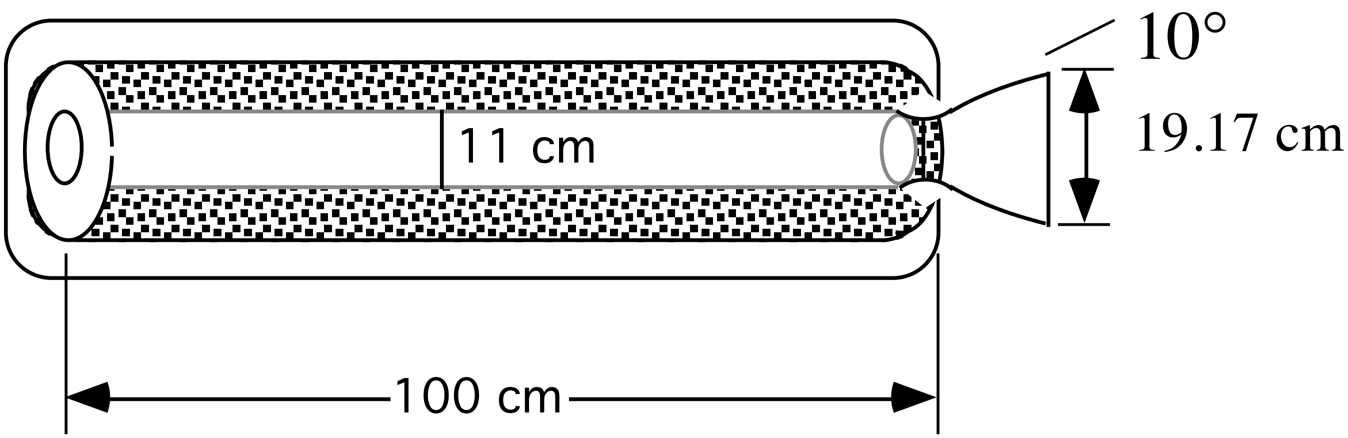
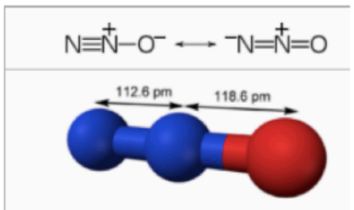
Homework 3.1

- Nitrous Oxide HTPB Hybrid Rocket design
- Desired Thrust of *8 kNt*
- Operate near optimal mixture ratio (based on **c***)
- Nozzle $A/A^*=16.4$, exit diameter = *19.17 cm*,
- Nozzle Exit Divergence angle = 10°
- Single Circular Grain Port, Initial Diameter *11 cm*
- Grain length *100 cm*
- Ambient pressure *60 kPa*
- Assume Isentropic Flow in Nozzle
- **D_{throat} = 4.734 cm**



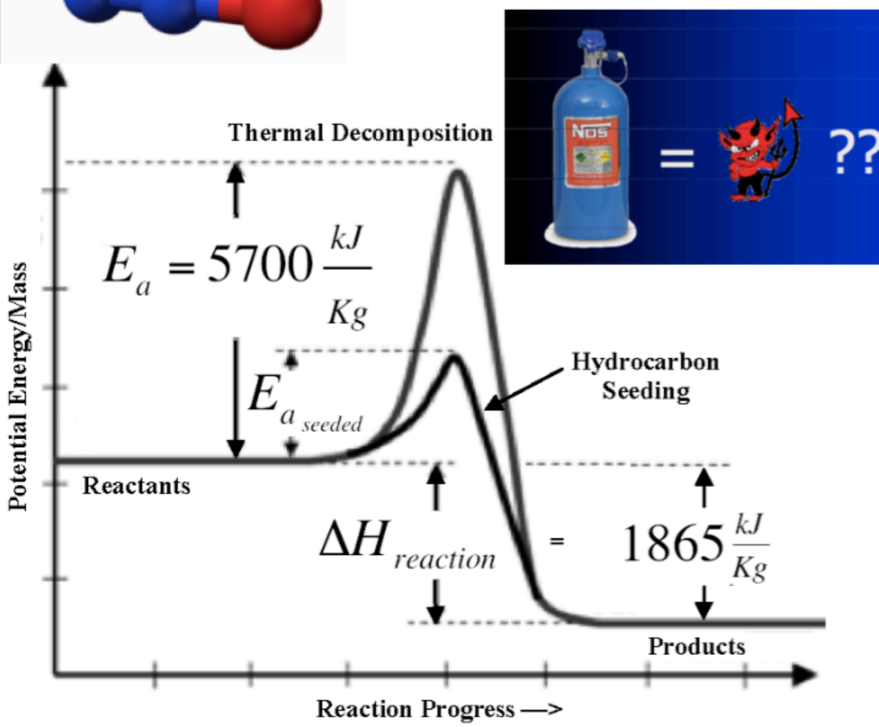
Homework 3.1 (2)

Nitrous Oxide (Oxidizer) Density Properties



N_2O

- **Can present dangerous decomposition hazard in vapor form**



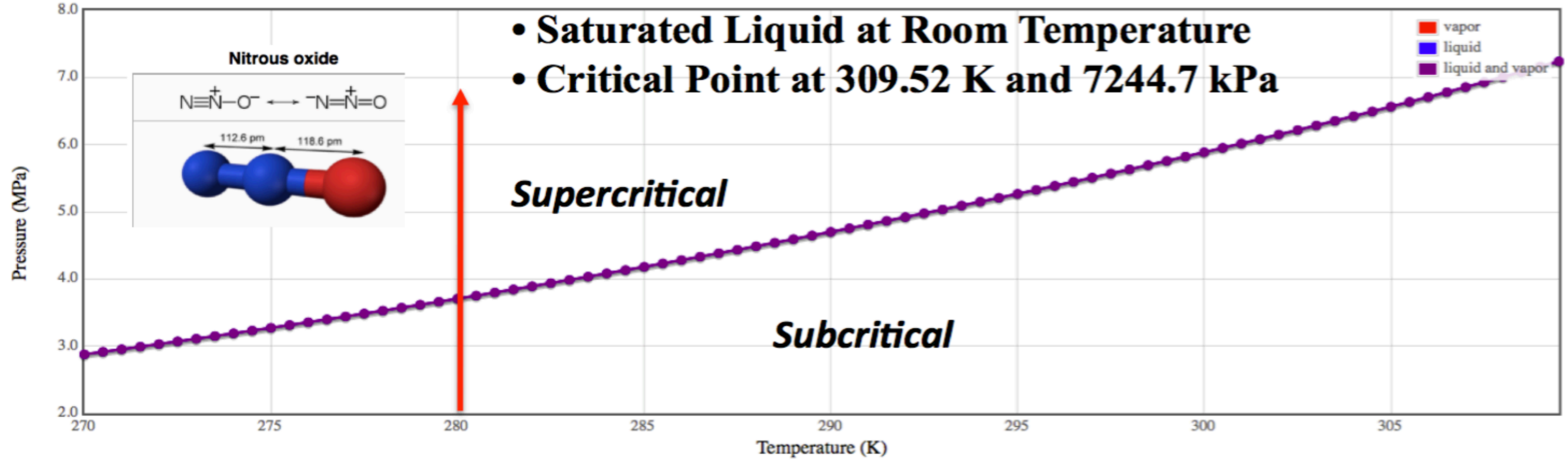
As a safety precaution we'll use a significant "top pressure" to keep Nitrous in a Supersaturated liquid form purely liquid phase until it hits injector, ~ 7000 kPa

Homework 3.1 (3)

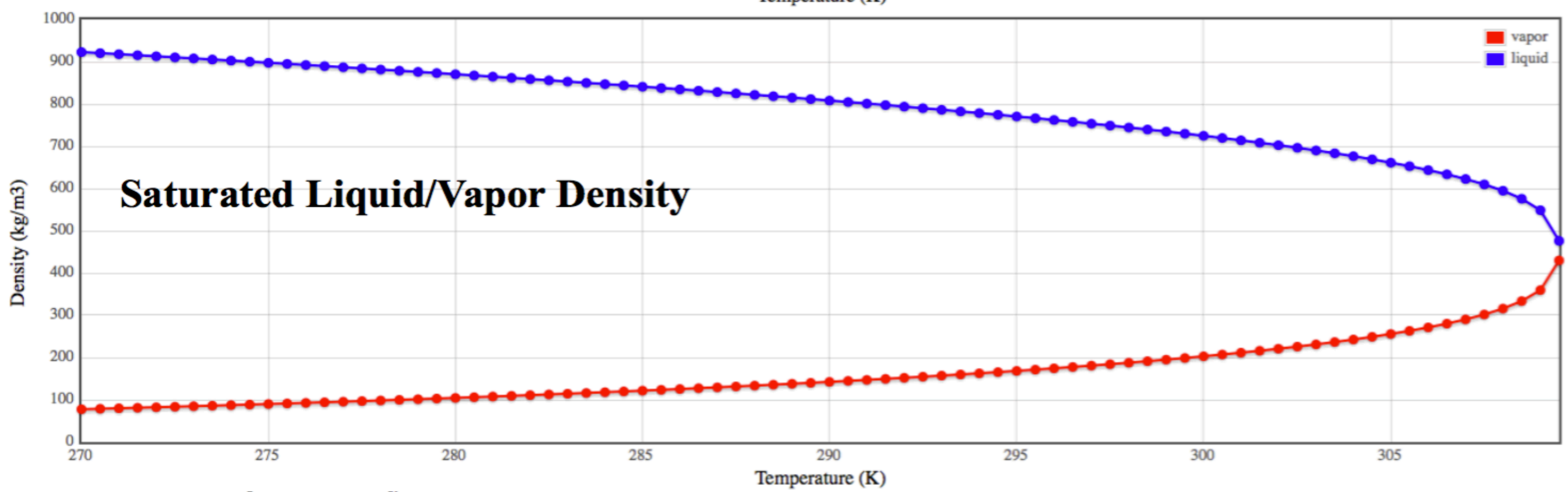
N_2O

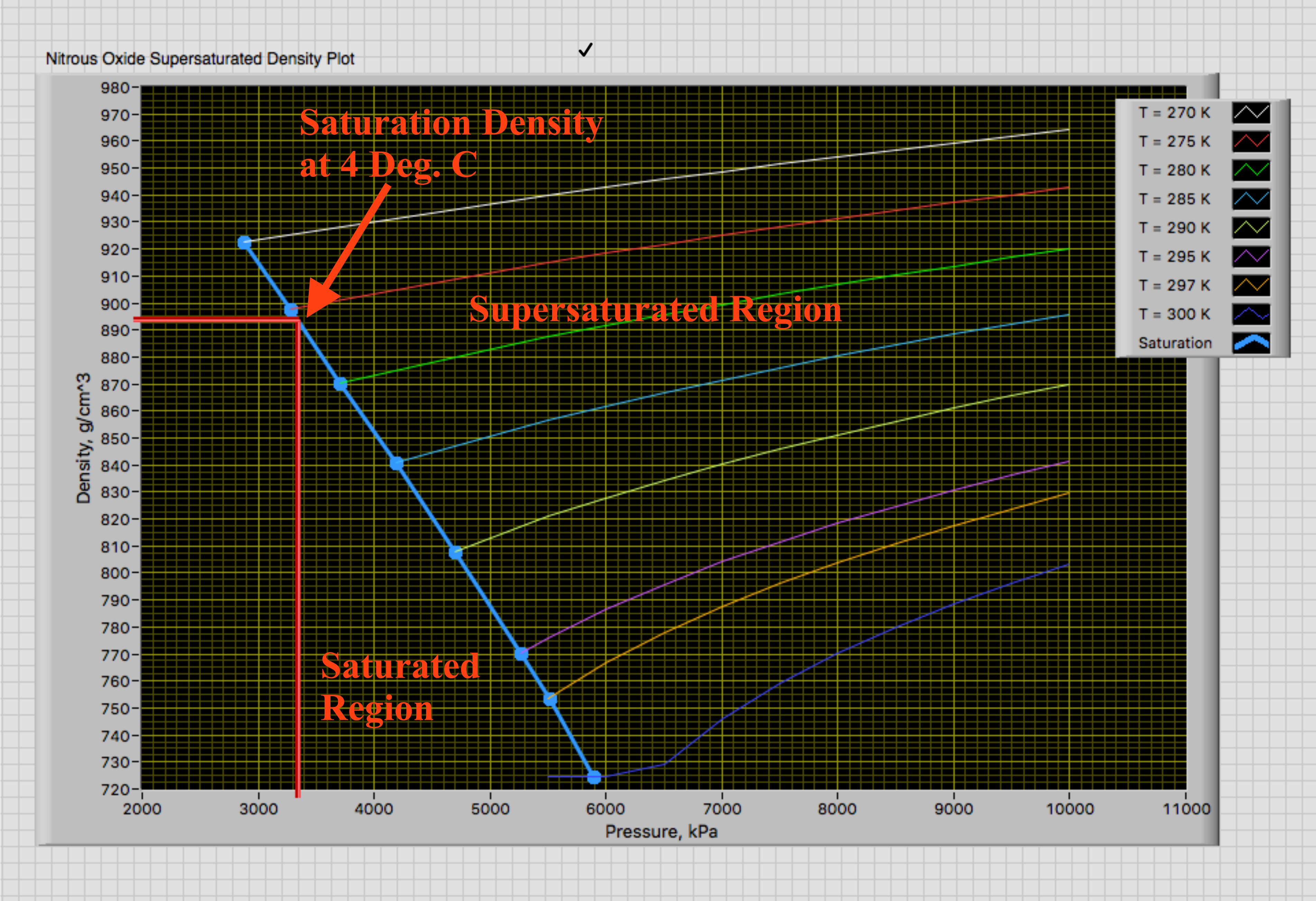
Saturation Vapor Pressure

- **Saturated Liquid at Room Temperature**
- **Critical Point at 309.52 K and 7244.7 kPa**



Saturated Liquid/Vapor Density




Nitrous Oxide Supersaturated Density Plot
Saturation Density
at 4 Deg. C
Supersaturated Region
Saturated
Region
Density, g/cm^3
Pressure, kPa
980
970
960
950
940
930
920
910
900
890
880
870
860
850
840
830
820
810
800
790
780
770
760
750
740
730
720
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
T = 270 K
T = 275 K
T = 280 K
T = 285 K
T = 290 K
T = 295 K
T = 297 K
T = 300 K
Saturation

Homework

- CEA Mean Combustion Property Calculations, $P_0 \simeq 3000$ kPa

N20/HTPB Ideal combustion properties

O/F Ratio	T0 K	γ	Mol.Wt kg/kg-mol
2.000	2019.400	1.251	20.494
3.000	2329.500	1.256	20.823
3.500	2643.200	1.280	21.653
4.000	2895.800	1.256	22.779
5.000	3225.500	1.208	24.613
5.500	3315.000	1.185	25.332
6.000	3366.300	1.167	25.925
7.000	3399.400	1.148	26.807
8.000	3388.300	1.142	27.421
9.000	3359.400	1.141	27.871

$CEA \rightarrow N_2O / C_4H_6 @ 3000_{kPa}$

CEA Data Table

2.000	2019.400	1.251	20.494
3.000	2329.500	1.256	20.823
3.500	2643.200	1.280	21.653
4.000	2895.800	1.256	22.779
5.000	3225.500	1.208	24.613
5.500	3315.000	1.185	25.332
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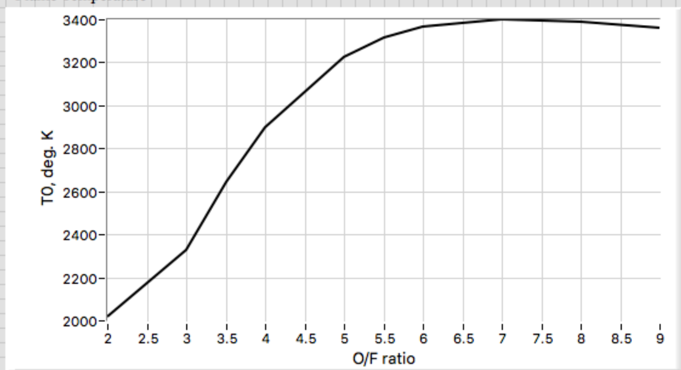
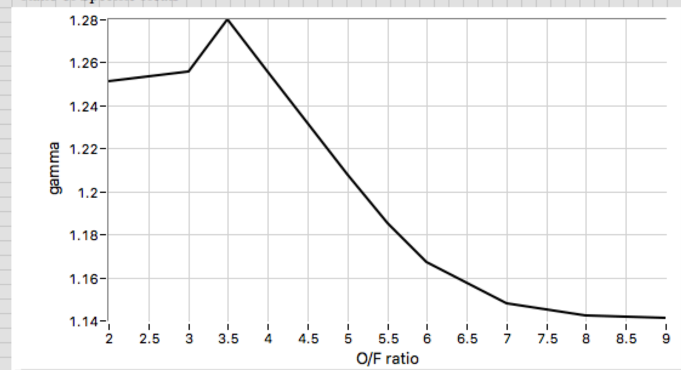
7.000	3399.400	1.148	26.807
8.000	3388.300	1.142	27.421
9.000	3359.400	1.141	27.871

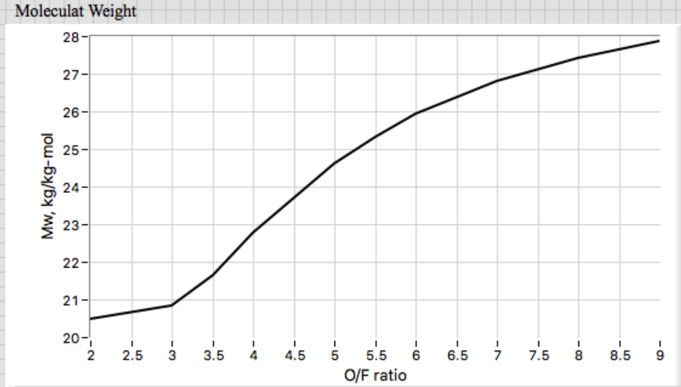
Flame Temperature



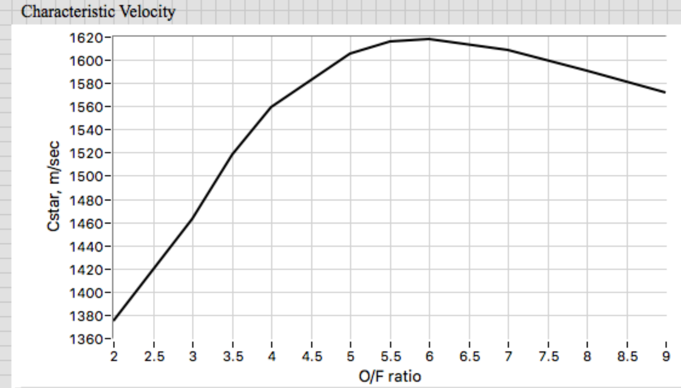
Ratio of Specific Heats



Moleculat Weight



Characteristic Velocity

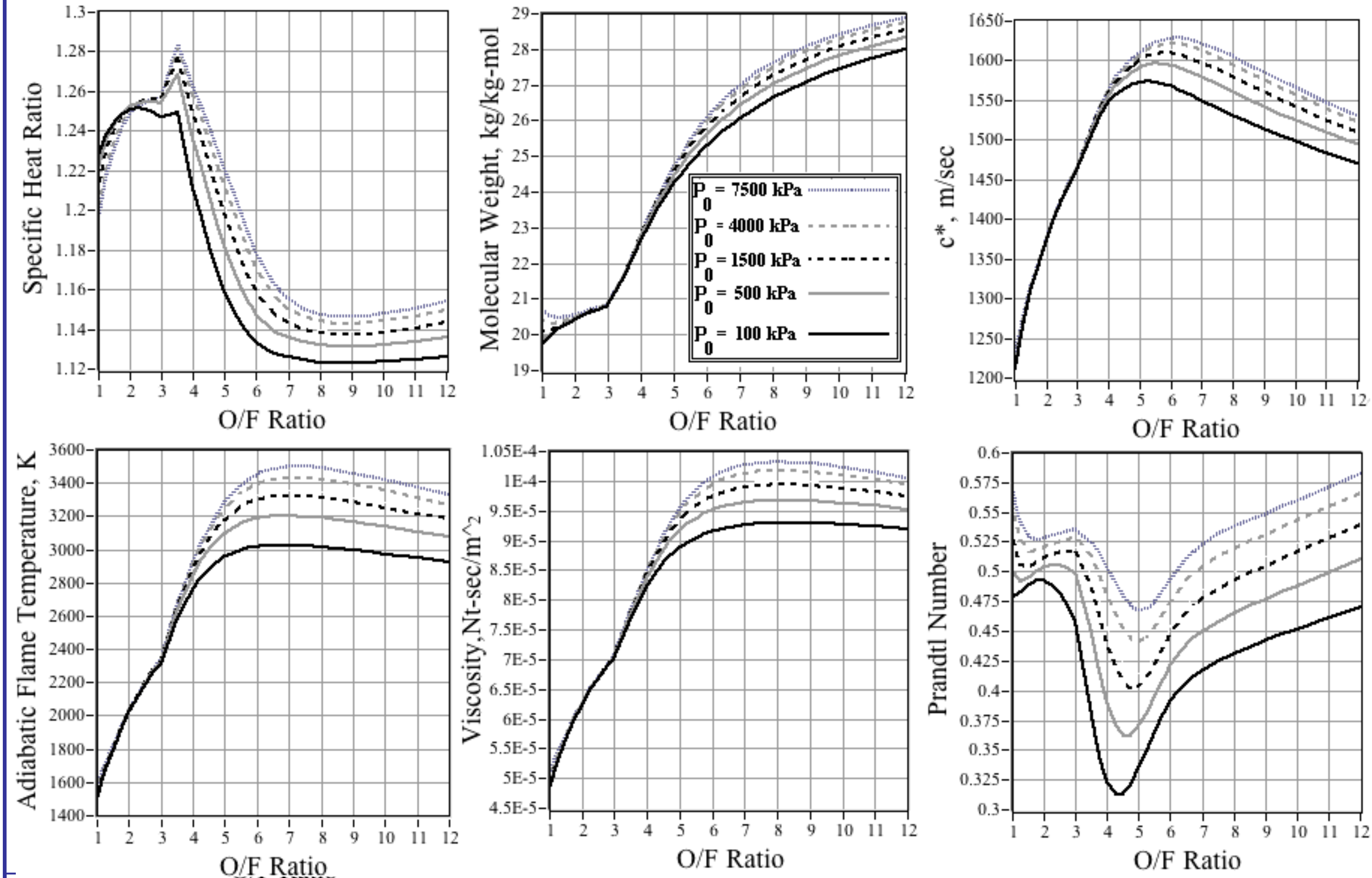


$$\left(c^*\right)_{actual} = \left(\frac{P_0 A^*}{\dot{m}_{exit}}\right)$$

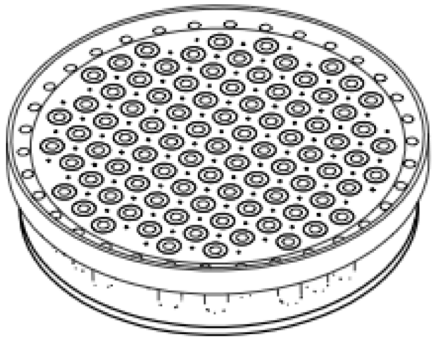
$$\left(c^*\right)_{ideal} = \frac{1}{\gamma}\cdot\sqrt{\left(\gamma\cdot R_g\cdot T_0\right)\cdot\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$\rightarrow \eta^* = \frac{\left(c^*\right)_{actual}}{\left(c^*\right)_{ideal}}$$

Full CEA Table for Nitrous Oxide and HTPB



Homework



- N_2O Injector / Oxidizer properties
- 50 injector ports, 2.0 mm in diameter
- Assume each port has a discharge coefficient C_d=0.81
- Assume Super-Saturation Liquid N_2O density 892 kg/M^3
  Chilled to 4 deg. C (276.15 K)

Gaseous N_2O Viscosity properties

Use Southerland's Law (semi-empirical fit) for Viscosity terms

$$\mu(T_0) = \mu_s \left(\frac{T_0}{T_s}\right)^{3/2} \left(\frac{T_s + C_s}{T_0 + C_s}\right)$$

$$C_s = 240^o K$$

$$T_s = 300^o K$$

$$\mu_s = 1.4889 \times 10^{-5} \, pa - \sec$$

$$T_0 = actual\ flame\ temperature$$

- Assume Boundary Layer Prandtl Number of 0.5
- Assume combustor efficiency (C^*_{actual}/C^*_{ideal}) = 0.99

Homework **3.1 (8)**

- **HTPB Properties**

- Solid propellant density of 930 kg/M^3
- Latent heat of vaporization (h_v), 1.8 MJ/kg

- Solid Grain temperature (assume constant) 300°K

Homework **3.1 (9)**

- **Use Isentropic analysis for Nozzle**

- **Build Dynamic Response Model ….. Assume averaged properties are constant as a function of distance along the propellant grain length**

$$\dot{r} = \frac{0.047}{P_r^{2/3}\,\rho_{fuel}} \left(\frac{c_p \left[T_0 - T_{fuel} \right]}{h_{v_{fuel}}} \right)^{0.23} \left[\frac{A_{ox} C_{d_{ox}}}{A_{c_{chamber}}} \sqrt{2\rho_{ox}\left(p_{ox} - P_0\right)} \right]^{\frac{4}{5}} \left(\frac{\mu_{ox}}{L} \right)^{\frac{1}{5}}$$

[T_0, R_g, M_W, γ] = $f(M_R)$

$$M_{O/F}(t) = 5.58244 \cdot P_r^{2/3} \cdot \left(\frac{h_{v_{solid\ fuel}}}{\Delta h_{flame\ fuel}} \right)^{0.23} \cdot \left(\frac{\dot{m}_{ox}}{\mu_e \cdot L} \right)^{1/5} \left(\frac{D_0 + 2 \cdot \int_0^t \dot{r}_{(\tau)} \cdot d\tau}{L} \right)^{3/5}$$

$$\dot{m}_{propellant} = \dot{m}_{ox} + \dot{m}_{fuel} = \frac{M_{O/F} + 1}{M_{O/F}} A_{ox} C_{d_{ox}} \sqrt{2\rho_{ox}\left(p_{ox} - P_0\right)}$$

- Use integrator of your choice

Homework 3.1 (10)

- 'TWEAK' **Injector pressure to give an approximate steady burn Thrust of 8 kNt** , START WITH Pinj ~ 32500–3500 KPA

Hint: See Slide 4 .. Saturation density plot

- **Choose optimal propellant masses to give a 6 second burn time**

- **Plot predicted linear regression rate as a function of N_2O mass velocity and total mass velocity**

$$\dot{r} \text{ vs } \left. \begin{array}{l} \rho_e \cdot U_e = \dfrac{\dot{m}_{ox}}{A_c} \\ \rho_t \cdot U_t = \dfrac{\dot{m}_{total}}{A_c} \end{array} \right|$$

Repeat Analysis Using Total Massflux Enhancement for regression rate

Required Plots

- **Classic Marxman**
  - Thrust vs. Time
  - Injector and Chamber Pressure vs. Time
  - Massflow vs Time
    - Oxidizer
    - Fuel
    - Total
    - Throat Exit
  - O/F Ratio vs. Time
  - Regression Rate vs. Time
  - Accumulated Impulse Vs. Time
  - Remaining Mass vs. Time
    - Oxidizer
    - Fuel
    - Total
  - I_{sp} vs. Time (Impulse / g_0·Total Mass Consumed)
  - Regression Rate vs Gox ($\dot{m}_{ox}/A_c$)
  - Theoretical c^* (from table, see slides 5 & 19) vs O/F, Actual c^* & O/F from simulation

Required Plots

- **Repeat with Classic Marxman with Total Massflow Enhancement for regression rate**
  - Thrust vs. Time
  - Injector and Chamber Pressure vs. Time
  - Massflow vs Time
    - Oxidizer
    - Fuel
    - Total
    - Throat Exit
  - O/F Ratio vs. Time
  - Regression Rate vs. Time
  - Accumulated Impulse Vs. Time
  - Remaining Mass vs. Time
    - Oxidizer
    - Fuel
    - Total
  - I_{sp} vs. Time (Impulse / $g_0 \cdot$Total Mass Consumed)
  - Regression Rate vs Gox ($\dot{m}_{total}/A_c$)
  - Theoretical c^* (from table, see slides 5 & 19) vs O/F, Actual c^* & O/F from simulation

Homework **3.1 (11)**

- *State Equations*

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn}\dot{r}_{fuel}}{V_c}\left[\rho_{fuel}R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] + \frac{R_g T_0}{V_c}A_{ox}C_{d_{ox}}\sqrt{2\rho_{ox}\left(p_{ox}-P_0\right)}$$

$$\frac{\partial R_{chamber}}{\partial t} = \dot{r}_{fuel}$$

$$\frac{\partial M_{LOX}}{\partial t} = A_{ox}C_{d_{ox}}\sqrt{2\rho_{ox}\left(p_{ox}-P_0\right)}$$

$$\frac{\partial M_{HTPB}}{\partial t} = \rho_{fuel}A_{burn}\dot{r}_{fuel}$$

- *Port Cross Section and Volume*

$$\begin{bmatrix} A_{burn} = 2\pi R_{chamber}L_{prop} \\ V_c = \pi R_{chamber}^{\;2}L_{prop} \end{bmatrix}$$

- *State Vector*

$$\begin{bmatrix} P_0 \\ R_{chamber} \\ M_{LOX} \\ M_{HTPB} \end{bmatrix}$$

- *Port Regression Rate*

$$\dot{r} = \left(\frac{0.0376}{P_r^{\,2/3}\cdot\rho_{\text{fuel}}}\right)\left(\frac{\Delta h_{flame}}{h_f}\right)^{0.23} G_{ox}^{\;4/5}\cdot\left(\frac{\mu}{x}\right)^{1/5}$$

$$\left[\,T_0, R_g, M_W, \gamma\,\right] = f(M_R)$$

$$R_{(t)} = R_0 + s = R_0 + \int_0^t \dot{r}_{(t)}dt$$

Homework

- Calculation Summary

$$\begin{array}{|c|} \text{State} \\ \text{Vector} \end{array} \rightarrow X(t) = \begin{bmatrix} P_{0_{(t)}} \\ r_{(t)} \\ M_{ox_{(t)}} \\ M_{Fuel_{(t)}} \end{bmatrix} = \begin{bmatrix} \text{Chamber pressure} \\ \text{Mean fuel port radius} \\ \text{Current oxidizer mass} \\ \text{Current fuel mass} \end{bmatrix}$$

$$\begin{array}{|c|} \text{State} \\ \text{Equations} \end{array} \rightarrow \dot{X}_{(t)} = \frac{\partial}{\partial t}\begin{bmatrix} P_0 \\ r \\ M_{ox} \\ M_{fuel} \end{bmatrix} = \begin{bmatrix} \frac{A_{burn}\dot{r}_{fuel}}{V_c}\left[\rho_{fuel}R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right] + \frac{R_g T_0}{V_c}A_{ox}C_{d_{ox}}\sqrt{2\rho_{ox}\left(p_{ox}-P_0\right)} \\ \frac{0.047}{\mathrm{P_r}^{2/3}\rho_{fuel}}\left(\frac{c_p\left[T_0 - T_{fuel}\right]}{h_{\nu_{fuel}}}\right)^{0.23}\left[\frac{A_{ox}C_{d_{ox}}}{A_c}\sqrt{2\rho_{ox}\left(p_{ox}-P_0\right)}\right]^{\frac{4}{5}}\left(\frac{\mu_{ox}}{L}\right)^{\frac{1}{5}} \\ A_{ox}C_{d_{ox}}\sqrt{2\rho_{ox}\left(p_{ox}-P_0\right)} \\ \rho_{fuel}A_{burn}\dot{r} \end{bmatrix} = F\left[X_{(t)}\right]$$

$$\dot{X}_{(t)} = F\left[X_{(t)}\right]$$

Homework 2.1 (13))

- Calculation Summary (2)

$$\dot{X}_{(t)} = F\left[X_{(t)}\right] \rightarrow \underline{\begin{array}{l} Integrate\ Trapezoidal\ Rule \rightarrow \\ sample\ interval = \Delta t \end{array}} \rightarrow$$

$$\boxed{\begin{array}{c} \hat{X}_{k+1} = X_k + \Delta t \cdot F\left[X_k\right] \\ X_{k+1} = X_k + \Delta t \cdot \dfrac{F\left[X_k\right] + F\left[\hat{X}_{k+1}\right]}{2} \end{array}}$$

Geometry Calculations $\rightarrow$

$$D_{port(t)} = \frac{1}{2} r_{(t)}$$

$$A_{burn(t)} = \pi \cdot D_{port(t)} \cdot L_{port}$$

$$A_{c(t)} = \frac{\pi}{4} \cdot D^2_{port(t)}$$

$$V_{c(t)} = A_{c(t)} \cdot L_{port}$$

$$A_{ox} = N_{inj} \cdot \frac{\pi}{4} \cdot D^2_{inj}$$

$$A^* = \frac{\pi}{4} \cdot D_{throat}$$

$$G_{ox}(t) = \frac{\frac{\partial}{\partial t} M_{ox\ (t)}}{A_{c(t)}} = \dot{m}_{ox}(t) / A_{c(t)}$$

Homework 2.1 (14)

- Calculation Summary (3)

$$\dot{m}_{nozzle}(t) = \dot{m}_{exit}(t) = P_0(t)\cdot A^* \cdot \sqrt{\frac{\gamma}{R_g \cdot T_{0_{actual}}} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$M_{exit} \rightarrow \frac{A_{exit}}{A^*} = \frac{1}{M_{exit}}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M_{exit}{}^2\right)\right]^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$p_{exit}(t) = P_0(t) \Big/ \left(1+\frac{\gamma-1}{2}M_{exit}{}^2\right)^{\frac{\gamma}{\gamma-1}}$$

De Laval Nozzle Calculations

$$\lambda = \frac{1}{2}\left(1+\cos\theta_{exit}\right)$$

$$T_{exit} = T_{0_{actual}} \Big/ \left(1+\frac{\gamma-1}{2}M_{exit}{}^2\right)$$

$$\rightarrow F_{vac}(t) = \lambda_{exit}\cdot \dot{m}_{exit}(t)\cdot M_{exit}\cdot\sqrt{\gamma \cdot R_g \cdot T_{exit}} + A_{exit}\cdot p_{exit}(t)$$

$$F_{SL}(t) = F_{vac}(t) - A_{exit}\cdot p_{SL}$$

$$I_{sp_{vac}}(t) = \frac{F_{vac}(t)}{g_0 \cdot \dot{m}_{exit}(t)} \rightarrow I_{sp_{SL}}(t) = \frac{F_{SL}(t)}{g_0\cdot \dot{m}_{exit}(t)} \rightarrow I_{sp_{mean}}(T) = \frac{\int_0^T F_{vac}(t)\cdot dt}{g_0 \cdot \left[M_{ox}(T) + M_{fuel}(T)\right]}$$

Homework 2.1 (15)

- Calculation Summary (4)

End of Frame Calculations $\rightarrow\rightarrow$

$$O/F_{(t)} = \frac{M_{ox\ (t)}}{M_{fuel\ (t)}}$$

$$\{\gamma, M_W, C_p, C_v, R_g\}_{(t)} \rightarrow CEA\left(P_0(t), O/F_{(t)}, \eta^*\right)$$

$$\eta^* = \sqrt{\frac{T_{0_{act}}}{T_{0_{ideal}}}}$$

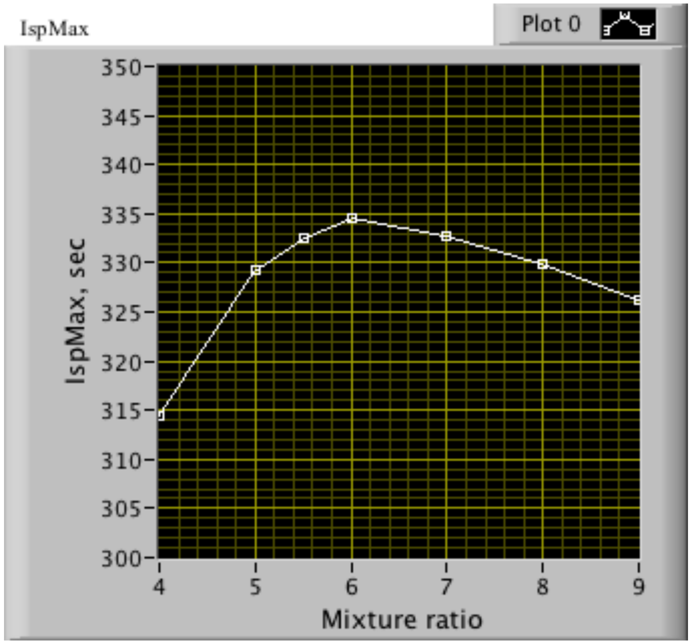
- Enhancement Option

4/5

Total Flux Mean Regression Rate $\rightarrow$

$$\dot{r} = \left(\frac{0.047}{P_r^{2/3}\cdot\rho_{fiel}}\right)\cdot\left(\frac{\Delta h_{\substack{flame\\surface}}}{h_v}\right)^{0.23}\cdot\left(G_{ox}^{\ 1/5} + \frac{5}{9}\cdot\left(\frac{0.047}{P_r^{2/3}}\right)\cdot\left(\frac{\Delta h_{\substack{flame\\surface}}}{h_v}\right)^{0.23}\cdot\left(\frac{\mu}{L}\right)^{1/5}\cdot\left(\frac{L}{D_{port}}\right)\right)\cdot\left(\frac{\mu}{L}\right)^{1/5}$$

Homework



$C^*_{max} \sim M_{O/F} = 6.0$

Initial Conditions

…. Start with this value

… Pick $P_0^{(o)} \sim 2\ P_{amb}$

… Pick $T_0^{(o)}$, $\gamma^{(o)}$, $M_R^{(0)}$

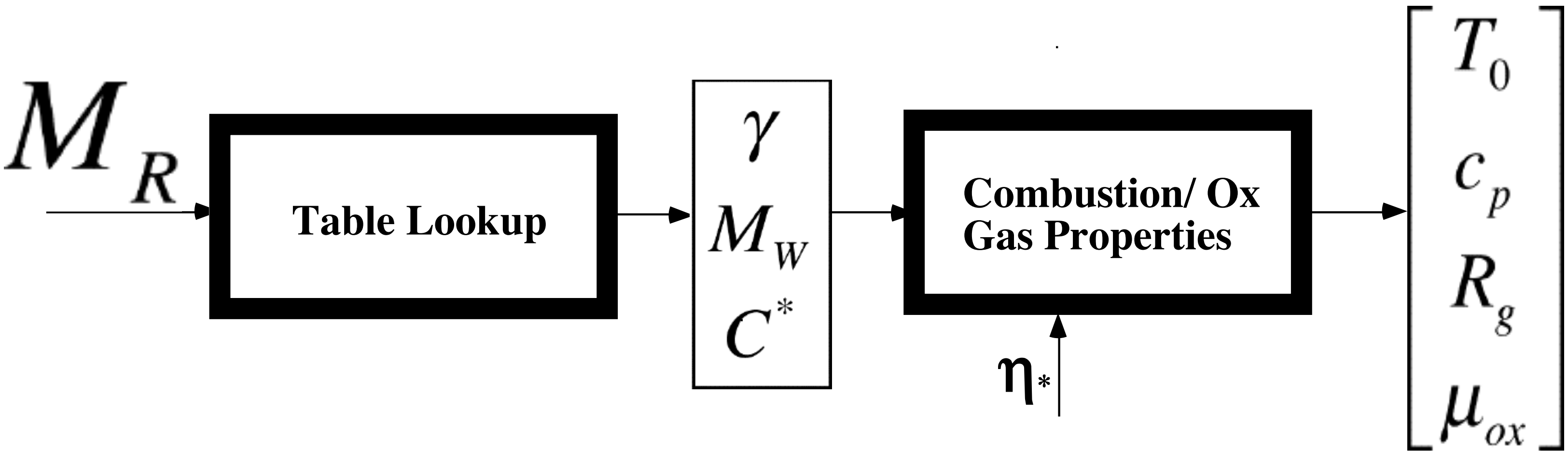
… corresponding to $M_R = 6.0$ (Table Lookup)

… compute initial γ, c_p, μ, etc.

$$[\ T_0, R_g, M_W, \gamma\ ] = f(M_R)$$

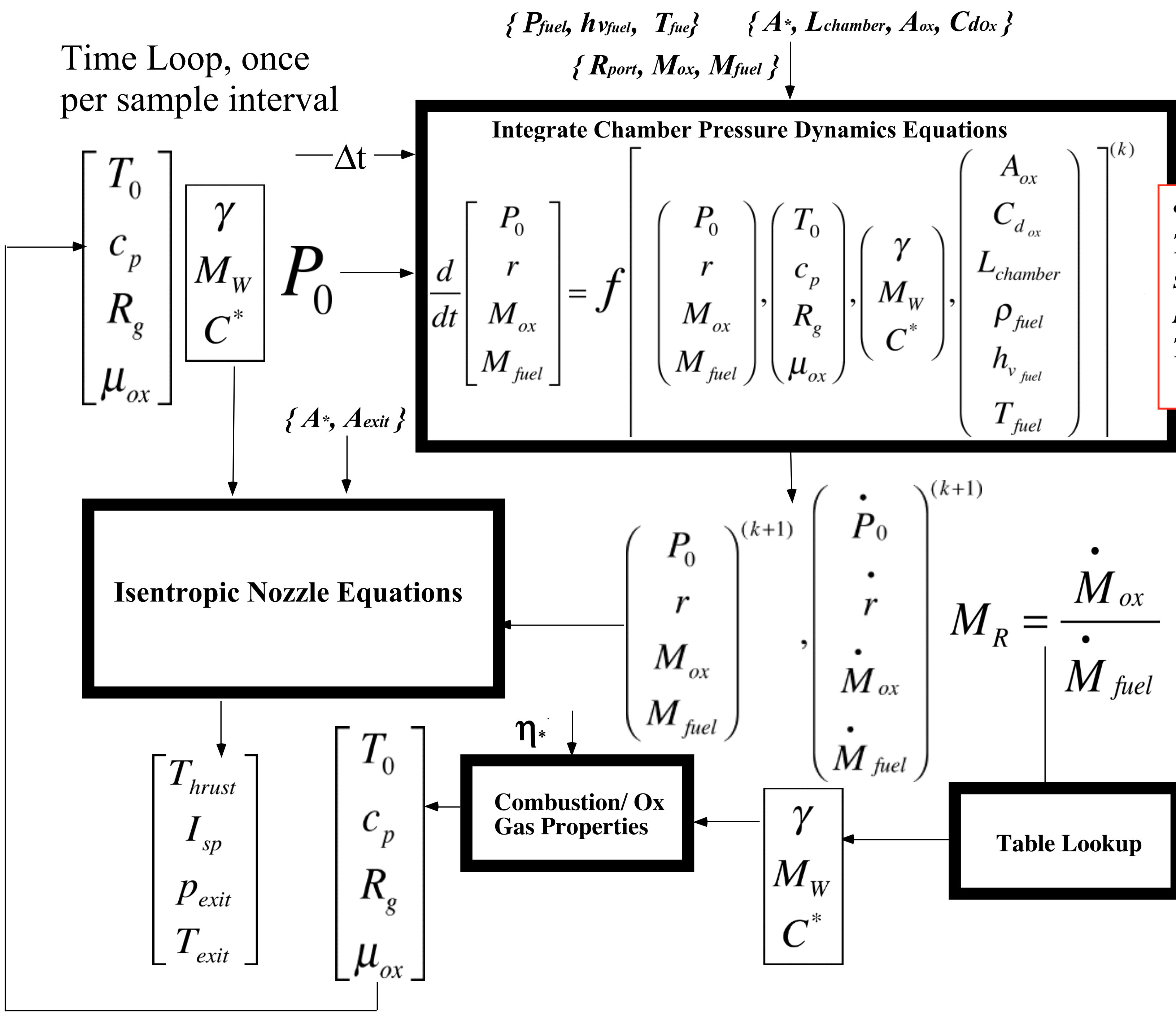
Homework **2.1 (17)**

INITIAL CONDITIONS: M_R~6, P_0 ~ 2 p_{amb}
(enough to choke nozzle)



Startup Initial Geometry / Mass Properties ***{ R_{port}, M_{ox}, M_{fuel} }***

Time Loop, once per sample interval



- *Use Variable Time step Be sure to Capture Initial Startup Transient*

- *Evaluate New Gas properties at End of each frame*

  … use new props. and state vector For each new frame