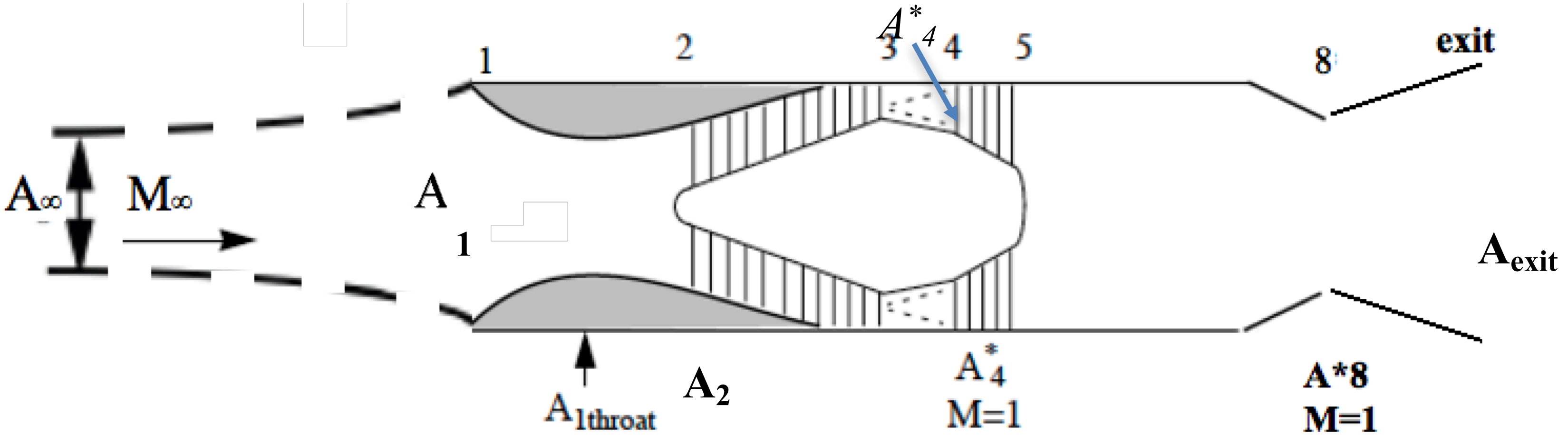
Homework 5.4



- Engine operates at a free stream Mach number, $M_\infty = 0.8$.
- Cruise Altitude is in the stratosphere, 11 km so T_∞ = 216.65 K.
- The design turbine inlet temperature, $T_{04} = 1944\ K$
- The design compressor ratio, $\pi_c = 20$.
- Relevant area ratios are $A_2 / A^*_4 = 10$ and $A_2/A_{1throat} = 1.2$.
- Inlet throat area $A_{1Throat} = 20\ cm^2$
- *Assume the compressor, burner and turbine all operate ideally.*
- *Nozzle is of a simple converging type with choked throat,* $A^*_8 = A_{exit}$
- *Stagnation pressure losses due to wall friction in the inlet and nozzle are negligible.*

$f \underset{-}{\sim} 50$

- ***Calculate Associated Fuel Enthalpy***

→ *CALCULATE*

→ *a) Correct Compressor Massflow and* M_2 *at compressor face*

→ *b) Normalized exit pressure thrust, momentum thrust, and total thrust*

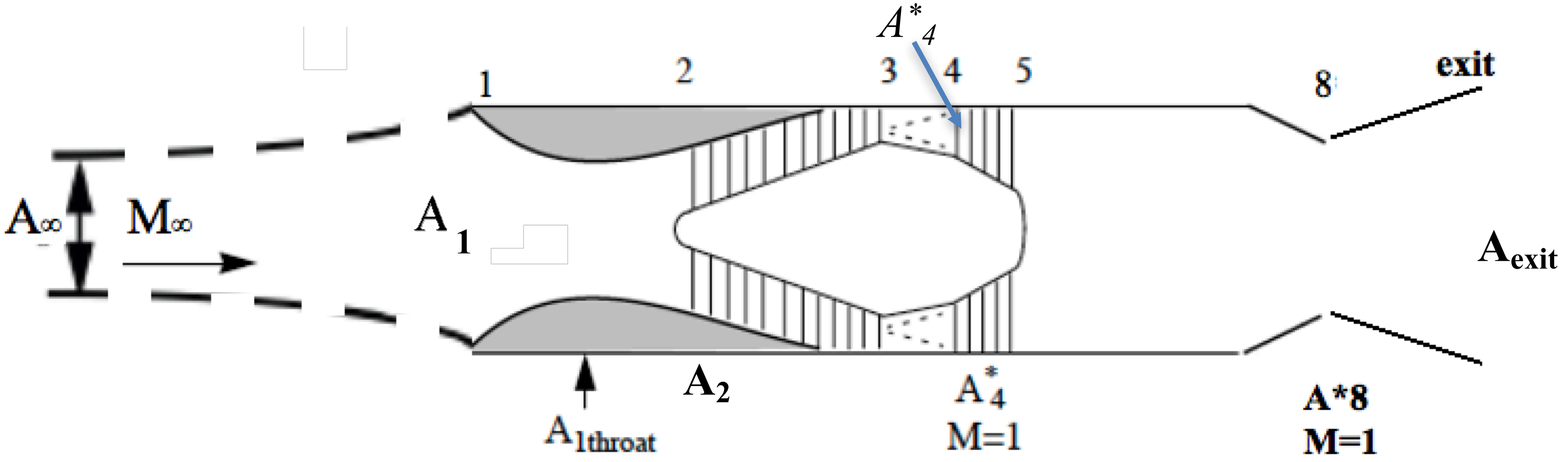
→ *c) Velocity ratio across Engine* V_{exit}/V_∞

→ *d) Mach number at diffuser throat,* $M_{1throat}$

→ *e) Inlet capture area*

→ *f) Total Thrust, Isp, TSFC*

Homework 5.4 (2)



- Now allow an expandable *Nozzle where,* $A_{exit} \geq A^*_8$

→ *CALCULATE*

→ *a) Optimal expansion ratio for nozzle* A_{exit}/A^*_8

→ *b) c) Velocity ratio across Engine* V_{exit}/V_∞

→ *c) thrust, Isp, TSFC of optimal nozzle,*

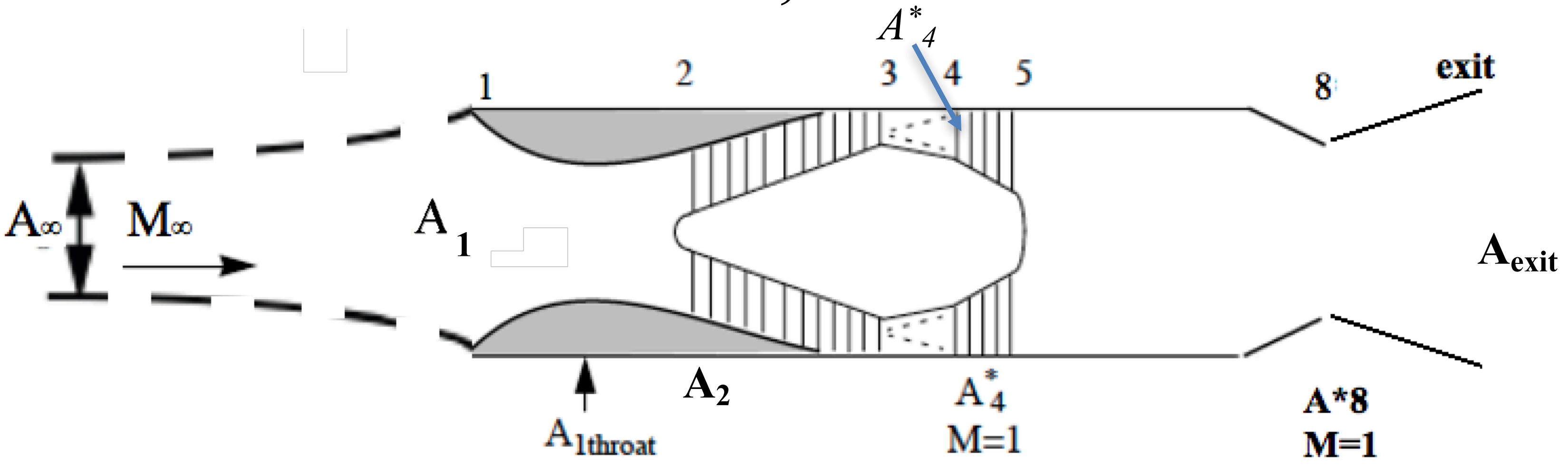
→ *d) Assuming the same combustor temperature and inlet throat area as previous page*

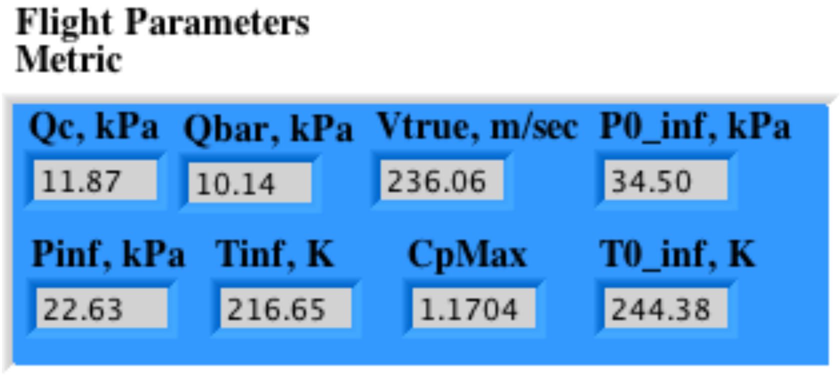
  → *At what compressor demand* π_c *does the inlet throat choke (@* $A1_{throat}$*)*

  → *Plot the Compressor operating line* → π_c *vs corrected massflow,* $f(M_2)$ *for* $1 \leq \pi_c < \pi_{c\ @\ choke}$

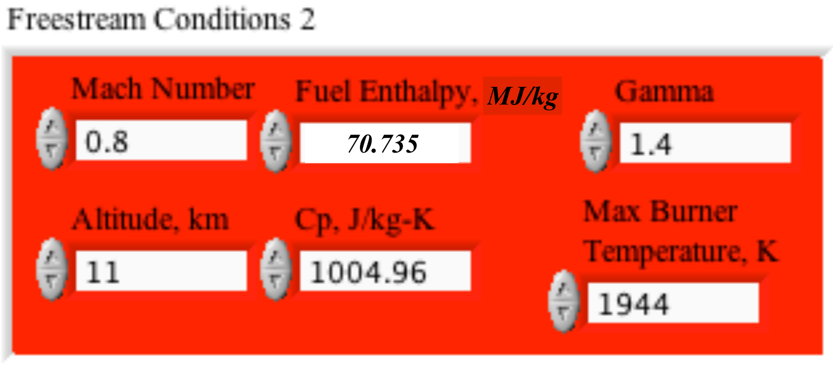
  → *Plot the capture area* A_∞ *vs corrected massflow,* $f(M_2)$ *for* $1 \leq \pi_c < \pi_{c\ @\ choke}$

Homework 5.4, SOLUTION

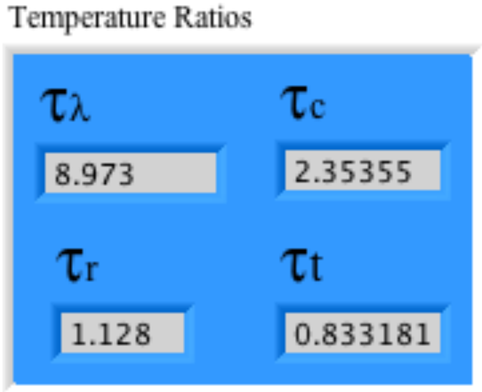



Flight Parameters
Metric

Qc, kPa 11.87
Qbar, kPa 10.14
Vtrue, m/sec 236.06
P0_inf, kPa 34.50
Pinf, kPa 22.63
Tinf, K 216.65
CpMax 1.1704
T0_inf, K 244.38

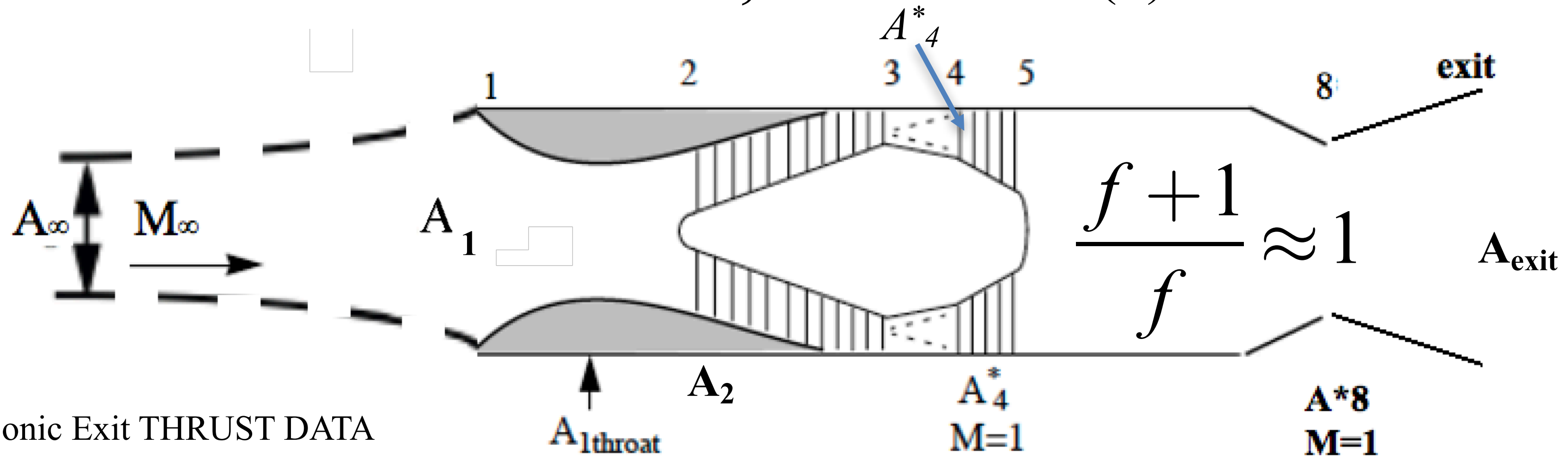

Freestream Conditions 2

Mach Number 0.8
Fuel Enthalpy, *MJ/kg* *70.735*
Gamma 1.4
Altitude, km 11
Cp, J/kg-K 1004.96
Max Burner Temperature, K 1944


Temperature Ratios

τ_λ 8.973
τ_c 2.35355
τ_r 1.128
τ_t 0.833181

Homework 5.4, SOLUTION (2)



$$\frac{f+1}{f} \approx 1$$

Sonic Exit THRUST DATA

→ *CALCULATE*

→ *a) Corrected Compressor massflow and M_2 at compressor face*

mdot_COR g/SEC
402.505

FM2
0.695209

M2
0.454202

$$f \simeq 50$$

→ *b) Normalized exit pressure thrust, momentum thrust, and total thrust*

Norm Pressure Thrust
1.79721

Normalized Momentum Thrust
1.95546

Norm Total Thrust
3.75267

Norm Isp
167.53

True Isp, sec
5040.76

→ *c) Velocity ratio across Engine V_{exit}/V_∞*

Ve/V∞
3.12003

→ *d) Mach number at diffuser throat, $M_{1throat}$*

M1 t
0.591318

→ *e) Inlet capture area*

A∞, cm^2
17.3229

$Fuel-to-Airflow\_Matching$

$$\frac{1}{f}=\frac{\tau_\lambda-\tau_r\cdot\tau_c}{\tau_f-\tau_\lambda}$$ **_Calculate Fuel Enthalpy_**

$Solve\ for\ \tau_f$

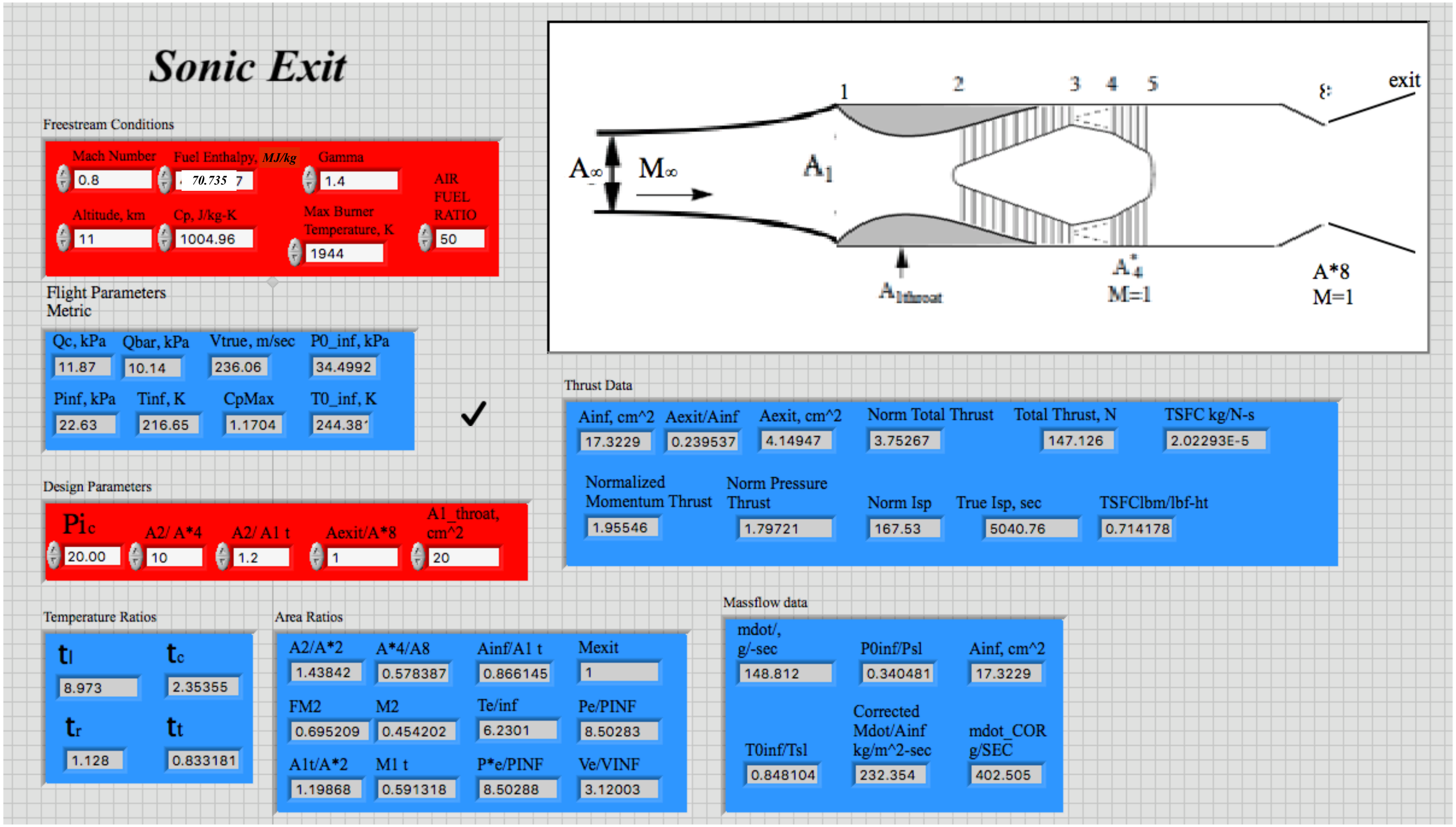
$$\rightarrow\frac{h_f}{c_p\cdot T_\infty}=\tau_f=\tau_\lambda+f\cdot\left(\tau_\lambda-\tau_r\cdot\tau_c\right)$$

$Solve\ for\ h_f$

$$\rightarrow h_f=c_p\cdot T_\infty\cdot\left[\tau_\lambda+f\cdot\left(\tau_\lambda-\tau_r\cdot\tau_c\right)\right]=$$

$$1004.96_{J/kg-K}\cdot216.65_K\cdot\left[8.973+50\cdot\left(9.973-1.128\cdot2.25355\right)\right]=$$

$70.735_{MJ/kg}$ **_Wow! hot stuff!_**



Sonic Exit

Freestream Conditions

Mach Number: 0.8
Fuel Enthalpy, *MJ/kg*: 70.735
Gamma: 1.4
Altitude, km: 11
Cp, J/kg-K: 1004.96
Max Burner Temperature, K: 1944
AIR FUEL RATIO: 50

Flight Parameters
Metric

Qc, kPa: 11.87
Qbar, kPa: 10.14
Vtrue, m/sec: 236.06
P0_inf, kPa: 34.4992
Pinf, kPa: 22.63
Tinf, K: 216.65
CpMax: 1.1704
T0_inf, K: 244.381

✓

Design Parameters

Pic: 20.00
A2/ A*4: 10
A2/ A1 t: 1.2
Aexit/A*8: 1
A1_throat, cm^2: 20

Temperature Ratios

tl: 8.973
tc: 2.35355
tr: 1.128
tt: 0.833181

Area Ratios

A2/A*2: 1.43842
A*4/A8: 0.578387
Ainf/A1 t: 0.866145
Mexit: 1
FM2: 0.695209
M2: 0.454202
Te/inf: 6.2301
Pe/PINF: 8.50283
A1t/A*2: 1.19868
M1 t: 0.591318
P*e/PINF: 8.50288
Ve/VINF: 3.12003

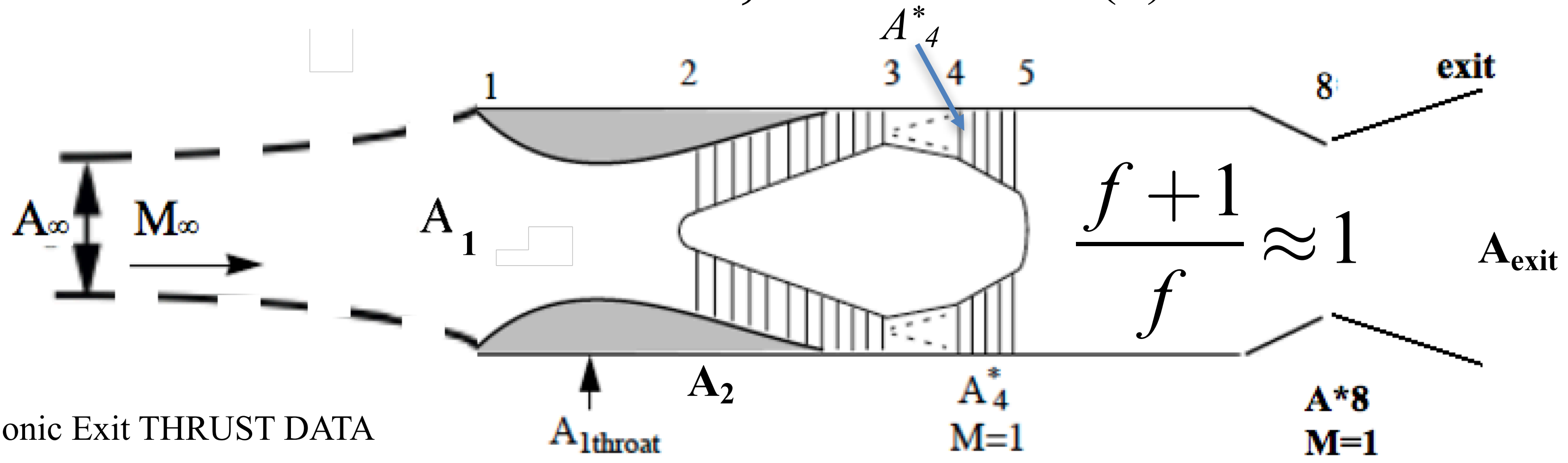
Thrust Data

Ainf, cm^2: 17.3229
Aexit/Ainf: 0.239537
Aexit, cm^2: 4.14947
Norm Total Thrust: 3.75267
Total Thrust, N: 147.126
TSFC kg/N-s: 2.02293E-5
Normalized Momentum Thrust: 1.95546
Norm Pressure Thrust: 1.79721
Norm Isp: 167.53
True Isp, sec: 5040.76
TSFClbm/lbf-ht: 0.714178

Massflow data

mdot/, g/-sec: 148.812
P0inf/Psl: 0.340481
Ainf, cm^2: 17.3229
T0inf/Tsl: 0.848104
Corrected Mdot/Ainf kg/m^2-sec: 232.354
mdot_COR g/SEC: 402.505

Homework 5.4, SOLUTION (2)



$$\frac{f+1}{f} \approx 1$$

Sonic Exit THRUST DATA

→ *CALCULATE*

→ *a) Corrected Compressor massflow and M_2 at compressor face*

mdot_COR g/SEC	FM2	M2
410.547	0.709099	0.466242

$$f \simeq \infty$$

→ *b) Normalized exit pressure thrust, momentum thrust, and total thrust*

Norm Pressure Thrust	Normalized Momentum Thrust	Norm Total Thrust
1.75516	1.89401	3.64917

→ *c) Velocity ratio across Engine V_{exit}/V_∞*

Ve/V∞

3.11379

→ *d) Mach number at diffuser throat, $M_{1throat}$*

M1 t

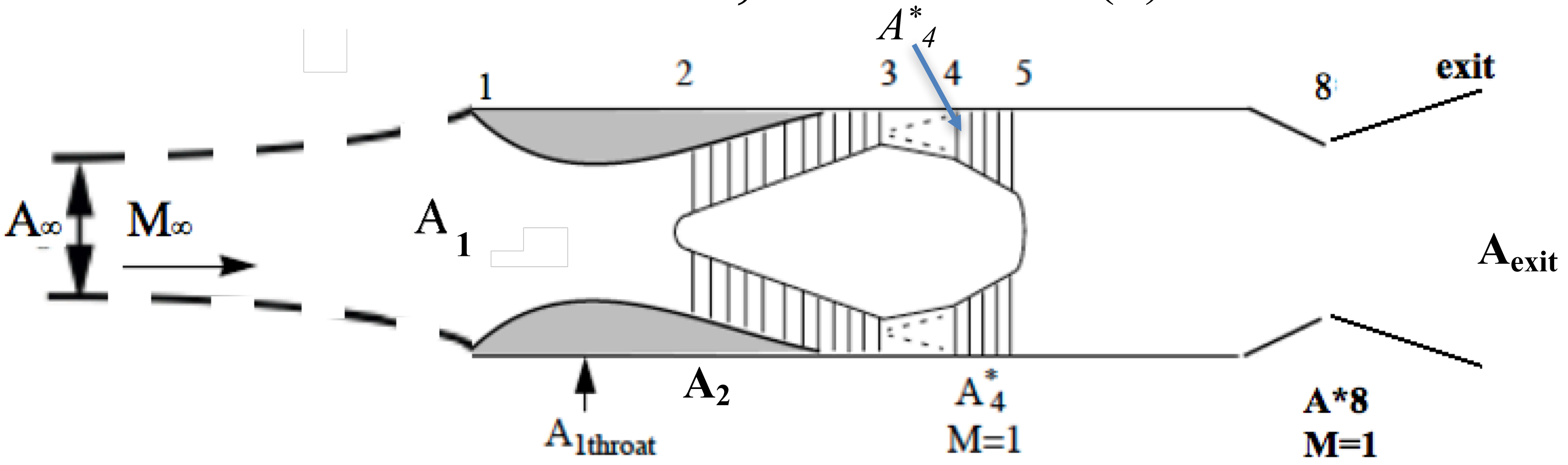
0.611294

→ *e) Inlet capture area*

A∞, cm^2

17.669

Homework 5.4, SOLUTION (3)



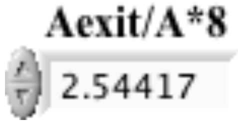
- Now allow an expandable *Nozzle where,* $A_{exit} \geq A^*_8$

→ *CALCULATE*

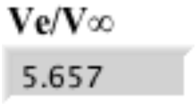
→ *a) Optimal expansion ratio for nozzle* A_{exit}/A^*_8


Pe/P∞
1

→


Aexit/A*8
2.54417

→ *b) c) Velocity ratio across Engine* V_{exit}/V_∞

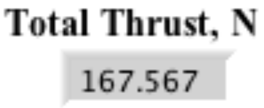

Ve/V∞
5.657

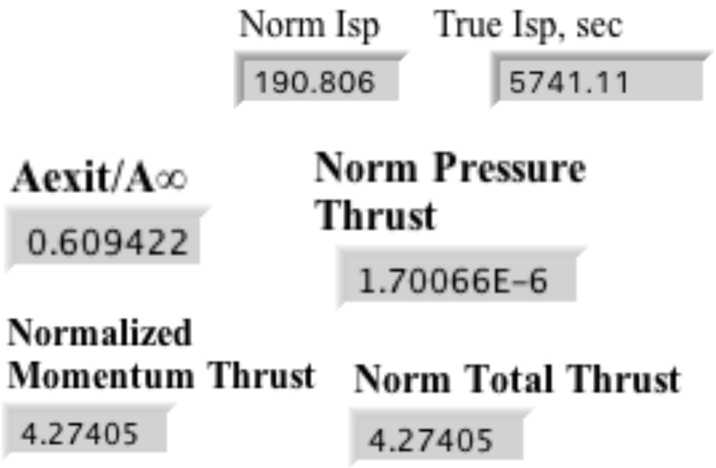
→

→ *c) thrust of optimal nozzle*

OPTIMAL THRUST DATA

f ≃ 50


Total Thrust, N
167.567


Norm Isp
190.806

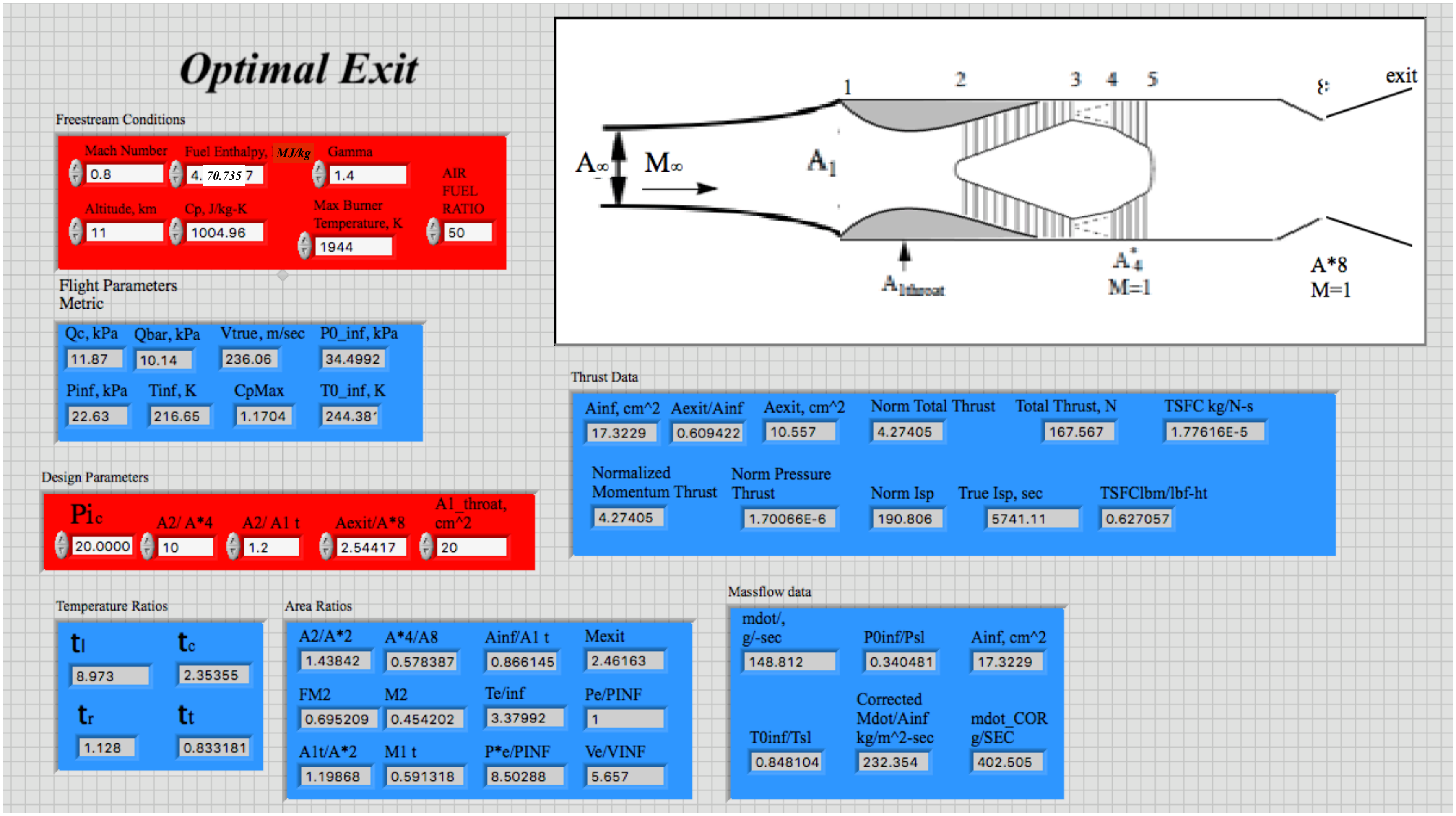
True Isp, sec
5741.11

Aexit/A∞
0.609422

Norm Pressure Thrust
1.70066E-6

Normalized Momentum Thrust
4.27405

Norm Total Thrust
4.27405



Optimal Exit

Freestream Conditions

Mach Number: 0.8
Fuel Enthalpy, MJ/kg: 4. 70.735 7
Gamma: 1.4
Altitude, km: 11
Cp, J/kg-K: 1004.96
Max Burner Temperature, K: 1944
AIR FUEL RATIO: 50

Flight Parameters
Metric

Qc, kPa: 11.87
Qbar, kPa: 10.14
Vtrue, m/sec: 236.06
P0_inf, kPa: 34.4992
Pinf, kPa: 22.63
Tinf, K: 216.65
CpMax: 1.1704
T0_inf, K: 244.38

Design Parameters

Pic: 20.0000
A2/ A*4: 10
A2/ A1 t: 1.2
Aexit/A*8: 2.54417
A1_throat, cm^2: 20

Thrust Data

Ainf, cm^2: 17.3229
Aexit/Ainf: 0.609422
Aexit, cm^2: 10.557
Norm Total Thrust: 4.27405
Total Thrust, N: 167.567
TSFC kg/N-s: 1.77616E-5
Normalized Momentum Thrust: 4.27405
Norm Pressure Thrust: 1.70066E-6
Norm Isp: 190.806
True Isp, sec: 5741.11
TSFClbm/lbf-ht: 0.627057

Temperature Ratios

tl: 8.973
tc: 2.35355
tr: 1.128
tt: 0.833181

Area Ratios

A2/A*2: 1.43842
A*4/A8: 0.578387
Ainf/A1 t: 0.866145
Mexit: 2.46163
FM2: 0.695209
M2: 0.454202
Te/inf: 3.37992
Pe/PINF: 1
A1t/A*2: 1.19868
M1 t: 0.591318
P*e/PINF: 8.50288
Ve/VINF: 5.657

Massflow data

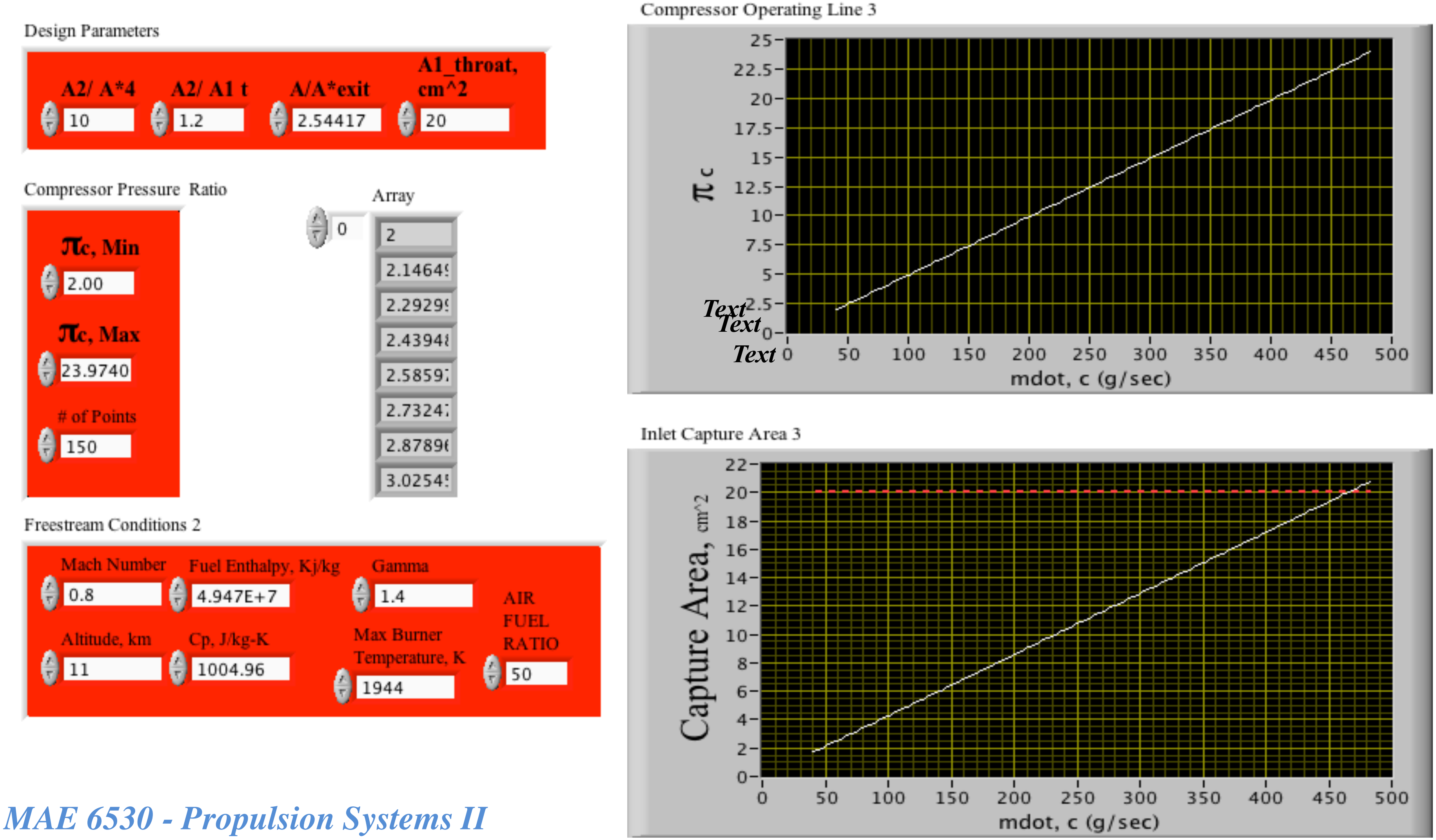
mdot/, g/-sec: 148.812
P0inf/Psl: 0.340481
Ainf, cm^2: 17.3229
T0inf/Tsl: 0.848104
Corrected Mdot/Ainf kg/m^2-sec: 232.354
mdot_COR g/SEC: 402.505

Homework 5.4, SOLUTION (4)

→ *d) Assuming the same combustor temperature and inlet throat area*

  → *Plot the Compressor operating line* → *π_c vs corrected massflow for $1 \leq \pi c <$ Choke*

  → *Plot the capture area A_∞ vs corrected massflow for $1 \leq \pi c <$ Choke*



Design Parameters

A2/ A*4: 10

A2/ A1 t: 1.2

A/A*exit: 2.54417

A1_throat, cm^2: 20

Compressor Pressure Ratio

πc, Min: 2.00

πc, Max: 23.9740

# of Points: 150

Array: 0

2
2.1464
2.2929
2.4394
2.5859
2.7324
2.8789
3.0254

Freestream Conditions 2

Mach Number: 0.8

Fuel Enthalpy, Kj/kg: 4.947E+7

Gamma: 1.4

Altitude, km: 11

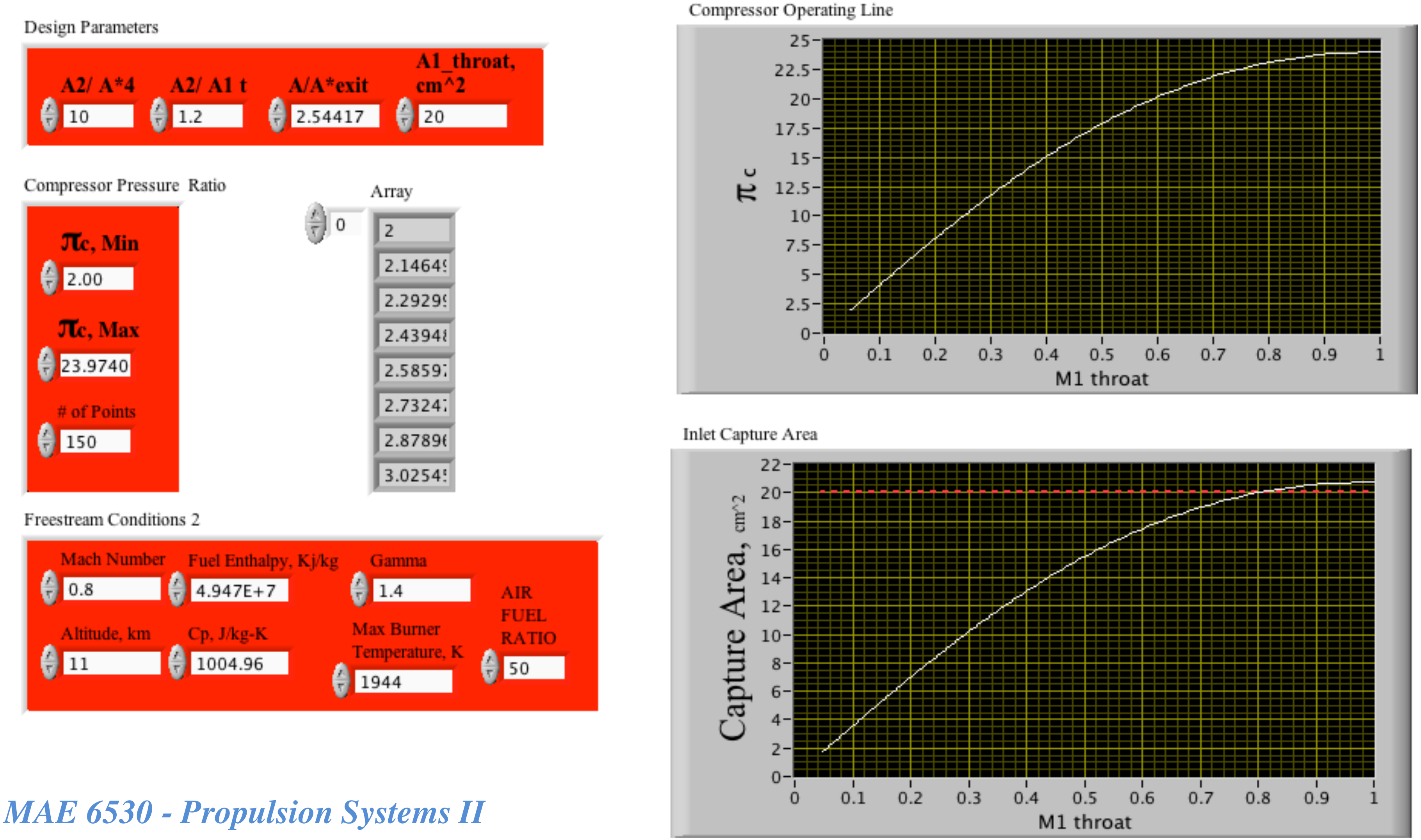
Cp, J/kg-K: 1004.96

Max Burner Temperature, K: 1944

AIR FUEL RATIO: 50

Homework 5.4, SOLUTION (4)

- → *d) Assuming the same combustor temperature and inlet throat area*
  - → *Plot the Compressor operating line* → π_c *vs corrected massflow for* $1 \leq \pi c < \text{Choke}$
  - → *Plot the capture area* A_∞ *vs corrected massflow for* $1 \leq \pi c < \text{Choke}$



Design Parameters

A2/ A*4	A2/ A1 t	A/A*exit	A1_throat, cm^2
10	1.2	2.54417	20

Compressor Pressure Ratio

πc, Min: 2.00
πc, Max: 23.9740
# of Points: 150

Array
0

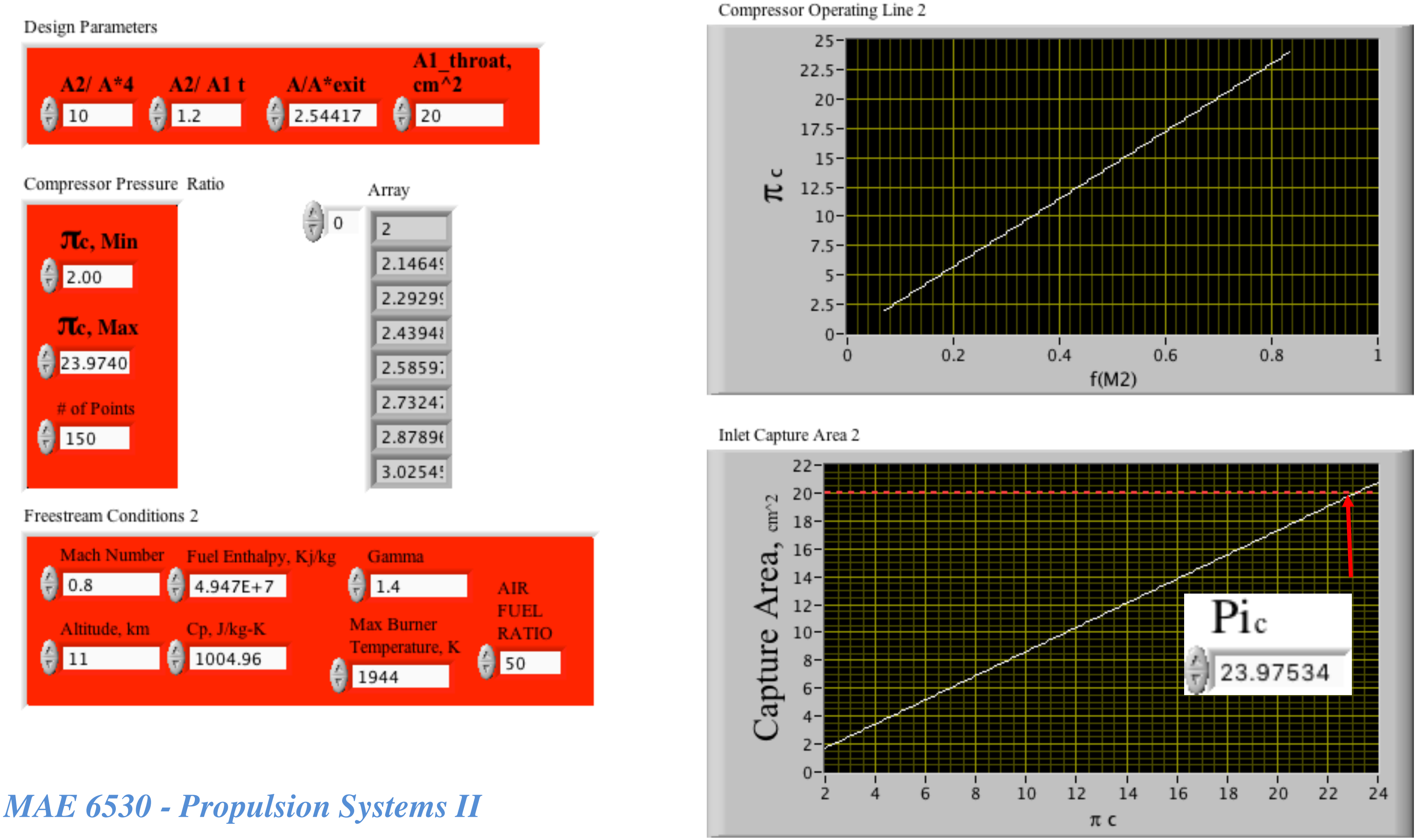
2
2.1464
2.2929
2.4394
2.5859
2.7324
2.8789
3.0254

Freestream Conditions 2

Mach Number: 0.8
Fuel Enthalpy, Kj/kg: 4.947E+7
Gamma: 1.4
Altitude, km: 11
Cp, J/kg-K: 1004.96
Max Burner Temperature, K: 1944
AIR FUEL RATIO: 50

Homework 5.4, SOLUTION (4)

- → *d) Assuming the same combustor temperature and inlet throat area*
  - → *Plot the Compressor operating line* → π_c *vs corrected massflow for* $1 \leq \pi c <$ *Choke*
  - → *Plot the capture area* A_∞ *vs corrected massflow for* $1 \leq \pi c <$ *Choke*



Design Parameters

A2/ A*4: 10
A2/ A1 t: 1.2
A/A*exit: 2.54417
A1_throat, cm^2: 20

Compressor Pressure Ratio

πc, Min: 2.00
πc, Max: 23.9740
# of Points: 150

Array: 0

2
2.1464
2.2929
2.4394
2.5859
2.7324
2.8789
3.0254

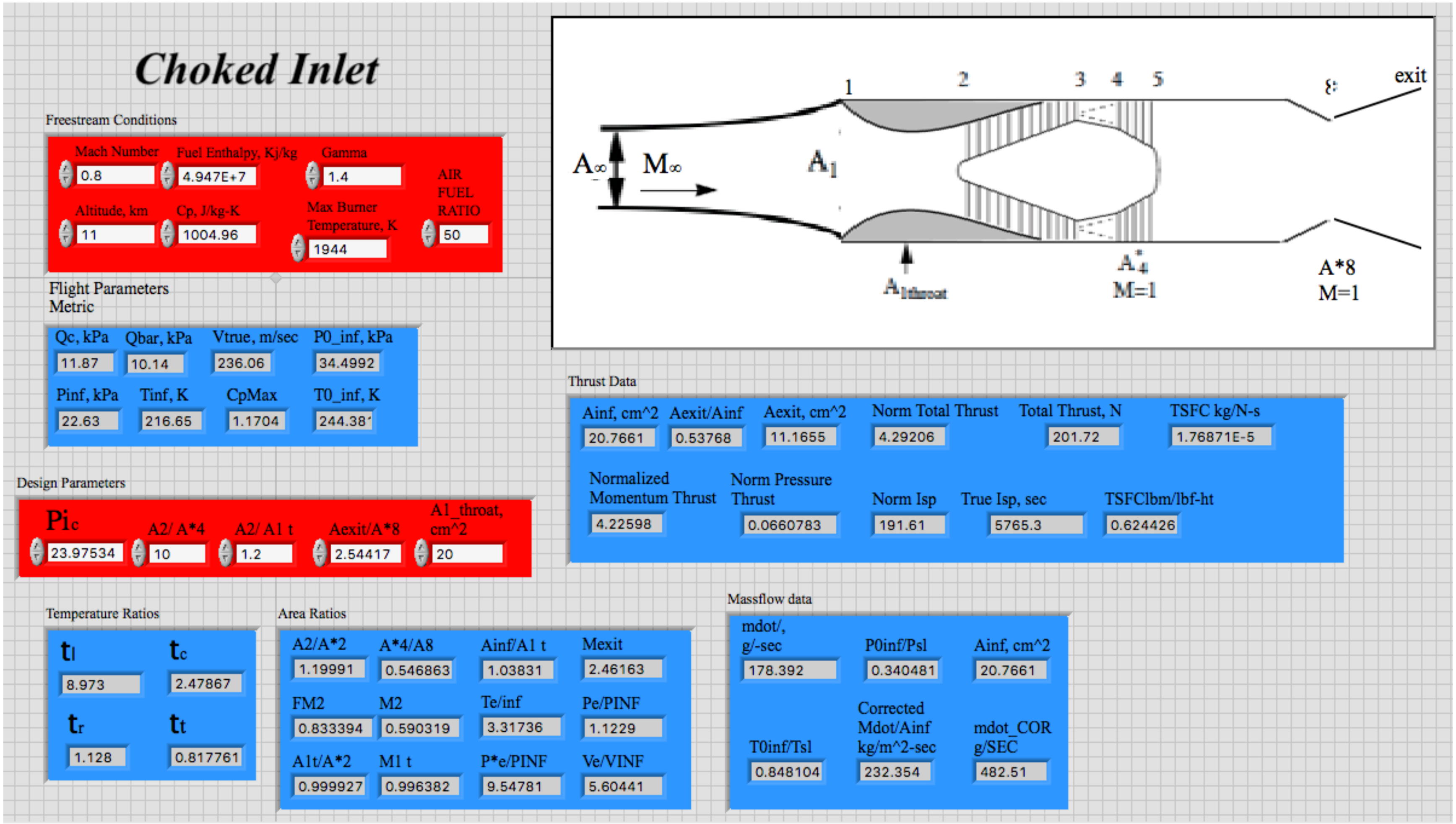
Freestream Conditions 2

Mach Number: 0.8
Fuel Enthalpy, Kj/kg: 4.947E+7
Gamma: 1.4
Altitude, km: 11
Cp, J/kg-K: 1004.96
Max Burner Temperature, K: 1944
AIR FUEL RATIO: 50

Compressor Operating Line 2

Inlet Capture Area 2

Pic: 23.97534



Choked Inlet

Freestream Conditions

Mach Number	Fuel Enthalpy, Kj/kg	Gamma	AIR FUEL RATIO
0.8	4.947E+7	1.4	50

Altitude, km	Cp, J/kg-K	Max Burner Temperature, K
11	1004.96	1944

Flight Parameters Metric

Qc, kPa	Qbar, kPa	Vtrue, m/sec	P0_inf, kPa
11.87	10.14	236.06	34.4992

Pinf, kPa	Tinf, K	CpMax	T0_inf, K
22.63	216.65	1.1704	244.38

Design Parameters

Pi c	A2/ A*4	A2/ A1 t	Aexit/A*8	A1_throat, cm^2
23.97534	10	1.2	2.54417	20

Thrust Data

Ainf, cm^2	Aexit/Ainf	Aexit, cm^2	Norm Total Thrust	Total Thrust, N	TSFC kg/N-s
20.7661	0.53768	11.1655	4.29206	201.72	1.76871E-5

Normalized Momentum Thrust	Norm Pressure Thrust	Norm Isp	True Isp, sec	TSFClbm/lbf-ht
4.22598	0.0660783	191.61	5765.3	0.624426

Temperature Ratios

t_l	t_c
8.973	2.47867

t_r	t_t
1.128	0.817761

Area Ratios

A2/A*2	A*4/A8	Ainf/A1 t	Mexit
1.19991	0.546863	1.03831	2.46163

FM2	M2	Te/inf	Pe/PINF
0.833394	0.590319	3.31736	1.1229

A1t/A*2	M1 t	P*e/PINF	Ve/VINF
0.999927	0.996382	9.54781	5.60441

Massflow data

mdot/, g/-sec	P0inf/Psl	Ainf, cm^2
178.392	0.340481	20.7661

T0inf/Tsl	Corrected Mdot/Ainf kg/m^2-sec	mdot_COR g/SEC
0.848104	232.354	482.51

Questions??