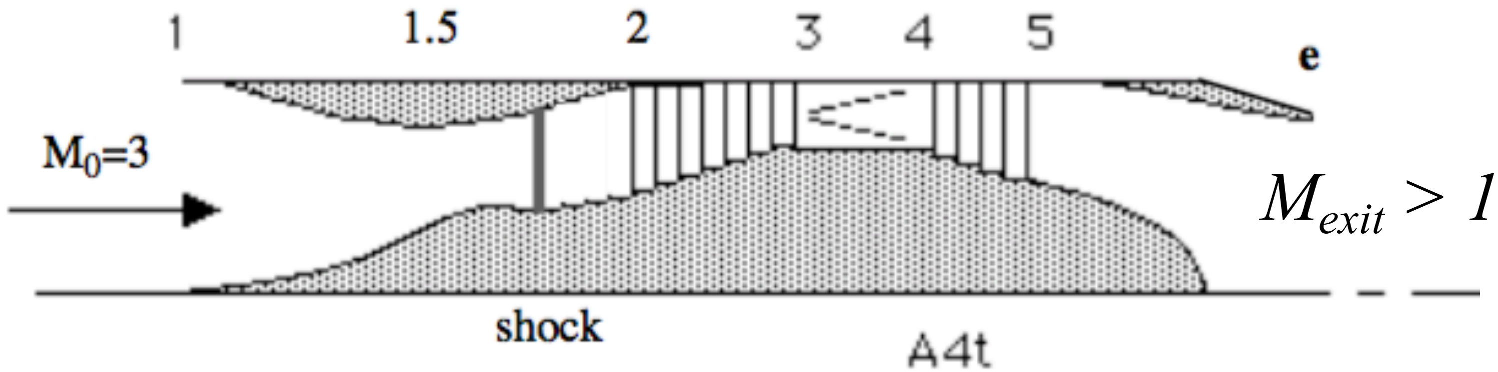
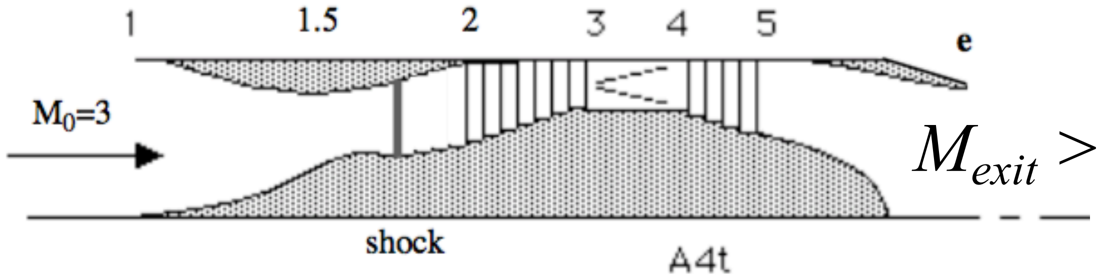
Homework 5.2



- Recall that this analysis assumes a sonic nozzle
- How would an Expanded (Supersonic) Nozzle Buy in Terms of Performance
- Find the Optimal Expansion Ratio and Exit Mach Number
- By What ratio does this Optimal expansion ratio Increase the thrust and specific Impulse of the Engine

Homework 5.2 (2)



$M_{exit} > 1$

- Hints:

$$\frac{F_{thrust}}{p_\infty \cdot A_\infty} = \frac{F_{thrust}}{p_\infty \cdot A_\infty} = \gamma \cdot M_\infty^2 \cdot \left(\frac{V_{exit}}{V_\infty} - 1\right) + \frac{A_{exit}}{A_\infty} \cdot \left(\frac{p_{exit}}{p_\infty} - 1\right) = \gamma \cdot M_\infty^2 \cdot \left(\frac{M_{exit}}{M_\infty}\sqrt{\frac{T_{exit}}{T_\infty}} - 1\right) + \frac{A_{exit}}{A_\infty} \cdot \left(\frac{p_{exit}}{p_\infty} - 1\right)$$

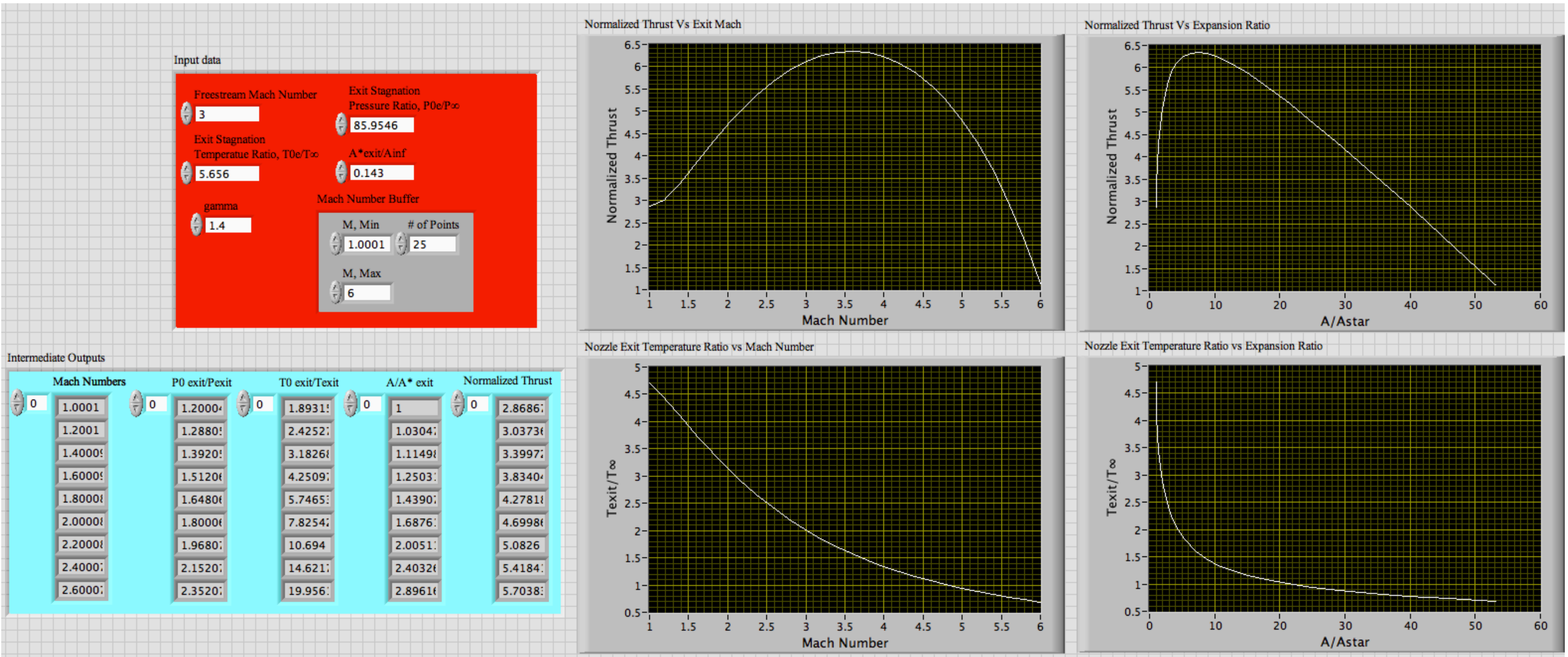
$$\frac{T_{exit}}{T_\infty} = \frac{T_{0_{exit}}}{T_\infty}\frac{T_{exit}}{T_{0_{exit}}} \qquad \frac{p_{exit}}{p_\infty} = \frac{P_{0_{exit}}}{p_\infty}\frac{p_{exit}}{P_{0_{exit}}}$$

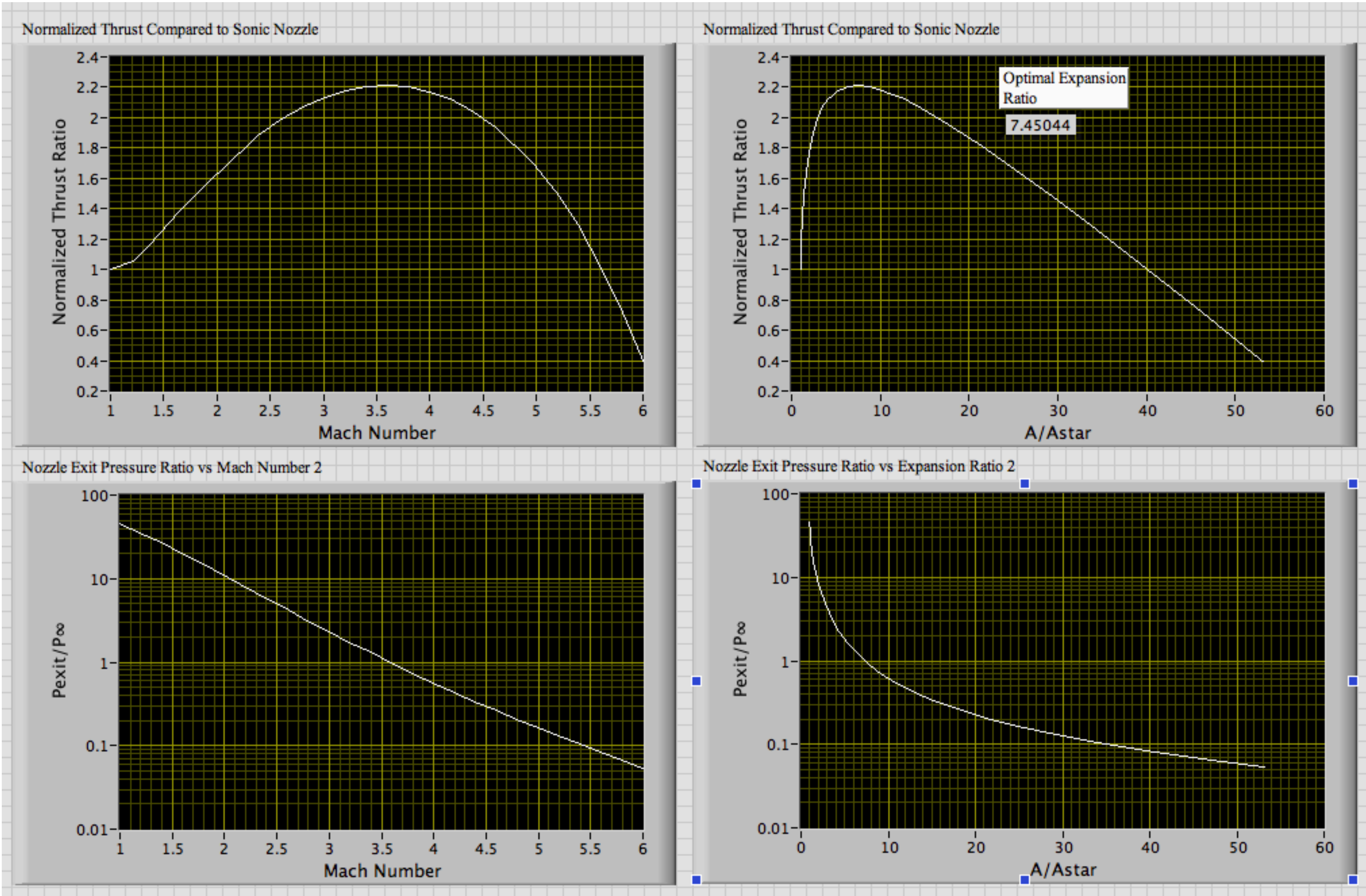
$$\frac{T_{exit}}{T_{0_{exit}}} = \frac{1}{\left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)}$$

$$\frac{T_{exit}}{T_{0_{exit}}} = \frac{1}{\left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)^{\frac{\gamma}{\gamma - 1}}}$$

$$\frac{A_{exit}}{A^*_{throat}} = \frac{1}{M_{exit}} \cdot \left[\left(\frac{2}{\gamma - 1}\right) \cdot \left(1 + \frac{\gamma - 1}{2} M_{exit}^{\ 2}\right)\right]^{\frac{\gamma + 1}{2 \cdot (\gamma - 1)}}$$

- Making these substitutions the normalized thrust can be written in terms of exit Mach number
- Graph Normalized Thrust and Exit expansion ratio as a function of exit Mach Number
- Verify that $p_{exit}/p_\infty = 1$ at the optimal performance condition?


Input data
Freestream Mach Number
3
Exit Stagnation Pressure Ratio, P0e/P∞
85.9546
Exit Stagnation Temperatue Ratio, T0e/T∞
5.656
A*exit/Ainf
0.143
gamma
1.4
Mach Number Buffer
M, Min
1.0001
# of Points
25
M, Max
6
Intermediate Outputs
Mach Numbers
1.0001
1.2001
1.4000
1.6000
1.8000
2.0000
2.2000
2.4000
2.6000
P0 exit/Pexit
1.2000
1.2880
1.3920
1.5120
1.6480
1.8000
1.9680
2.1520
2.3520
T0 exit/Texit
1.8931
2.4252
3.1826
4.2509
5.7465
7.8254
10.694
14.621
19.956
A/A* exit
1
1.0304
1.1149
1.2503
1.4390
1.6876
2.0051
2.4032
2.8961
Normalized Thrust
2.8686
3.0373
3.3997
3.8340
4.2781
4.6998
5.0826
5.4184
5.7038
Normalized Thrust Vs Exit Mach
Normalized Thrust
Mach Number
Normalized Thrust Vs Expansion Ratio
Normalized Thrust
A/Astar
Nozzle Exit Temperature Ratio vs Mach Number
Texit/T∞
Mach Number
Nozzle Exit Temperature Ratio vs Expansion Ratio
Texit/T∞
A/Astar


Normalized Thrust Compared to Sonic Nozzle
Normalized Thrust Ratio
Mach Number
Normalized Thrust Compared to Sonic Nozzle
Optimal Expansion Ratio
7.45044
Normalized Thrust Ratio
A/Astar
Nozzle Exit Pressure Ratio vs Mach Number 2
Pexit/P∞
Mach Number
Nozzle Exit Pressure Ratio vs Expansion Ratio 2
Pexit/P∞
A/Astar