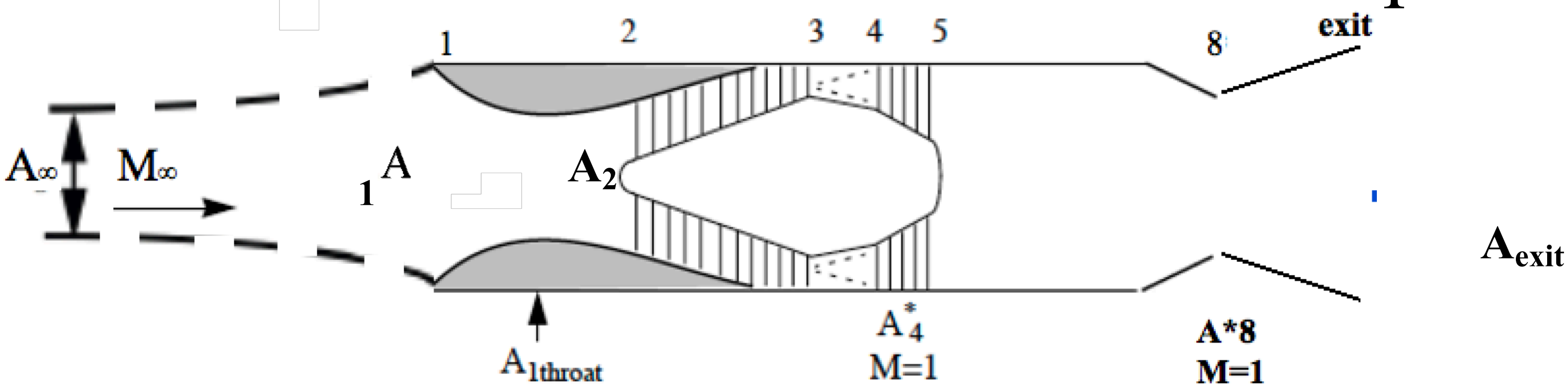
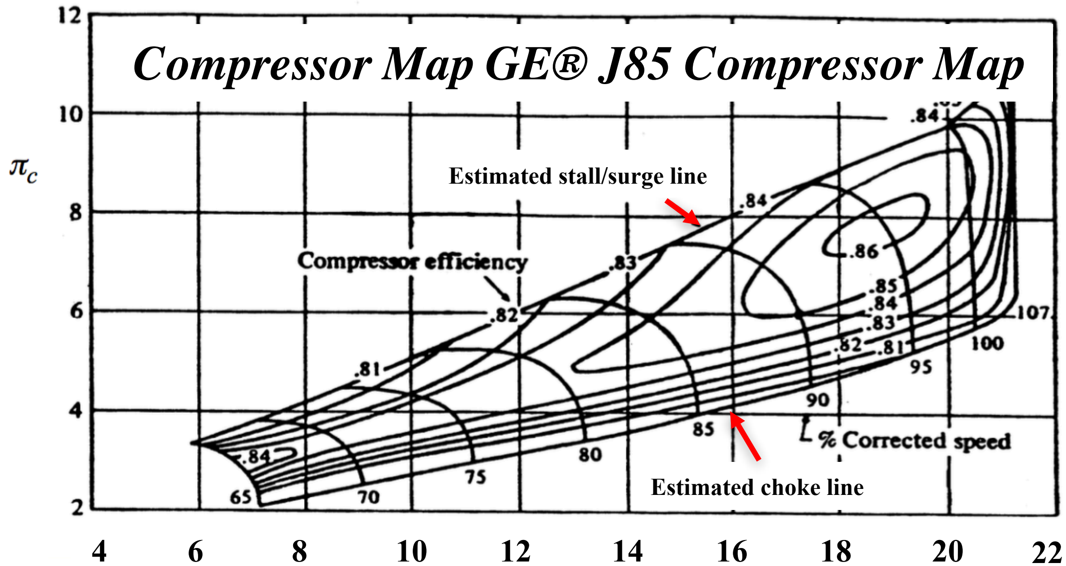
HW5.1 Turbojet, Matching Example

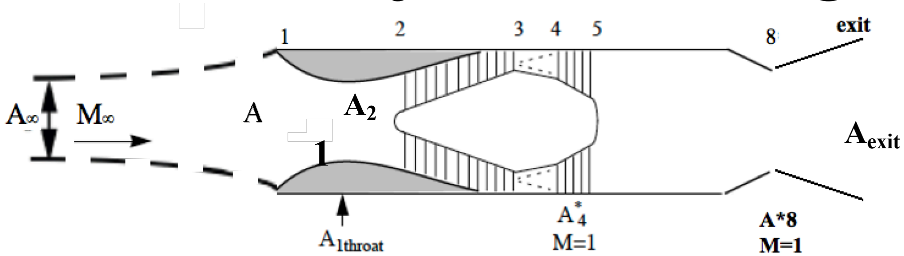


- Consider Jet Engine Built Around ***GE® J85 Compressor***



$$\dot{m}_c = \dot{m}_{air} \cdot \left(\sqrt{\Theta}/\Delta\right), kg/\sec$$

Turbojet, Matching Example (2)



- Engine operates at a free stream Mach number, $M_\infty = 0.80$
- Cruise Altitude is in the stratosphere, 11 km so $T_\infty = 216.65\ K$, $p_\infty = 22.63\ kPa$.
- The design turbine inlet temperature, $T_{04} = 2000\ K$ ($1726.85\ ^oC$)
- The design compressor ratio range, $\pi_c = 2\text{-}10$.
- Relevant area ratios are $A_2 / A^*_4 = 9.65$ and $A_2/A_{1throat} = 1.45$.
- Inlet throat area $A_{1Throat} = 2000\ cm^2$ (50.463 cm, 19.87" diameter)
- *Assume the compressor, burner and turbine all operate ideally.*
- *Converging/Diverging type Nozzle with choked throat*
- *Stagnation pressure losses due to wall friction in the inlet and nozzle are negligible.*
- *Octane (Gasoline) Fuel,* $h_f = 49.47\ MJ/kg$

Part 1. Assume sonic nozzle exit, π_c=10.0 …. CALCULATE

a) *Compressor Operating Line, Plot π_c vs. Corrected massflow, $\pi_c = 2 \rightarrow 10$*

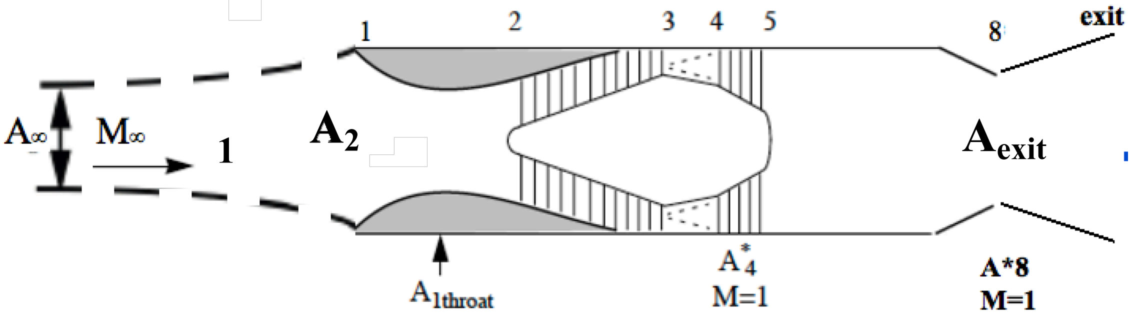
b) *Overlay Operating Line on J-85 Compressor Map*
   - *You can manually plot Operating line on Map Image or Use .xls file link for Compressor map*
   - *What is the Design Operating Condition at 100% Rotor Speed*
     - *(corrected massflow, compression ratio)*
   - *Plot the Engine Surge and Choke Margins as a function of % Rotor Speed*
     - *Surge Margin =*
     - *Choke margin =*

$$100\% \times \left(\frac{\dot{m}_w - \dot{m}_{surge,\,choke}}{\dot{m}_w} \right)$$

c) *Plot Diffuser Throat, Compressor face Mach numbers, AND Inlet Capture Area vs π_c*

d) *Plot Fuel-to-Air Ratio (1/f) vs. π_c, as required to maintain T_{04} at 2000 K*

Turbojet, Matching Example (3)



Incoming Air to Turbojet (@ to station 3)

- Molecular weight = 28.96443 *kg/kg-mole*
- γ = 1.40
- R_g = 287.058 *J/kg-K*
- T_∞ = 216.65 *K*
- p_∞ = 22.63 *kPa.*
- V_∞ = 220 *m/sec*
- Universal Gas Constant: R_u = 8314.4612 *J/kg-K*

Calorically Perfect Gas

$$\gamma = \frac{c_p}{c_v}$$

$$R_g = c_p - c_v$$

$$c_p = \frac{\gamma}{\gamma - 1} \cdot R_g$$

$$c_v = \frac{1}{\gamma - 1} \cdot R_g$$

Assume γ, c_p, M_w are constant across engine

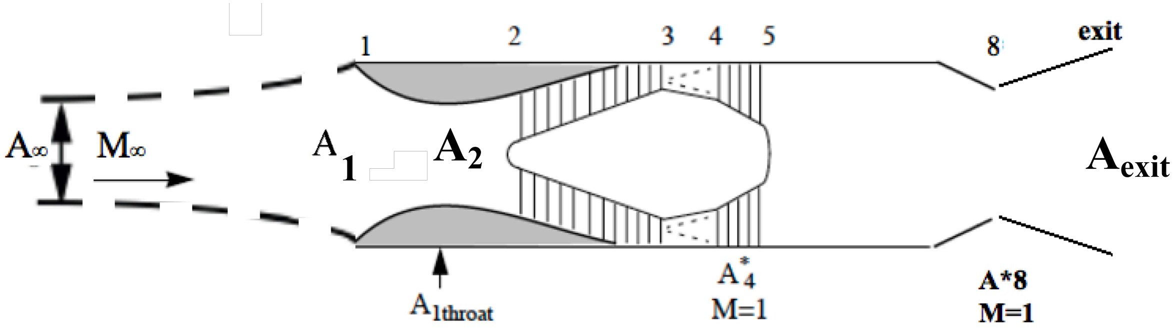
For …Isentropic Conditions →

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Ideal Gas

$$p = \rho \cdot R_g \cdot T$$

Turbojet, Matching Example (4)



Parameter Definitions

$$\{\tau_r, \tau_c, \tau_\lambda, \tau_f, \gamma\} \rightarrow$$

$$\tau_r = \frac{T_{0_\infty}}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \rightarrow \textit{Freestream Mach number reference conditions}$$

$$\tau_c = \frac{T_{0_3}}{T_{0_2}} \rightarrow \textit{Compressor stagnation temperature ratio measure of compressor work input}$$

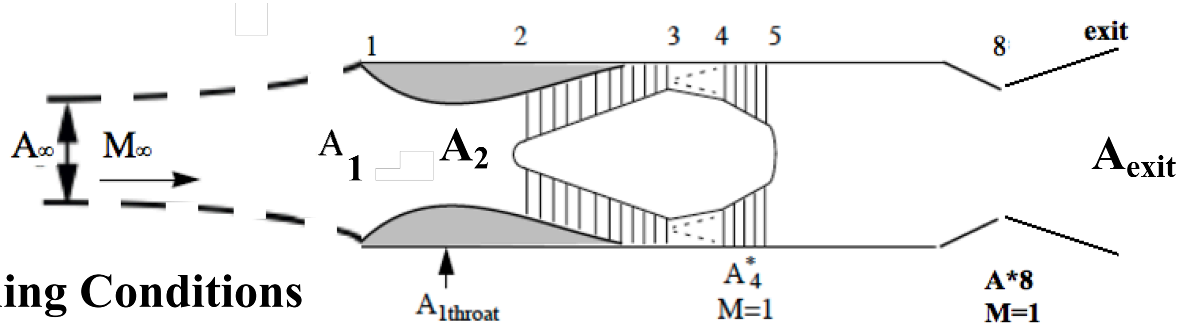
$$\tau_\lambda = \frac{T_{0_4}}{T_\infty} \rightarrow \textit{Combustor flame temperature...Optimized up to Material limits of combustor, turbine}$$

$$\tau_f = \frac{h_f}{h_\infty} \rightarrow \textit{Fuel enthalpy of combustion relative to incoming air stream total enthalpy}$$

$$\gamma = \frac{C_p}{C_v} \rightarrow \textit{Ratio of specific heats}$$

Choice of Fuel

Turbojet, Matching Example (5)



Matching Conditions

Nozzle /Turbine:(massflow)

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}} \qquad \pi_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2\cdot\gamma}{\gamma+1}}$$

Compressor/Turbine:(power)

$$\tau_t = 1 - \left(\frac{f}{f+1}\right)\frac{\tau_r \cdot (\tau_c - 1)}{\tau_\lambda} \rightarrow (\tau_c - 1) = \left(\frac{f+1}{f}\right)\cdot\left(\frac{\tau_\lambda}{\tau_r}\right)\cdot(1-\tau_t)$$

Compressor/Turbine:(massflow)

$$\frac{M_2}{\left(\left(1+\frac{\gamma-1}{2}M_2^2\right)\left(\frac{2}{\gamma+1}\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \left(\frac{f}{f+1}\right)\cdot\frac{(\pi_c)}{\sqrt{\tau_\lambda/\tau_r}}\cdot\frac{A_4^*}{A_2}$$

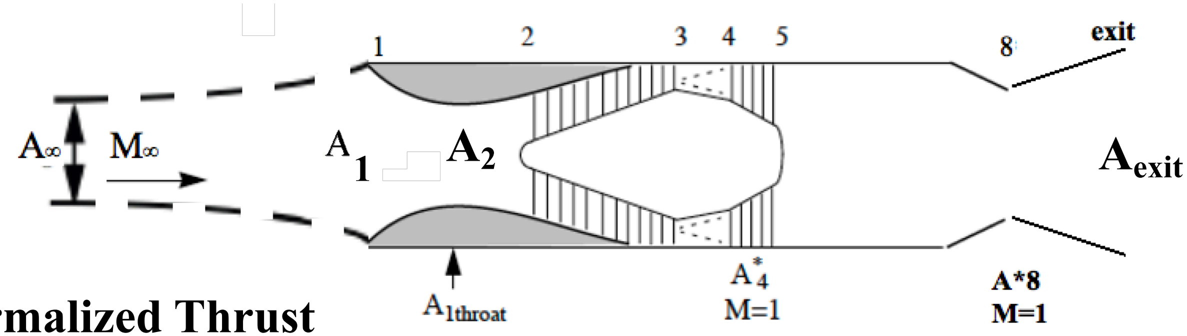
Inlet/Compressor:(massflow)

$$\frac{1}{\pi_d}\frac{A_\infty}{A_2}\frac{M_\infty}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{M_2}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Air/Fuel Ratio:(design burner exit temperature)

$$f = \frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r \cdot \tau_c}$$

Turbojet, Matching Example (6)



Normalized Thrust

$$\mathbb{T}_{Opt} = \frac{2\cdot\gamma}{\gamma-1}\cdot\left(\tau_r - 1\right)\cdot\left[\left(\frac{f+1}{f}\right)\cdot\sqrt{\left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\cdot\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)}-1\right]$$

$$\left(\frac{V_{exit}}{V_\infty}\right)^2 = \left(\frac{\left(\tau_r\cdot\tau_c\cdot\tau_t\right)-1}{\left(\tau_r-1\right)}\right)\left(\frac{\tau_\lambda}{\tau_c\tau_r}\right)$$

$$\mathbb{T}_{Non\text{-}Opt} = \frac{F_{thrust}}{p_\infty\cdot A_\infty} = \gamma\cdot M_\infty^2\cdot\left(\frac{V_{exit}}{V_\infty}-1\right)+\frac{A_{exit}}{A_\infty}\cdot\left(\frac{p_{exit}}{p_\infty}-1\right)$$

Momentum Thrust **Pressure Thrust**

Preliminaries, Freestream Properties

Freestream Conditions

Mach Number	Fuel Enthalpy, Kj/kg	Gamma
0.8	4.947E+7	1.4
Altitude, km	**Cp, J/kg-K**	**Max Burner Temperature, K**
11	1004.96	2000

Flight Parameters, Metric

Qc, kPa	Qbar, kPa	Vtrue, m/sec	P0_inf, kPa
11.87	10.14	236.06	34.4992
Pinf, kPa	**Tinf, K**	**CpMax**	**T0_inf, K**
22.63	216.65	1.1704	244.381

$$T_\lambda = \frac{T_{04}}{T_\infty} = \frac{2000_K}{216.65_K} = 9.23148$$

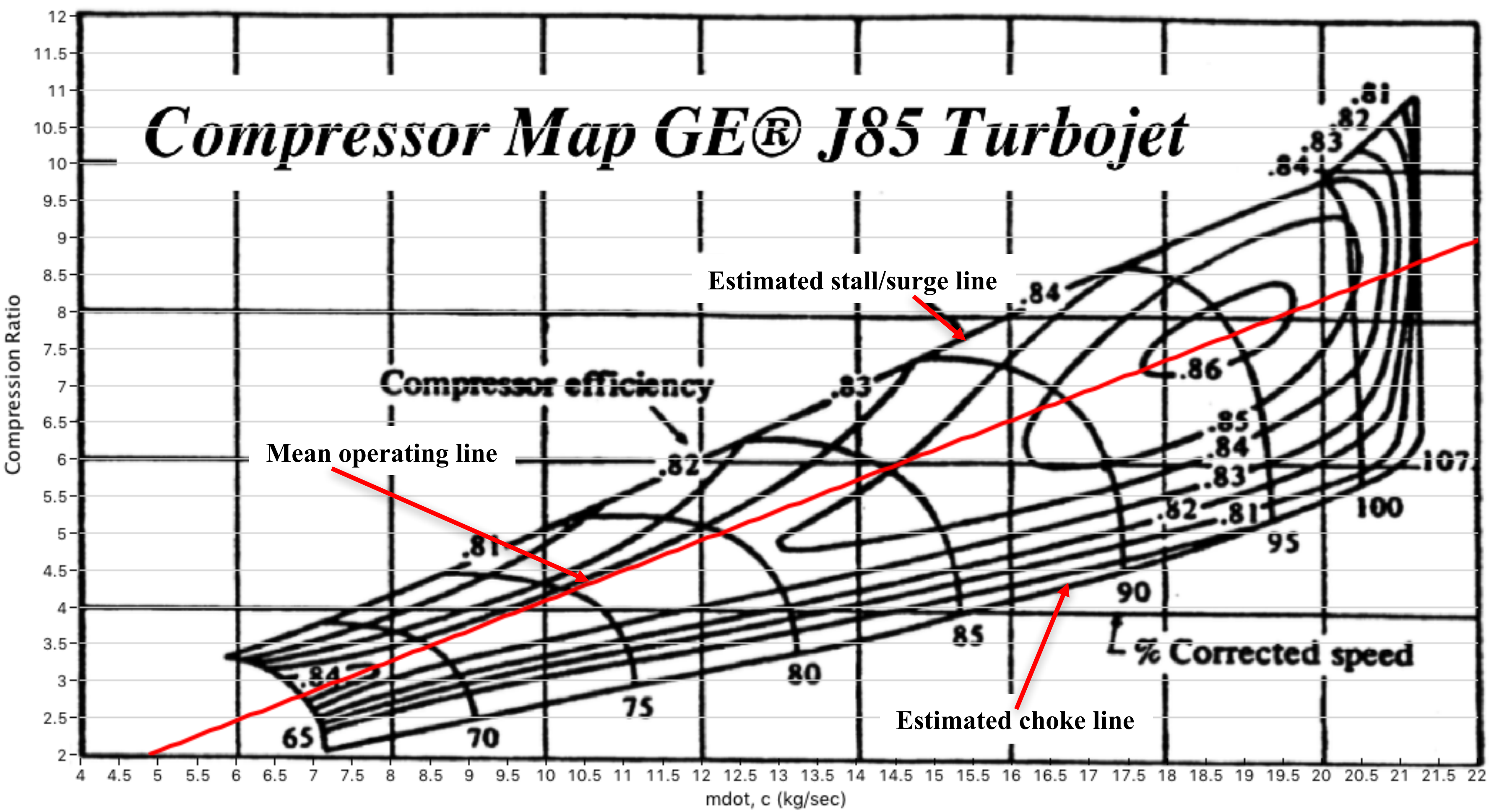
$$\tau_r = \frac{T_{01}}{T_\infty} = \frac{244.38_K}{216.65_K} = 1.128$$

$$h_\infty = c_p \cdot T_\infty = 1004.7_{j/kg-K} \cdot 216.65_K = 217{,}669_{j/kg}$$

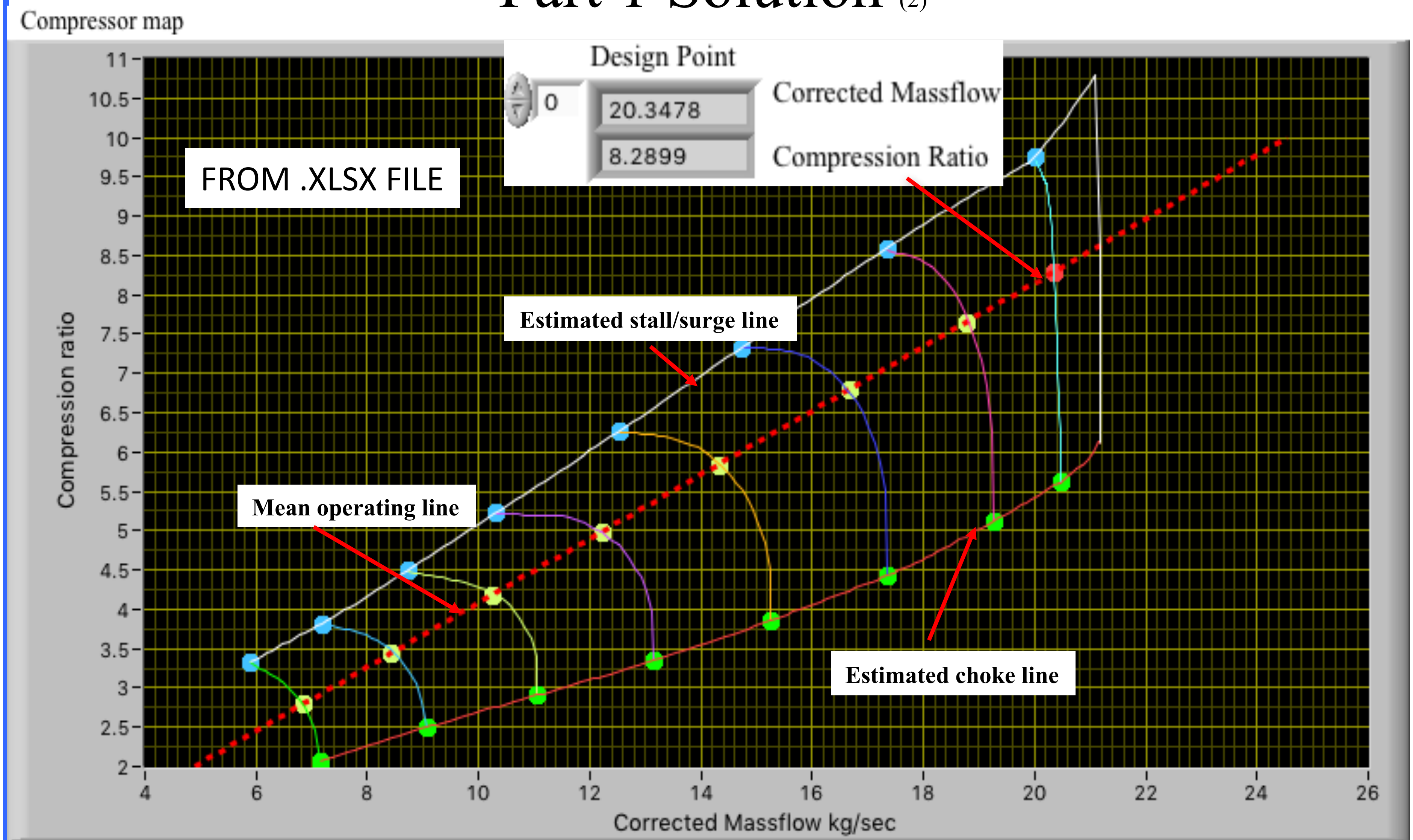
$$\tau_{fuel} = \frac{h_f}{h_\infty} = \frac{49.47 \cdot 10^6_{j/kg}}{217{,}669_{j/kg}} = 2277.272$$

Part 1 Solution (1)

Compressor Operating Line

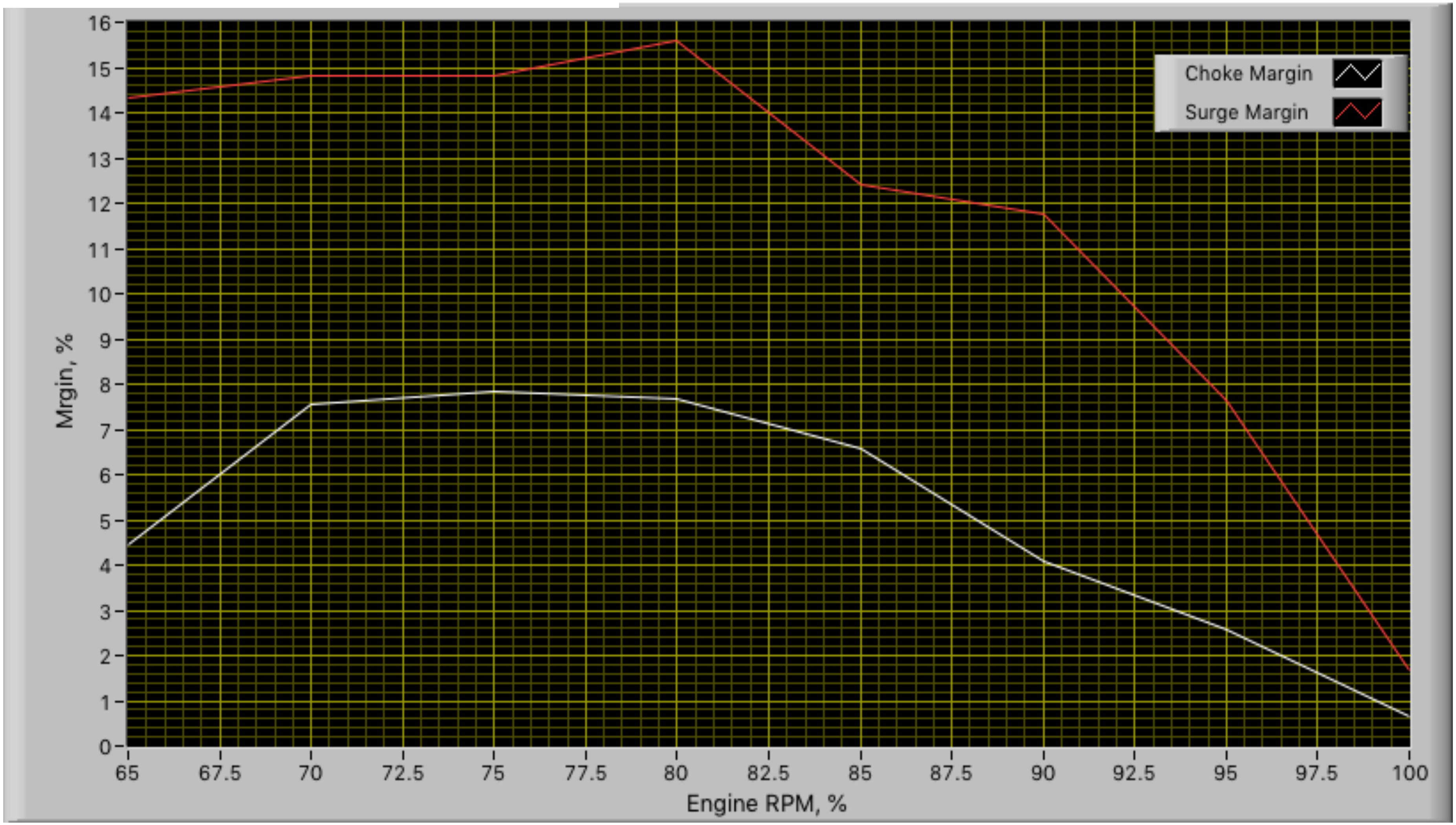


Part 1 Solution (2)



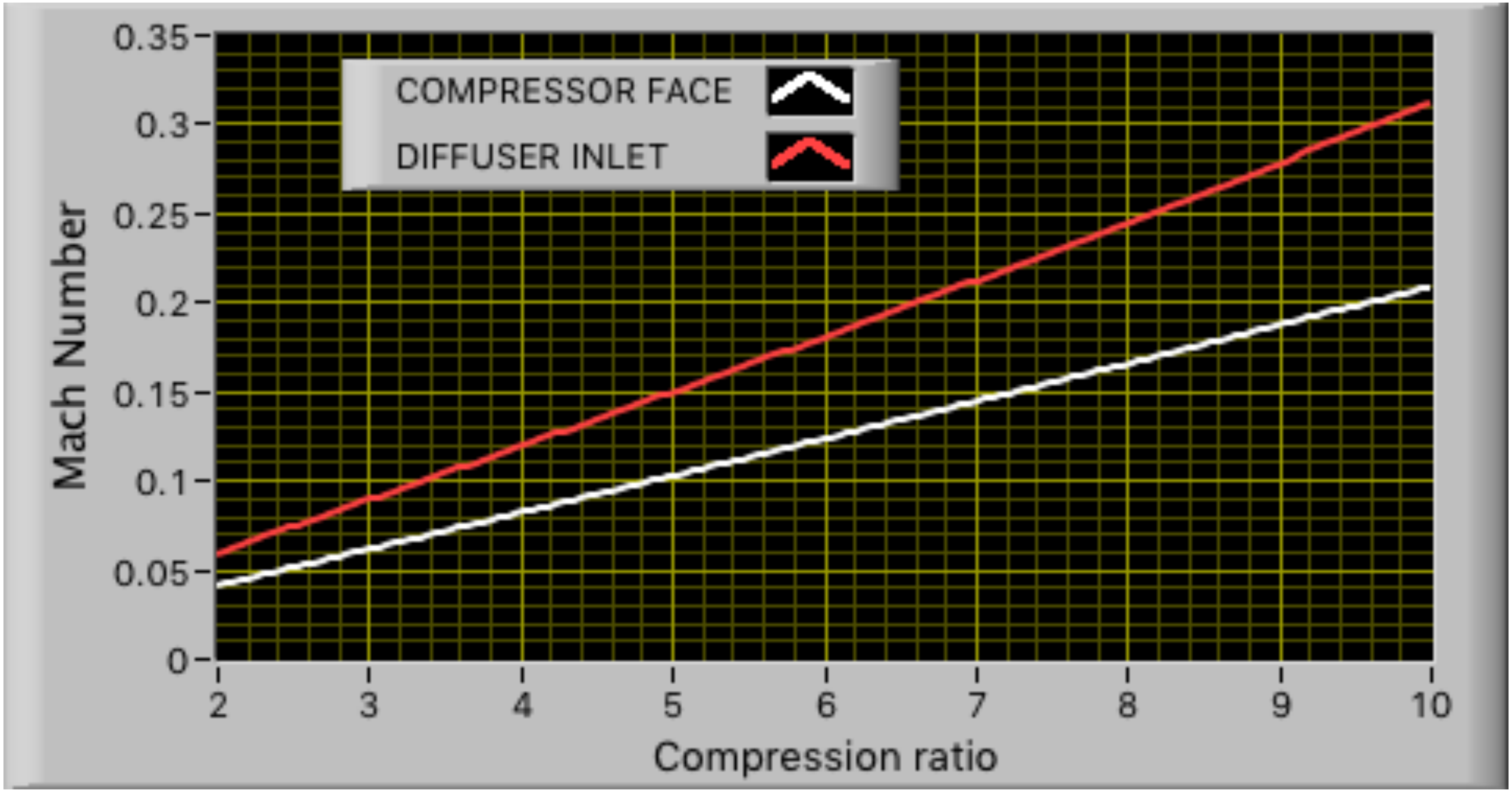
Part 1 Solution (3)

Stall/Surge, Choke Margins



Part 1 Solution (4)

Compressor Face/Diffuser Throat Mach Number

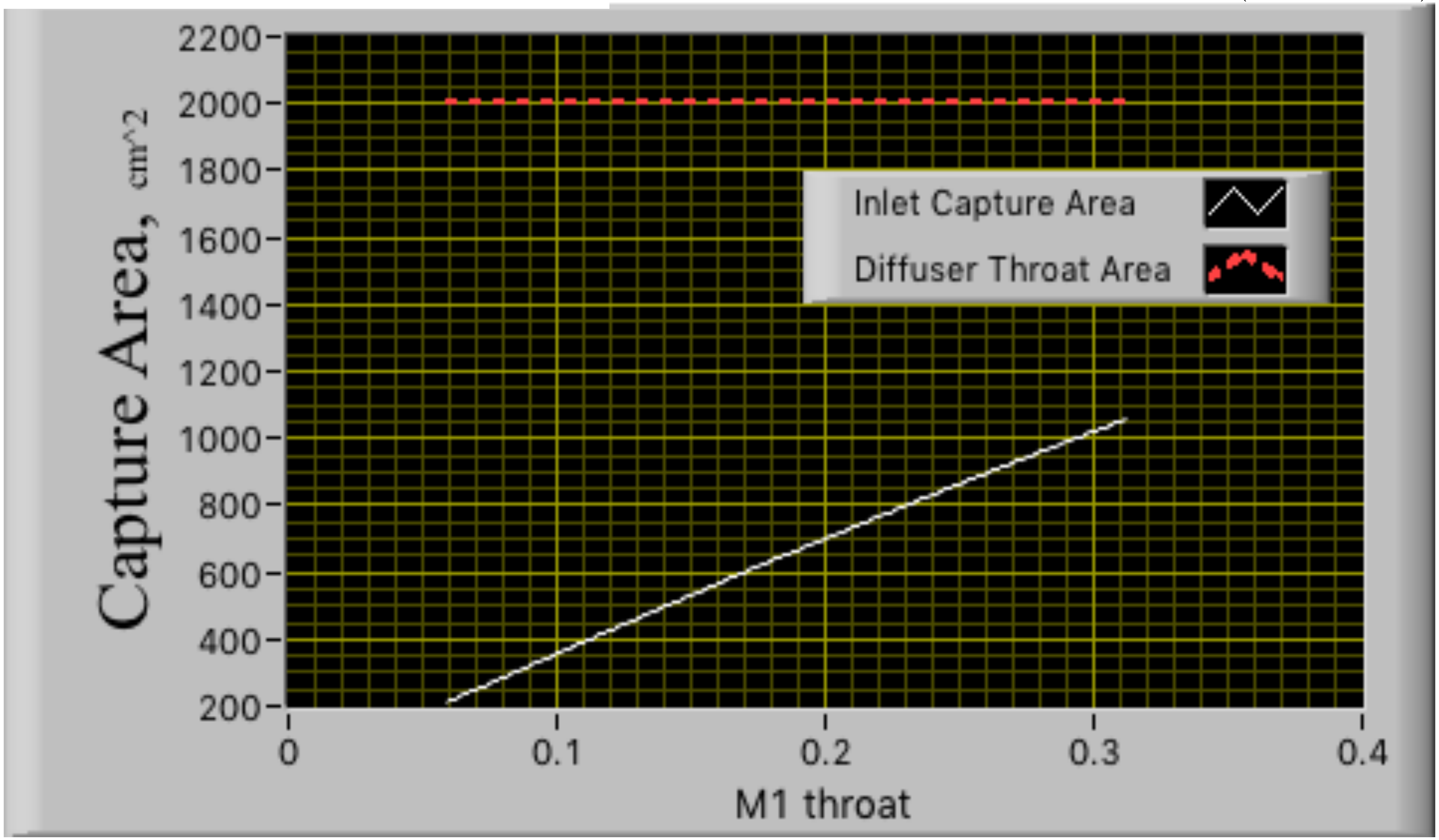


Part 1 Solution (5)

$$\pi_d = 1$$

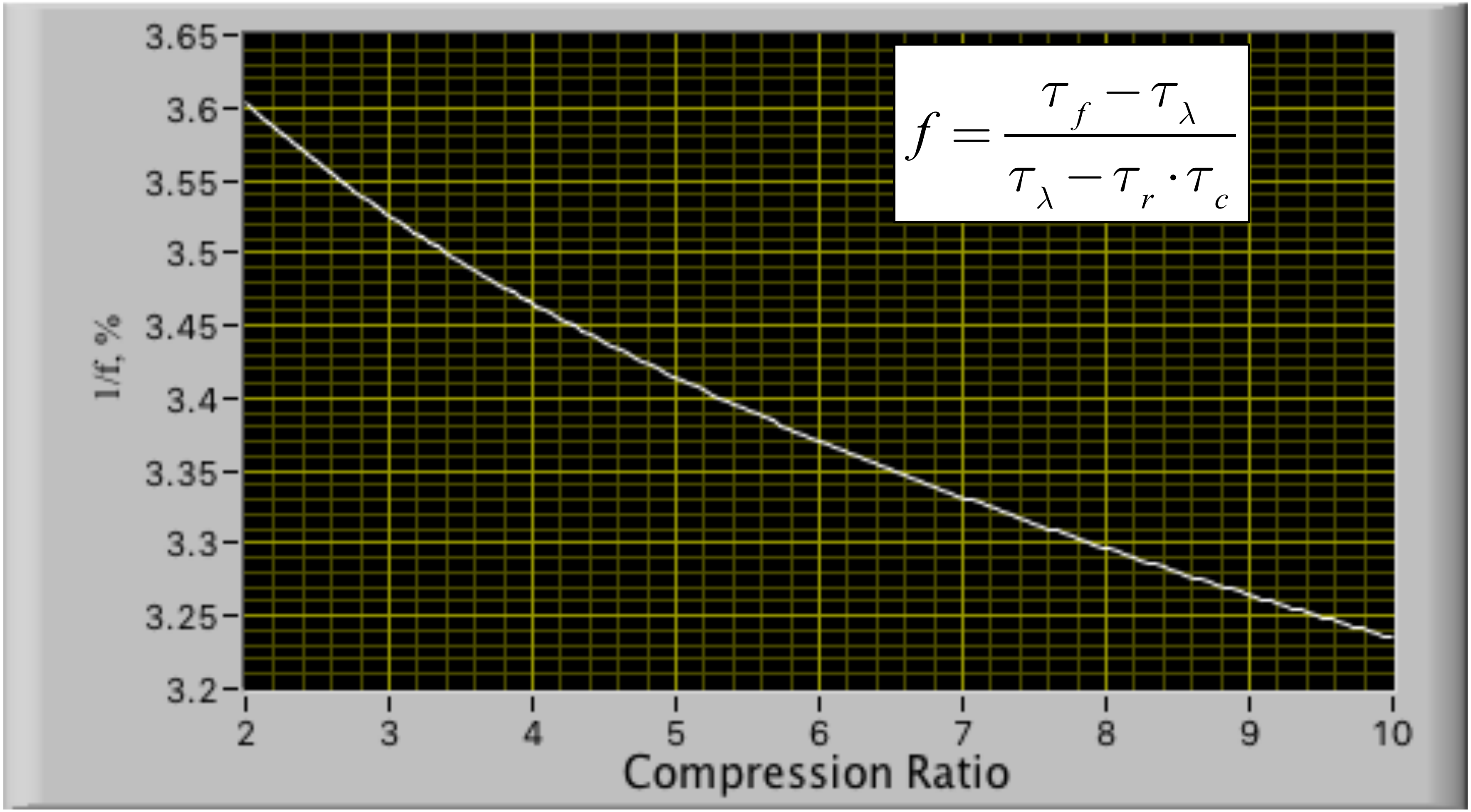
$$\frac{A_\infty}{A_{1throat}} = \frac{A_2}{A_{1throat}} \frac{A_\infty}{A_2} = \frac{A_2}{A_{1throat}} \cdot \pi_d \cdot \frac{\dfrac{\sqrt{\gamma} \cdot M_2}{\left(1 + \dfrac{\gamma - 1}{2} M_2^{\ 2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}{\dfrac{\sqrt{\gamma} \cdot M_\infty}{\left(1 + \dfrac{\gamma - 1}{2} M_\infty^{\ 2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}}$$

Inlet Capture Area



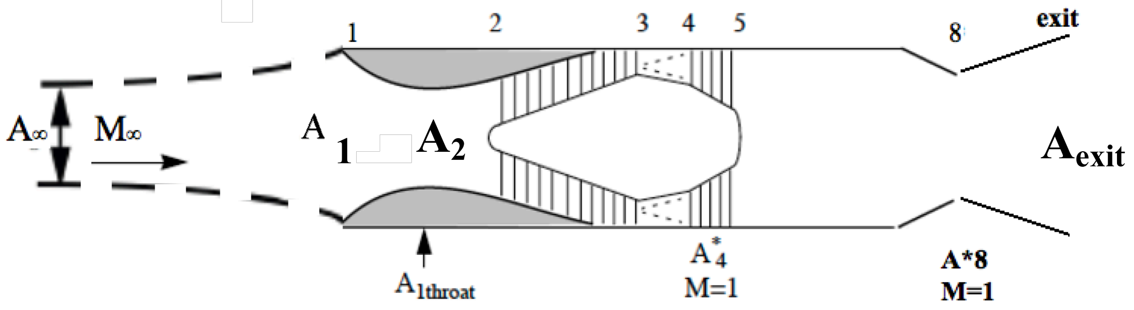
Part 1 Solution (6)

Fuel-to-Air Ratio



$$f = \frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r \cdot \tau_c}$$

Turbojet, Matching Example (2)



Part 2. Optimal Design

$\boldsymbol{Op}$ = 100% N_1 Design Operating Point $\boldsymbol{\pi_c}$

a) *For π_c=**Op** .... CALCULATE to Optimal Nozzle expansion ratio, A_{exit}/A^*_8*
   - *Using Optimal Nozzle expansion ratio (@ π_c=**Op**), Plot Nozzle Exit Pressure Ratio, p_{exit}/p_∞, vs. vs. π_c*

b) *Using Optimal Nozzle expansion ratio (@ π_c=**Op**), Plot Normalized Thrust vs. π_c*
   - *Normalized Momentum Thrust*
   - *Normalized Pressure Thrust*
   - *Normalized Total Thrust*

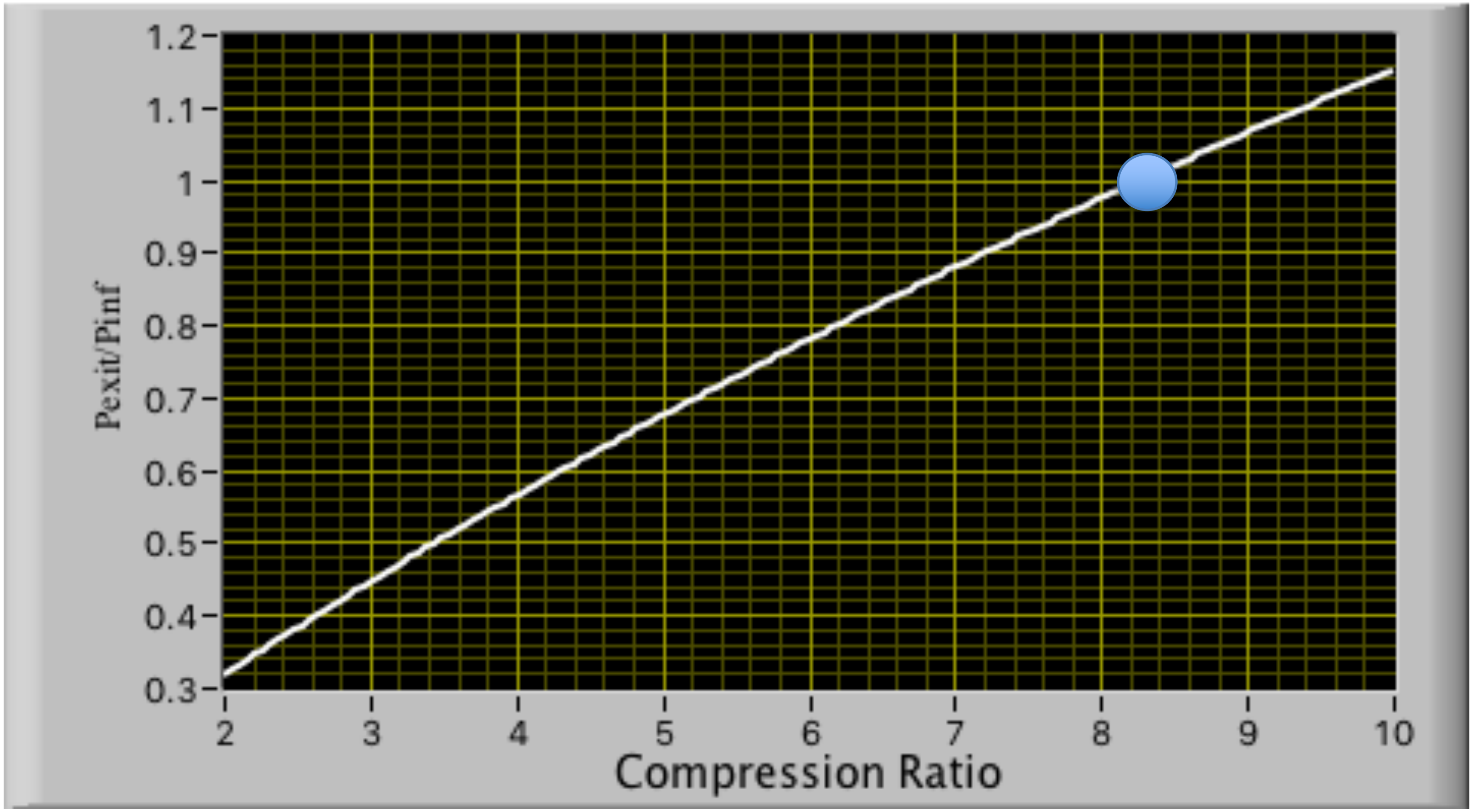
c) *Using Optimal Nozzle expansion ratio (@ π_c=**Op**), Plot*
   - *True Thrust vs. π_c*
   - *Corrected and True Compressor Massflow vs. vs. π_c*
   - *Nozzle Exit Massflow vs. vs. π_c*

Part 2 Solution (1)

A/A*exit Optimal

1.7986

Nozzle Exit Pressure Ratio

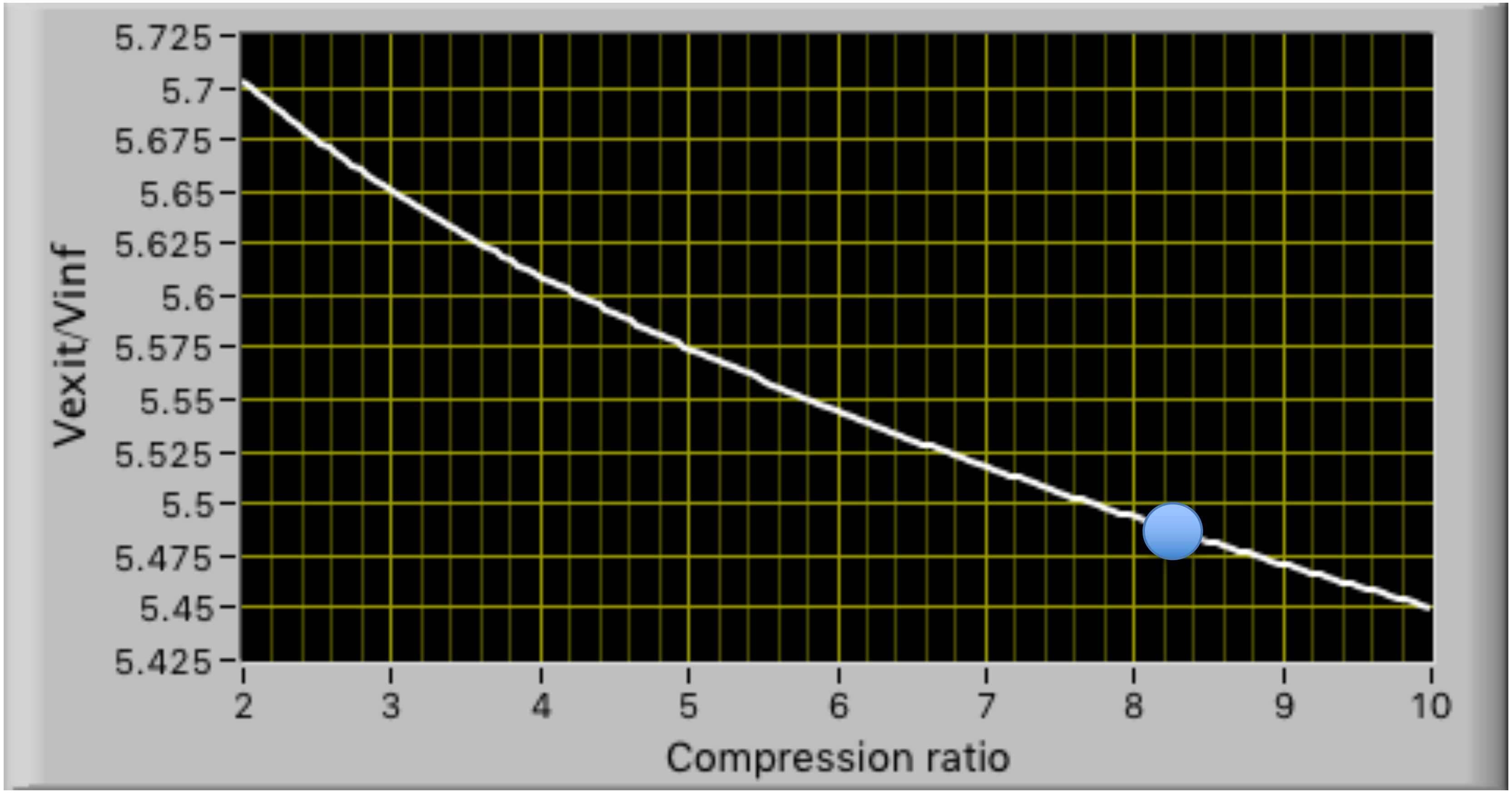


Part 2 Solution (2)

A/A*exit Optimal

1.7986

Engine Velocity Ratio

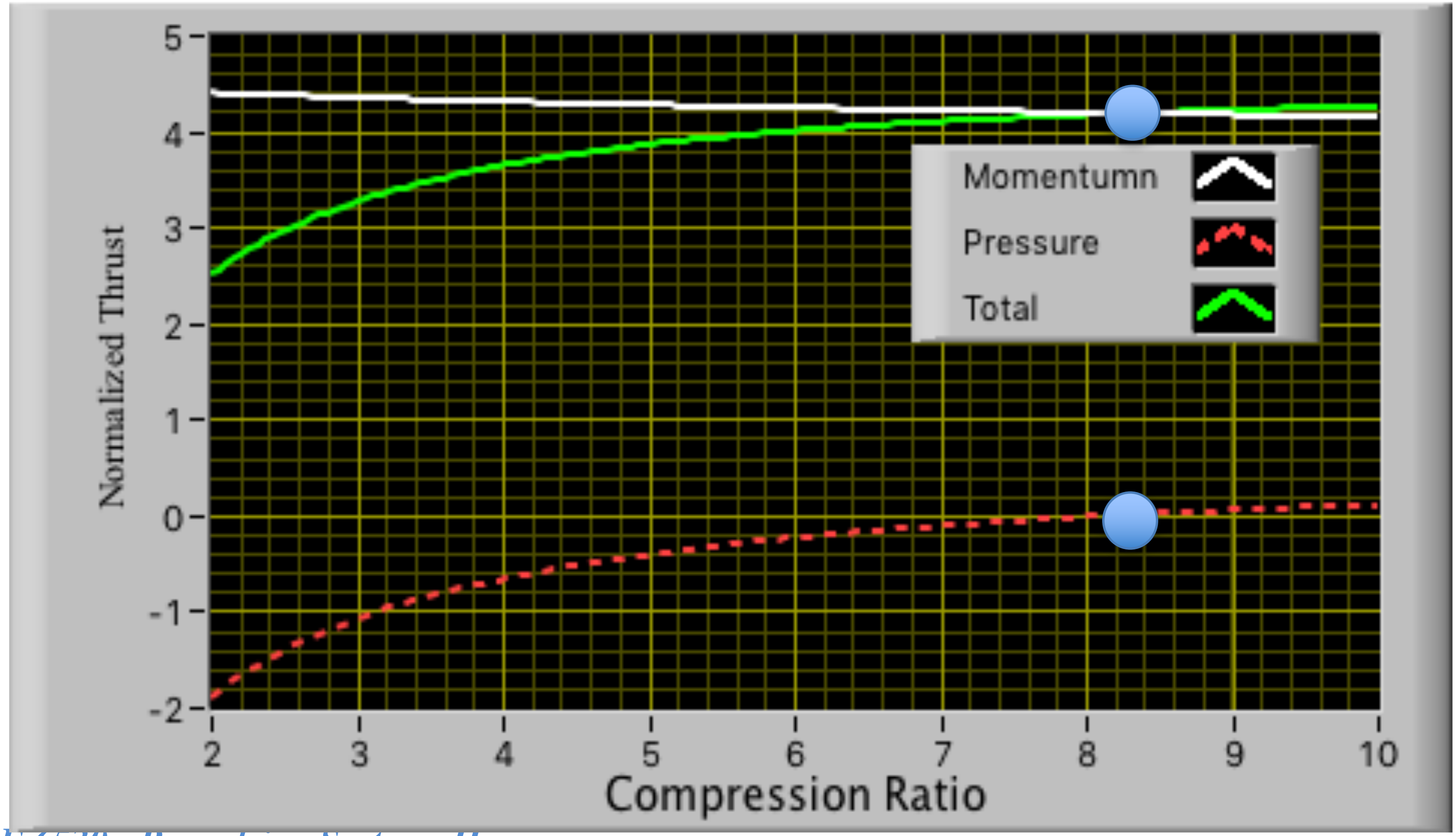


Part 2 Solution (3)

A/A*exit Optimal

1.7986

Normalized Thrust

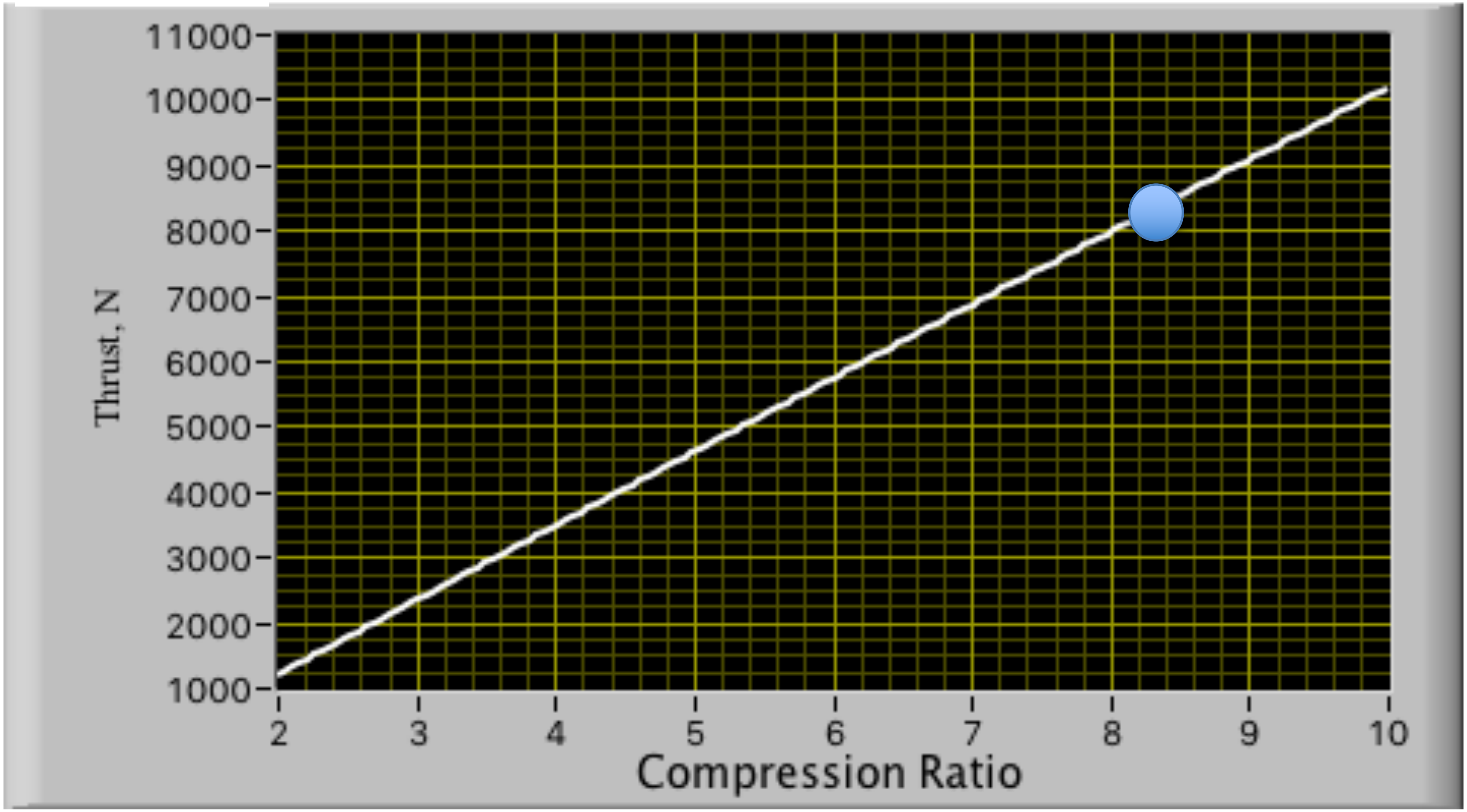


Part 2 Solution (4)

A/A*exit Optimal

1.7986

True Thrust

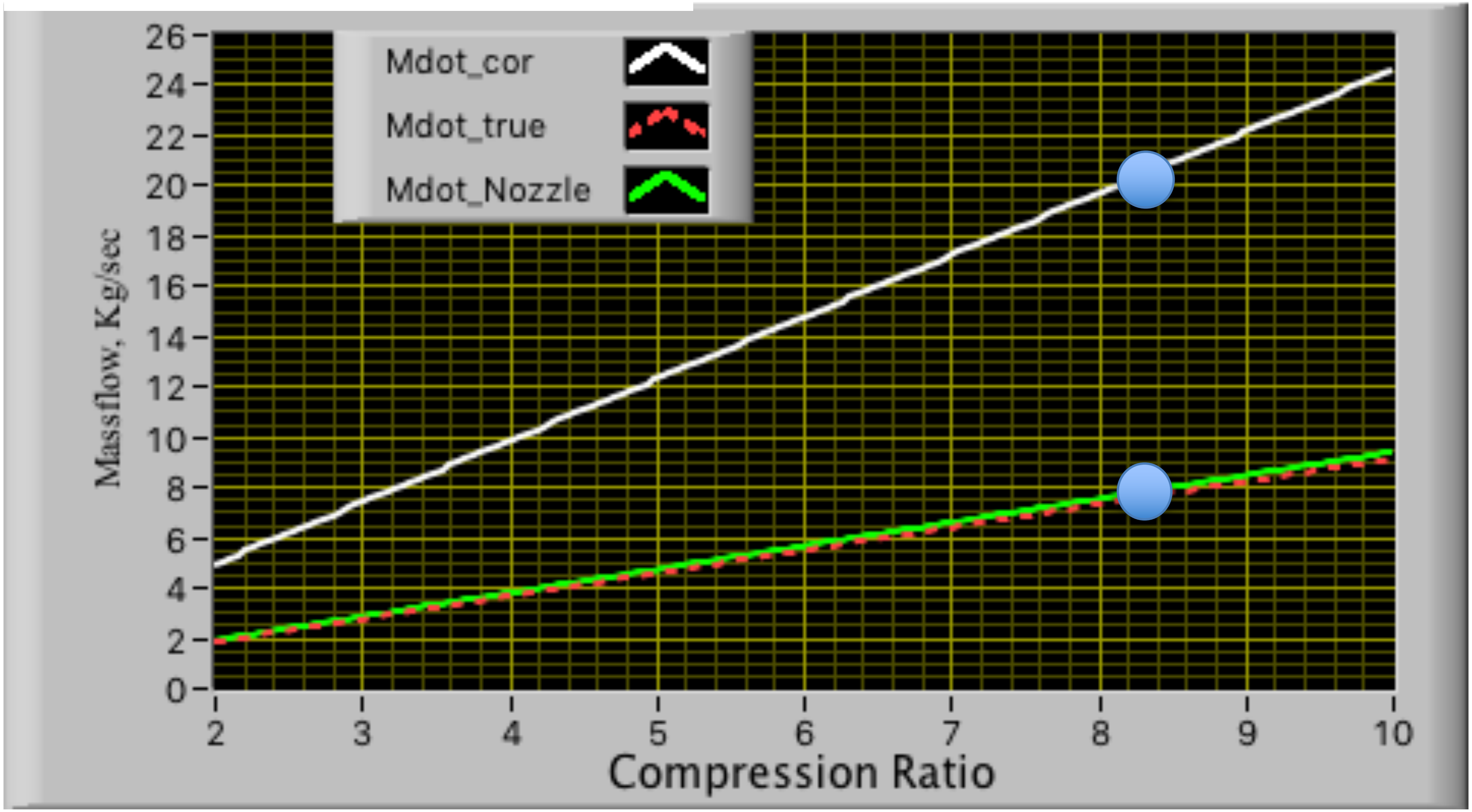


Part 2 Solution (5)

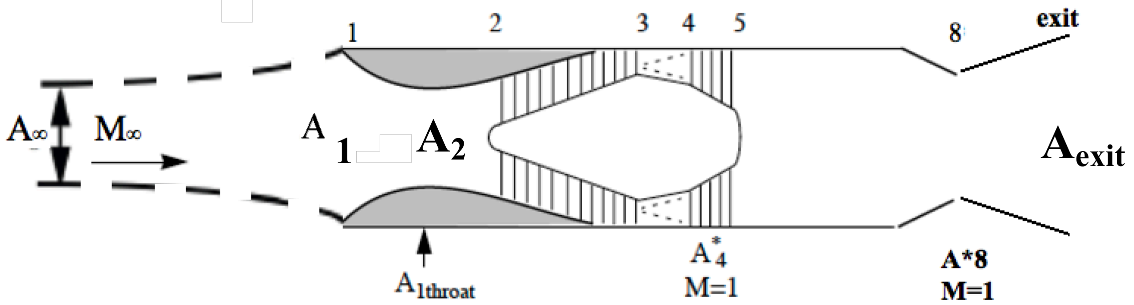
A/A*exit Optimal

1.7986

Massflows



Turbojet, Matching Example (2)



Part 3. Efficiencies

a) *For Optimal Expansion ratio at π_c=**Op** .... Plot*

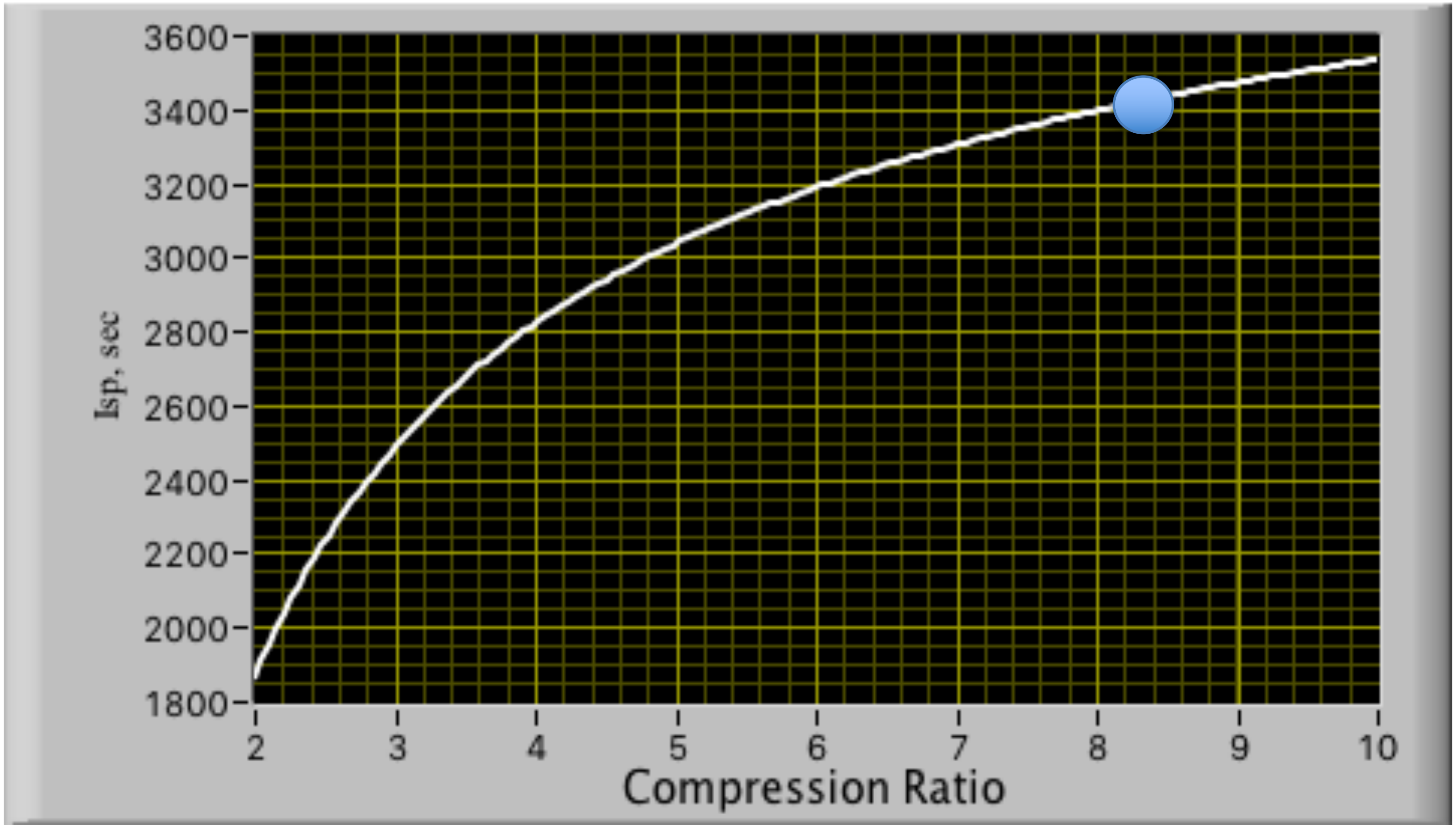
- *Specific Impulse vs. π_c*
- *TSFC vs. π_c*
- *Propulsive (Mechanical) Efficiency vs π_c*
- *Thermal Efficiency vs π_c*
- *Total Efficiency vs π_c*
  - *Include effects of air-to-fuel ratio*
  - *See section4.1, slides 19 & 21*

Part 3 Solution (1)

A/A*exit Optimal

1.7986

True Specific Impulse



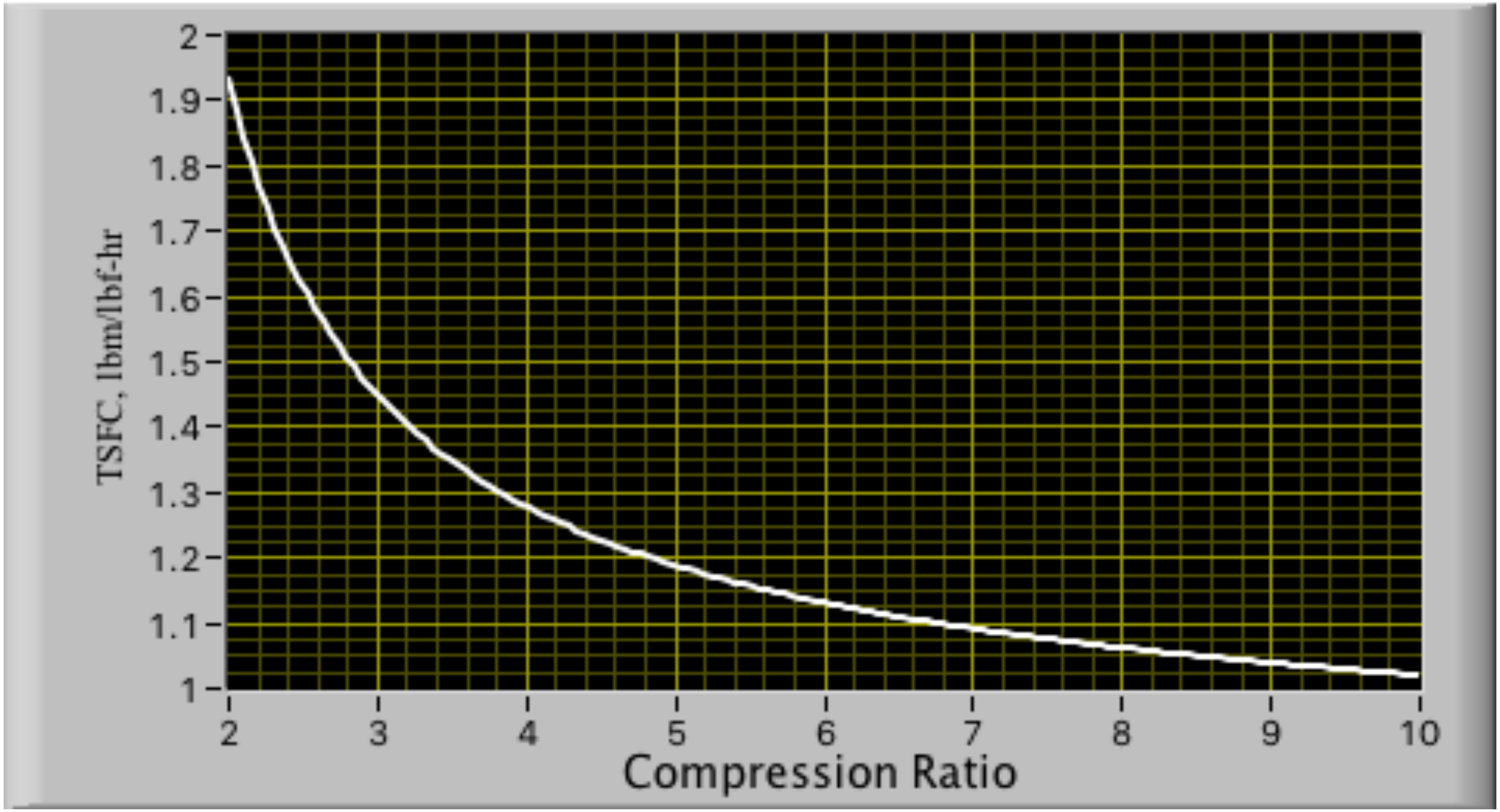
Part 3 Solution (2)

A/A*exit Optimal

1.7986

$$TSFC_{lbm/lbf-hr} = \frac{\dot{m}_{fuel}}{T_{hrust}} = \frac{1}{I_{sp} \cdot g_o} \times 35{,}304.1_{\frac{lbm/lbf\text{-}hr}{kg/N-s}}$$

Thrust Specific Fuel Consumption



Part 3 Solution (3)

A/A*exit Optimal

1.7986

$$\eta_{propulsive} = \frac{\dot{m}_{air} \cdot \left(\left(\frac{f+1}{f} \right) \cdot V_{exit} - V_\infty \right) \cdot V_\infty}{\dot{m}_{air} \cdot \left(\frac{1}{2} \left(\frac{f+1}{f} \right) \cdot V_{exit}^2 - \frac{1}{2} V_\infty^2 \right)} =$$

$$\frac{2 \cdot \left(\left(\frac{f+1}{f} \right) \cdot V_{exit} - V_\infty \right) \cdot \frac{1}{V_\infty^2} \cdot V_\infty}{\left(\left(\frac{f+1}{f} \right) \cdot V_{exit}^2 - V_\infty^2 \right) \cdot \frac{1}{V_\infty^2}} = \frac{2 \cdot \left(\left(\frac{f+1}{f} \right) \cdot \left(\frac{V_{exit}}{V_\infty} \right) - 1 \right)}{\left(\left(\frac{f+1}{f} \right) \cdot \left(\frac{V_{exit}}{V_\infty} \right)^2 - 1 \right)}$$

$$\eta_{thermal} = \frac{\frac{1}{2} \cdot \left(\frac{f+1}{f} \right) \cdot V_{exit}^2 - \frac{1}{2} \cdot V_\infty^2}{\frac{1}{f} h_{fuel}} = \frac{\frac{1}{2} \cdot V_\infty^2 \left[\left(\frac{f+1}{f} \right) \cdot \left(\frac{V_{exit}}{V_\infty} \right)^2 - 1 \right]}{\frac{1}{f} h_{fuel}} =$$

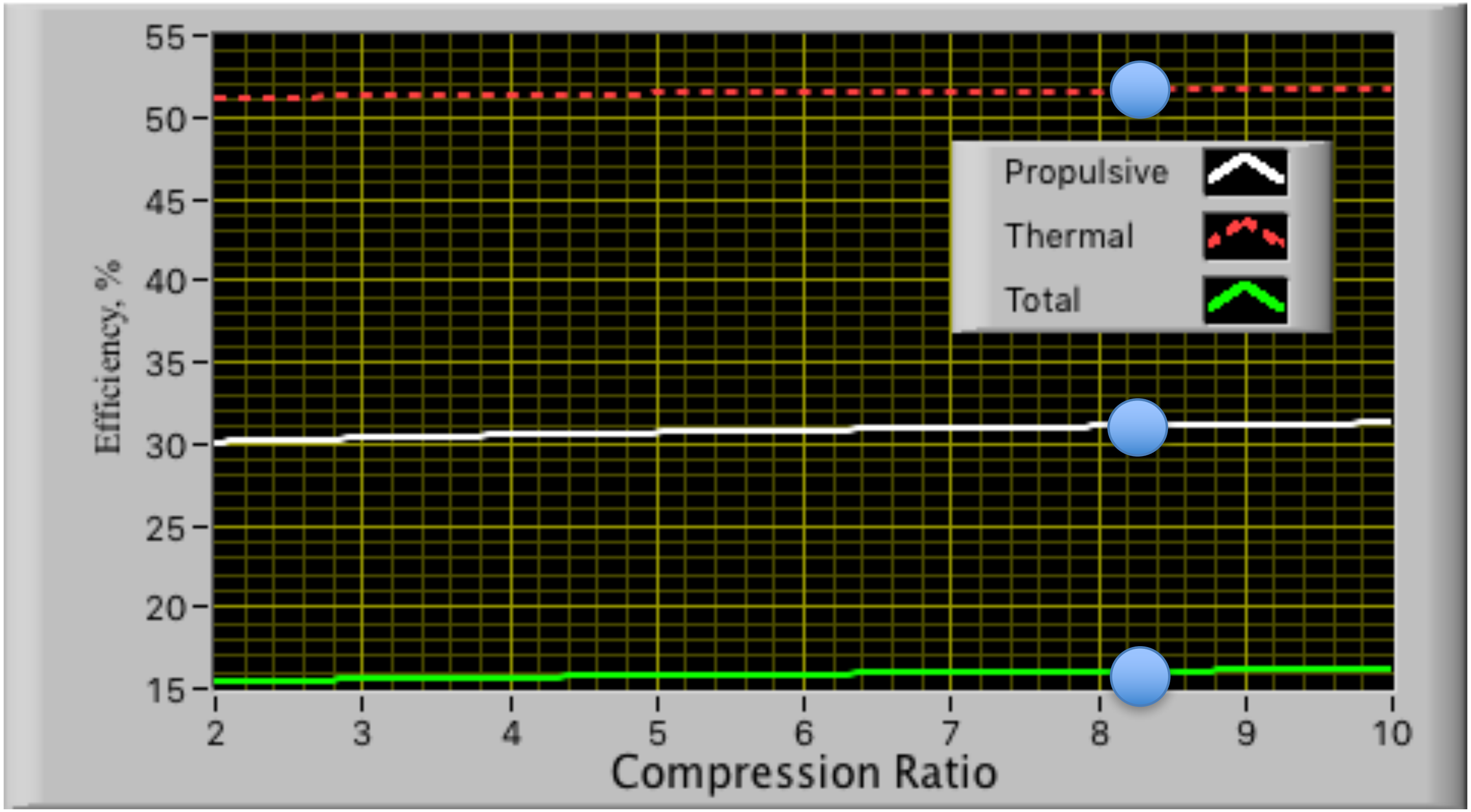
$$\frac{\frac{1}{2} \cdot V_\infty^2 \cdot f \cdot \left[\left(\frac{f+1}{f} \right) \cdot \left(\frac{V_{exit}}{V_\infty} \right)^2 - 1 \right]}{h_{fuel}} = \frac{\frac{1}{2} \cdot V_\infty^2 \cdot f \cdot \left[\left(\frac{f+1}{f} \right) \cdot \left(\frac{V_{exit}}{V_\infty} \right)^2 - 1 \right]}{h_{fuel}}$$

Part 3 Solution (4)

A/A*exit Optimal

1.7986

Efficiencies




Questions??