Homework 4.2

KGW-1 (later re-designated as LTV-N-2) was the US Navy's version of American flying bomb *JB-2 Loon*. It was developed to be carried on the aft deck of submarines in watertight containers. The first submarine to employ them was the SS-348 *Cusk* which successfully launched its first Loon on 12 February 1947 in Point Mugu, California. It has the following data:

- Static thrust 2200 N with air inlet speed of 180 m/s @ Sea Level
- Intake area 0.145 m^2
- Fuel is standard 80-octane gasoline having heating value $Q_R = 40$ MJ/kg
- Burner efficiency 0.90
- Typical flight duration is 1800 s
- Exhaust temperature 735 K

Assume Nozzle Optimized for Sea Level

Homework 4.2 (2)

Assume specific heat of air $Cp_a = 1.005 \frac{\text{kJ}}{\text{kgK}}$ and specific heat of hot gases $Cp_h = 1.12 \frac{\text{kJ}}{\text{kgK}}$

$$h_{fuel} = \eta_{combustor} \cdot Q_R$$

Calculate

1. Air mass flow rate into engine
2. Exhaust velocity
3. Maximum temperature inside the engine
4. Maximum pressure
5. Thrust specific fuel consumption (TSFC)
6. Average range *Launch Weight = 2,150 kg*

Assume Stagnation (items 3–4)

7. Mean L/D for (Sea Level) Cruise Conditions

Solution

1976 US Standard Atmosphere:

$$p_{\substack{sea \\ level}} = 101.325_{kpa}$$

$$T_{\substack{sea \\ levl}} = 288.15_{K}$$

$$\rho_{\substack{sea \\ level}} = 1.225_{kg/m^3}$$

1) Inlet Air Massflow:

$$\dot{m}_{air} = \rho_{sl} \cdot A_{inlet} \cdot V_{\infty} = 1.225 \cdot 0.145 \cdot 180 = 31.9725 \text{ kg/sec}$$

2) Exhaust Velocity:

$$F_{thrust} = \dot{m}_{air} \cdot \left(V_{exit} - V_{\infty}\right) \rightarrow V_{exit} = \frac{F_{thrust}}{\dot{m}_{air}} + V_{\infty} = \frac{2200}{31.9725} + 180$$

$$= 248.81 \text{ m/sec}$$

Solution (2)

3) Maximum Temperature Inside Engine:

First use given Exhaust Temperature to Calculate Engine air/fuel Ratio

$$\dot{m}_{fuel} \cdot \eta_{burner} \cdot Q_R + \dot{m}_{air} \cdot \left(C_{p_{air}} T_\infty + \frac{V_\infty^2}{2} \right) = \left(\dot{m}_{fuel} + \dot{m}_{air} \right) \left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right)$$

$$\eta_{burner} \cdot Q_R + \frac{\dot{m}_{air}}{\dot{m}_{fuel}} \cdot \left(C_{p_{air}} T_\infty + \frac{V_\infty^2}{2} \right) = \left(\frac{\dot{m}_{fuel} + \dot{m}_{air}}{\dot{m}_{fuel}} \right) \left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right)$$

$$f \equiv \frac{\dot{m}_{air}}{\dot{m}_{fuel}} \rightarrow \eta_{burner} \cdot Q_R + f \cdot \left(C_{p_{air}} T_\infty + \frac{V_\infty^2}{2} \right) = \left(f + 1 \right) \left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right)$$

$$f \left[\left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right) - \left(C_{p_{air}} T_\infty + \frac{V_\infty^2}{2} \right) \right] = \eta_{burner} \cdot Q_R - \left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right)$$

$$f = \frac{\eta_{burner} \cdot Q_R - \left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right)}{\left[\left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right) - \left(C_{p_{air}} T_\infty + \frac{V_\infty^2}{2} \right) \right]}$$

Solution (3)

3) Maximum Temperature Inside Engine:

Use given Exhaust Temperature to Calculate Engine air/fuel Ratio

$$f = \frac{\eta_{burner} \cdot Q_R - \left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right)}{\left[\left(C_{p_h} T_{exhaust} + \frac{V_{exhaust}^2}{2} \right) - \left(C_{p_{air}} T_\infty + \frac{V_\infty^2}{2} \right) \right]} \rightarrow$$

$$\eta_{burner} = 0.9$$

$$Q_R = 40 \cdot 10^6 \; {}_{J/kg}$$

$$C_{p_{air}} = 1005 \; {}_{J/kg-K}$$

$$C_{p_{air}} = 1120 \; {}_{J/kg-K}$$

$$T_\infty = 288.15 \; {}_K$$

$$V_\infty = 180 \; {}_{m/\sec}$$

$$T_{exit} = 735 \; {}_K$$

$$V_{exit} = \cancel{265.482}\ \mathbf{248.81} \; {}_{m/\sec}$$

$$\frac{0.9 \cdot 40 \cdot 10^6 - \left(1120 \cdot 735 + \frac{248.81^2}{2} \right)}{\left(1120 \cdot 735 + \frac{248.81^2}{2} \right) - \left(1005 \cdot 288.15 + \frac{180^2}{2} \right)} = 64.0924$$

Solution (4)

3) Maximum Temperature Inside Engine:

Now use Combustor Enthalpy Balance to Calculate Combustor Stagnation Temperature

$$\eta_{burner} \cdot Q_R + f \cdot \left(C_{p_{air}} T_\infty + \frac{V_\infty^2}{2} \right) = (f+1)\left(C_{p_h} T_{0_{combustor}} \right)$$

$$T_{0_{combustor}} = \frac{\eta_{burner} \cdot Q_R + f \cdot \left(C_{p_{air}} T_\infty + \frac{V_\infty^2}{2} \right)}{(f+1) \cdot C_{p_h}} =$$

$$\frac{0.9 \cdot 40 \cdot 10^6 + 64.0924 \left(1005 \cdot 288.15 + \frac{180^2}{2} \right)}{(64.092 + 1)\ 1120} = 762.64 \text{ K}$$

Solution (5)

3) Maximum Temperature Inside Engine:

Alternate Method .. Assume isentropic properties down stream of combustor

$$T_{0_{combustor}} = T_{0_{exit}} = T_{exit} + \frac{V_{exit}^2}{2} = 735 + \frac{248.81^2}{2 \cdot 1120} = 762.64 \text{ K}$$

4) Maximum Pressure Inside Engine:

→ **Stagnation Pressure Loss Across Combustor ….**

→ **Maximum Pressure is freestream stagnation pressure**

$$P_{0_\infty} = p_\infty \left(1 + \frac{\gamma - 1}{2} \frac{V_\infty^2}{\gamma \cdot R_g \cdot T_\infty}\right)^{\frac{\gamma}{\gamma-1}} =$$

$$101.325 \left(1 + \frac{1.4 - 1}{2} \frac{180^2}{1.4 \cdot 287.056 \cdot 288.15}\right)^{\frac{1.4}{1.4-1}} = 122.597 \text{ kPa}$$

Solution (6)

5) Thrust Specific Fuel Consumption:

$$TSFC = \frac{\dot{m}_f}{F_{thrust}} = \frac{\dot{m}_{air} / f}{F_{thrust}} =$$

$$TSFC = \frac{\dot{m}_f}{F_{thrust}} = \frac{\dot{m}_{air} / f}{F_{thrust}} = \frac{31.9725}{2200 \cdot 64.0924} = \mathbf{0.00022675} \text{ kg/N-sec}$$

$$TSFC = \mathbf{0.00022675} \text{ kg/N-sec} \times \frac{}{2.204_{lbm/kg}} \times 4.4495_{N/lbf} \times 3600_{sec/hr} = \mathbf{8.00521} \text{ lbm-hr/lbf}$$

6) Average Range:

$$R = V_\infty \cdot T_{burn} = \frac{180_{m/\sec} \cdot 1800_{\sec}}{1000_{m/km}} = 324_{km}$$

Solution (7)

7) Mean L/D for (Sea Level) Cruise Conditions

$$R = V_\infty \cdot T_{burn} = \frac{180_{m/\sec} \cdot 1800_{\sec}}{1000_{m/km}} = 324_{km}$$

Breguet Range Equation

$$R = I_{sp} \cdot \left(\frac{L}{D} \cdot V_\infty \right) \cdot \ln \left(\frac{M_{initial}}{M_{final}} \right)$$

$$I_{sp} = \frac{F_{thrust}}{g_0 \cdot \dot{m}_{fuel}} = \frac{F_{thrust}}{g_0 \cdot \frac{1}{f} \dot{m}_{air}} = \frac{2200}{9.8067 \frac{31.9725}{64.0924}} = 449.707 \text{ sec}$$

Solution (8)

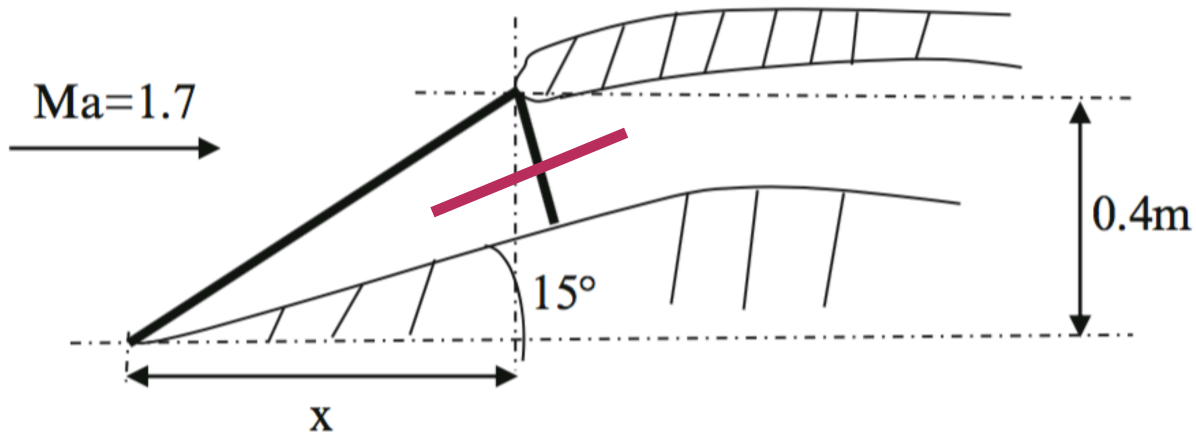
$$M_{final} = M_{initial} - t_{burn} \cdot \frac{1}{f} \dot{m}_{air} = 2150 - 1800 \frac{31.9725}{64.0924}$$ = 1252.07 kg

$$R = I_{sp} \cdot \left(\frac{L}{D} \cdot V_{\infty} \right) \cdot \ln\left(\frac{M_{initial}}{M_{final}} \right) \rightarrow \frac{L}{D} = \frac{R}{I_{sp} \cdot V_{\infty} \cdot \ln\left(\frac{M_{initial}}{M_{final}} \right)} =$$

$$\frac{324 \cdot 1000}{449.707 \cdot 180 \left(\ln\left(\frac{2150}{1252.07} \right) \right)} = 7.40310$$

Homework 4.3, Part 1

A ramjet operates at an altitude of 10,000 m ($T_a = 223\ K$, $P_a = 0.26\ atm$, $\gamma = 1.4$) at a Mach number of 1.7. The external diffusion is based on an oblique shock and on a normal shock, as described in the shown figure.



Assume ∞ = a

Calculate

- Stagnation pressure recovery, $\frac{P_{02}}{P_{0a}}$?
- At what Mach number does the oblique shock become detached?
- What is the distance x, from the cone tip to the outer inlet lip, for the condition described in the figure?
- What is the best turning angle θ in terms of highest pressure ratio, $\frac{P_{02}}{P_{0a}}$?

Homework 4.3, Part 1 (2)

- Oblique Shock wave, $\theta = 15^o$, $M_\infty = 1.7$ …. Compute shock wave angle β, *Properties behind oblique shock*

Strong Shock Solution 2

Beta (strong shock), deg.
73.839:

M1n (strong shock), deg.
1.6328:

M1t (strong shock), deg.
0.4732

Weak Shock Solution

Beta (weak shock), deg.
55.9840

M1n (weak shock), deg.
1.4091

M1t (weak shock), deg.
0.9510

Output Data

Rho2/Rho1
1.705426

P2/P1
2.149819

T2/T1
1.260576

P02/P01
0.955896

n
0.735909

P02/P1
3.081197

M2 (weak shock), deg. 2
1.122071

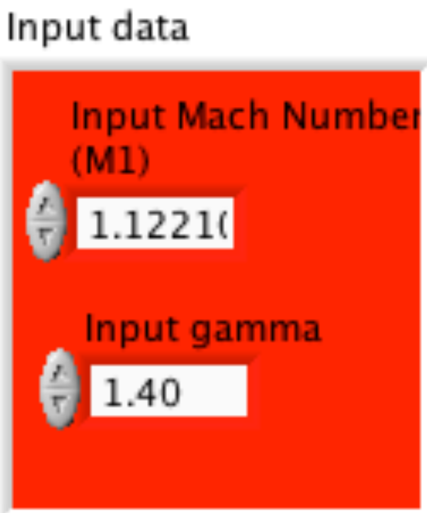
$$\beta = 55.98^o$$

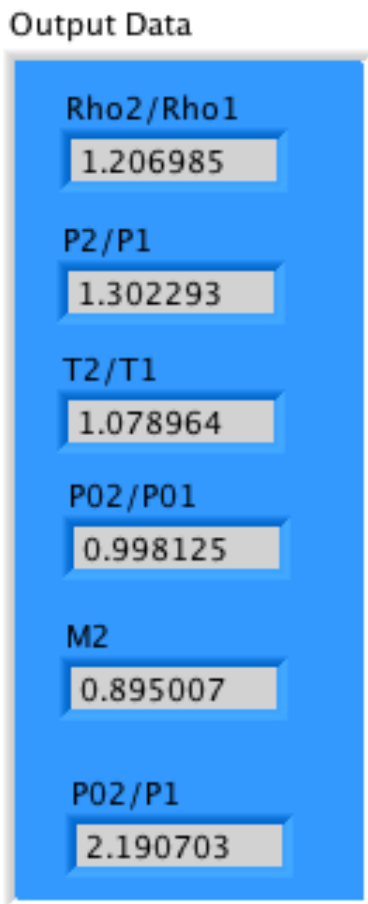
$$M_1 = 1.1221$$

$$\frac{P_{0_1}}{P_{0_\infty}} = 0.955896$$

Homework 4.3, Part 1 (3)

- Now compute properties across normal shock wave at cowl throat





$$\frac{P_{0_1}}{P_{0_\infty}} = 0.955896$$

$$\frac{P_{0_2}}{P_{0_1}} = 0.998125$$

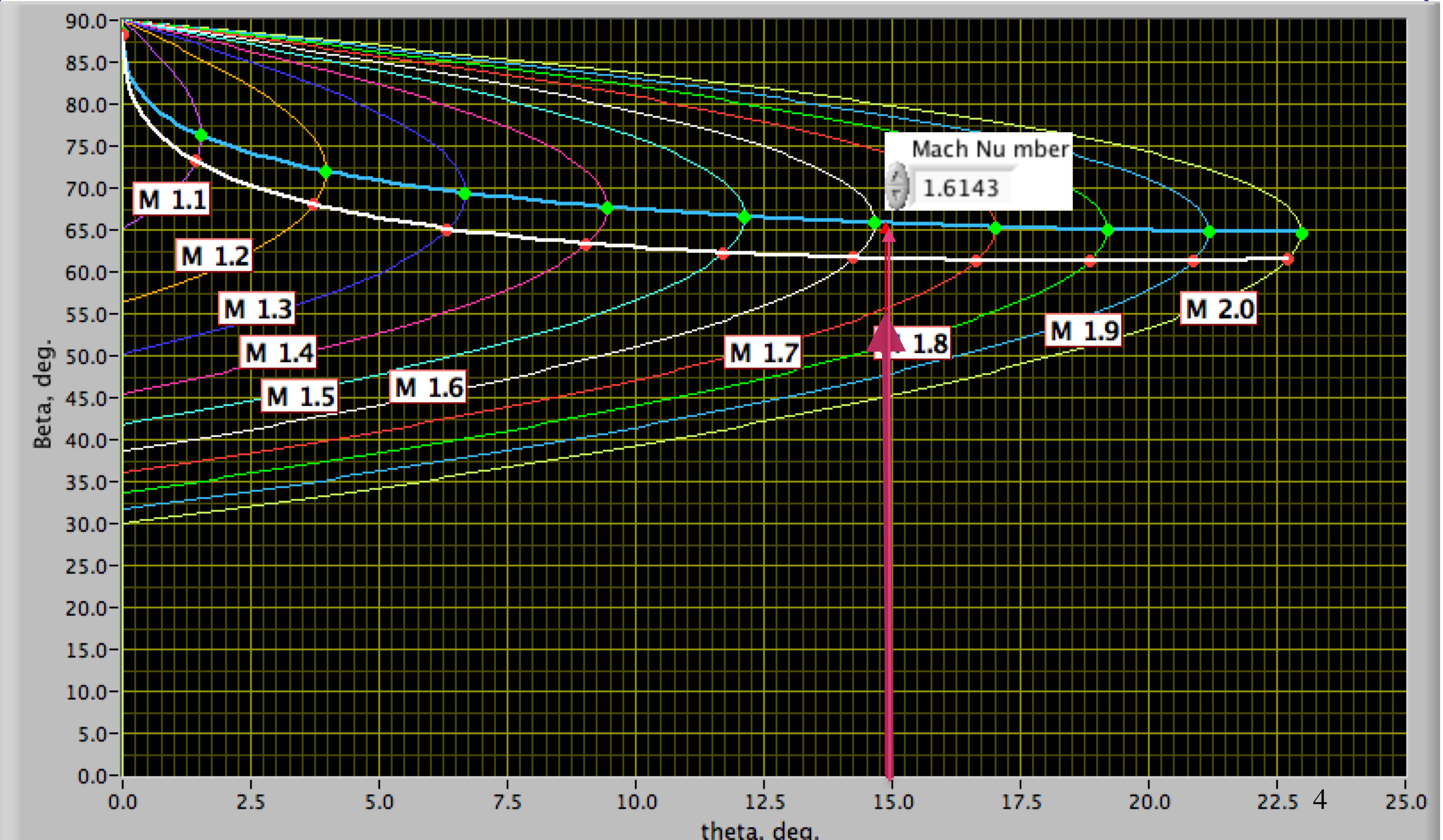
$$\rightarrow \frac{P_{0_2}}{P_{0_\infty}} = \frac{P_{0_1}}{P_{0_\infty}} \cdot \frac{P_{0_2}}{P_{0_1}} =$$

$$0.955896 \cdot 0.998125 = 0.954104$$

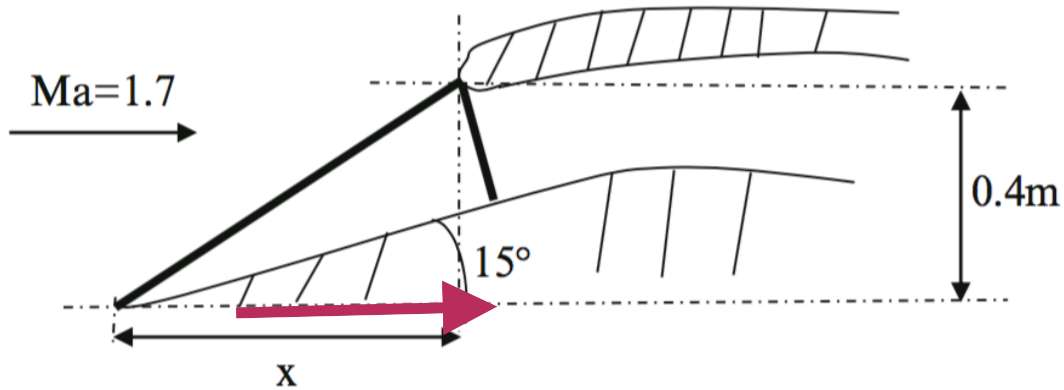
Stagnation Pressure Recovery

Homework 4.3, Part 1 (4)

- What is detachment Mach number of 15 degree ramp: 1.6143



Homework 4.3, Part 1 (5)



- What is the distance *x*, from the cone tip to the outer inlet lip, for the condition described in the figure?

$$z = r \cdot \sin\beta \rightarrow r = \frac{z}{\sin\beta}$$

$$\beta = 55.98^{o}$$

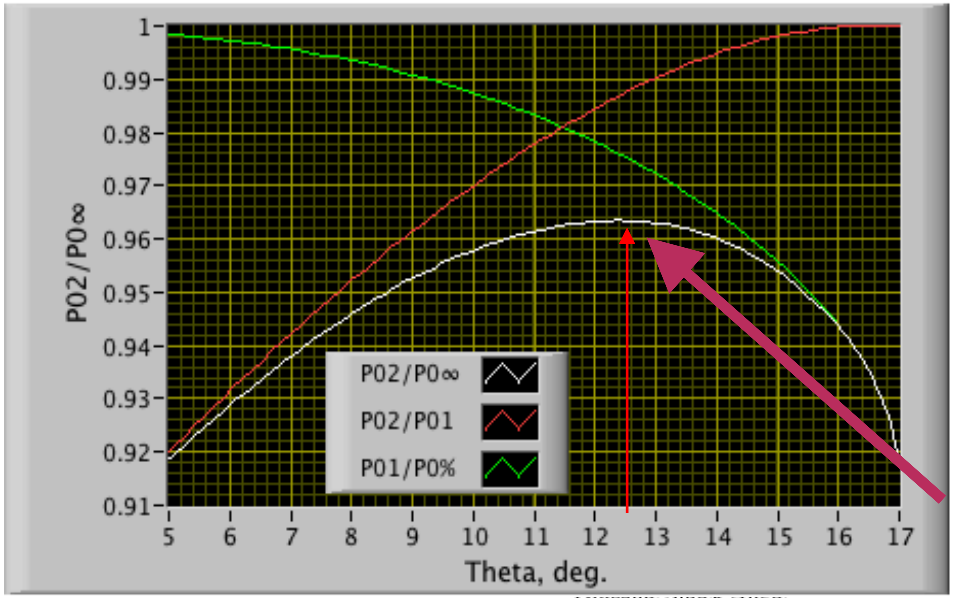
$$x = r \cdot \cos\beta = \frac{z}{\sin\beta} \cdot \cos\beta = \frac{z}{\tan\beta}$$

$$= \frac{40}{\tan\left(\frac{\pi}{180} 55.98\right)} = 27.0007 \text{ cm}$$

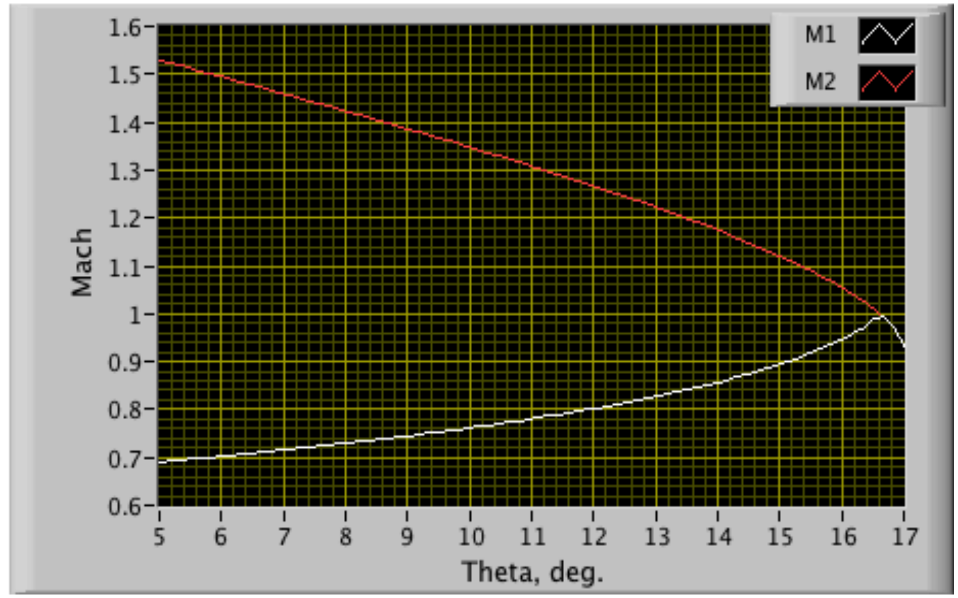
Homework 4.3, Part 1 (6)

- What is the best turning angle θ in terms of highest pressure ratio, $\frac{P_{02}}{P_{0\infty}}$?

Stagnation Pressure Ratio



Mach Number



$$\theta_{OPT} \approx 12.4^{O}$$