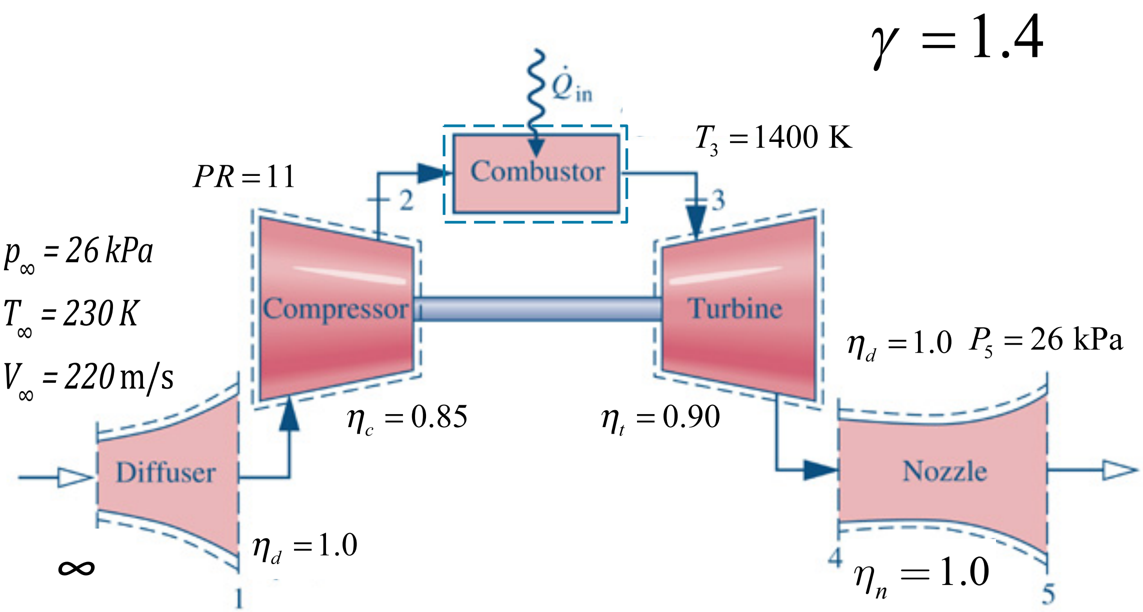
Section 4.1 Homework

Given: A turbojet engine operating as shown below



- Assume Isentropic Diffuser, Nozzle
- Compressible, Combustor Turbine NOT! Isentropic
- Assume Constant C_p, C_v across cycle
- Air massflow >> fuel massflow

Calculate :

(a) The properties at all the state points in the cycle
(b) The heat transfer rate in the combustion chamber (*kW*)
(c) The velocity at the nozzle exit (*m/s*)
(d) The propulsive force (*lbf*)
(e) The propulsive power developed (*kW*)
(f) Propulsive Efficiency
(g) Thermal Efficiency
(h) Total Efficiency
(i) Draw *T-s* diagram
(j) Draw p-v diagram

Section 4.1 Homework (2)

Given: A turbojet engine operating as shown below

Incoming Air to Turbojet (@ to station *3*)

- Molecular weight = 28.96443 *kg/kg-mole*
- γ = 1.40
- R_g = 287.058 *J/kg-K*
- T_∞ = 230 K
- p_∞ = 26 kPa
- V_∞ = 220 *m/sec*
- Universal Gas Constant: R_u = 8314.4612 *J/kg-K*

Calorically Perfect Gas

$$\gamma = \frac{c_p}{c_v}$$

$$R_g = c_p - c_v$$

$$c_p = \frac{\gamma}{\gamma - 1} \cdot R_g$$

$$c_v = \frac{1}{\gamma - 1} \cdot R_g$$

For …Isentropic Conditions →

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Ideal Gas

$$p = \rho \cdot R_g \cdot T$$

Section 4.1 Homework (3)

Given: Across Components

Isentropic Diffuser

Assume $D_{inlet} = 60.96$ cm (24 in.)

$D_{outlet} = 1.5 \times D_{inlet}$

$$h_{0_1} \equiv h_1 + \frac{V_1^2}{2} = h_\infty + \frac{V_\infty^2}{2}$$

$$h_{0_1} \approx C_{p1} \cdot T_{0_1}$$

Compressor

- ASSUME COMPRESSOR EXIT MACH ~ 0

$$\eta_c = \frac{\text{isentropic power input}}{\text{actual power input}}$$

$$\eta_c = \frac{h_{0_{2|s=0}} - h_{0_1}}{h_{0_2} - h_{0_1}} \rightarrow$$

$$h_{0_1} = C_{p_{air}} \cdot T_{0_1}$$

$$h_{0_2} = C_{p_{air}} \cdot T_{0_{1\,actual}}$$

$$h_{0_{2|s=0}} = C_{p_{air}} \cdot T_{0_{2\,ideal}}$$

$$\frac{\dot{w}_c}{\dot{m}} = h_{0_2} - h_{0_1}$$

$$\frac{p_2}{p_1} \approx \frac{P_{0_2}}{P_{0_1}} = 11$$

$$s_2 - s_1 = C_p \ln\left(\frac{T_{2_{actual}}}{T_1}\right) - R_g \ln\left(\frac{p_2}{p_1}\right)$$

$$\frac{h_{0_{2|s=0}}}{h_{0_1}} = \frac{C_p \cdot T_{0_{2|s=0}}}{C_p \cdot T_{0_1}} \approx \frac{T_{0_{2|s=0}}}{T_{0_1}} = \left(\frac{P_{0_2}}{P_{0_1}}\right)^{\frac{\gamma-1}{\gamma}}$$

Section 4.1 Homework (4)

Given: Across Components

Combustor

$$contant\ pressure, \dot{m}_{air} >> \dot{m}_{air}$$

$$C_p, \gamma \sim const, \ \mathrm{T}_3 = T_{flame} = 1400K$$

$$s_3 - s_2 = C_p \ln\left(\frac{T_{flame}}{T_{2_{actual}}}\right)$$

Assume combustor Inlet/ outlet Mach numbers are essentially zero

$$\frac{p_3}{p_2} \approx \frac{P_{0_3}}{P_{0_2}} = 1$$

Turbine

$$\eta_t = \frac{\text{actual power output}}{\text{isentropic power poutput}}$$

$$\eta_t = \frac{h_{0_3} - h_{0_4}}{h_{03} - h_{0_{4_{s=0}}}} \rightarrow \begin{array}{l} h_{0_3} = C_{p_{air}} \cdot T_{0_3} \\ h_{0_4} = C_{p_{air}} \cdot T_{0_{4actual}} \\ h_{0_{4_{s=0}}} = C_{p_{air}} \cdot T_{0_{4ideal}} \end{array}$$

$$Assume \rightarrow \frac{\dot{w}_t}{\dot{m}} = \frac{\dot{w}_c}{\dot{m}} = h_{0_3} - h_{0_4}$$ Actual !

$$\frac{P_{0_4}}{P_{0_3}} = \left(\frac{T_{0_{4|s=0}}}{T_{0_3}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{h_{0_3} - \frac{1}{\eta_t} \cdot \frac{\dot{w}}{\dot{m}}}{h_{0_3}}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 - \frac{1}{\eta_t \cdot h_{0_3}} \cdot \frac{\dot{w}}{\dot{m}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$s_4 - s_3 = C_p \ \ln\left(\frac{T_{0_{4actual}}}{T_{0_3}}\right) - R_g \ln\left(\frac{P_{0_4}}{P_{0_3}}\right)$$

Section 4.1 Homework (5)

Given: Across Components

Nozzle   Assumed Optimized Nozzle → $p_{exit} = p_\infty$ $T_{exit} = T_4 \cdot \left(\frac{P_4}{p_{exit}}\right)^{\frac{\gamma-1}{\gamma}}$

$$\dot{m}\left(h_4 + \frac{V_4^2}{2}\right) = \dot{m}\left(h_{exit} + \frac{V_{exit}^2}{2}\right) \rightarrow V_4 \approx 0 \rightarrow V_{exit} = \sqrt{2\left(h_4 - h_{exit}\right)}$$

$$F = \dot{m}\left(V_{exit} - V_\infty\right) \qquad \dot{W}_p = F \cdot V_\infty$$

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._\infty\right)} \qquad \eta_{thermal} = \frac{\left(K.E._{exit} - K.E._\infty\right)}{\dot{m}_{fuel} \cdot h_{fuel}}$$

$$\eta_{total} = \eta_{prop} \cdot \eta_{thermal} = \frac{F \cdot V_\infty}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Section 4.1 Homework (8)

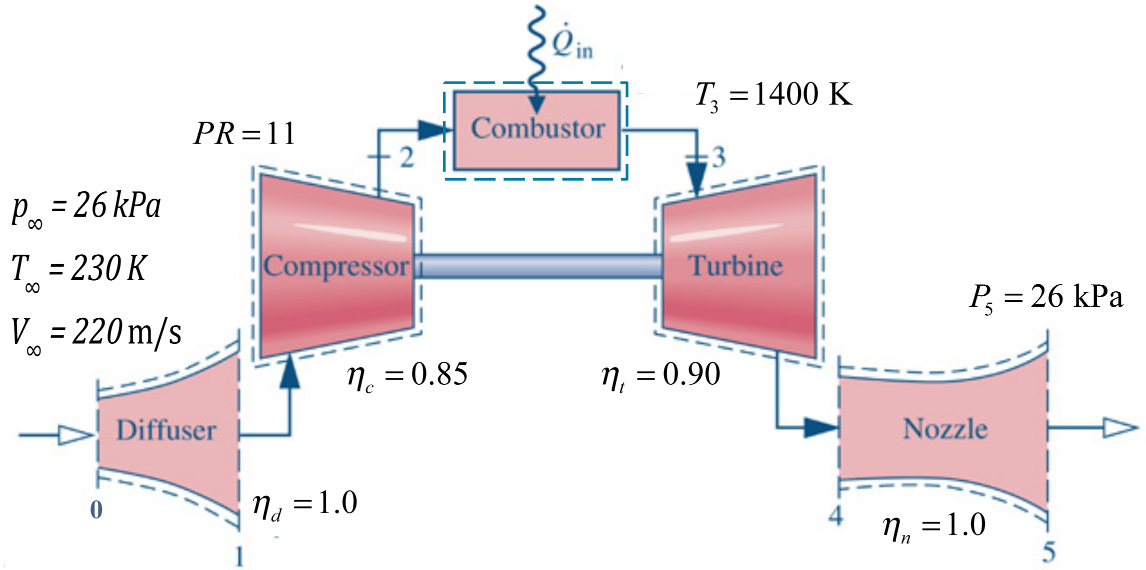
Summary

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} = \frac{2\left(\left(\frac{V_{exit}}{V_{\infty}}\right) - \left(\frac{V_{exit}}{V_{\infty}}\right)^2\right)}{\left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^2\right)}$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}} = \frac{\left(\frac{1}{2}V^2{}_{exit}\right) \cdot \left(1 - \left(\frac{V_{\infty}}{V_{exit}}\right)^2\right)}{\left(h_{0_3} - h_{02}\right)}$$

$$K.E._{\substack{out \\ excess}} = K.E._{net} - P_{prop} = \frac{1}{2}\dot{m} \cdot V^2{}_{exit} \cdot \left(1 - \left(\frac{V_{exit}}{V_{\infty}}\right)\right)$$

State Table Example



Efficiencies

Propulsive

Thermal

Total

State Table

Station	Pi, kPa	Ti, K	h0 KJ/kg	si, KJ/kg-K	Rho, kg/m^3	vol m^3/kg
0						
1						
2						
3						
4						
5						

Problem Solution

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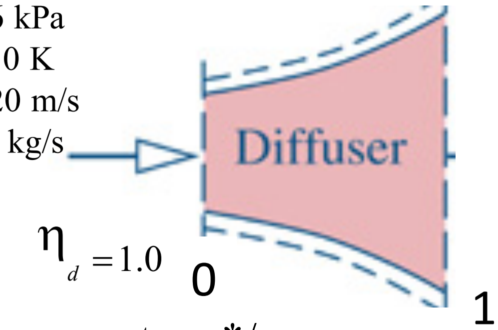
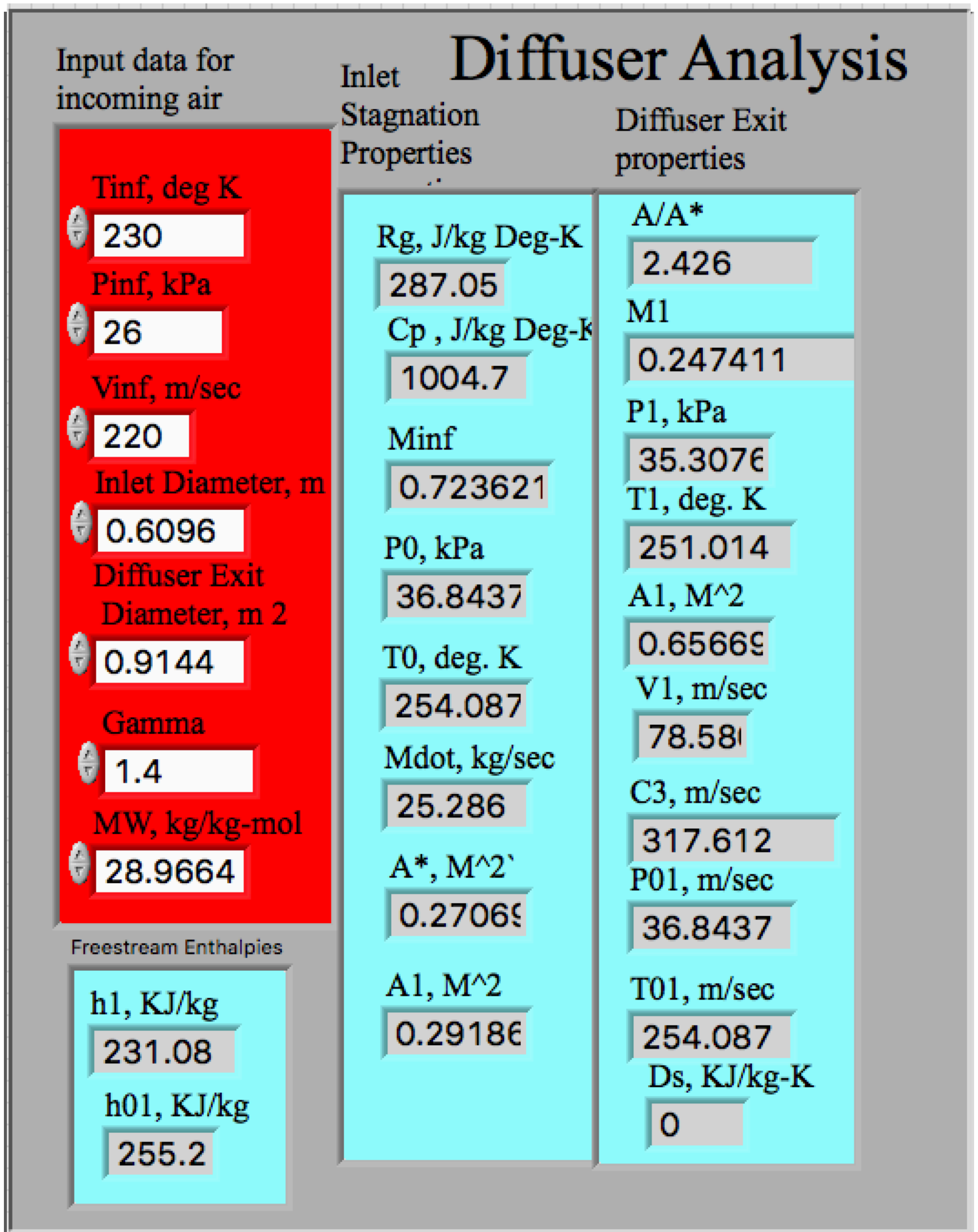
Diffuser Analysis

P_0 = 26 kPa
T_0 = 230 K
V_0 = 220 m/s
$\dot{m}$ = 25 kg/s





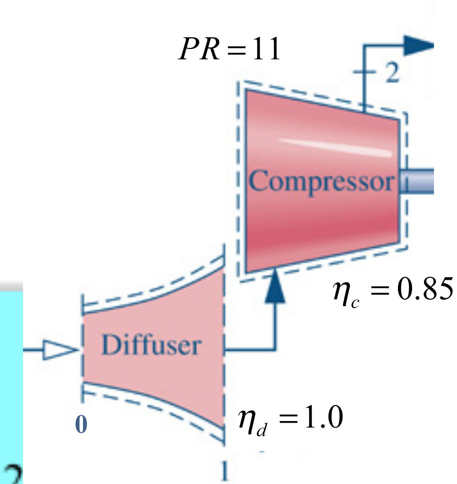

```
/* Calculate stagnation temperature */
T01=T1 + (V1**2)/(2*Cp1);

/* Calculate Mach number */
term2 = sqrt(gamma*Rg1*T1);
Minf = V1/sqrt(gamma*Rg1*T1);

/* Calculate stagnation pressure */
expn = gamma/(gamma-1);
P01 = P1*( 1 + ( ( gamma-1)/2
)*(Minf**2))**(expn);

/* calculate inlet massflow */
A1 = (pi/4)*(D1**2);
mdot = ( (P1*1000)/(Rg1*T1) )*V1*A1;

/* calculate Inlet specific enthalpies *
/h1 = Cp1*T1/1000;
h01 = Cp1*T01/1000;
```

Compressor Analysis

```
/* calculate exit pressure */
p2 = P01*Pr;

/* Ideal (ISENTROPIC) Stagnation Temperature */
expn = (gamma-1)/gamma;
T02_i=  T01*(Pr**expn);

/* Ideal DEMAND stagnation specific enthalpy */
h02_i = Cp1*T02_i/1000;

/* true DEMAND stagnation specific enthalpy */
h02 =h01+(h02_i - h01)/eta;

/* True Stagnation Temperature */
T02 = 1000*h02/Cp1;

/* change in entropy */
DS2 = (Cp1*ln(T02/T01) - Rg*ln(Pr) )/1000;

/* actual compressor work */
Wdot = h02-h01;
```

Assume compressor outlet Mach number is essentially zero

$$\eta_c = \frac{h_{0_2|_{s=0}} - h_{0_1}}{h_{0_2} - h_{0_1}}$$



Compressor Exit

P02, kPa
405.29

T02_I, deg. K 2
504.11

T02, deg. K
548.246

Ds2, KJ/kg-K
0.084326

Outputput Enthalpies 2

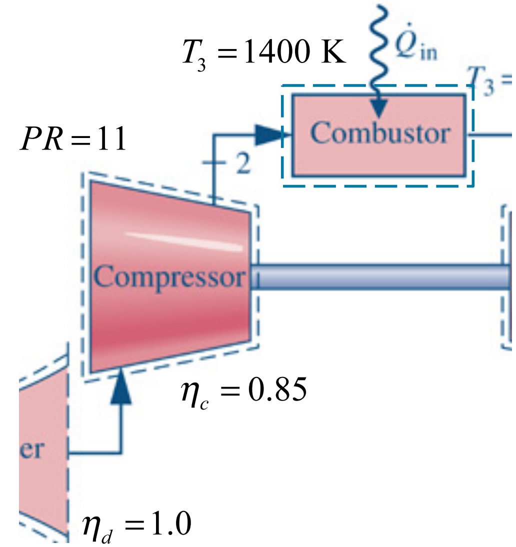
h02_i, KJ/kg
506.4

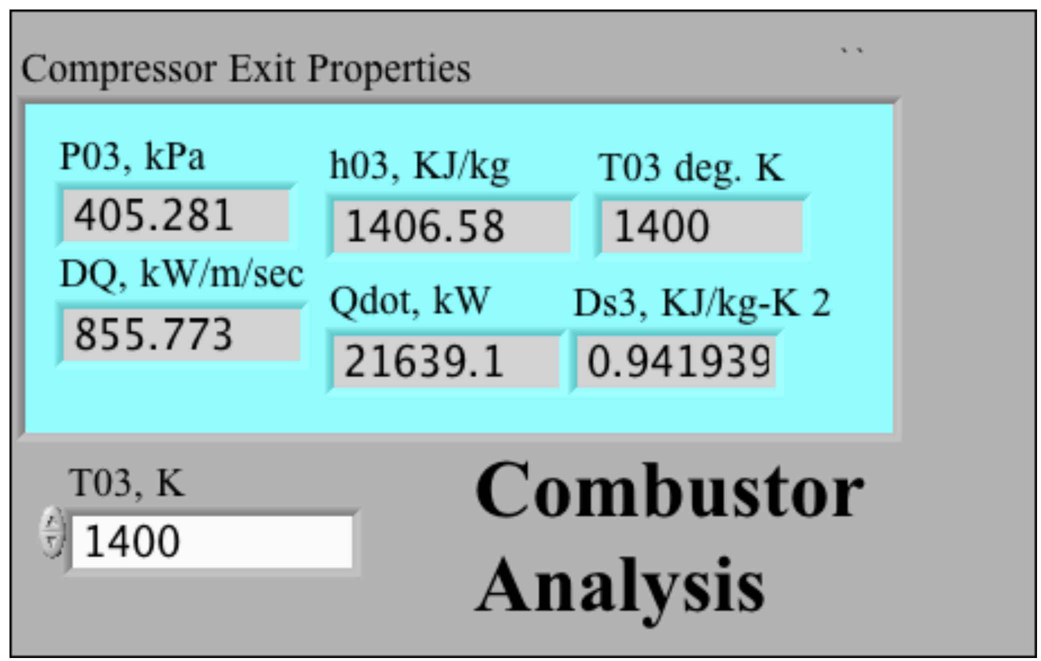
h02, KJ/kg
550.8

Compressor DEMAND specific Power kW/kg/sec
295.523

Combustor Analysis

```
/* calculate outlet enthalpy */
h03 = Cp*T03/1000;

/* calculate heat input
per unit massflow */
DQ = (h03-h02);

/* calculate total heat  input */
qdot = DQ*mdot;

/* calculate change in enthalpy */
DS =
Cp*ln(T03/T02) /1000;
```

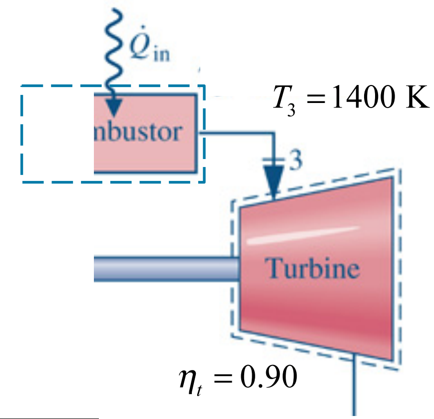




Turbine Analysis

```
/* calculate idealized REQUIRED
Output enthalpy */
h04_i= h03-(Wdot)/eta;
T04_i = 1000*h04_i/Cp;

/* calculate actual REQUIRED output
enthalpy from turbine */
h04= h03-Wdot;

/* calculate output stagnation temperature */
T04=T03+(h04-h03)/(Cp/1000);

expn = gamma/(gamma-1);
P04=P03*( ( h04_i/h03)**expn);

/* change in entropy */
DS = (Cp*ln(h04/h03) - Rg*ln( P04/P03  ) )/1000;
```

$$\eta_t = \frac{h_{0_3} - h_{0_4}}{h_{03} - h_{0_{4_{s=0}}}}$$



Turbine Analysis

Turbine Exit properties

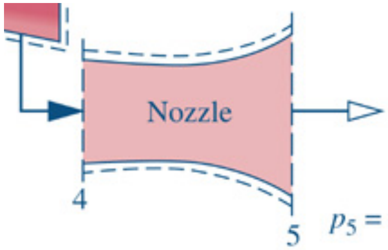
h04_i, KJ/kg	1078.22
T04_i, K	1073.18
h04, KJ/kg 2	1111.05
T04, K	1105.86
P04, kPa	159.829
Ds3, KJ/kg-K 2	0.0301403

Turbine Efficiency 0.9

Compressor DEMAND specific Power kW/kg/sec

295.523

Nozzle Analysis

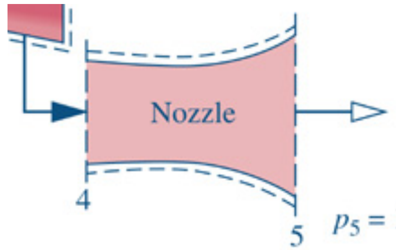


$\eta_n = 1.0$

$P_5 = 26$ kPa

```
/* calculate exit temperature */
expn = (gamma-1)/gamma;
Pratio = P0/pinf;
Texit = T4*( (1/Pratio) **expn );
hexit = Cp*Texit/1000.;

/* calculate exit velocity */
Vexit = sqrt( 2*( h04*1000- Cp*Texit ) );
h0exit = hexit+0.5*(Vexit**2);

/* calculate exit sonic velocity.Mach */
Cexit = sqrt(gamma*Rg*Texit);
Mexit1 = Vexit/Cexit;

/* calculate output mach */
expn = (gamma-1)/gamma;
Pratio = P0/pinf;
Mach =sqrt( ( Pratio**expn - 1)*(2/(gamma-1) ) );

/* Calculate Thrust */
Thrust = mdot*(Vexit-Vinf)/1000;

/* Propulsive Power */
PF = Thrust*Vinf;

/* Net kinetic energy rate leaving engine */
DKE = 0.001*mdot*( Vexit**2 - Vinf**2)/2.0;

/* propulsive efficiency */
Peff = PF/DKE;

/* shed excess heat */
Qdotout=mdot*( Cp*Texit  -1000*h1)/1000;

/* shed excess kinetic energy */
ShedKE = DKE-PF;

/* Thermal efficiency */
Teff = 0.0005*(Vexit**2)*
(1- ( Vinf/Vexit)**2)/(h03-h02);

/* Total imported energy */
TE = mdot*(h03-h02);
```

Nozzle Analysis (2)



$\eta_n = 1.0$

$P_5 = 26$ kPa

Nozzle Analysis

Nozzle Exit Properties

Pexit, kPa
26

Texit, deg K
658.20

Vexit, m/sec
948.425

Cexit, m/sec 2
514.314

Mexit
1.84406

Mach (alt)
1.84406

Momentum Thrust (KN)
18.419

Propulsive Power (kW)
4052.18

Efficiencies

Propulsive
0.37657

Thermal
0.49727

Total
0.18726

Net K.E. Rate (kW)
10760.6

Shet Excess Heat (KW)
10878.5

Shet Excess Kinetic Energy (KW) 2
6708.44

Total Exported Enegy (KW) Rate
21639.1

Total Iput Enegy (KW) Rate 2
21639.1

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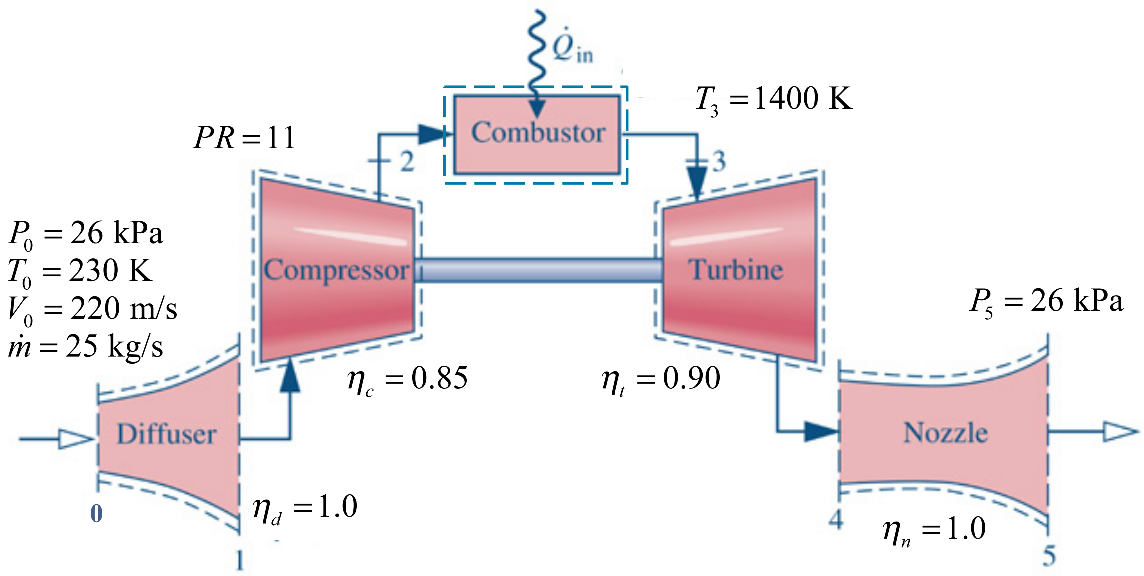
End-to-End State Table



Efficiencies

Propulsive: 0.37657

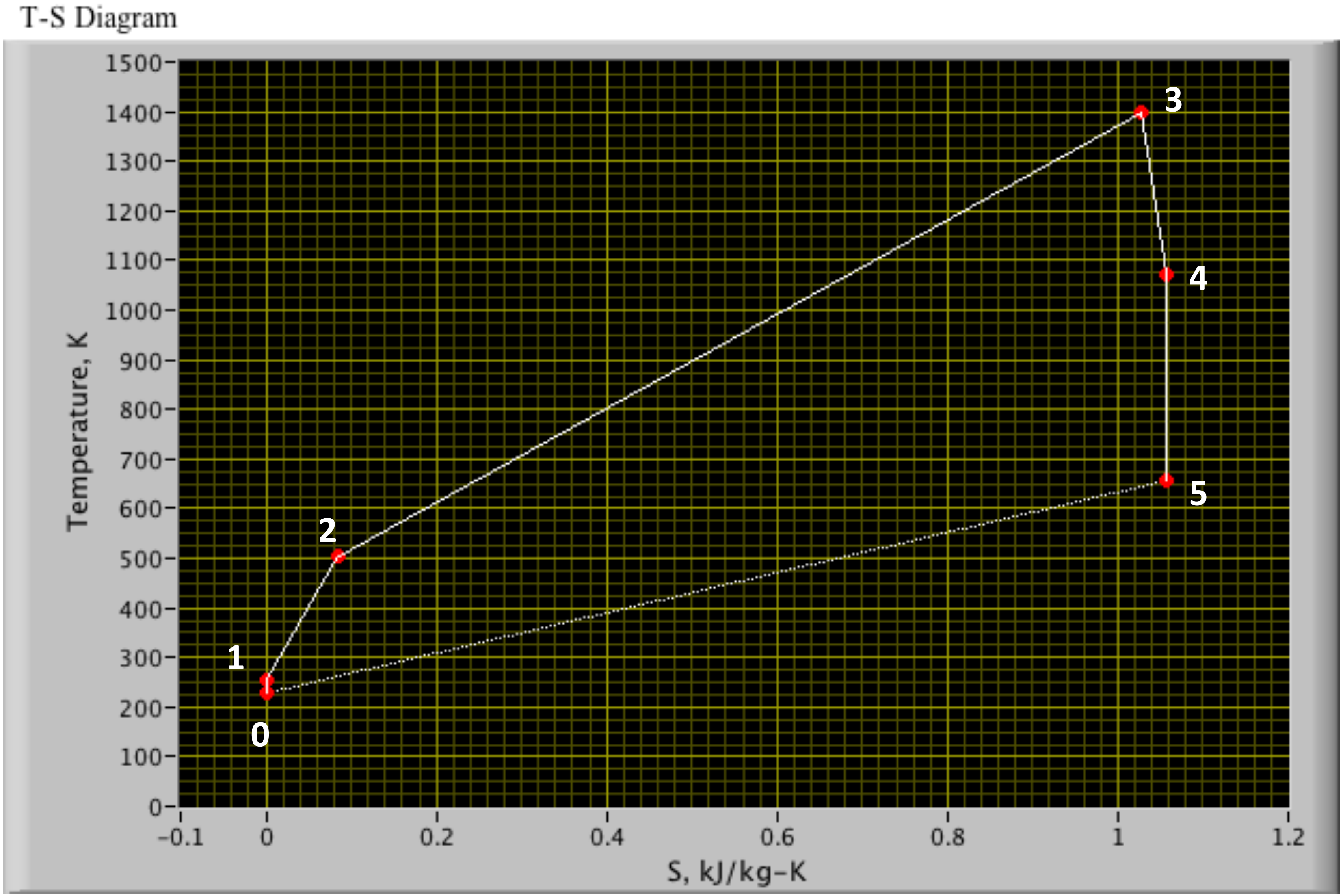
Thermal: 0.49727

Total: 0.18726

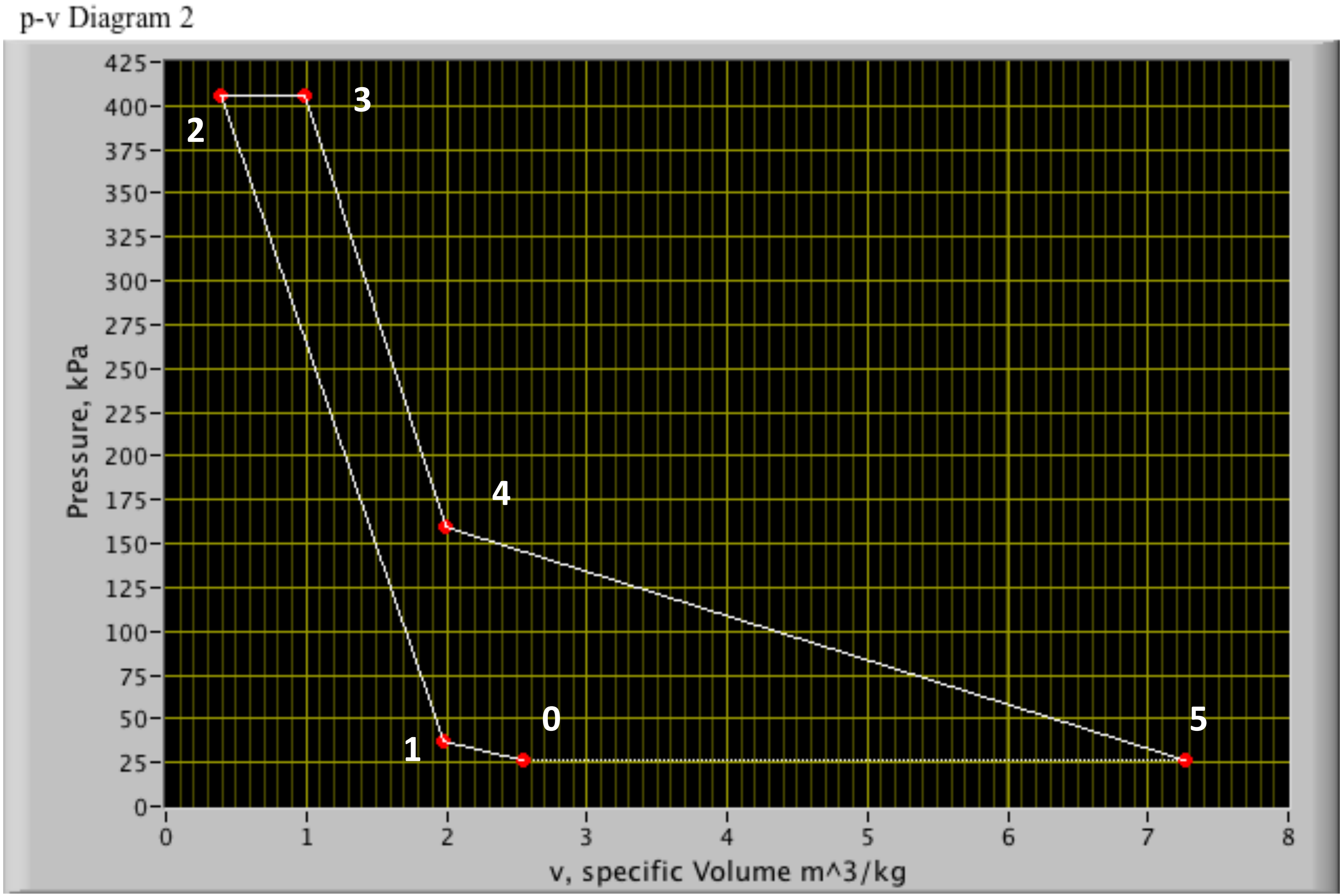
State Table

Station	Pi, kPa	Ti, K	h, KJ/kg	si, KJ/kg-K	Rho, kg/m^3	vol m^3/kg
0	26	230	231.08	0	0.393803	2.53934
1	36.8437	254.087	255.28	0	0.505144	1.97964
2	405.281	548.229	550.804	0.0842973	2.5753	0.388304
3	405.281	1400	1406.58	1.02624	1.00847	0.991604
4	159.829	1105.86	1111.05	1.05638	0.503489	1.98614
5	26	658.205	661.297	1.05638	0.137608	7.26699

T-S Diagram


T-S Diagram
Temperature, K
S, kJ/kg-K
0
1
2
3
4
5
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
-0.1
0
0.2
0.4
0.6
0.8
1
1.2

P-v Diagram


p-v Diagram 2
Pressure, kPa
v, specific Volume m^3/kg
425
400
375
350
325
300
275
250
225
200
175
150
125
100
75
50
25
0
0
1
2
3
4
5
6
7
8
2
3
4
1
0
5

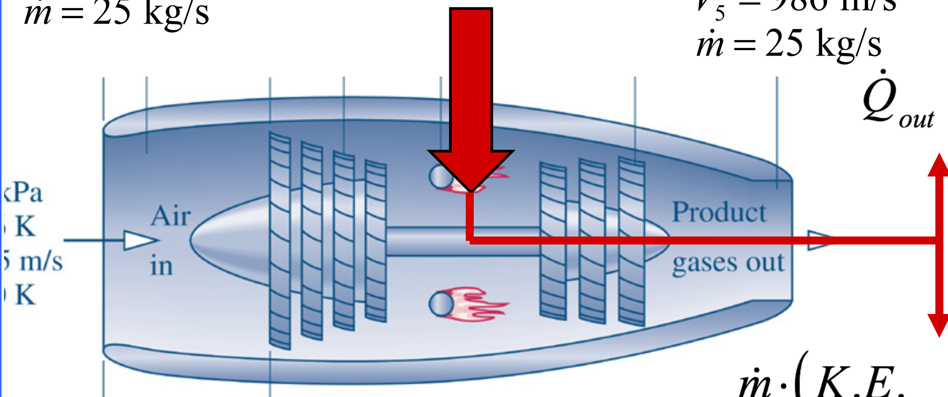
Energy Decomposition

How is the energy input to this engine distributed?

$P_0 = 26$ kPa
$T_0 = 230$ K
$V_0 = 220$ m/s
$\dot{m} = 25$ kg/s

$\dot{Q}_{in} = 21{,}639.1$ kW

$P_5 = 26$ kPa
$T_5 = 719.5$ K
$V_5 = 986$ m/s
$\dot{m} = 25$ kg/s



excess thermal energy transfer

$$\dot{Q}_{out} = \dot{m}\cdot\left(h_{exit} - h_{\infty}\right) = 10{,}878.5 \text{ kW} \quad (50.3\%)$$

kinetic energy production rate

$$\dot{m}\cdot\left(K.E._{net}\right) = \frac{\dot{m}}{2}\left(V_{exit}^2 - V_{\infty}^2\right) = 10{,}760.6 \text{ kW} \quad (49.7\%)$$

$$\dot{m}\cdot\left(K.E._{excess}\right) = 6708.4 \text{ kW} \quad (62.3\%)$$

$$\dot{W}_{prop} = 4{,}052.2 \text{ kW} \quad (37.7\%)$$

Excess Enthalpy Transfer Rate	Thrust Power Output	Excess K.E. Lost	Total Heat Input
10878.5 +	4052.2 +	6708.4	= 21639.1 KW

Energy Decomposition (2)

Calculate Efficiencies

- **Total Heat Input at Combustor**

$$\dot{Q}_{comb} = \dot{m}\cdot\left(h_3 - h_2\right) = 21{,}639.1 \text{ kW}$$

- **Excess thermal energy transfer**

$$\dot{Q}_{out} = \dot{m}\cdot\left(h_{exit} - h_{\infty}\right) = 10{,}878.5 \text{ kW}$$

- **Kinetic energy production rate**

$$\dot{m}\cdot\left(K.E._{net}\right) = \frac{\dot{m}}{2}\left(V_{exit}^2 - V_{\infty}^2\right) = 10{,}760.6 \text{ kW}$$

- **Propulsive power generation**

$$\dot{W}_{prop} = F_{thrust}\cdot V_{\infty} = 4{,}052.2 \text{ kW}$$

Efficiencies

Propulsive	0.49037
Thermal	0.49721
Total	0.24381

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)} =$$

$$\frac{4{,}052.2_{KW}}{10{,}760.6_{KW}} = 0.3766\ (37.66\%)$$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel}\cdot h_{fuel}} =$$

$$\frac{10{,}760.6_{KW}}{21{,}639.1_{KW}} = 0.4973\ (49.73\%)$$

$$\eta_{total} = \eta_{prop}\cdot\eta_{thermal} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)}\cdot\frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel}\cdot h_{fuel}}$$

$$= \frac{4{,}052{,}2_{KW}}{21{,}639.1_{KW}} = 0.1872\ (18.73\%)$$