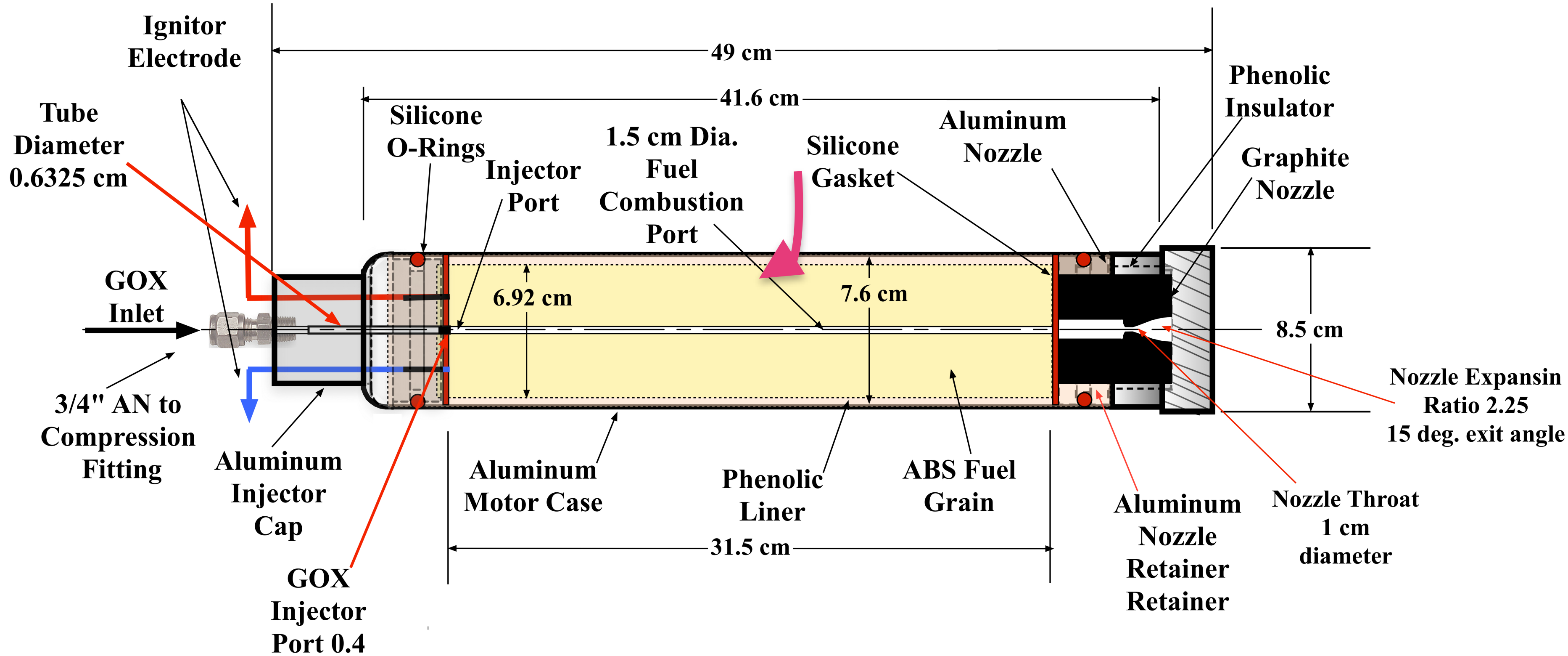
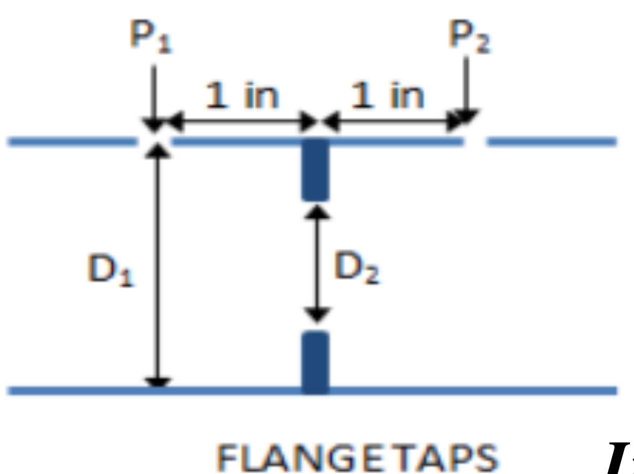
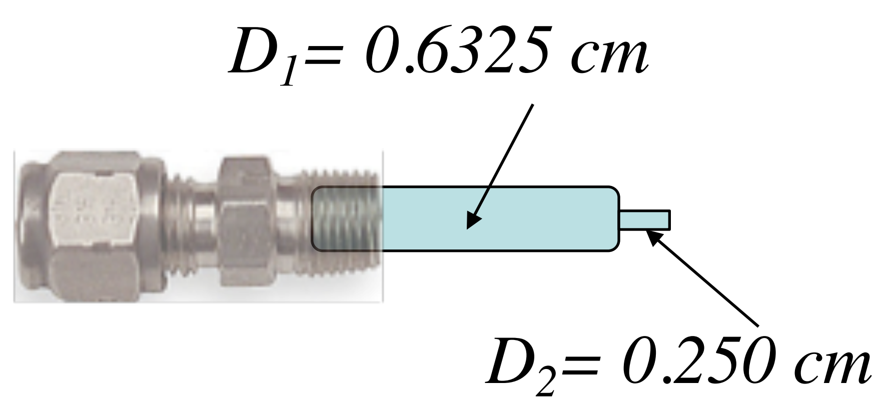
Homework 3.2

- Gaseous Oxygen (GOX)/ABS Hybrid Rocket
- Injector Feed Pressure, 2500 kPa
- Single Port Injector, Port Diameter 0.25 cm
- GOX Feed Pipe to Injector, Diameter 0.6325
- Nozzle A/A^*=2.25, Throat diameter = 1 cm,
- Nozzle Exit Divergence angle = 15°
- Single Circular Grain Port, Initial Diameter 1.5 cm
- Grain length 31.5 cm
- Ambient pressure 60 kPa
- Assume Isentropic Flow in Nozzle
- Allow for 20 second burn time
- Calculate regression rate Using Marxman parameters, corrected for total port massflux

Homework 2.2 (2)







Injector Geometry Model

Homework 2.2 (3)

- Compare Performance Calculations .. using

→ Incompressible Injector Equation, Cd calculated per *ASME_MFC_14M_2001*

→ Compressible Injector Equation, Cd calculated per method *of D. A. Jobson, https://doi.org/10.1243/PIME_PROC_1955_169_077_02*

→ Base compressibility correction on mean injector pressure ratio, $p_{inj}/pc_{\_mean}$

→ Hold Injector C_d constant for entire burn time, Assume Flange Injector Geometry

→ May need to iterate runs a couple of times, adjusting compressible Cd bwtween runs

Time History Plots:

i. Chamber Pressure
ii. Thrust
iii. Massflow (Ox, Fuel, Total, Choke)
iv. O/F Ratio
v. Consumed mass (Ox, Fuel, Total)
vi. Injector pressure ratio

Compare Injector Model Results:

i. Mean Thrust
ii. Total Impulse
iii. Isp
iv. Consumed mass (Ox, Fuel, Total)
v. Mean Injector pressure ratio

Homework 2.2 (4)

- **ABS Combustion Properties**

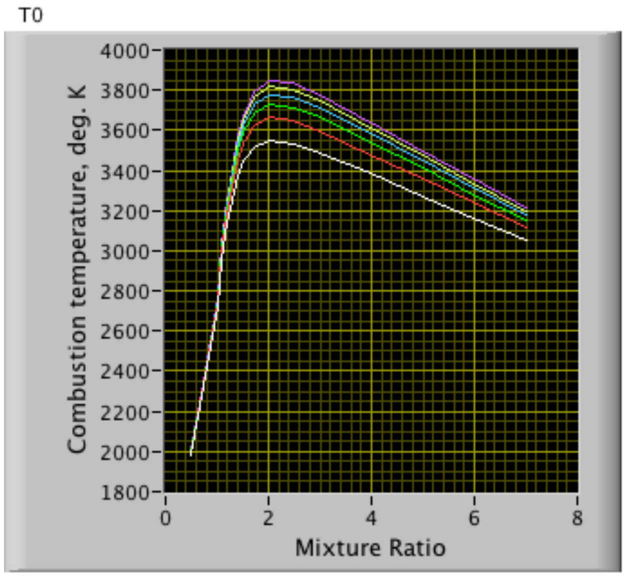
- *Use 2-D Linear Table Lookup of Properties Based on Current O/F and Pc Values*

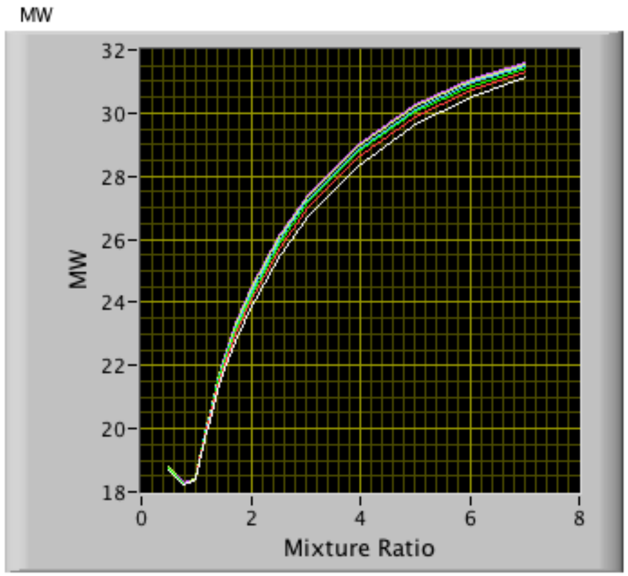
- Assume combustor efficiency (C^*_{actual}/C^*_{ideal}) = 0.90
- Assume Boundary Layer Prandtl Number of 0.5

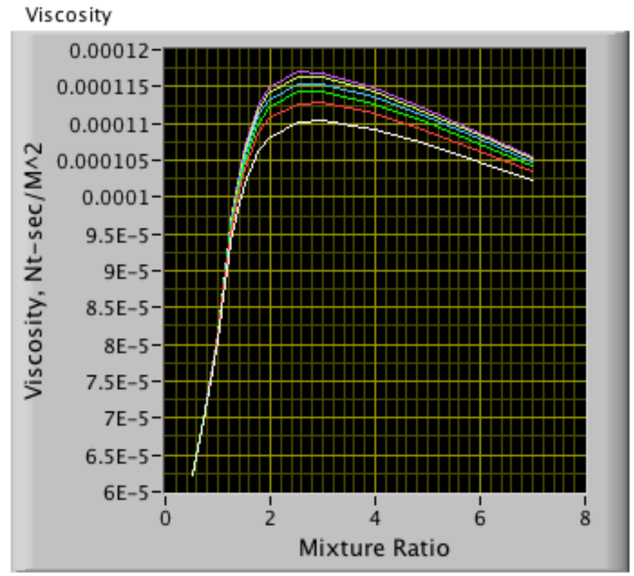
- Solid ABS propellant density of 975 kg/M^3
- Latent heat of vaporization (h_v), 3.0 MJ/kg
- Solid Grain temperature (assume constant) 293.15°K

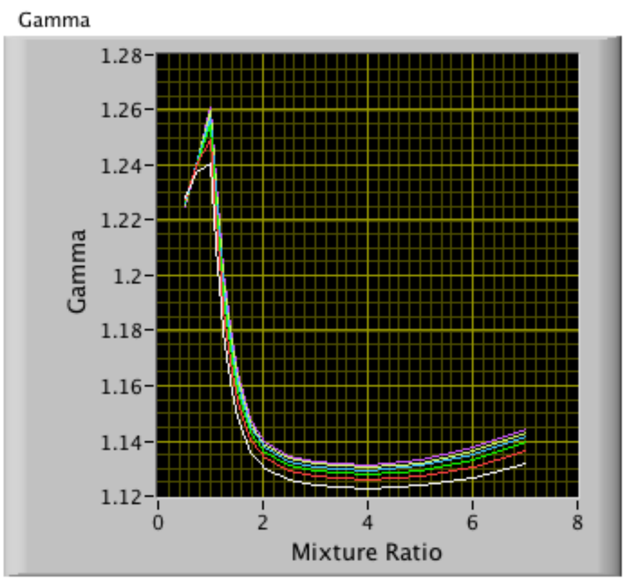
Homework 2.2 (5)

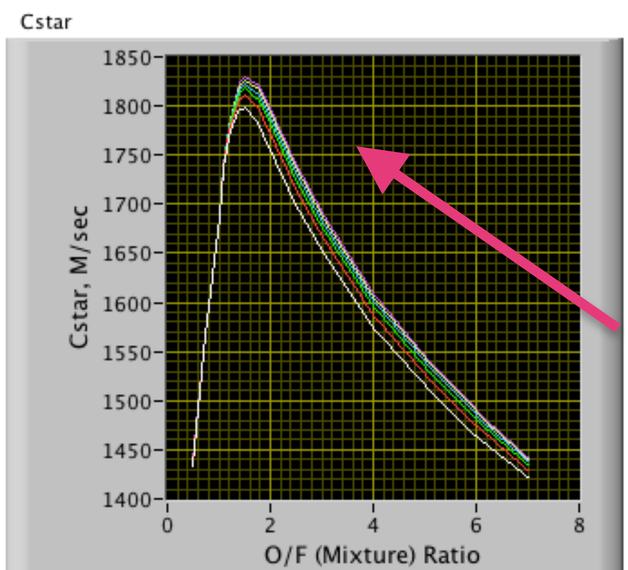
- CEA GOX/ABS Combustion Properties

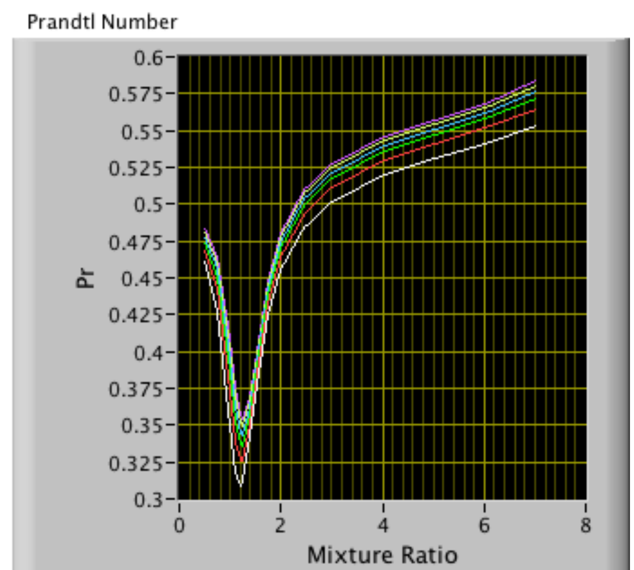




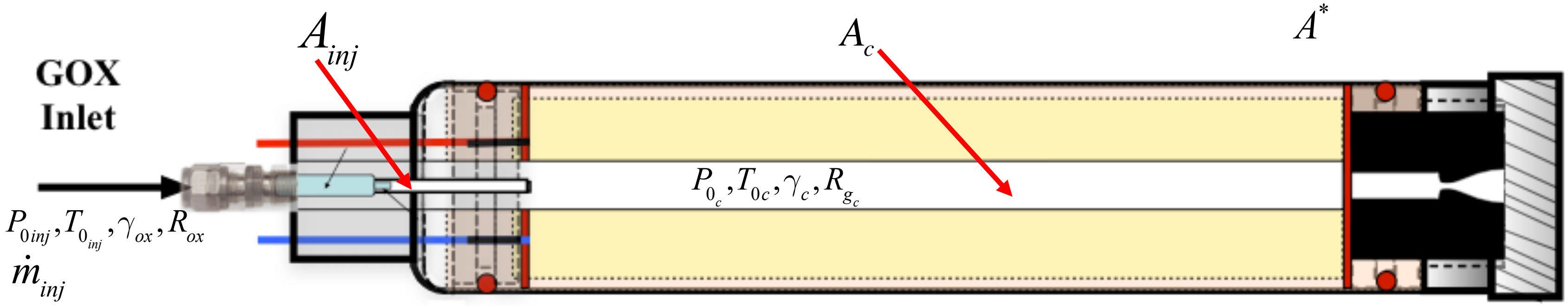








Hybrid Ballistic Equations for Compressible Oxidizer



$$Subcritical: \left(\frac{P_{0inj}}{P_{0_c}}\right) < \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

$\rightarrow$ *Injector Not Choked*

$$K_n = \sqrt{\frac{2\cdot\gamma_{ox}}{\gamma_{ox}-1}\cdot\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)^{\frac{2}{\gamma_{ox}}}\left[1-\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)^{\frac{\gamma_{ox}-1}{\gamma_{ox}}}\right]} \rightarrow \dot{m}_{ox} = \left(K_n\cdot C_d\cdot A\right)_{inj}\cdot\frac{P_{0inj}}{\sqrt{R_{g_{ox}}T_{0_{inj}}}}$$

$$C_d = \frac{1}{2\cdot f\cdot\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)^{\frac{1}{\gamma_{ox}}}}\cdot\left[1-\sqrt{\left\{1-\left(2\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)^{\frac{1}{\gamma_{ox}}}\right)^2\left(1-\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right)\cdot f\Big/K_n^2\right\}}\right]$$

Chamber Pressure :

$$\frac{\partial P_{0c}}{\partial t} = \frac{A_{burn}\dot{r}_{fuel}}{V_c}\left[\rho_{fuel}R_{g_c}T_{0c}-P_{0c}\right]-P_{0c}\left[\frac{A^*}{V_c}\sqrt{\gamma_c R_{g_c}T_0\left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c+1}{(\gamma_c-1)}}}\right]+\frac{R_{g_c}T_{0_c}}{V_c}\cdot\left\{\left(K_n\cdot C_d\cdot A_{inj}\right)_{inj}\cdot\frac{P_{0inj}}{\sqrt{R_{g_{ox}}\cdot T_{0_{inj}}}}\right\}$$

Regression :

0.23

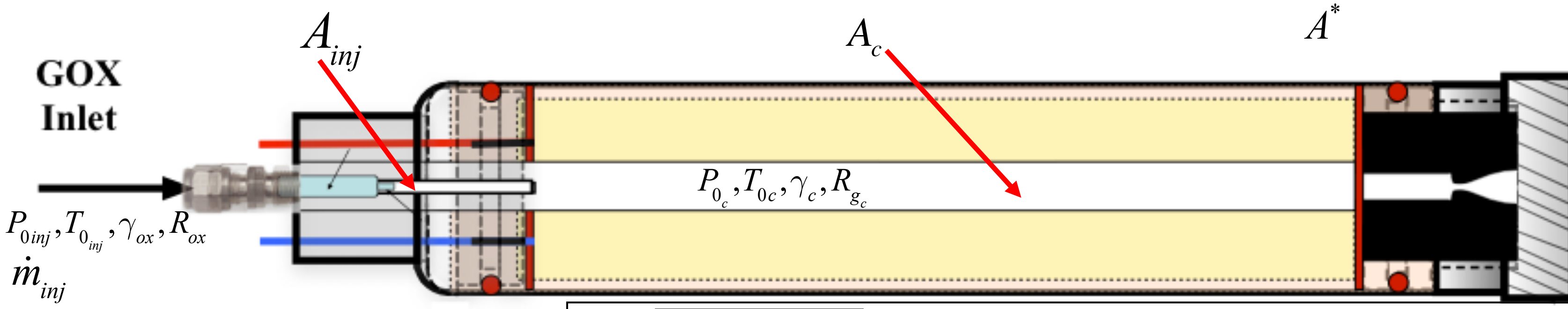
$$\dot{r}_{fuel} = \left(\frac{0.047}{\rho_{fuel}\cdot\left(P_{r_c}\right)^{2/3}}\right)\cdot\left(\frac{C_{P_c}\cdot\left(T_{0_c}-T_{fuel_{surf}}\right)}{h_{v_{fuel}}}\right)\cdot\left(\frac{\dot{m}_{ox}}{A_c}\right)^{4/5}\cdot\left(\frac{\mu_c}{L}\right)^{1/5}$$

$$\dot{m}_{fuel} = \rho_{fuel}\cdot A_{burn}\cdot\dot{r}_{fuel} = \rho_{fuel}\cdot\pi\cdot(D\cdot L)_{port}\cdot\dot{r}_{fuel} \rightarrow O/F = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{\left(K_n\cdot C_d\cdot A\right)_{inj}\cdot\frac{P_{0inj}}{\sqrt{R_{g_{ox}}T_{0_{inj}}}}}{\rho_{fuel}\cdot\pi\cdot(D\cdot L)_{port}\cdot\dot{r}_{fuel}}$$

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Hybrid Ballistic Equations for Compressible Oxidizer



$$K_n = \sqrt{\gamma_{ox} \cdot \left(\frac{2}{\gamma_{ox}+1}\right)^{\frac{\gamma_{ox}+1}{\gamma_{px}-1}}} \rightarrow \dot{m}_{ox} = \left(K_n \cdot C_d \cdot A\right)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0_{inj}}}}$$

$$C_d = \left(\frac{1}{2 \cdot f \cdot r_c^{\frac{1}{\gamma_{ox}}}}\right) \cdot \left[\left\{1 + \frac{\left(r_c - \left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right) \cdot r_c^{\frac{1}{\gamma_{ox}}}}{K_n^2}\right\} - \sqrt{1 + \frac{\left(r_c - \left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right) \cdot r_c^{\frac{1}{\gamma_{ox}}}}{K_n^2} - \frac{\left(2 \cdot r_c^{\frac{1}{\gamma_{ox}}}\right)^2 \cdot \left(1 - \left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right) \cdot f}{K_n^2}}\right]$$

$$Supercritical: \left(\frac{P_{0inj}}{P_{0_c}}\right) \geq \left(\frac{\gamma_{ox}+1}{2}\right)^{\frac{\gamma_{ox}}{\gamma_{px}-1}}$$

$$\rightarrow Injector\ Choked \rightarrow r_c = \left(\frac{2}{\gamma_{0x}+1}\right)^{\frac{\gamma_{ox}}{\gamma_{ox}-1}}$$

Chamber Pressure:

$$\frac{\partial P_{0c}}{\partial t} = \frac{A_{burn} \dot{r}_{fuel}}{V_c}\left[\rho_{fuel} R_{g_c} T_{0c} - P_{0c}\right] - P_{0c}\left[\frac{A^*}{V_c}\sqrt{\gamma_c R_{g_c} T_0 \left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c+1}{(\gamma_c-1)}}}\right] + \frac{R_{g_c} T_{0_c}}{V_c} \cdot \left\{\left(K_n \cdot C_d \cdot A_{inj}\right)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} \cdot T_{0_{inj}}}}\right\}$$

Regression:

0.23

$$\dot{r}_{fuel} = \left(\frac{0.047}{\rho_{fuel} \cdot \left(P_{r_c}\right)^{2/3}}\right) \cdot \left(\frac{C_{P_c} \cdot \left(T_{0_c} - T_{fuel_{surf}}\right)}{h_{v_{fuel}}}\right) \cdot \left(\frac{\dot{m}_{ox}}{A_c}\right)^{4/5} \cdot \left(\frac{\mu_c}{L}\right)^{1/5}$$

$$\dot{m}_{fuel} = \rho_{fuel} \cdot A_{burn} \cdot \dot{r}_{fuel} = \rho_{fuel} \cdot \pi \cdot (D \cdot L)_{port} \cdot \dot{r}_{fuel} \rightarrow O/F = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{\left(K_n \cdot C_d \cdot A\right)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0_{inj}}}}}{\rho_{fuel} \cdot \pi \cdot (D \cdot L)_{port} \cdot \dot{r}_{fuel}}$$