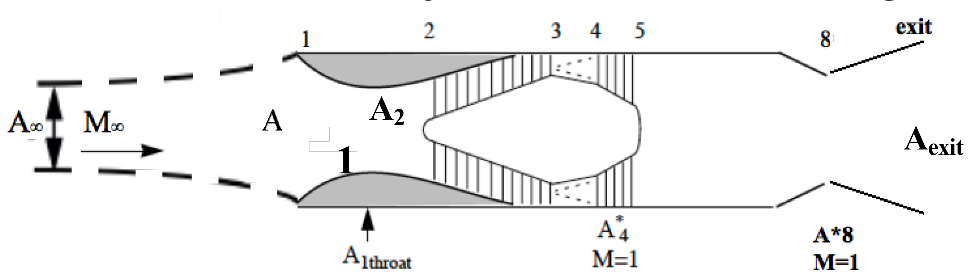
Utah State University

Mechanical & Aerospace Engineering

# Turbojet, Matching Example (2)




- Engine operates at a free stream Mach number, $M_\infty = 0.80$
- Cruise Altitude is in the stratosphere, 11 km so $T_\infty = 216.65$ *K*, $p_\infty = 22.63$ *kPa*.
- The design turbine inlet temperature, $T_{04} = 2000$ *K (1726.85 °C)*
- The design compressor ratio range, $\pi_c$ = 2-*10* .
- Relevant area ratios are $A_2 / A^*_4 = 9.65$ and $A_2/A_{1throat} = 1.45$.
- Inlet throat area $A_{1Throat} = 2000$ *cm²* *(50.463 cm, 19.87" diameter)*
- *Assume the compressor, burner and turbine all operate ideally.*
- *Converging/Diverging type Nozzle with choked throat*
- *Stagnation pressure losses due to wall friction in the inlet and nozzle are negligible.*
- *Octane (Gasoline) Fuel,* $h_f = 49.47$ *MJ/kg*

## *Part 1. Assume sonic nozzle exit, …. CALCULATE*

a) *Compressor Operating Line, Plot* $\pi_c$ *vs. Corrected massflow,* $\pi_c = 2 \rightarrow 10$

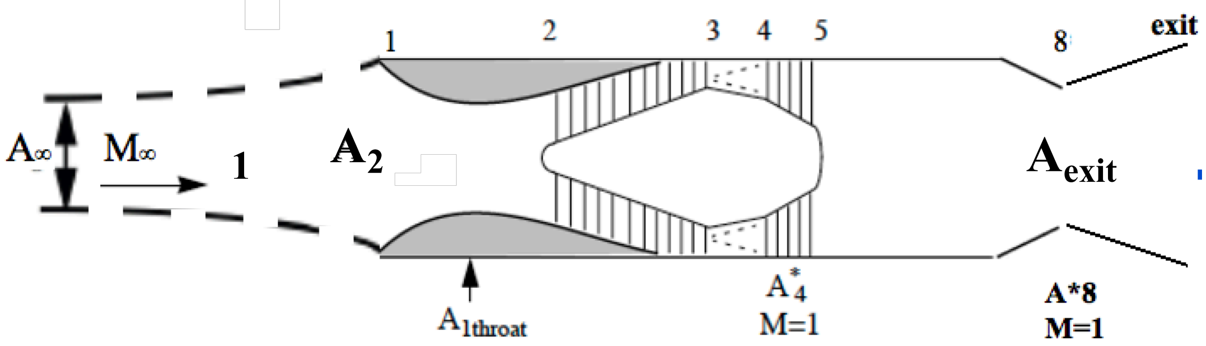
b) *Overlay Operating Line on J-85 Compressor Map*
  - *You can manually plot Operating line on Map Image or Use .xls file link for Compressor map*
  - *What is the Design Operating Condition at 100% Rotor Speed*
    - *(corrected massflow, compression ratio)*
  - *Plot the Engine Surge and Choke Margins as a function of % Rotor Speed*
    - *Surge Margin =*
    - *Choke margin =*

$$100\% \times \left( \frac{\dot{m}_w - \dot{m}_{surge,\,choke}}{\dot{m}_w} \right)$$

c) *Plot Diffuser Throat, Compressor face Mach numbers, AND Inlet Capture Area vs* $\pi_c$

d) *Plot Fuel-to-Air Ratio (1/f) vs.* $\pi_c$*, as required to maintain* $T_{04}$ *at 2000 K*

# Turbojet, Matching Example (3)




**Incoming Air to Turbojet (@ to station *3*)**

- Molecular weight = 28.96443 *kg/kg-mole*
- $\gamma$ = 1.40
- $R_g$ = 287.058 *J/kg-K*
- $T_\infty$ = *216.65 K*
- $p_\infty$ = *22.63 kPa.*
- $V_\infty$ = 220 *m/sec*
- Universal Gas Constant: $R_u$ = 8314.4612 *J/kg-K*

**Calorically Perfect Gas**

$$\gamma = \frac{c_p}{c_v}$$

$$R_g = c_p - c_v$$

$$c_p = \frac{\gamma}{\gamma - 1} \cdot R_g$$

$$c_v = \frac{1}{\gamma - 1} \cdot R_g$$

Assume $\gamma$, $c_p$, $M_w$ are constant across engine

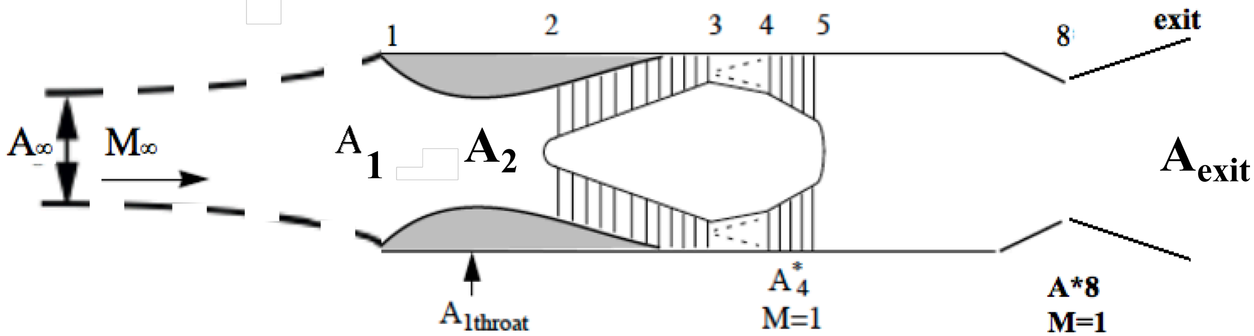
**For ...Isentropic Conditions →**

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

**Ideal Gas**

$$p = \rho \cdot R_g \cdot T$$

# Turbojet, Matching Example (4)




**Parameter Definitions**

$$\left\{\tau_r, \tau_c, \tau_\lambda, \tau_f, \gamma\right\} \rightarrow$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \rightarrow \textit{Freestream Mach number reference conditions}$$

$$\tau_c = \frac{T_{0_3}}{T_{0_2}} \rightarrow \textit{Compressor stagnation temperature ratio measure of compressor work input}$$

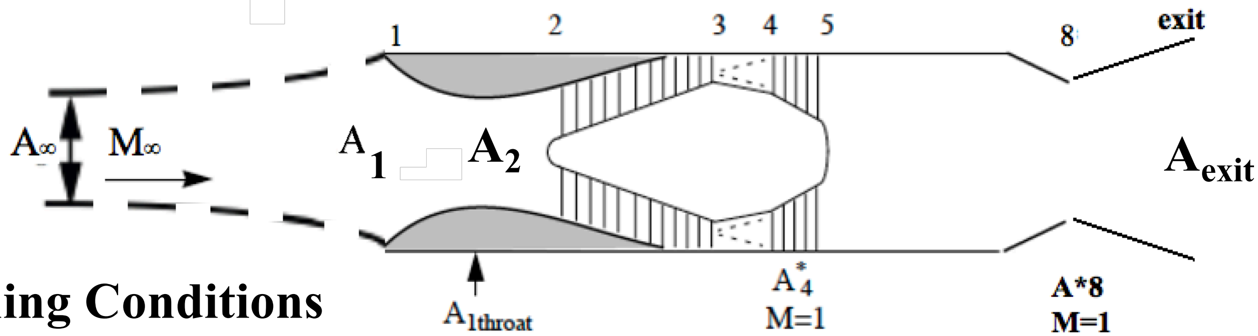
$$\tau_\lambda = \frac{T_{0_4}}{T_\infty} \rightarrow \textit{Combustor flame temperature...Optimized up to Material limits of combustor, turbine}$$

$$\tau_f = \frac{h_f}{h_\infty} \rightarrow \textit{Fuel enthalpy of combustion relative to incoming air stream total enthalpy}$$

*Choice of Fuel*

$$\gamma = \frac{C_p}{C_v} \rightarrow \textit{Ratio of specific heats}$$

# Turbojet, Matching Example (5)




**Matching Conditions**

*Nozzle /Turbine:(massflow)*

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}} \qquad \pi_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2\cdot\gamma}{\gamma+1}}$$

*Compressor/Turbine:(power)*

$$\tau_t = 1 - \left(\frac{f}{f+1}\right)\frac{\tau_r\cdot(\tau_c - 1)}{\tau_\lambda} \rightarrow (\tau_c - 1) = \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot(1-\tau_t)$$

*Compressor/Turbine:(massflow)*

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^{\ 2}\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^*}{A_2}$$

*Inlet/Compressor:(massflow)*

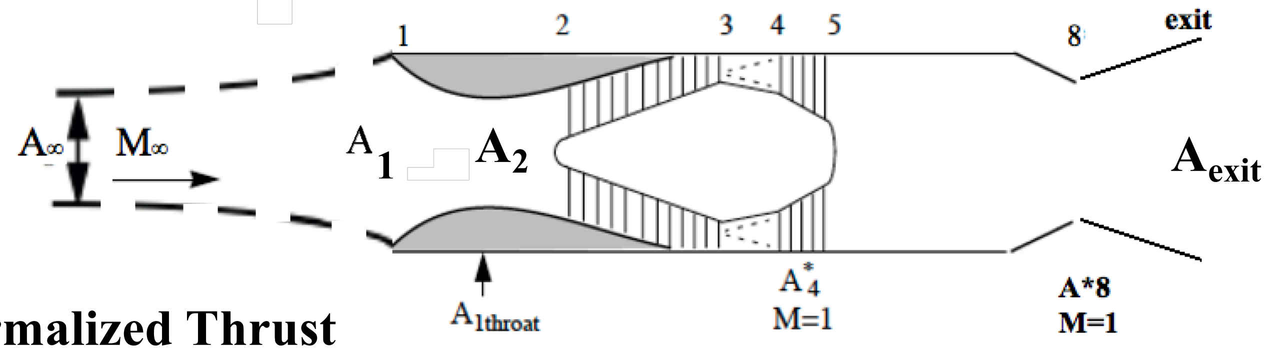
$$\frac{1}{\pi_d}\frac{A_\infty}{A_2}\frac{M_\infty}{\left(1+\frac{\gamma-1}{2}M_\infty^{\ 2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{M_2}{\left(1+\frac{\gamma-1}{2}M_2^{\ 2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

*Air/Fuel Ratio:(design burner exit temperature)*

$$f = \frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r\cdot\tau_c}$$

# Turbojet, Matching Example (6)






**Normalized Thrust**

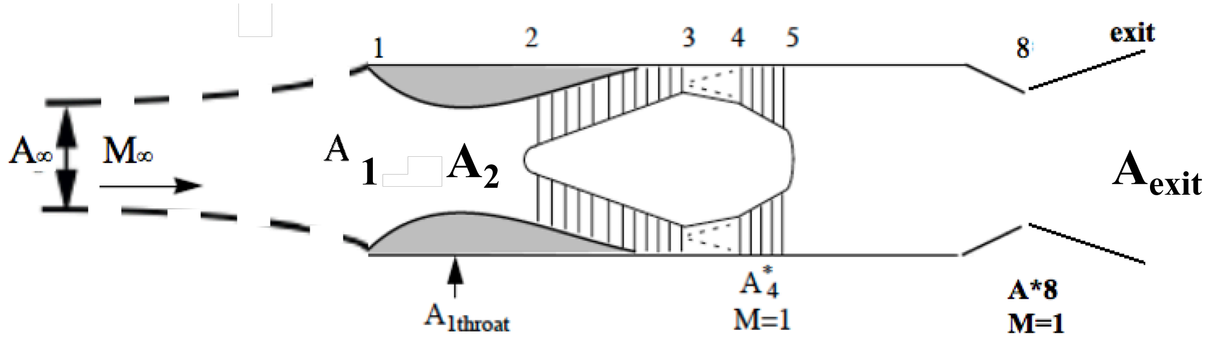
$$\underset{\text{Opt}}{\mathbb{T}} = \frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r-1\right)\cdot\left[\left(\frac{f+1}{f}\right)\cdot\sqrt{\left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\cdot\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)}-1\right]$$

$$\left(\frac{V_{exit}}{V_\infty}\right)^2 = \left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)$$

$$\underset{\text{Non-Opt}}{\mathbb{T}} = \frac{F_{thrust}}{p_\infty\cdot A_\infty} = \gamma\cdot M_\infty^2\cdot\left(\frac{V_{exit}}{V_\infty}-1\right)+\frac{A_{exit}}{A_\infty}\cdot\left(\frac{p_{exit}}{p_\infty}-1\right)$$

**Momentum Thrust** **Pressure Thrust**

# Turbojet, Matching Example (2)




***Part 2. Optimal Design***

***Op*** = 100% $N_1$ Design Operating Point $\pi_c$

*a)* *For $\pi_c$=**Op** .... CALCULATE to Optimal Nozzle expansion ratio, $A_{exit}/A^*_8$*

- *Using Optimal Nozzle expansion ratio ( @ $\pi_c$=**Op** ), Plot Nozzle Exit Pressure Ratio, $p_{exit}/p_\infty$, vs. vs. $\pi_c$*

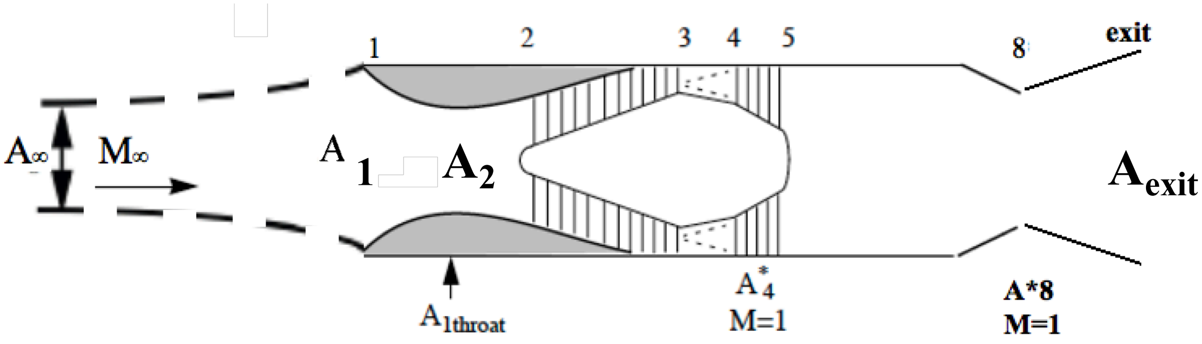
*b)* *Using Optimal Nozzle expansion ratio ( @ $\pi_c$=**Op** ), Plot Normalized Thrust vs. $\pi_c$*

- *Normalized Momentum Thrust*
- *Normalized Pressure Thrust*
- *Normalized Total Thrust*

*c)* *Using Optimal Nozzle expansion ratio ( @ $\pi_c$=**Op** ), Plot*

- *True Thrust vs. $\pi_c$*
- *Corrected and True Compressor Massflow vs. vs. $\pi_c$*
- *Nozzle Exit Massflow vs. vs. $\pi_c$*

# Turbojet, Matching Example (2)




***Part 3. Efficiencies***

a) *For Optimal Expansion ratio at $\pi_c$=**Op** .... Plot*
- *Specific Impulse vs. $\pi_c$*
- *TSFC vs. $\pi_c$*
- *Propulsive (Mechanical) Efficiency vs $\pi_c$*
- *Thermal Efficiency vs $\pi_c$*
- *Total Efficiency vs $\pi_c$*
  - *Include effects of air-to-fuel ratio*
  - *See section4.1, slides 19 & 21*

$$\eta_{propulsive} = \frac{\dot{m}_{air} \cdot \left( \left( \frac{f+1}{f} \right) \cdot V_{exit} - V_\infty \right) \cdot V_\infty}{\dot{m}_{air} \cdot \left( \frac{1}{2} \left( \frac{f+1}{f} \right) \cdot V_{exit}^2 - \frac{1}{2} V_\infty^2 \right)} =$$

$$\frac{2 \cdot \left( \left( \frac{f+1}{f} \right) \cdot V_{exit} - V_\infty \right) \cdot \frac{1}{V_\infty^2} \cdot V_\infty}{\left( \left( \frac{f+1}{f} \right) \cdot V_{exit}^2 - V_\infty^2 \right) \cdot \frac{1}{V_\infty^2}} = \frac{2 \cdot \left( \left( \frac{f+1}{f} \right) \cdot \left( \frac{V_{exit}}{V_\infty} \right) - 1 \right)}{\left( \left( \frac{f+1}{f} \right) \cdot \left( \frac{V_{exit}}{V_\infty} \right)^2 - 1 \right)}$$

$$\eta_{thermal} = \frac{\frac{1}{2} \cdot \left( \frac{f+1}{f} \right) \cdot V_{exit}^2 - \frac{1}{2} \cdot V_\infty^2}{\frac{1}{f} h_{fuel}} = \frac{\frac{1}{2} \cdot V_\infty^2 \left[ \left( \frac{f+1}{f} \right) \cdot \left( \frac{V_{exit}}{V_\infty} \right)^2 - 1 \right]}{\frac{1}{f} h_{fuel}} =$$

$$\frac{\frac{1}{2} \cdot V_\infty^2 \cdot f \cdot \left[ \left( \frac{f+1}{f} \right) \cdot \left( \frac{V_{exit}}{V_\infty} \right)^2 - 1 \right]}{h_{fuel}} = \frac{\frac{1}{2} \cdot V_\infty^2 \cdot f \cdot \left[ \left( \frac{f+1}{f} \right) \cdot \left( \frac{V_{exit}}{V_\infty} \right)^2 - 1 \right]}{h_{fuel}}$$


Questions??