aerospike Nozzle
Truncation Point
3-D (Conical) Aerospike Nozzle
Spike Design Characteristics

... iii) Calculate design altitude for this expansion ratio and plot design mach number and pressure profile along spike, *assume* 15.2503:1 expansion ratio and chamber properties identical to RS-27A

![Design Altitude, km]

- Design Altitude, km: 8.16812
- Pressure, Pa, kPa: 34.74

![Design Spike Contour]

- X, cm: 0 to 300
- R(x), cm: -80 to 80
Compare RS-27A Nozzle to Minimum Length Nozzle

- Slight Thrust For Minimum Length Nozzle Due to Higher Exit Angle
- Launch Thrust, $I_{sp}$ Levels
Compare RS-27A Nozzle to Aerospike Nozzle

Spike Design Characteristics (1)

At Design Condition

Design Spike Contour

Vacuum

Sea Level
Spike Ramp Characteristics (1)
## Nozzle Comparison Summary

<table>
<thead>
<tr>
<th></th>
<th>Launch Thrust, kNt</th>
<th>Vacuum Thrust, kN</th>
<th>Design Thrust, kNt</th>
<th>Launch I&lt;sub&gt;sp&lt;/sub&gt;, sec</th>
<th>Vacuum I&lt;sub&gt;sp&lt;/sub&gt;, sec</th>
<th>Design I&lt;sub&gt;sp&lt;/sub&gt;, sec</th>
<th>Length, cm</th>
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<tbody>
<tr>
<td>RS-27A Normal Nozzle</td>
<td>890.0</td>
<td>1085.8</td>
<td>1018.7</td>
<td>247.0</td>
<td>301.3</td>
<td>282.7</td>
<td>107.3</td>
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<tr>
<td>RS-27A Minimum Length Nozzle</td>
<td>844.6</td>
<td>1040.4</td>
<td>973.3</td>
<td>234.4</td>
<td>288.7</td>
<td>270.3</td>
<td>83.8</td>
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<tr>
<td>RS-27A Full Aerospike Nozzle</td>
<td>965.7</td>
<td>1161.5</td>
<td>1094.4</td>
<td>268.0</td>
<td>322.3</td>
<td>303.7</td>
<td>276.6</td>
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<tr>
<td>RS-27A Truncated Aerospike Nozzle, 61.1% (RS-27A Length)</td>
<td>953.8</td>
<td>1148.8</td>
<td>1082.6</td>
<td>264.7</td>
<td>318.8</td>
<td>300.4</td>
<td>107.6</td>
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<tr>
<td>RS-27A Truncated Aerospike Nozzle, 69.7% (Minimum Length)</td>
<td>945.7</td>
<td>1140.7</td>
<td>1074.49</td>
<td>262.4</td>
<td>316.6</td>
<td>298.2</td>
<td>83.8</td>
</tr>
</tbody>
</table>

*Spike Wins all Around!*
Spike Performance Comparisons

- Thrust
- Specific Impulse

Graphs showing thrust and specific impulse variations with altitude for different spike configurations.
... iv) Plot delivered Thrust and $I_{sp}$ as a function of altitude for RS-27a Stage

stage 1a: 0 to 16.31 km altitude (RS-27A + 3 x Gem40)
stage 1b: 16.31 km altitude 105.52 km altitude (RS-27A)

... Assume conventional Nozzles for Gem-40 boosters, full aerospike for RS-27A

**Stage “1a”**

• Full Aerospike Nozzle Thrust, $I_{sp}$ at Sea Level:

  $965.7$ kNt, $268.0$ sec

• Aerospike Nozzle Thrust, $I_{sp}$ at 16.31 km ($P_{amb} = 9.797$ kPa):

  $1142.6$ kNt, $317.1$ sec

• Nozzle Mass Flow: $367.445$ kg/sec
vi) Effective Specific Impulse

(3 x Gem 40 + RS-27A/w aerospike over operating altitude range)

\[
(I_{sp})_{\text{launch}} = \frac{\left( F_{RS-27A} + 3 \cdot F_{Gem40} \right)_{\text{launch}}}{g_0 \cdot \left( \dot{m}_{RS-27A} + 3 \cdot \dot{m}_{Gem40} \right)} = \frac{965.7 + 3 \cdot 442.95}{367.447 + 3 \cdot 185.315} \cdot \frac{1000}{9.8067} = 253.4 \text{ sec}
\]

\[
(I_{sp})_{\text{burnout}} = \frac{\left( F_{RS-27A} + 3 \cdot F_{Gem40} \right)_{\text{burnout}}}{g_0 \cdot \left( \dot{m}_{RS-27A} + 3 \cdot \dot{m}_{Gem40} \right)} = \frac{1142.6 + 3 \cdot 493.761}{367.447 + 3 \cdot 185.315} \cdot \frac{1000}{9.8067} = 289.8 \text{ sec}
\]

\[
(I_{sp})_{\text{eff}} = \frac{2}{3} \left[ (I_{sp})_{\text{Rs-27A+3xGem40}}^{\text{Launch}} \right] + \frac{1}{3} \left[ (I_{sp})_{\text{Rs-27A+3xGem40}}^{\text{Burnout}} \right] = \frac{2}{3} \cdot 253.4 + \frac{1}{3} \cdot 289.8 = 265.5 \text{ sec}
\]
... iv) Plot delivered Thrust and $I_{sp}$ as a function of altitude for RS-27a Stage

stage 1a: 0 to 16.31 km altitude (RS-27A + 3 x Gem40)
stage 1b: 16.31 km altitude 105.52 km altitude (RS-27A)

... Assume conventional Nozzles for Gem-40 boosters, full aerospike for RS-27A

**Stage “1b”**

- Aerospike Nozzle Vacuum Thrust, $I_{sp}$:

  1161.5 kNt, 322.3.0 sec

\[
(I_{sp})_{eff} = \frac{2}{3} \left[ (I_{sp})_{R2-27A \text{ Gem40 Burnou}} \right] + \frac{1}{3} \left[ (I_{sp})_{R2-27A \text{ MECO}} \right]
\]

\[
\frac{2}{3} 317.1 + \frac{1}{3} 322.3 = 318.8 \text{ sec}
\]
Required versus Available Delta V

At $250/oz
You just saved
~$4.5 Mil! On Payload

Aerospike
2688 kg

Original
2175 kg
... iv) Plot delivered Thrust and $I_{sp}$ as a function of altitude for RS-27a Stage

stage 1a: 0 to 16.31 km altitude (RS-27A + 3 x Gem40)
stage 1b: 16.31 km altitude 105.52 km altitude (RS-27A)

... Assume conventional Nozzles for Gem-40 boosters, full aerospike for RS-27A

**Stage “1a”**

- 61.1% Aerospike Nozzle Thrust, $I_{sp}$ at Sea Level:
  
  $953.8 \text{ kNt, 264.7 sec}$

- Aerospike Nozzle Thrust, $I_{sp}$ at 16.31 km ($P_{amb} = 9.797 \text{ kPa}$):
  
  $1130.9 \text{ kNt, 313.825 sec}$

- Nozzle Mass Flow: $367.445 \text{ kg/sec}$
vi) Effective Specific Impulse for 61.1% Truncated Nozzle

\[ (3 \times \text{Gem 40} + \text{RS-27A/w aerospike over operating altitude range} ) \]

\[ (I_{sp})_{\text{launch}} = \frac{\left( F_{\text{RS-27A}} + 3 \cdot F_{\text{Gem 40}} \right)_{\text{launch}}}{g_0 \cdot \left( m_{\text{RS-27A}} + 3 \cdot m_{\text{Gem 40}} \right)} = \frac{953.8 + 3 \cdot 442.95}{367.447 + 3 \cdot 185.315 \cdot 9.8067} = 252.1 \text{ sec} \]

\[ (I_{sp})_{\text{burnout}} = \frac{\left( F_{\text{RS-27A}} + 3 \cdot F_{\text{Gem 40}} \right)_{\text{Gem 40 burnout}}}{g_0 \cdot \left( m_{\text{RS-27A}} + 3 \cdot m_{\text{Gem 40}} \right)} = \frac{1130.9 + 3 \cdot 493.761}{367.447 + 3 \cdot 185.315 \cdot 9.8067} = 288.5 \text{ sec} \]

\[ (I_{sp})_{\text{eff}} = \frac{2}{3} \left( (I_{sp})_{\text{Rs-27A+ 3x Gem 40 Launch}} + \frac{1}{3} \left( (I_{sp})_{\text{Rs-27A+ 3x Gem 40 Gem 40 Burnou}} \right) \right) = \frac{2}{3} \cdot 252.1 + \frac{1}{3} \cdot 288.5 \]

\[ = 264.2 \text{ sec} \]
Delta II Launch Analysis

... iv) Plot delivered Thrust and $I_{sp}$ as a function of altitude for RS-27a Stage

stage 1a: 0 to 16.31 km altitude (RS-27A + 3 x Gem40)
stage 1b: 16.31 km altitude 105.52 km altitude (RS-27A)

... Assume conventional Nozzles for Gem-40 boosters, full aerospike for RS-27A

**Stage “1b”**

• Aerospike Nozzle Vacuum Thrust, $I_{sp}$:

$$1148.8 \text{ kNt, } 318.8 \text{ sec}$$

61.1% truncated nozzle

$$\left( I_{sp} \right)_{\text{eff}} = \frac{2}{3} \left[ \left( I_{sp} \right)_{R2-27A}^{\text{Gem40 Burnou}} + \frac{1}{3} \left[ \left( I_{sp} \right)_{R2-27A}^{\text{MECO}} \right] R2-27A \right]$$

$$\frac{2}{3} \times 313.8 + \frac{1}{3} \times 318.8 = 315.5 \text{ sec}$$
Required vs. Available Delta V

- Original: 2175 kg
- 61.1% Aerospike: 2600 kg

Still a reasonable $4 mil gain!
Examples: Current (2007) Launch Costs to LEO

- **Soyuz booster (Soyuz spacecraft, Progress)**
  LEO (200 km) Payload: 7,000 kg; Launch vehicle price: $37.5 million; Payload cost: $5,357 per kilogram. **($151.9/oz)**

- **Space Transportation System (Space Shuttle)**
  LEO Payload: 28,803 kg; Launch vehicle price (estimated, pre-Columbia): $300 million, Payload cost: $10,416 per kilogram **($295.4/oz)**
  Post-Columbia, $1 billion per launch, Payload cost: $34,718 per kilogram

- **Proton (ISS modules)**
  LEO Payload: 19,760 kg; Launch vehicle price: $85 million; Payload cost: $4,302 per kilogram **($122.0/oz)**

- **Ariane 5ES (ATV vehicle)**
  LEO Payload: 21,00 kg; Launch vehicle price: $176 million; Payload cost: $9,778 per kilogram **($277.6/oz)**

- **Zenit 2**
  LEO Payload: 13,740 kg; Launch vehicle price: $42,5 million; Payload cost: $3,093 per kilogram. **($87.7/oz)**

- **Space-X Falcon 9**
  LEO (185 mi) mission from CCAFS, 9,900 kg payload; Launch vehicle price: $35 million, Payload cost: $3,535 per kilogram. **($100.3/oz)**

At $250/oz
You just saved

*Any way you slice it …*  
LEO Payload is worth **Far More than its weight in pure Silver!**