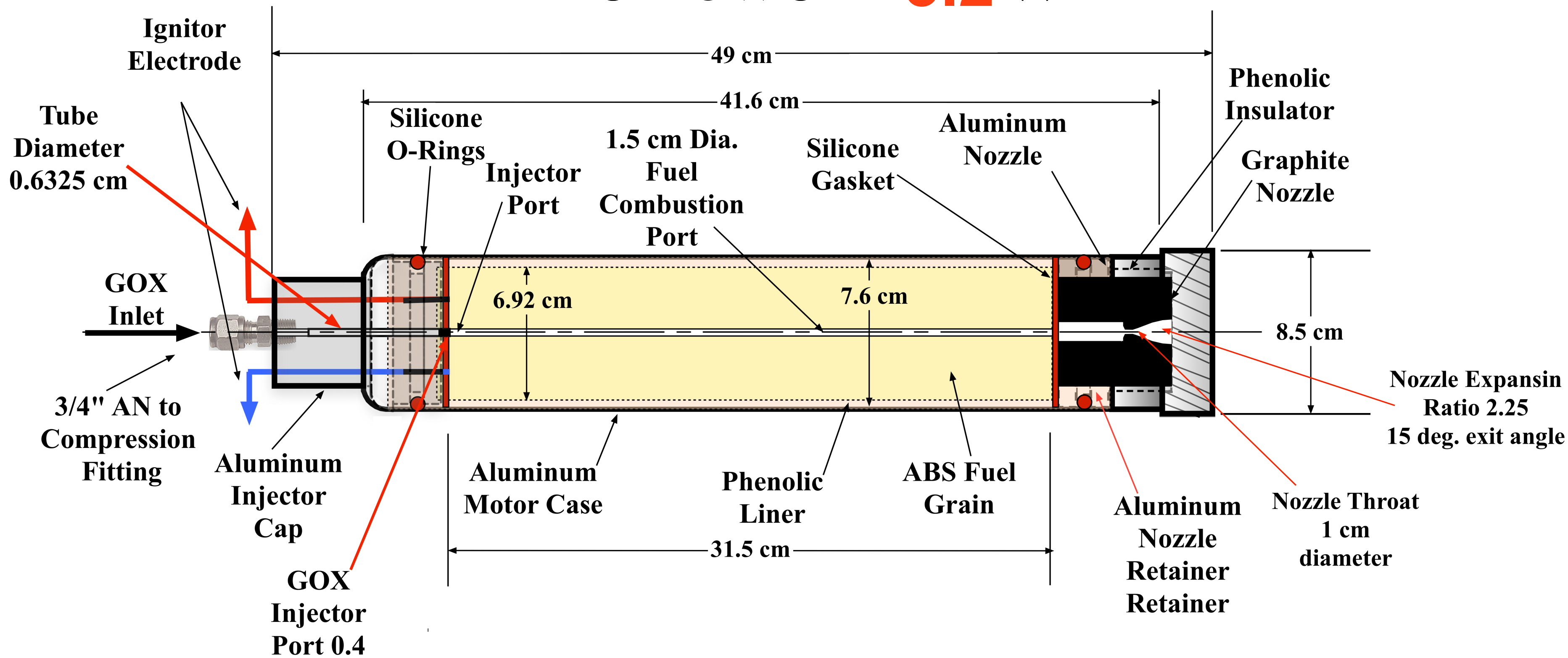
UtahState UNIVERSITY

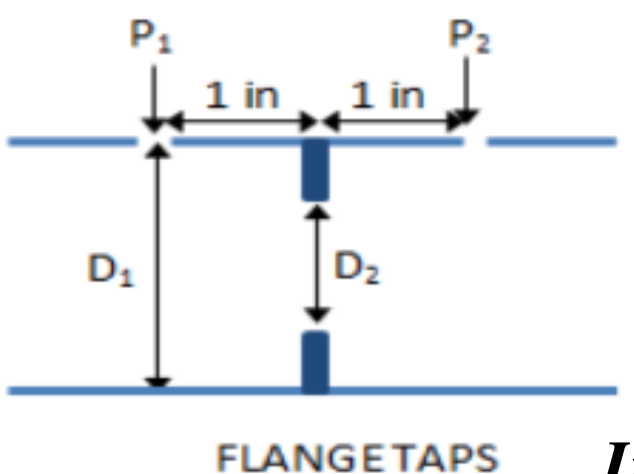
Mechanical & Aerospace Engineering

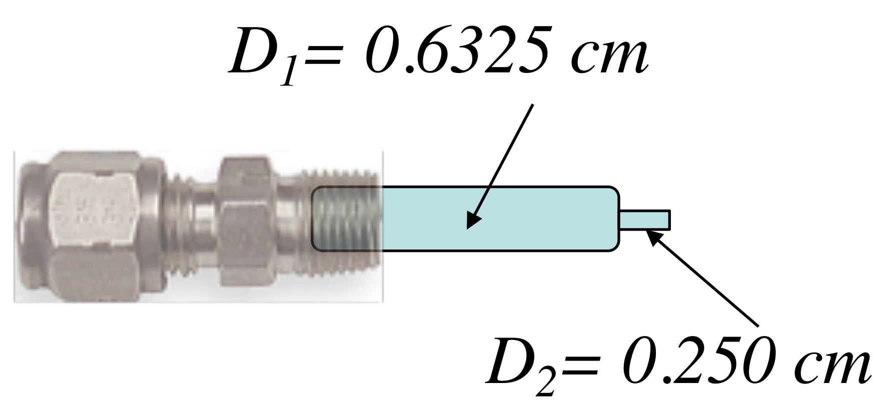
Homework 3.2

- Gaseous Oxygen (GOX)/ABS Hybrid Rocket
- Injector Feed Pressure, 2500 kPa
- Single Port Injector, Port Diameter 0.25 cm
- GOX Feed Pipe to Injector, Diameter 0.6325
- Nozzle $A/A^*=2.25$, Throat diameter = 1 cm,
- Nozzle Exit Divergence angle = 15°
- Single Circular Grain Port, Initial Diameter 1.5 cm
- Grain length 31.5 cm
- Ambient pressure 60 kPa
- Assume Isentropic Flow in Nozzle
- Allow for 20 second burn time
- Calculate regression rate Using Marxman parameters, corrected for total port massflux

Homework 3.2 (2)







Injector Geometry Model

Homework 3.2, (3)

- Compare Performance Calculations .. using

→ Incompressible Injector Equation, Cd calculated per *ASME_MFC_14M_2001*

→ Compressible Injector Equation, Cd calculated per method *of D. A. Jobson, https://doi.org/10.1243/PIME_PROC_1955_169_077_02*

→Base compressibility correction on mean injector pressure ratio, $p_{inj}/pc_{\_mean}$

→ Hold Injector C_d constant for entire burn time, Assume Flange Injector Geometry

→May need to iterate runs a couple of times, adjusting compressible Cd bwtween runs

Time History Plots:

i. *Chamber Pressure*
ii. *Thrust*
iii. *Massflow (Ox, Fuel, Total, Choke)*
iv. *O/F Ratio*
v. *Consumed mass (Ox, Fuel, Total)*
vi. *Injector pressure ratio*

Compare Injector Model Results:

i. *Mean Thrust*
ii. *Total Impulse*
iii. *Isp*
iv. *Consumed mass (Ox, Fuel, Total)*
v. *Mean Injector pressure ratio*

Homework 3.2 (4)

- **ABS Combustion Properties**

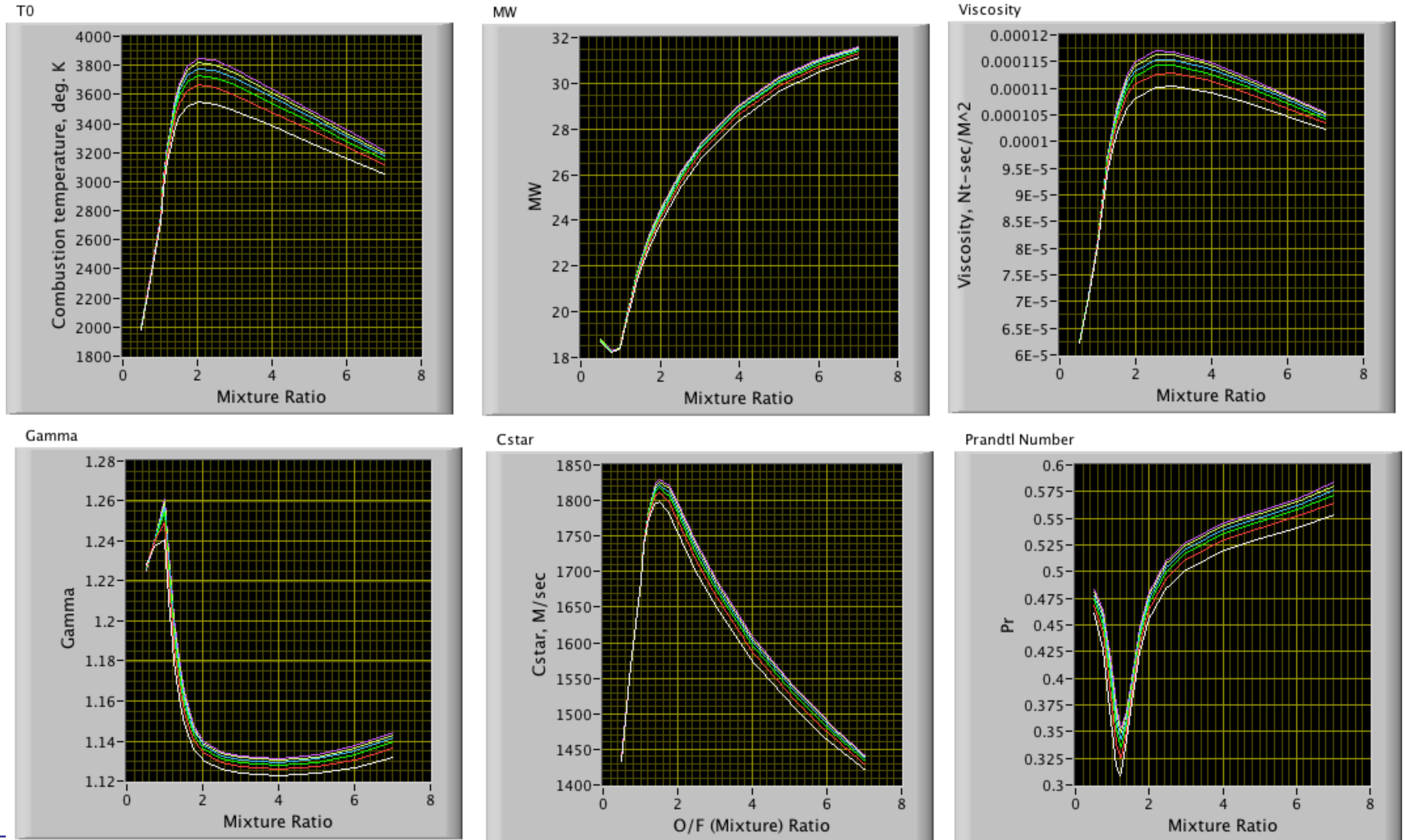
- *Use 2-D Linear Table Lookup of Properties Based on Current O/F and Pc Values*

- Assume combustor efficiency (C^*_{actual}/C^*_{ideal}) = 0.90
- Assume Boundary Layer Prandtl Number of 0.5

- Solid ABS propellant density of 975 kg/M^3
- Latent heat of vaporization (h_v), 3.0 MJ/kg
- Solid Grain temperature (assume constant) 293.15°K

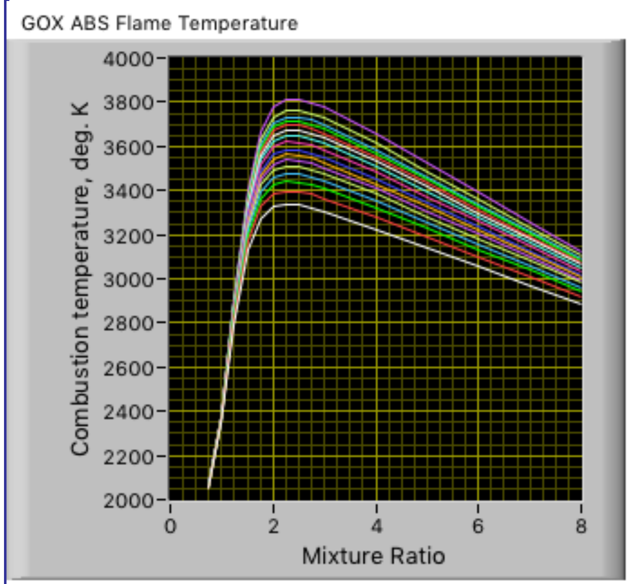
Homework 3.2 (5)

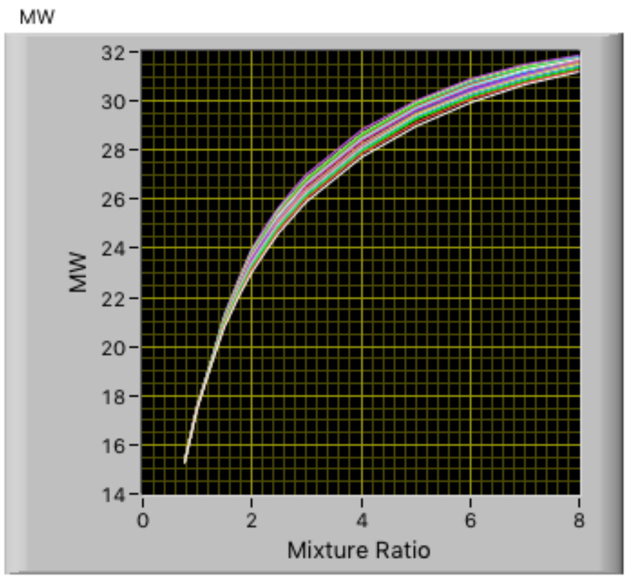
- CEA GOX/ABS Combustion Properties

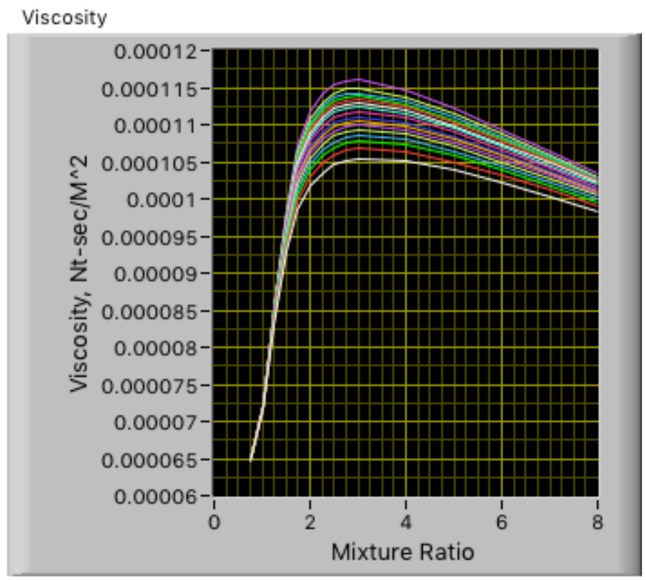


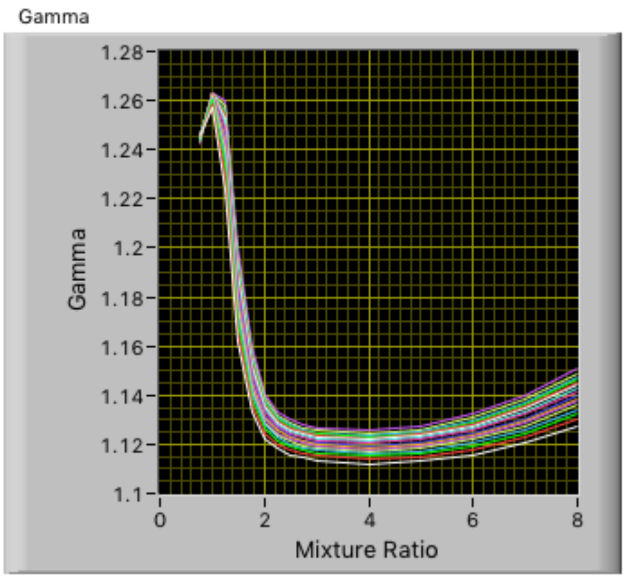
Homework 3.2 (5)

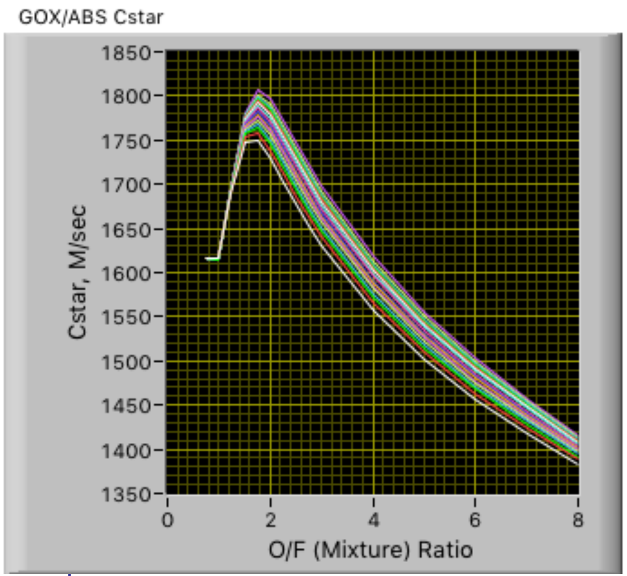
- CEA GOX/ABS Combustion Properties, Extended Pressures

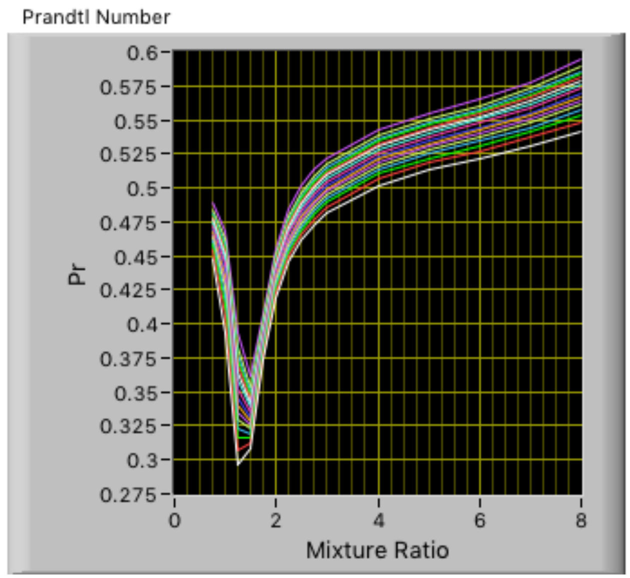




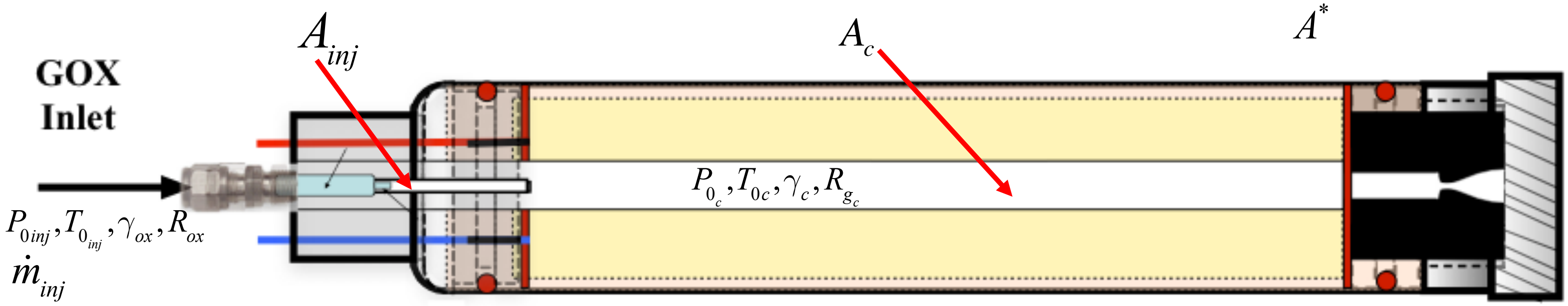








Hybrid Ballistic Equations for Compressible Oxidizer



$$Subcritical: \left(\frac{P_{0inj}}{P_{0_c}}\right) < \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

$\rightarrow$ *Injector Not Choked*

$$K_n = \sqrt{\frac{2\cdot\gamma_{ox}}{\gamma_{ox}-1}\cdot\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)^{\frac{2}{\gamma_{ox}}}\left[1-\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)^{\frac{\gamma_{ox}-1}{\gamma_{ox}}}\right]} \rightarrow \dot{m}_{ox} = \left(K_n\cdot C_d\cdot A\right)_{inj}\cdot\frac{P_{0inj}}{\sqrt{R_{g_{ox}}T_{0_{inj}}}}$$

$$C_d = \frac{1}{2\cdot f\cdot\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)^{\frac{1}{\gamma_{ox}}}}\cdot\left[1-\sqrt{\left\{1-\left(2\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)^{\frac{1}{\gamma_{ox}}}\right)^2\left(1-\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right)\cdot f \Big/ K_n^2\right\}}\right]$$

Chamber Pressure :

$$\frac{\partial P_{0c}}{\partial t} = \frac{A_{burn}\dot{r}_{fuel}}{V_c}\left[\rho_{fuel}R_{g_c}T_{0c}-P_{0c}\right]-P_{0c}\left[\frac{A^*}{V_c}\sqrt{\gamma_c R_{g_c}T_0\left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c+1}{(\gamma_c-1)}}}\right]+\frac{R_{g_c}T_{0_c}}{V_c}\cdot\left\{\left(K_n\cdot C_d\cdot A_{inj}\right)_{inj}\cdot\frac{P_{0inj}}{\sqrt{R_{g_{ox}}\cdot T_{0_{inj}}}}\right\}$$

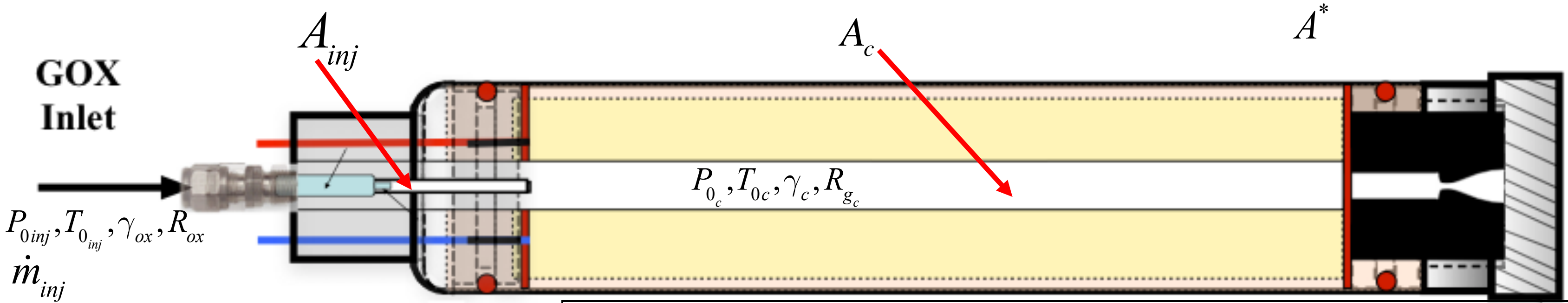
Regression :

0.23

$$\dot{r}_{fuel} = \left(\frac{0.047}{\rho_{fuel}\cdot\left(P_{r_c}\right)^{2/3}}\right)\cdot\left(\frac{C_{P_c}\cdot\left(T_{0_c}-T_{fuel_{surf}}\right)}{h_{v_{fuel}}}\right)\cdot\left(\frac{\dot{m}_{ox}}{A_c}\right)^{4/5}\cdot\left(\frac{\mu_c}{L}\right)^{1/5}$$

$$\dot{m}_{fuel} = \rho_{fuel}\cdot A_{burn}\cdot\dot{r}_{fuel} = \rho_{fuel}\cdot\pi\cdot(D\cdot L)_{port}\cdot\dot{r}_{fuel} \rightarrow O/F = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{\left(K_n\cdot C_d\cdot A\right)_{inj}\cdot\frac{P_{0inj}}{\sqrt{R_{g_{ox}}T_{0_{inj}}}}}{\rho_{fuel}\cdot\pi\cdot(D\cdot L)_{port}\cdot\dot{r}_{fuel}}$$

Hybrid Ballistic Equations for Compressible Oxidizer



$$Supercritical: \left(\frac{P_{0inj}}{P_{0_c}}\right) \ge \left(\frac{\gamma_{ox}+1}{2}\right)^{\frac{\gamma_{ox}}{\gamma_{px}-1}}$$

$$\rightarrow Injector\ Choked \rightarrow r_c = \left(\frac{2}{\gamma_{0x}+1}\right)^{\frac{\gamma_{ox}}{\gamma_{ox}-1}}$$

$$K_n = \sqrt{\gamma_{ox}\cdot\left(\frac{2}{\gamma_{ox}+1}\right)^{\frac{\gamma_{ox}+1}{\gamma_{px}-1}}} \rightarrow \dot{m}_{ox} = \left(K_n \cdot C_d \cdot A\right)_{inj} \cdot \frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0_{inj}}}}$$

$$C_d = \left(\frac{1}{2\cdot f\cdot r_c^{\frac{1}{\gamma_{ox}}}}\right)\cdot\left[\left\{1+\frac{\left(r_c-\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right)\cdot r_c^{\frac{1}{\gamma_{ox}}}}{K_n^2}\right\} - \sqrt{1+\frac{\left(r_c-\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right)\cdot r_c^{\frac{1}{\gamma_{ox}}}}{K_n^2} - \frac{\left(2\cdot r_c^{\frac{1}{\gamma_{ox}}}\right)^2\cdot\left(1-\left(\frac{P_{0_c}}{P_{0_{inj}}}\right)\right)\cdot f}{K_n^2}}\right]$$

Chamber Pressure :

$$\frac{\partial P_{0c}}{\partial t} = \frac{A_{burn}\,\dot{r}_{fuel}}{V_c}\left[\rho_{fuel} R_{g_c} T_{0c} - P_{0c}\right] - P_{0c}\left[\frac{A^*}{V_c}\sqrt{\gamma_c R_{g_c} T_0\left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c+1}{(\gamma_c-1)}}}\right] + \frac{R_{g_c} T_{0_c}}{V_c}\cdot\left\{\left(K_n\cdot C_d\cdot A_{inj}\right)_{inj}\cdot\frac{P_{0inj}}{\sqrt{R_{g_{ox}}\cdot T_{0_{inj}}}}\right\}$$

Regression :

0.23

$$\dot{r}_{fuel} = \left(\frac{0.047}{\rho_{fuel}\cdot\left(P_{r_c}\right)^{2/3}}\right)\cdot\left(\frac{C_{P_c}\cdot\left(T_{0_c} - T_{fuel_{surf}}\right)}{h_{v_{fuel}}}\right)\cdot\left(\frac{\dot{m}_{ox}}{A_c}\right)^{4/5}\cdot\left(\frac{\mu_c}{L}\right)^{1/5}$$

$$\dot{m}_{fuel} = \rho_{fuel}\cdot A_{burn}\cdot\dot{r}_{fuel} = \rho_{fuel}\cdot\pi\cdot(D\cdot L)_{port}\cdot\dot{r}_{fuel} \rightarrow O/F = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} = \frac{\left(K_n\cdot C_d\cdot A\right)_{inj}\cdot\frac{P_{0inj}}{\sqrt{R_{g_{ox}} T_{0_{inj}}}}}{\rho_{fuel}\cdot\pi\cdot(D\cdot L)_{port}\cdot\dot{r}_{fuel}}$$

Solution

Incompressible CV, CD

Injector inputs

Injector Port Diameter,cm (d): 0.25

Injector Tube Diameter,cm (D): 0.6325

Upstream Pressure (P1), kPa: 2500

Down Stream Pressure (P1), kPa: 790.38

Initial Cd Value: 1

Tinlet, C: 20

Cv, Cd Iteration Values, Incompressible (index 0)

	Iteration 1	Iteration 2	Iteration 3
Cvi Value 2	0.60351	0.603929	0.603929
Cdi Value	0.611012	0.611437	0.611437
Error	0.388988	0.00069557	1.10669E-6

Flow Calculations

Port FLow Calculations: Compressible / InCompressible

Incompressible Initial Mdot, g/sec: 30.2529

Fluid Properties Iteration Values (index 2)

Molecular Weight kg/kg-mol: 32

Gamma: 1.4

Viscosity, Nt-sec/M^2: 2.02299E-5

Rg, J/kg-K: 259.827

Rho1, kg/M^3: 32.8221

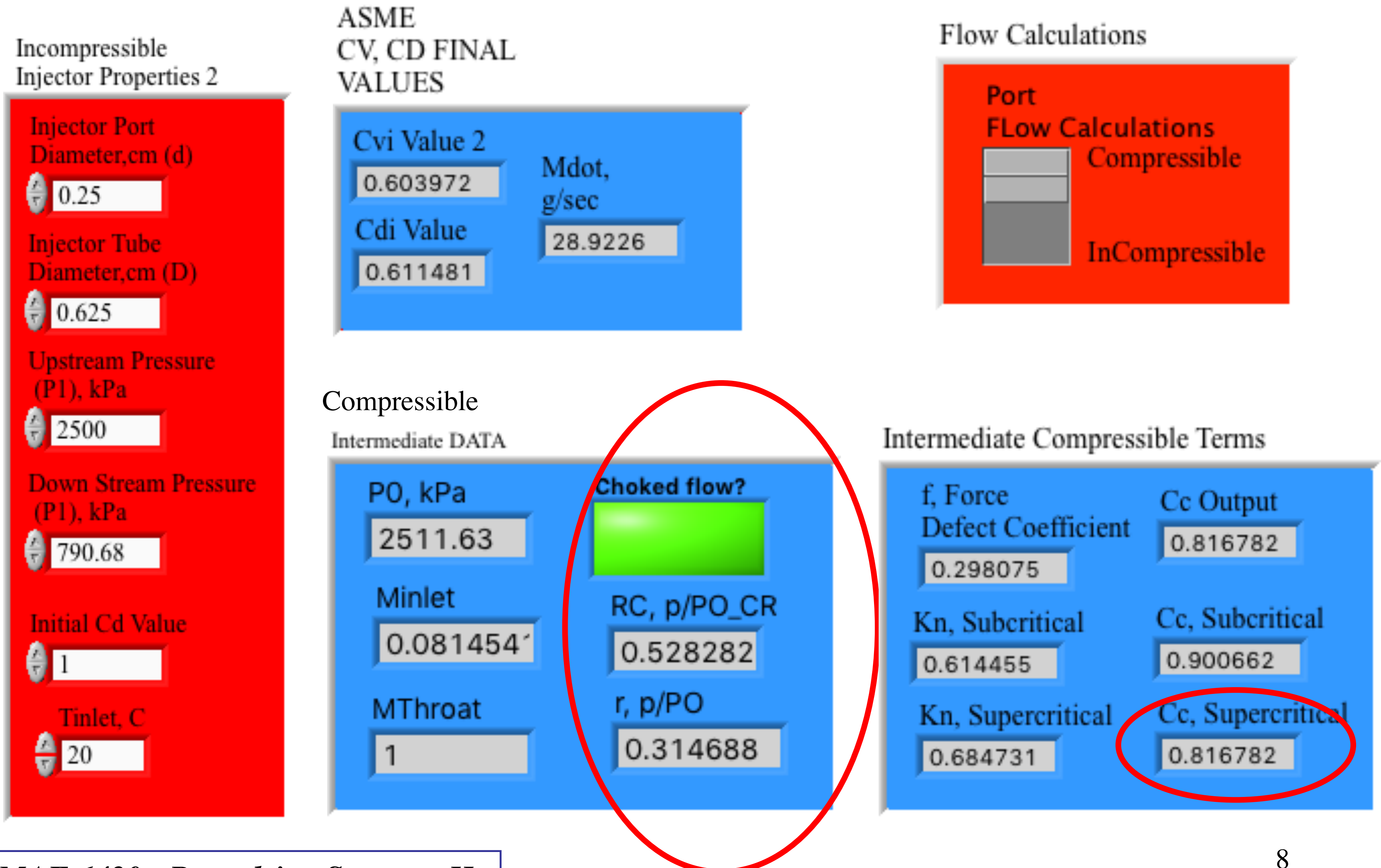
Tube Reynolds Number, ReD: 301040

Tube Velocity m/sec: 29.3353

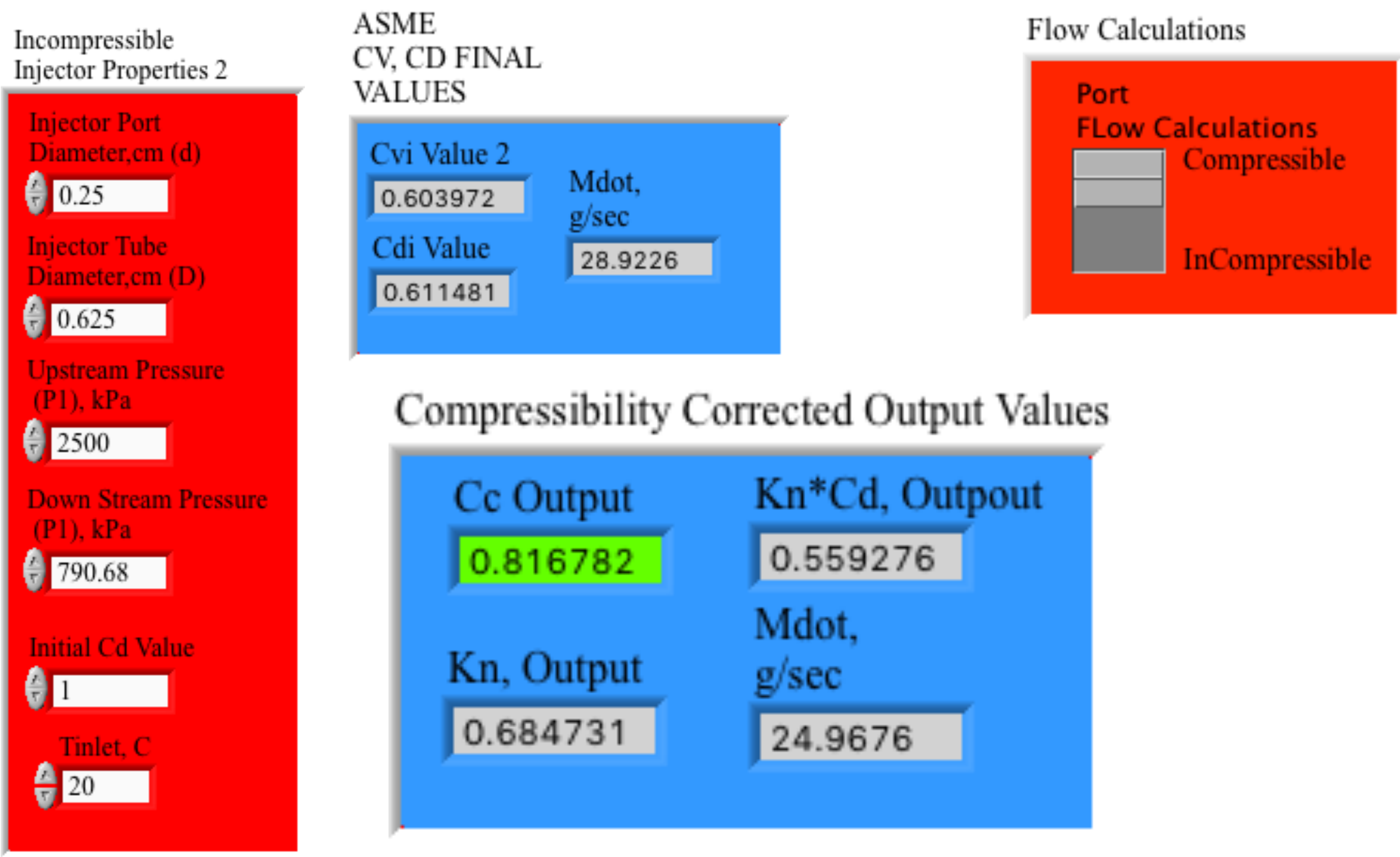
Qvol m^2/sec: 0.00092172

"Flange Tap

Compressible Adjustments (2)



Compressible Adjustments (2)



Compressible Hybrid Injector Model

Compressible Subsonic Injector Formula:

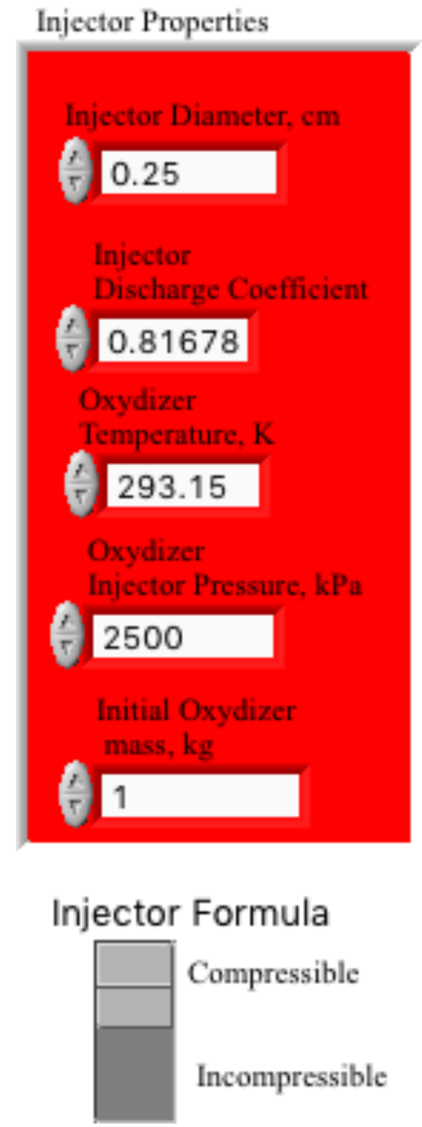
$$\ldots \left(\frac{P_{tank}}{P_{chamber}}\right) < \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\dot{m} = C_d \cdot A_{inj} \cdot \sqrt{\left(\frac{2\cdot\gamma}{\gamma-1}\right)\cdot \rho_{tank} \cdot P_{tank} \cdot \left[\left(\frac{P_{chamber}}{P_{tank}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{chamber}}{P_{tank}}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

Compressible Choked Injector Formula:

$$\ldots \left(\frac{P_{tank}}{P_{chamber}}\right) \geq \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

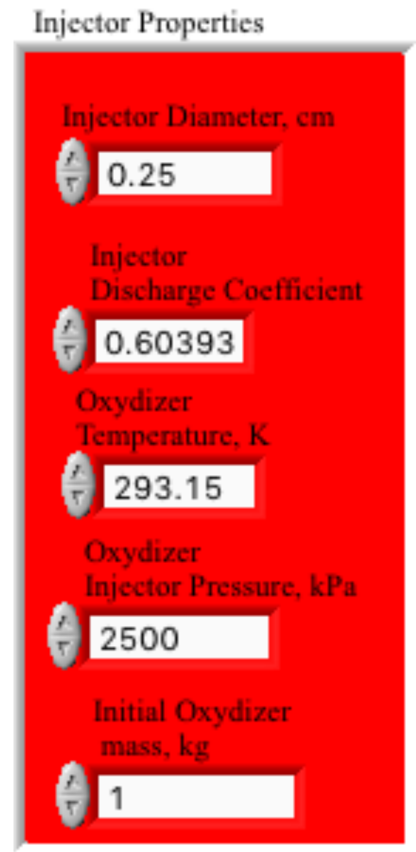
$$\dot{m} = C_d \cdot A_{inj} \cdot \sqrt{\gamma \cdot \rho_{tank} \cdot P_{tank} \cdot \left[\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]}$$

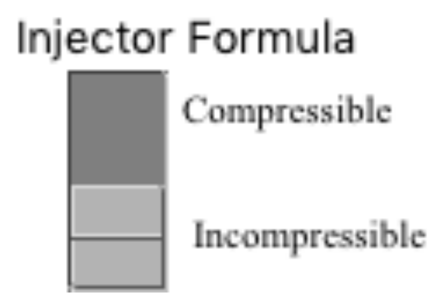


Incompressible Hybrid Injector Model

Incompressible Injector Formula:

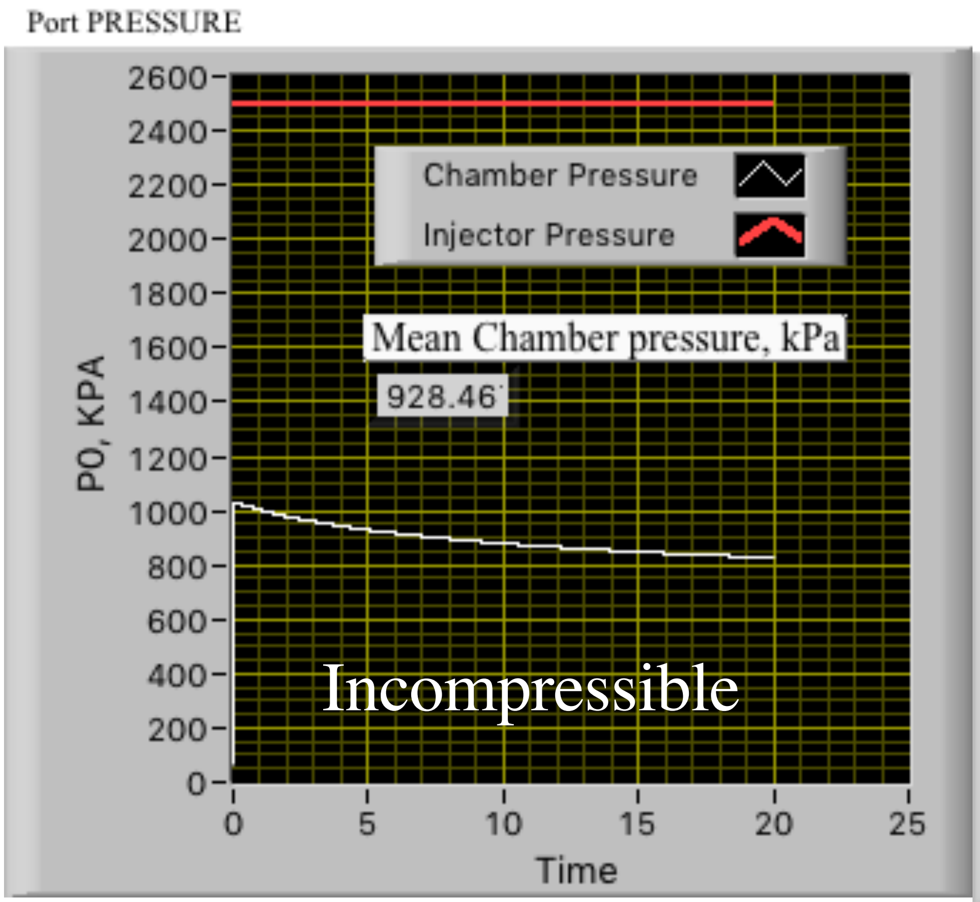
$$\dot{m} = C_{d_{incr}} \cdot A_{inj} \cdot \sqrt{2 \cdot \rho_{tank} \cdot \left(P_{tank} - P_{chamber}\right)}$$

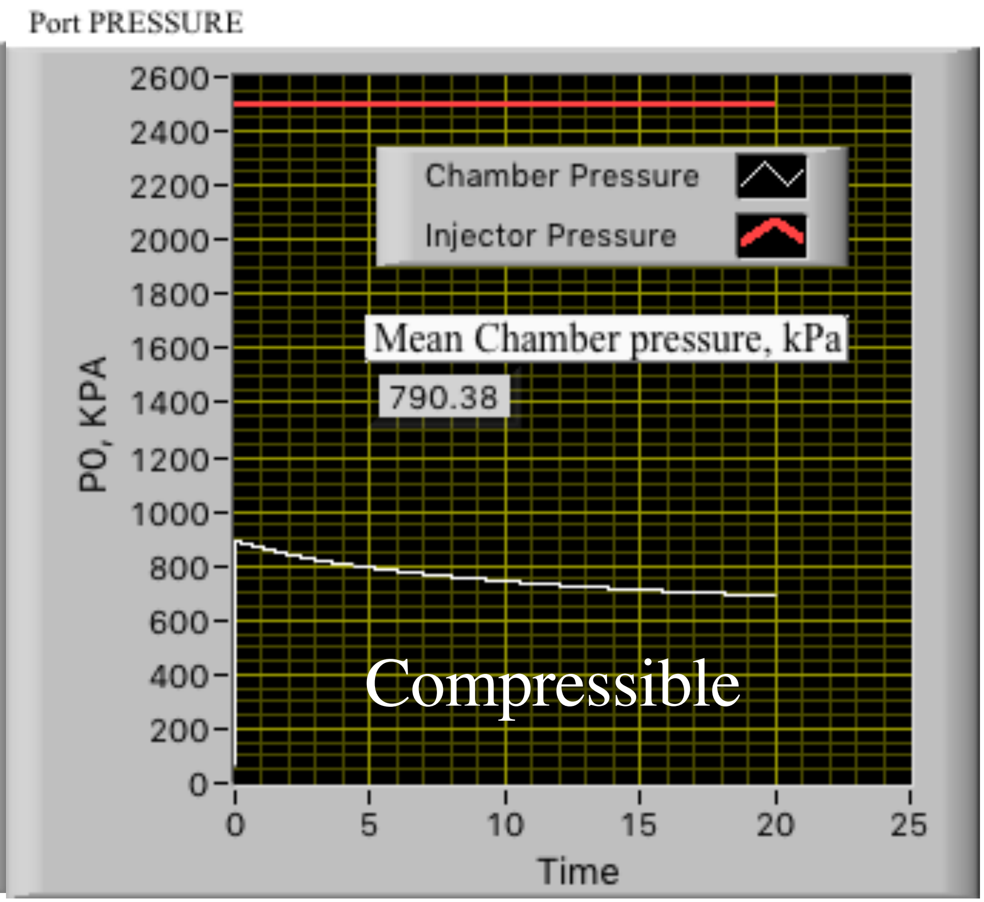




Incompressible/Compressible Model Comparisons

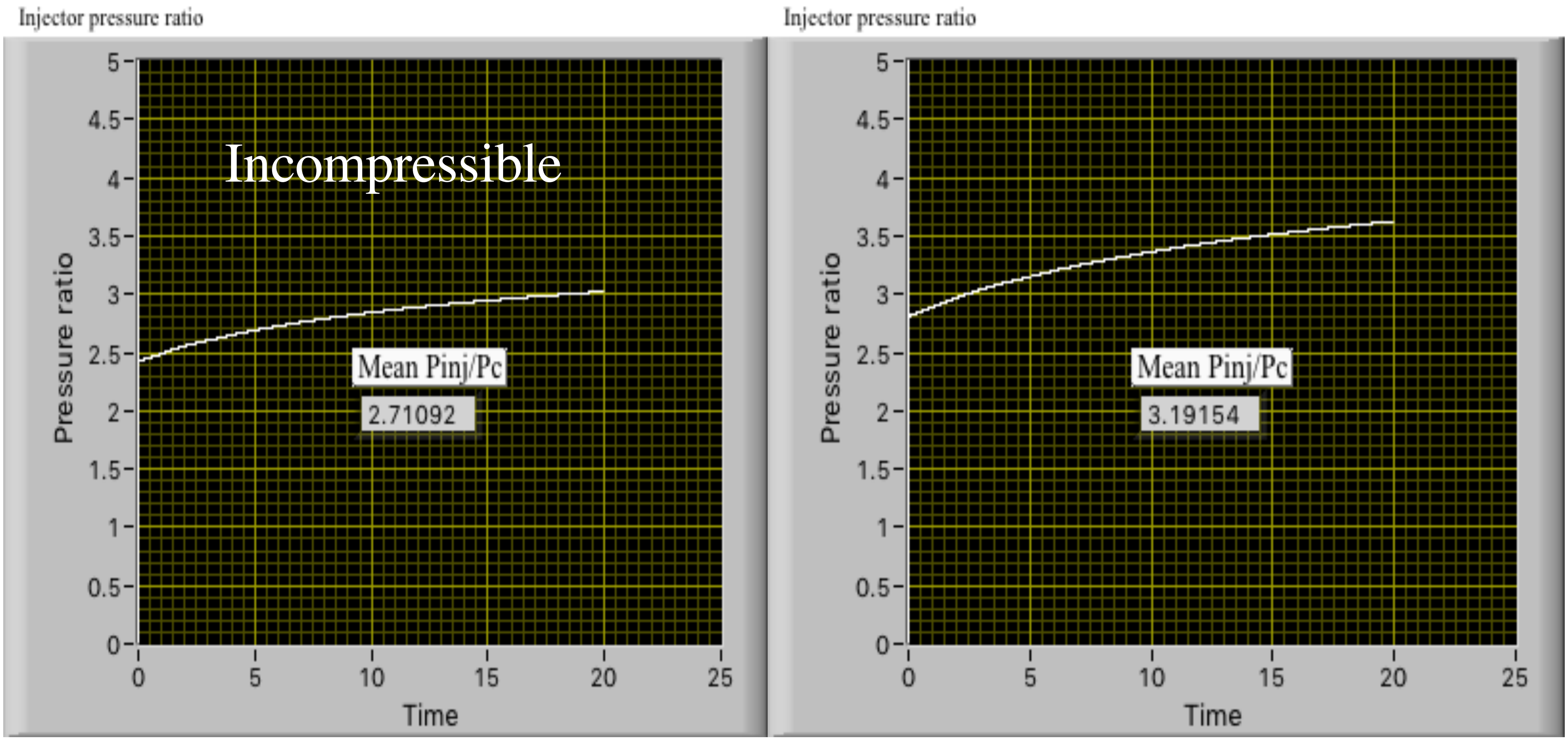
Chamber Pressure





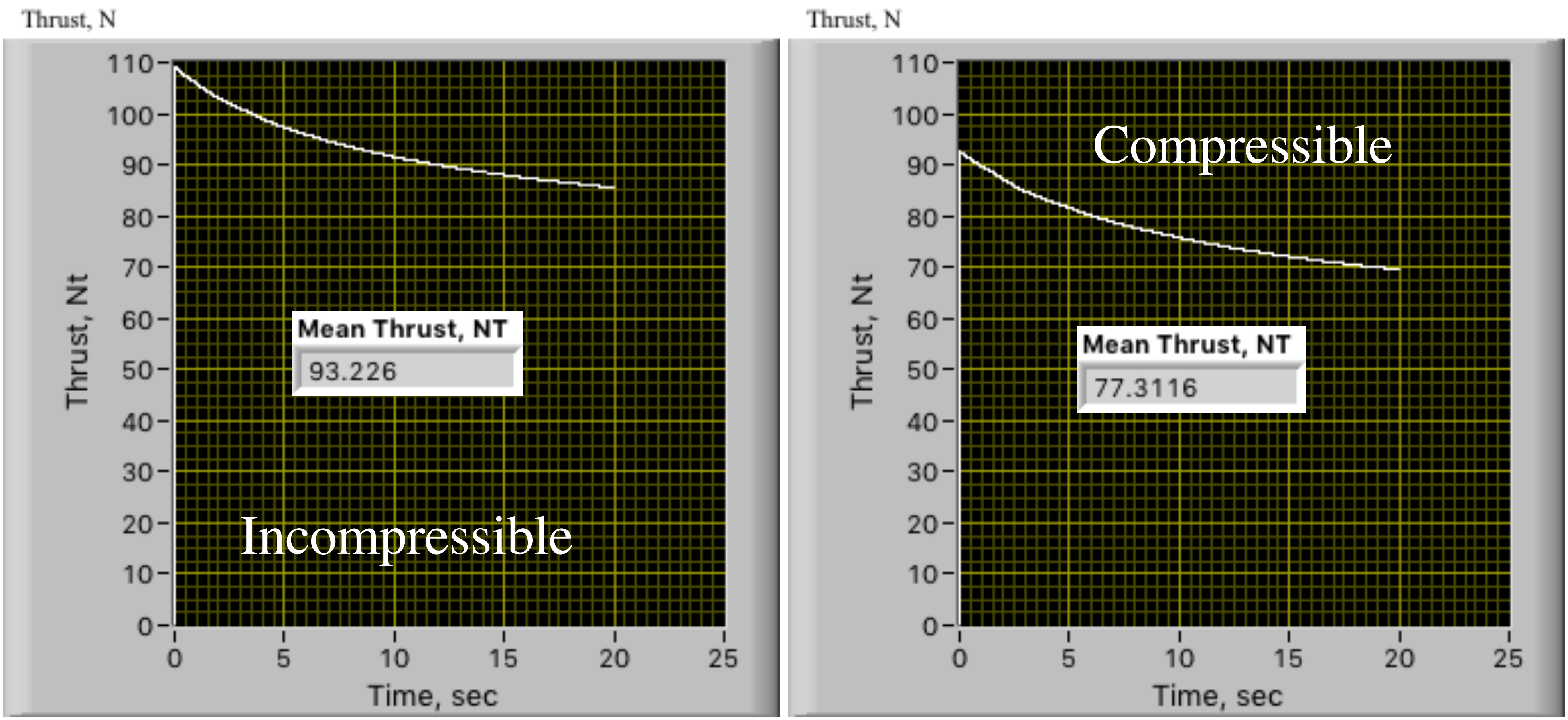
Incompressible/Compressible Comparisons (2)

Injector Pressure Ratio



Incompressible/Compressible Comparisons (3)

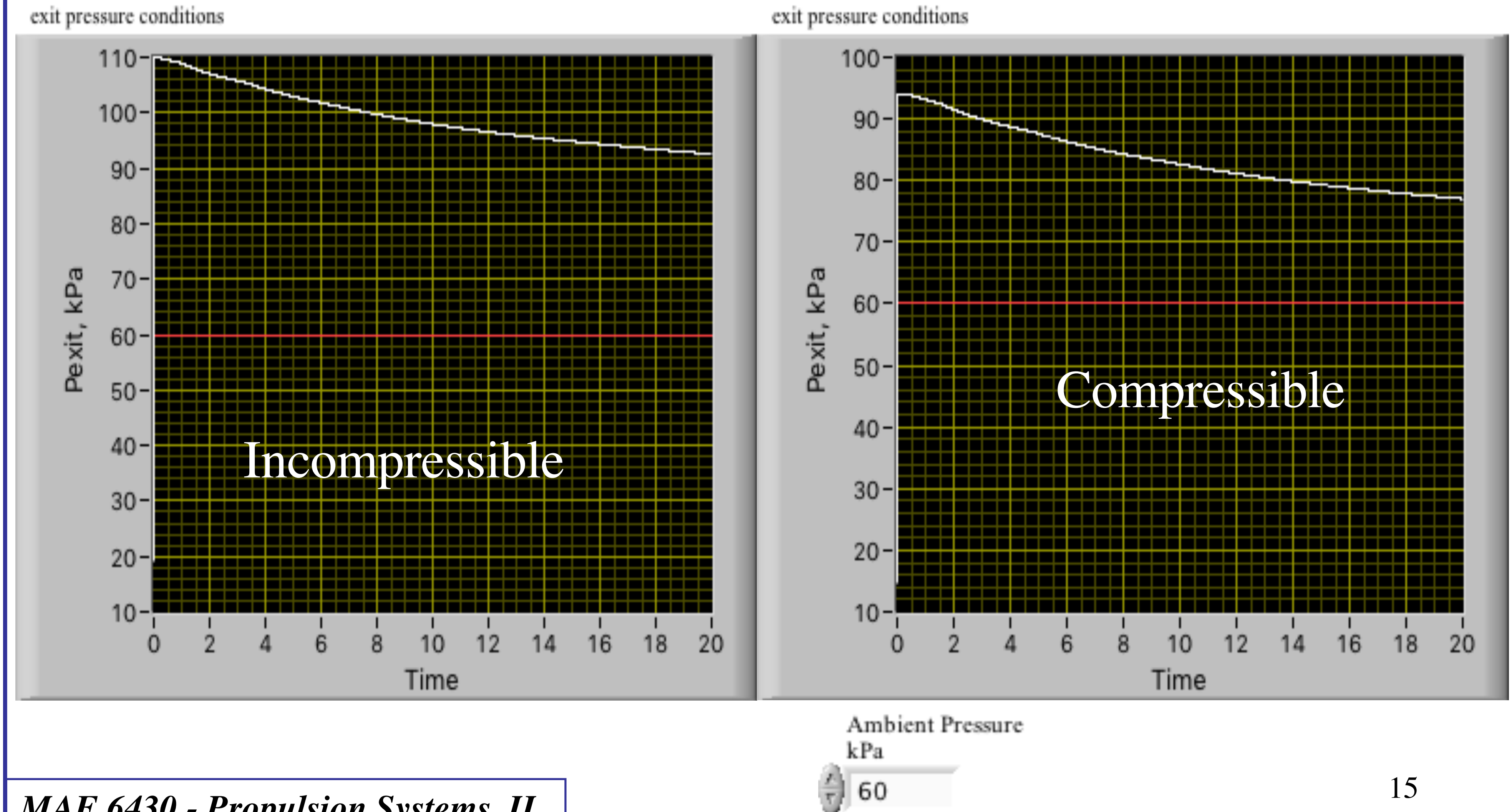
Thrust



Ambient Pressure
kPa
60

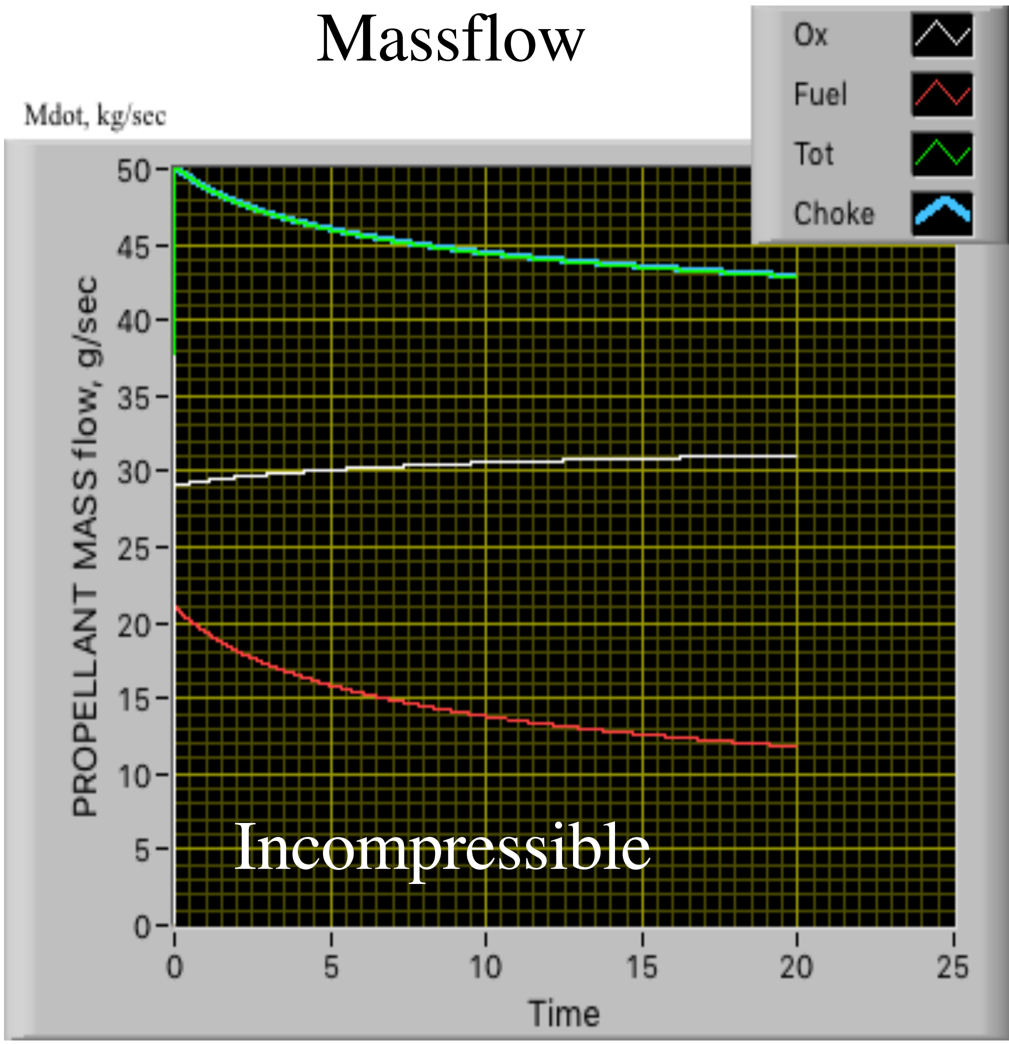
Incompressible/Compressible Comparisons (4)

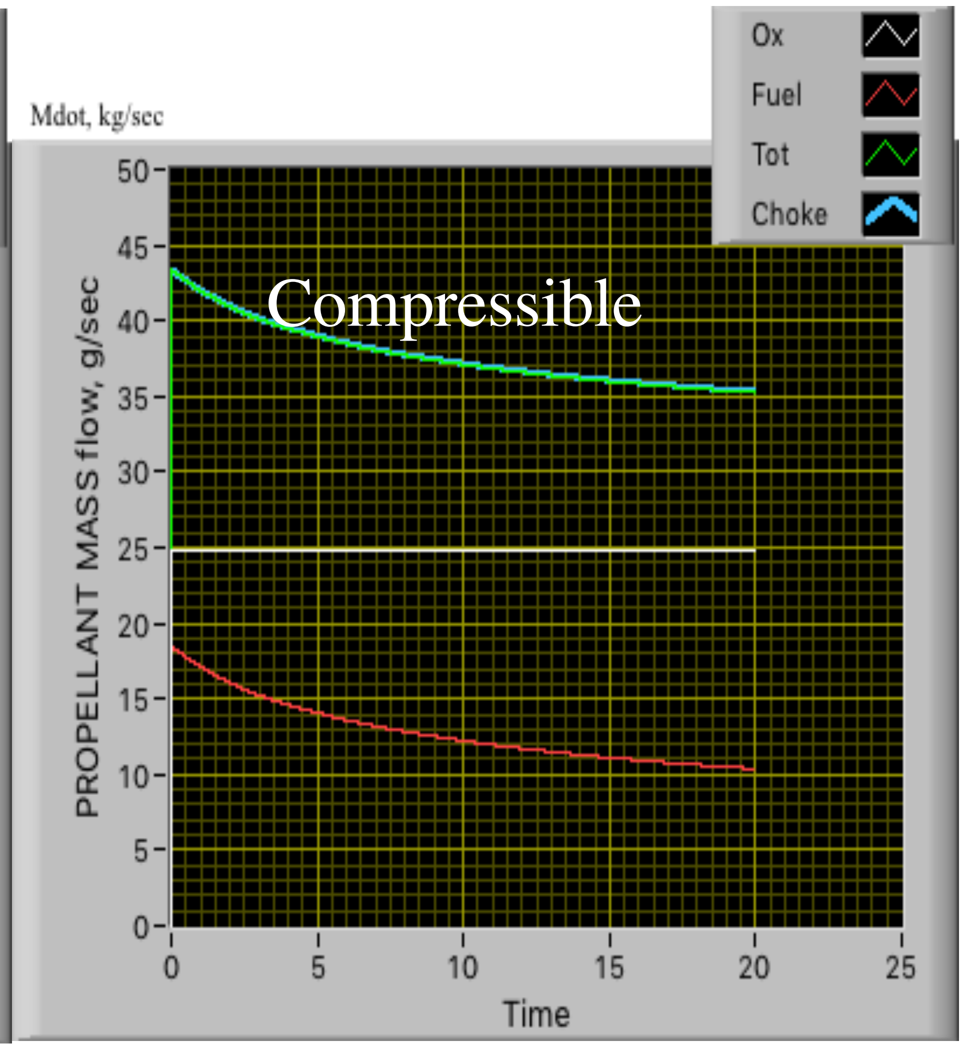
Pexit



Incompressible/Compressible Comparisons (5)

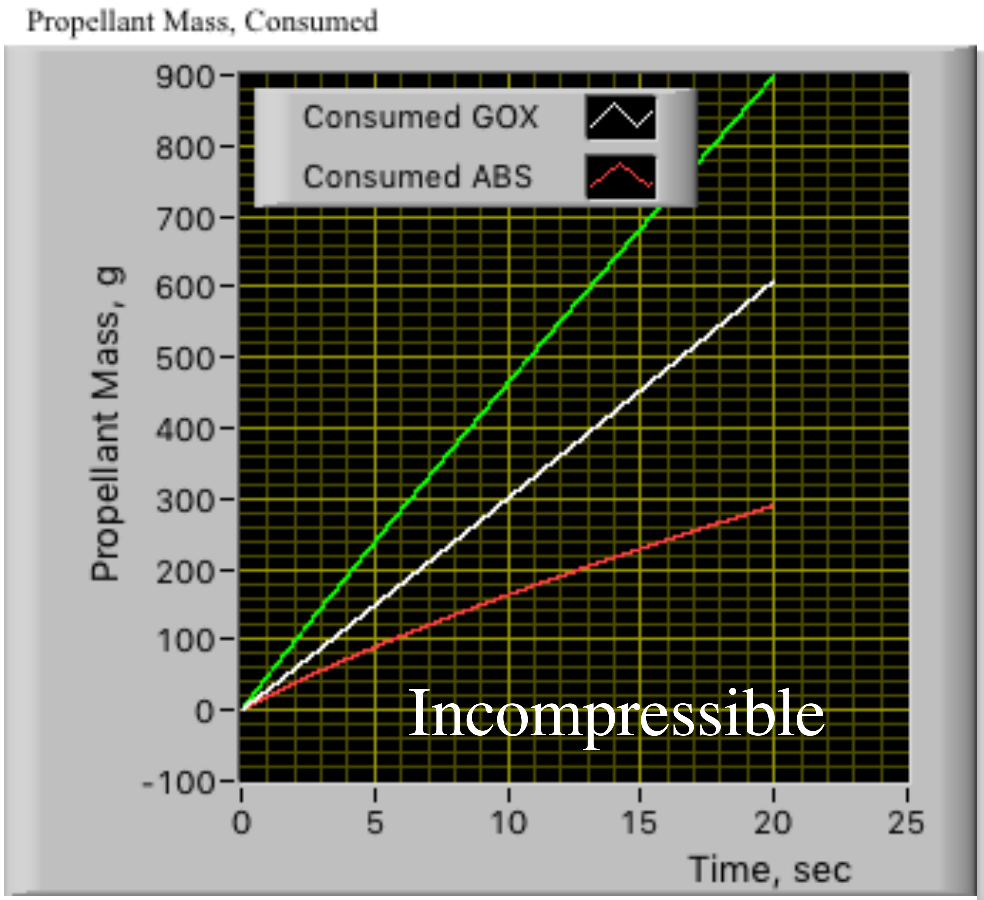
Massflow

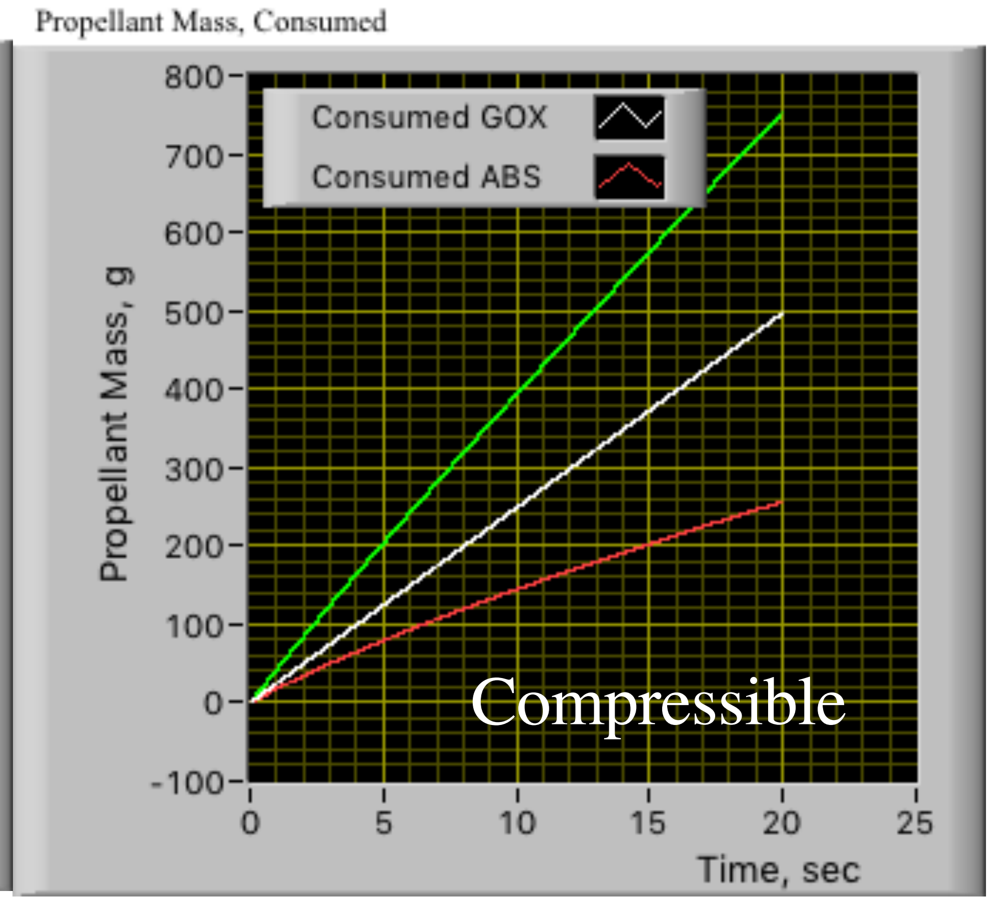




Incompressible/Compressible Comparisons (5)

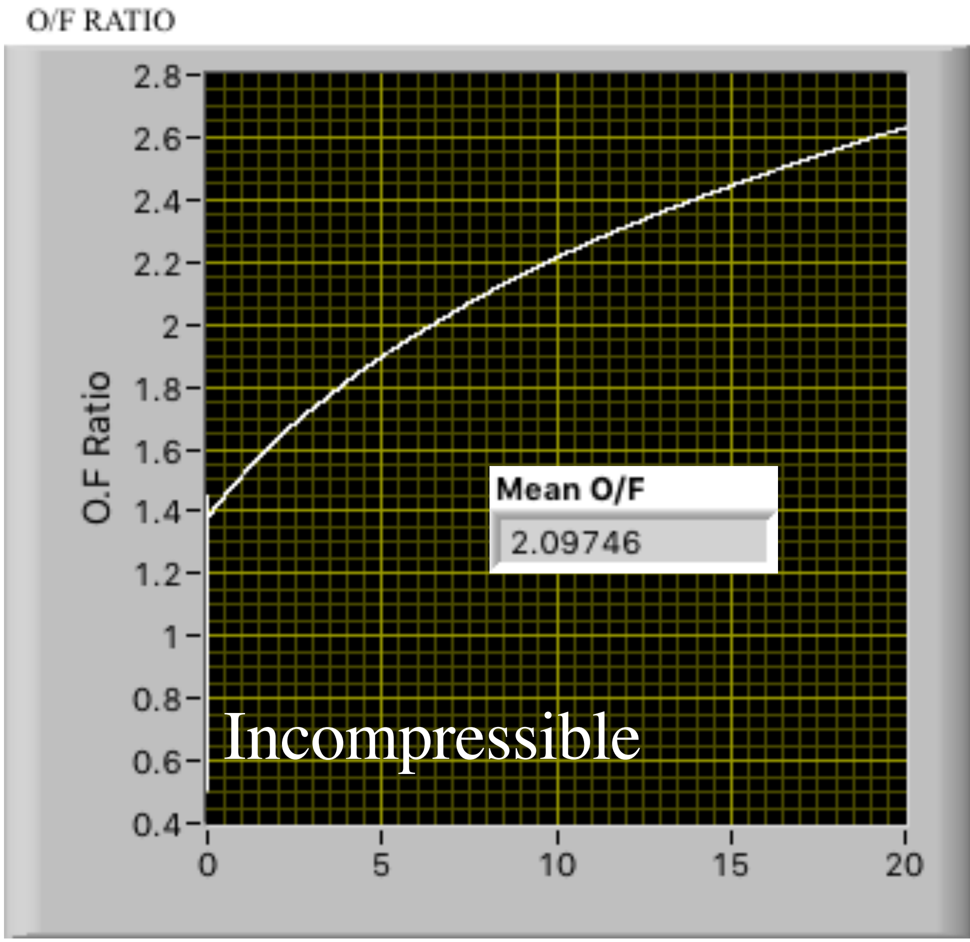
Consumed Propellant Mass

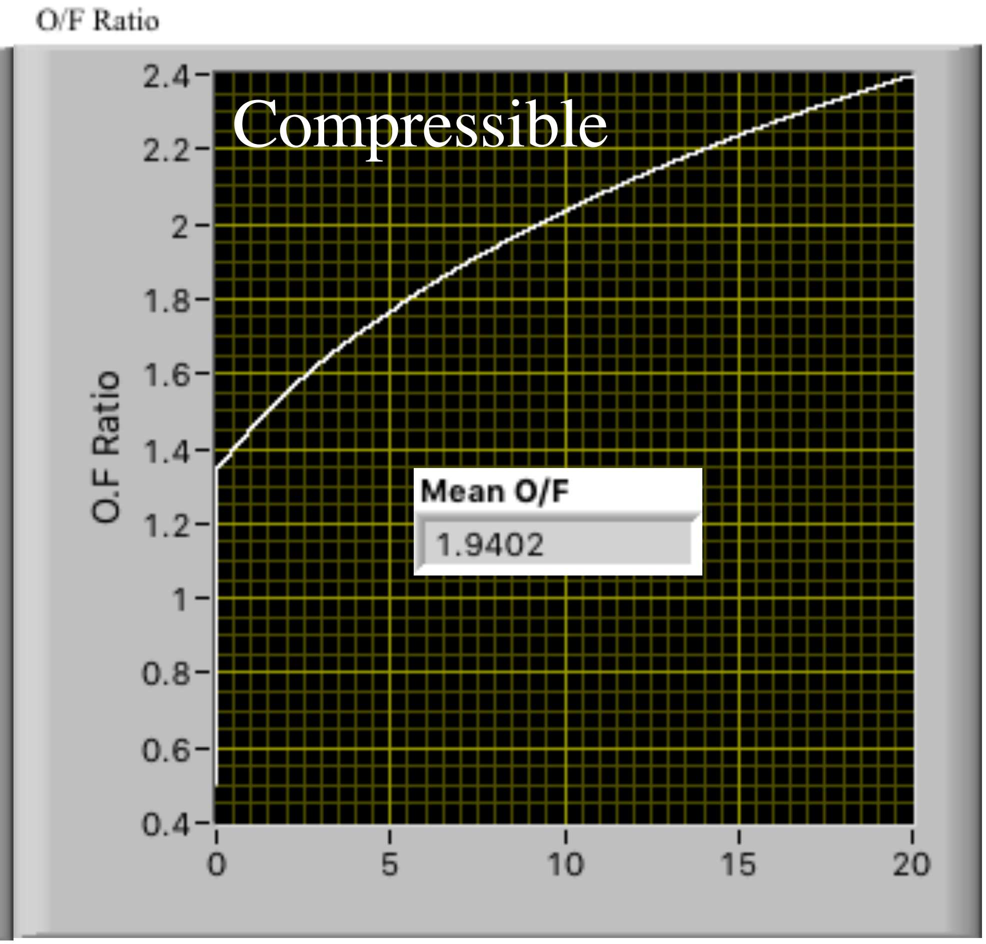




Incompressible/Compressible Comparisons (6)

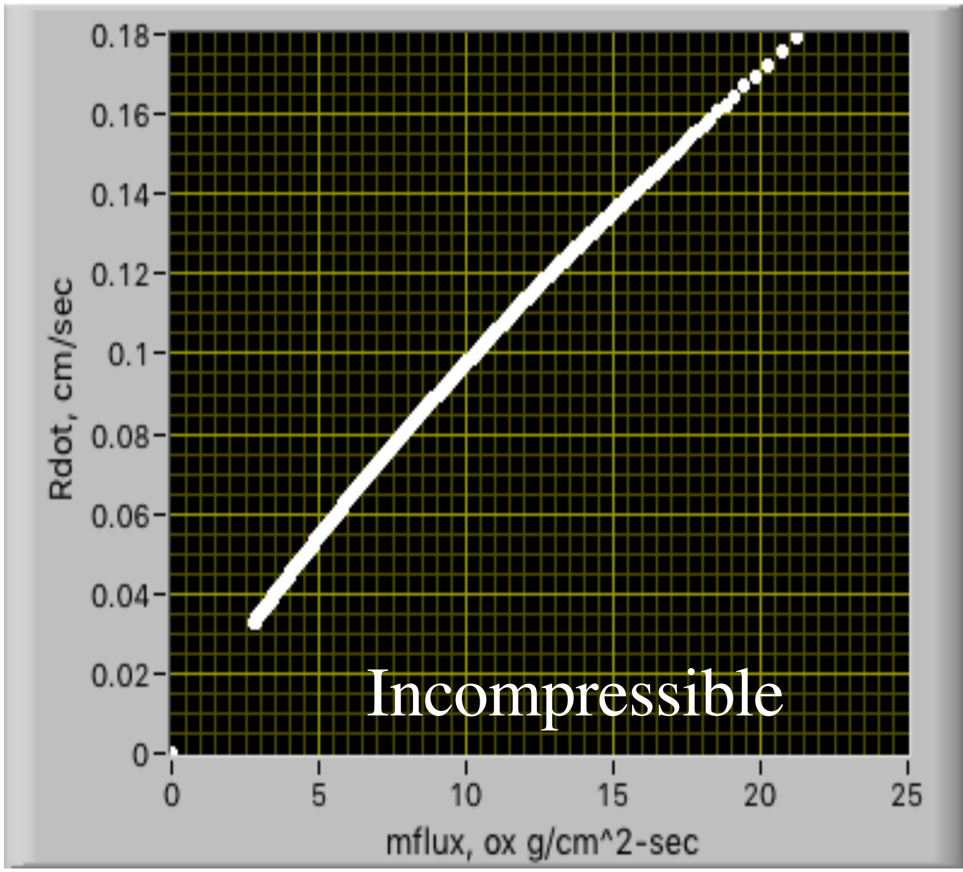
O/F Ratio



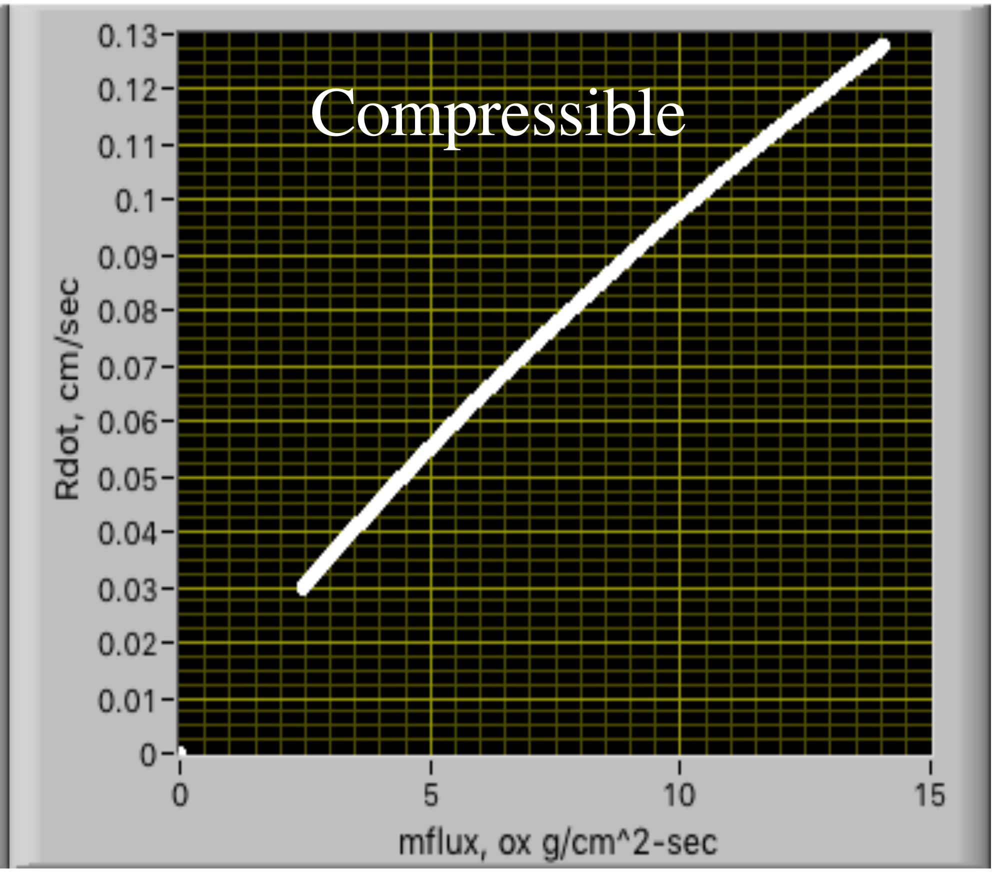


Incompressible/Compressible Comparisons (6)

rdot vs ox mass flux



rdot vs ox mass flux



Incompressible/Compressible Comparisons (6)

Data Summary

Incompressible

Impulse. Nt-sec	1864.56
Mean Thrust, NT	93.226
Mean Thrust, lbf	20.952
MEAN ISP BASED ON CONSUMED MASS, sec	211.524
Vacuum Isp Based on Consumed Propellant , sec	235.581
Consumed Propellant Load, kg	0.898864
Mean O/F	2.09746

Bottom Line: Using Incompressible Injector Model Over-estimates the system performance considerably

Compressible

Impulse. Nt-sec	1546.26
Mean Thrust, NT	77.3116
Mean Thrust, lbf	17.3753
MEAN ISP BASED ON CONSUMED MASS, sec	209.208
Vacuum Isp Based on Consumed Propellant , sec	237.9
Consumed Propellant Load, kg	0.75367
Mean O/F	1.9402