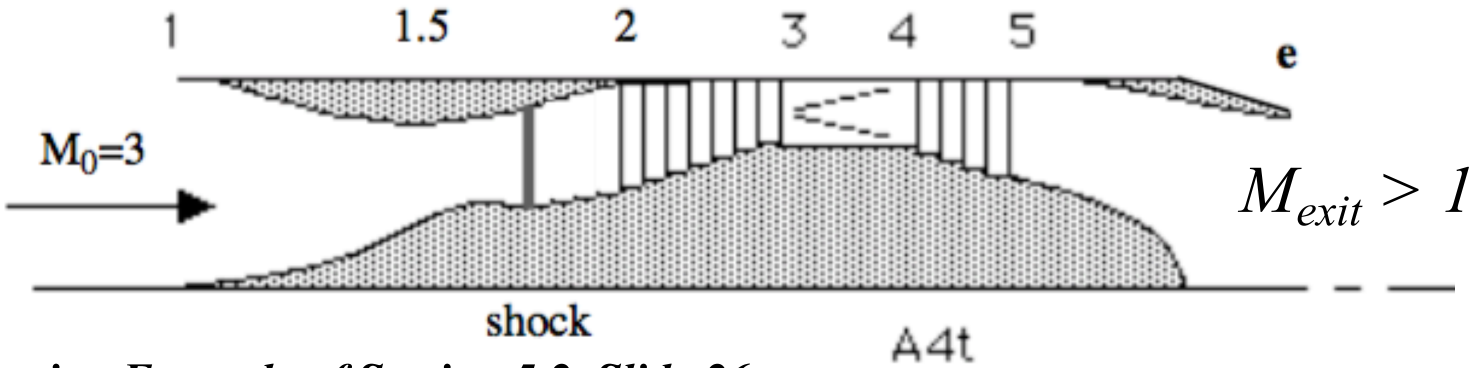
UtahState UNIVERSITY

Mechanical & Aerospace Engineering

Homework 5.2



$M_{exit} > 1$

Reviewing Example of Section 5.2, Slide 26,

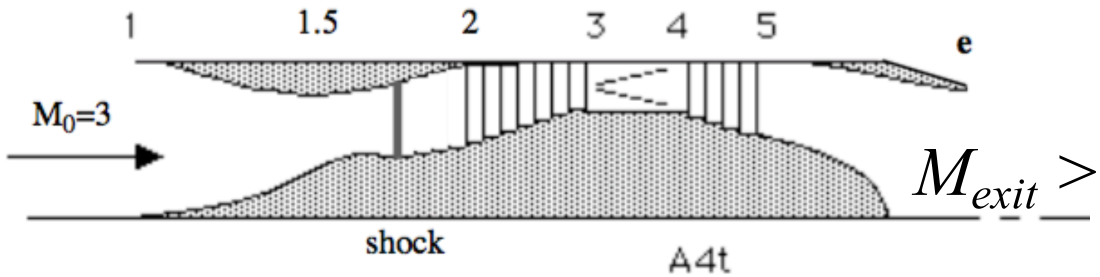
- Recall that this analysis assumes a sonic nozzle

- How would an Expanded (Supersonic) Nozzle Buy in Terms of Performance

- Find the Optimal Expansion Ratio and Exit Mach Number

- By What ratio does this Optimal expansion ratio Increase the thrust and specific Impulse of the Engine

Homework 5.2 (2)



$M_{exit} > 1$

- Hints:

$$\frac{F_{thrust}}{p_\infty \cdot A_\infty} = \frac{F_{thrust}}{p_\infty \cdot A_\infty} = \gamma \cdot M_\infty^2 \cdot \left(\frac{V_{exit}}{V_\infty} - 1\right) + \frac{A_{exit}}{A_\infty} \cdot \left(\frac{p_{exit}}{p_\infty} - 1\right) = \gamma \cdot M_\infty^2 \cdot \left(\frac{M_{exit}}{M_\infty}\sqrt{\frac{T_{exit}}{T_\infty}} - 1\right) + \frac{A_{exit}}{A_\infty} \cdot \left(\frac{p_{exit}}{p_\infty} - 1\right)$$

$$\frac{T_{exit}}{T_\infty} = \frac{T_{0_{exit}}}{T_\infty}\frac{T_{exit}}{T_{0_{exit}}} \qquad \frac{p_{exit}}{p_\infty} = \frac{P_{0_{exit}}}{p_\infty}\frac{p_{exit}}{P_{0_{exit}}}$$

$$\frac{T_{exit}}{T_{0_{exit}}} = \frac{1}{\left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)}$$

$$\frac{p_{exit}}{P_{0_{exit}}} = \frac{1}{\left(1 + \frac{\gamma - 1}{2} M_{exit}^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{A_{exit}}{A^*_{throat}} = \frac{1}{M_{exit}} \cdot \left[\left(\frac{2}{\gamma - 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)\right]^{\frac{\gamma+1}{2\cdot(\gamma-1)}}$$

- Making these substitutions the normalized thrust can be written in terms of exit Mach number

- Graph Normalized Thrust and Exit expansion ratio as a function of exit Mach Number

- Verify that $p_{exit}/p_\infty = 1$ at the optimal performance condition?