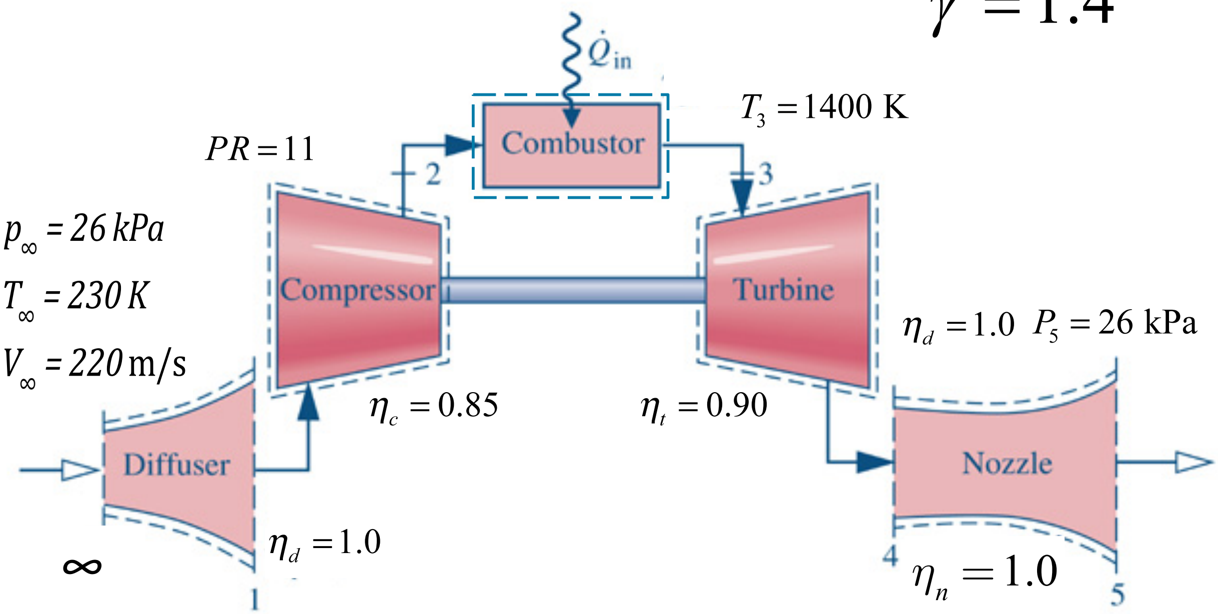
Section 4.1 Homework

Given: A turbojet engine operating as shown below

$\gamma = 1.4$



- Assume Isentropic Diffuser, Nozzle
- Compressible, Combustor Turbine NOT! Isentropic
- Assume Constant C_p, C_v across cycle
- Air massflow >> fuel massflow

Calculate :

(a) The properties at all the state points in the cycle
(b) The heat transfer rate in the combustion chamber (*kW*)
(c) The velocity at the nozzle exit (*m/s*)
(d) The propulsive force (*lbf*)
(e) The propulsive power developed (*kW*)
(f) Propulsive Efficiency
(g) Thermal Efficiency
(h) Total Efficiency
(i) Draw *T-s* diagram
(j) Draw p-v diagram

Section 4.1 Homework (2)

Given: A turbojet engine operating as shown below

Incoming Air to Turbojet (@ to station *3*)

- Molecular weight = 28.96443 *kg/kg-mole*
- γ = 1.40
- R_g = 287.058 *J/kg-K*
- T_∞ = 230 K
- p_∞ = 26 kPa
- V_∞ = 220 *m/sec*
- Universal Gas Constant: R_u = 8314.4612 *J/kg-K*

Calorically Perfect Gas

$$\gamma = \frac{c_p}{c_v}$$

$$R_g = c_p - c_v$$

$$c_p = \frac{\gamma}{\gamma - 1} \cdot R_g$$

$$c_v = \frac{1}{\gamma - 1} \cdot R_g$$

For ...Isentropic Conditions →

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Ideal Gas

$$p = \rho \cdot R_g \cdot T$$

Section 4.1 Homework (3)

Given: Across Components

Isentropic Diffuser

Assume D_{inlet} = 60.96 cm (24 in.)

$D_{outlet} = 1.5 \times D_{inlet}$

$$h_{0_1} \equiv h_1 + \frac{V_1^2}{2} = h_\infty + \frac{V_\infty^2}{2}$$

$$h_{0_1} \approx C_{p1} \cdot T_{0_1}$$

Compressor

- ASSUME COMPRESSOR EXIT MACH ~ 0

$$\eta_c = \frac{\text{isentropic power input}}{\text{actual power input}}$$

$$\eta_c = \frac{h_{0_{2|s=0}} - h_{0_1}}{h_{0_2} - h_{0_1}} \rightarrow$$

$$h_{0_1} = C_{p_{air}} \cdot T_{0_1}$$

$$h_{0_2} = C_{p_{air}} \cdot T_{0_1\,actual}$$

$$h_{0_{2|s=0}} = C_{p_{air}} \cdot T_{0_2\,ideal}$$

$$\frac{\dot{w}_c}{\dot{m}} = h_{0_2} - h_{0_1}$$

$$\frac{p_2}{p_1} \approx \frac{P_{0_2}}{P_{0_1}} = 11$$

$$s_2 - s_1 = C_p \ln\left(\frac{T_{2_{actual}}}{T_1}\right) - R_g \ln\left(\frac{p_2}{p_1}\right)$$

$$\frac{h_{0_{2|s=0}}}{h_{0_1}} = \frac{C_p \cdot T_{0_{2|s=0}}}{C_p \cdot T_{0_1}} \approx \frac{T_{0_{2|s=0}}}{T_{0_1}} = \left(\frac{P_{0_2}}{P_{0_1}}\right)^{\frac{\gamma-1}{\gamma}}$$

Section 4.1 Homework (4)

Given: Across Components

Combustor

$$contant\ pressure, \dot{m}_{air} >> \dot{m}_{air}$$

$$C_p, \gamma \sim const, \quad T_3 = T_{flame} = 1400K$$

$$s_3 - s_2 = C_p \ln\left(\frac{T_{flame}}{T_{2_{actual}}}\right)$$

Assume combustor Inlet/ outlet Mach numbers are essentially zero

$$\frac{p_3}{p_2} \approx \frac{P_{0_3}}{P_{0_2}} = 1$$

Turbine

$$\eta_t = \frac{\text{actual power output}}{\text{isentropic power poutput}}$$

$$\eta_t = \frac{h_{0_3} - h_{0_4}}{h_{03} - h_{0_{4_{s=0}}}} \rightarrow \begin{array}{l} h_{0_3} = C_{p_{air}} \cdot T_{0_3} \\ h_{0_4} = C_{p_{air}} \cdot T_{0_{4actual}} \\ h_{0_{4_{s=0}}} = C_{p_{air}} \cdot T_{0_{4ideal}} \end{array}$$

$$Assume \rightarrow \frac{\dot{w}_t}{\dot{m}} = \frac{\dot{w}_c}{\dot{m}} = h_{0_3} - h_{0_4}$$

Actual !

$$\frac{P_{0_4}}{P_{0_3}} = \left(\frac{T_{0_{4|s=0}}}{T_{0_3}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{h_{0_3} - \frac{1}{\eta_t} \cdot \frac{\dot{w}}{\dot{m}}}{h_{0_3}}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 - \frac{1}{\eta_t \cdot h_{0_3}} \cdot \frac{\dot{w}}{\dot{m}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$s_4 - s_3 = C_p \ln\left(\frac{T_{0_{4actual}}}{T_{0_3}}\right) - R_g \ln\left(\frac{P_{0_4}}{P_{0_3}}\right)$$

Section 4.1 Homework (5)

Given: Across Components

Nozzle

Assumed Optimized Nozzle → $p_{exit} = p_\infty$ $\quad T_{exit} = T_4 \cdot \left(\frac{P_4}{p_{exit}}\right)^{\frac{\gamma-1}{\gamma}}$

$$\dot{m}\left(h_4 + \frac{V_4^2}{2}\right) = \dot{m}\left(h_{exit} + \frac{V_{exit}^2}{2}\right) \rightarrow V_4 \approx 0 \rightarrow V_{exit} = \sqrt{2\left(h_4 - h_{exit}\right)}$$

$$F = \dot{m}\left(V_{exit} - V_\infty\right) \qquad \dot{W}_p = F \cdot V_\infty$$

$$\eta_{propulsive} = \frac{\dot{W}_p}{\dot{m}_{air}\left(K.E._{exit} - K.E._\infty\right)} \qquad \eta_{thermal} = \frac{\left(K.E._{exit} - K.E._\infty\right)}{\dot{m}_{fuel} \cdot h_{fuel}}$$

$$\eta_{total} = \eta_{prop} \cdot \eta_{thermal} = \frac{F \cdot V_\infty}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Section 4.1 Homework (8)

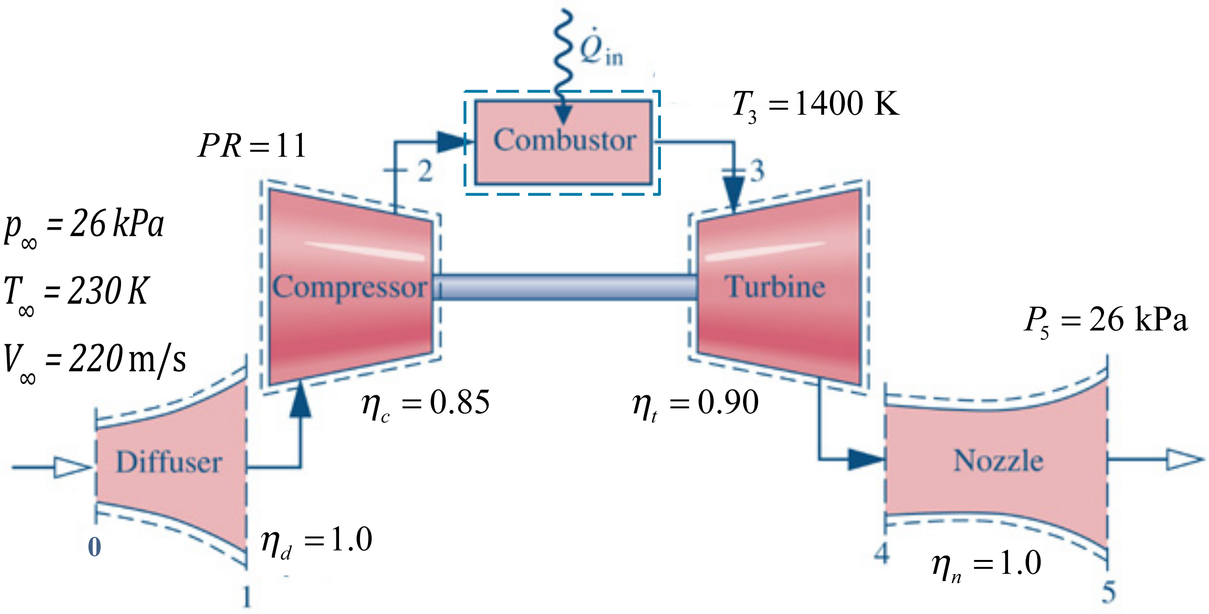
Summary

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{2\left(\left(\frac{V_{exit}}{V_\infty}\right) - \left(\frac{V_{exit}}{V_\infty}\right)^2\right)}{\left(1 - \left(\frac{V_\infty}{V_{exit}}\right)^2\right)}$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\left(\frac{1}{2}V^2{}_{exit}\right) \cdot \left(1 - \left(\frac{V_\infty}{V_{exit}}\right)^2\right)}{\left(h_{0_3} - h_{02}\right)}$$

$$K.E._{\substack{out\\excess}} = K.E._{net} - P_{prop} = \frac{1}{2}\dot{m} \cdot V^2{}_{exit} \cdot \left(1 - \left(\frac{V_{exit}}{V_\infty}\right)\right)$$

State Table Example



Efficiencies

Propulsive

Thermal

Total

State Table

Station	Pi, kPa	Ti, K	h0 KJ/kg	si, KJ/kg-K	Rho, kg/m^3	vol m^3/kg
0						
1						
2						
3						
4						
5						