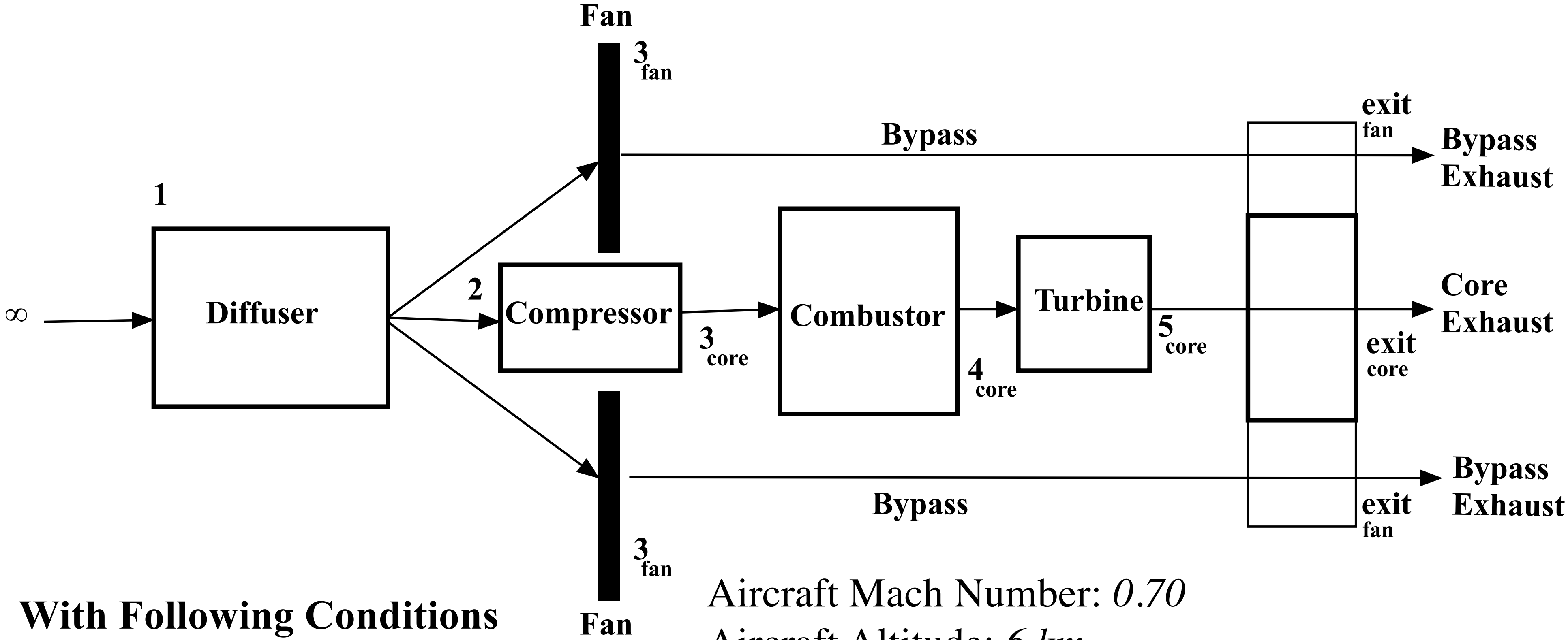
Homework 6.2

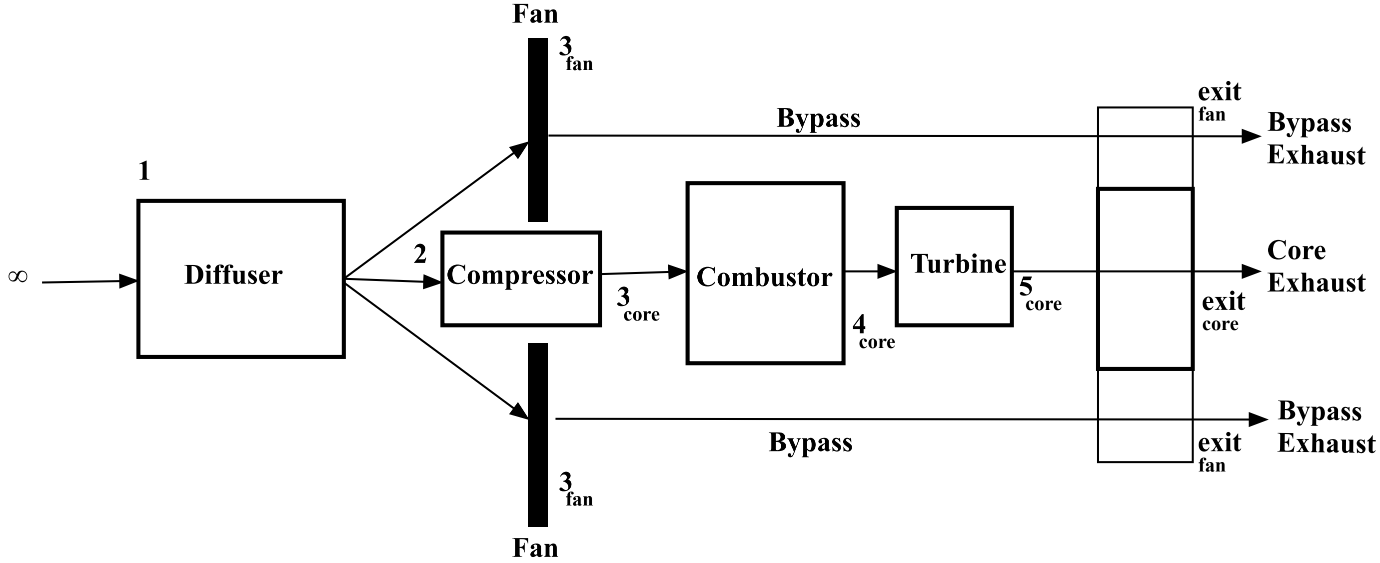
Consider the TurboFan Engine whose Block Diagram is Shown Below

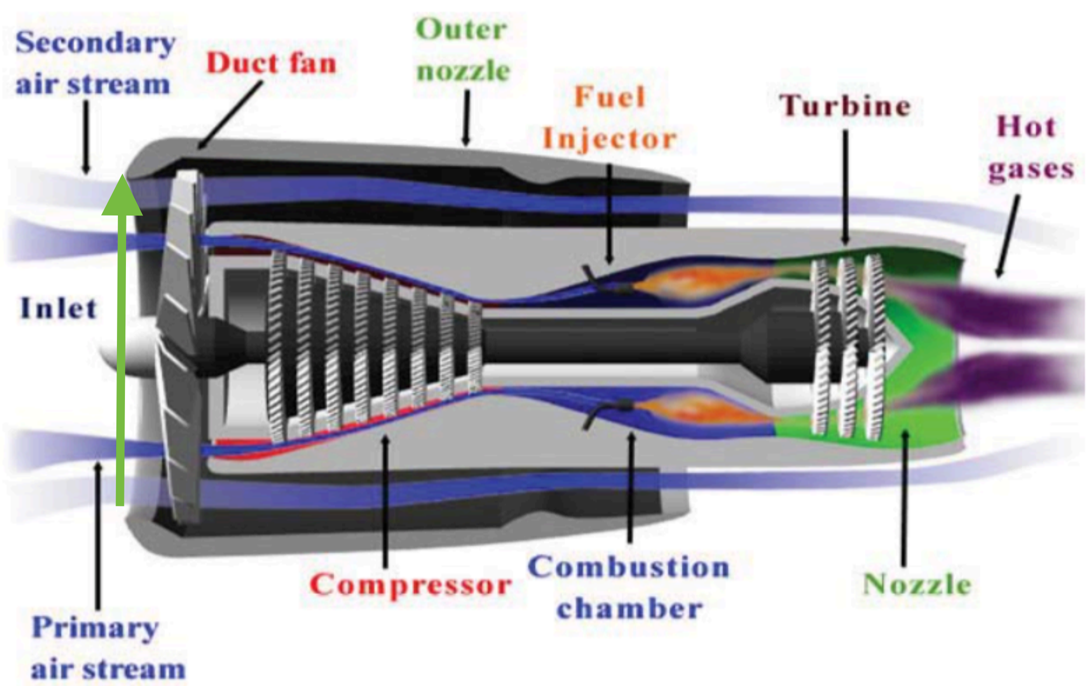


With Following Conditions

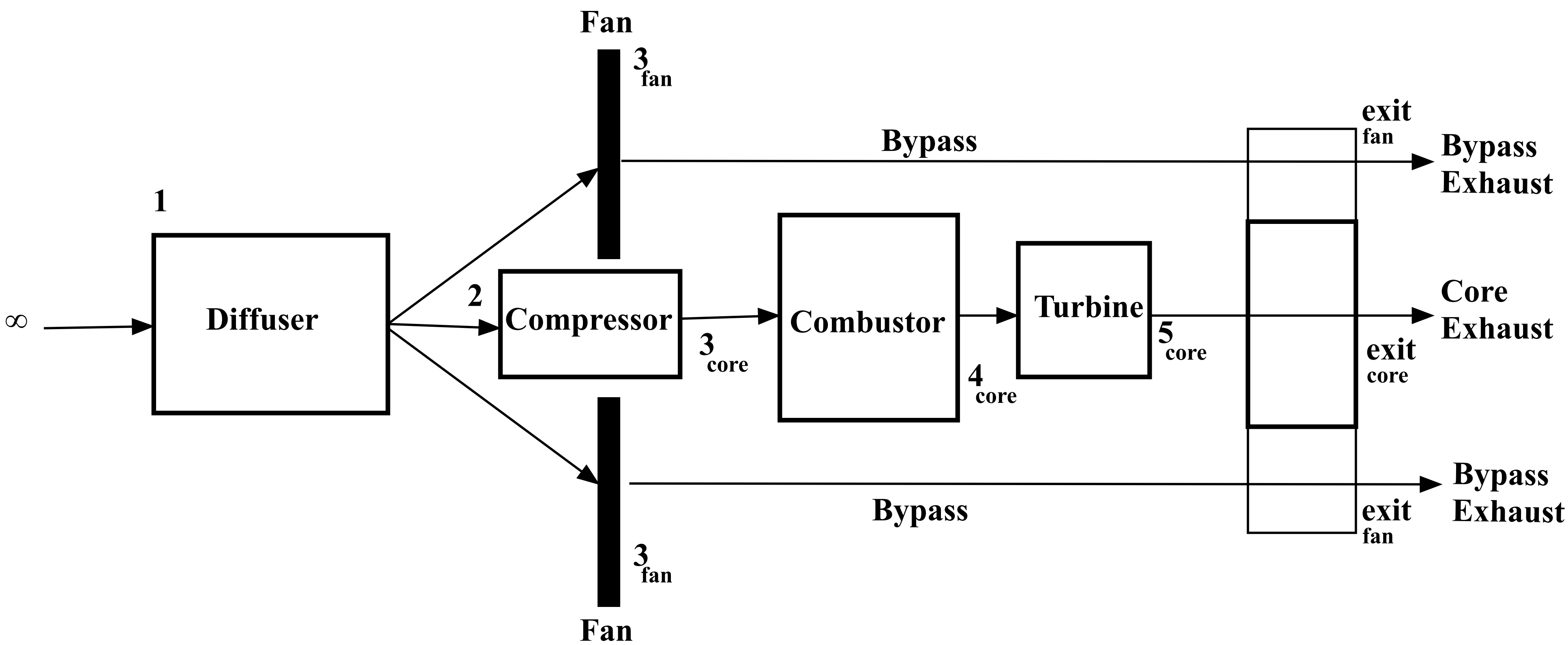
Aircraft Mach Number: *0.70*
Aircraft Altitude: *6 km*
Bypass Ratio: *2*
Fan Pressure Ratio: *2*
Compressor Ratio: *6*
Burner Outlet Temperature: *1700 K*
Fuel: *JP4* → $h_f = 42.68\ MJ/kg$

Homework 6.2 (2)





Homework 6.2 (3)



Assume the Following Component Properties

i) Diffuser, Compressor, Fan, Turbine, Nozzle ~ Isentropic

ii) Nozzle exit flow is NOT mixed

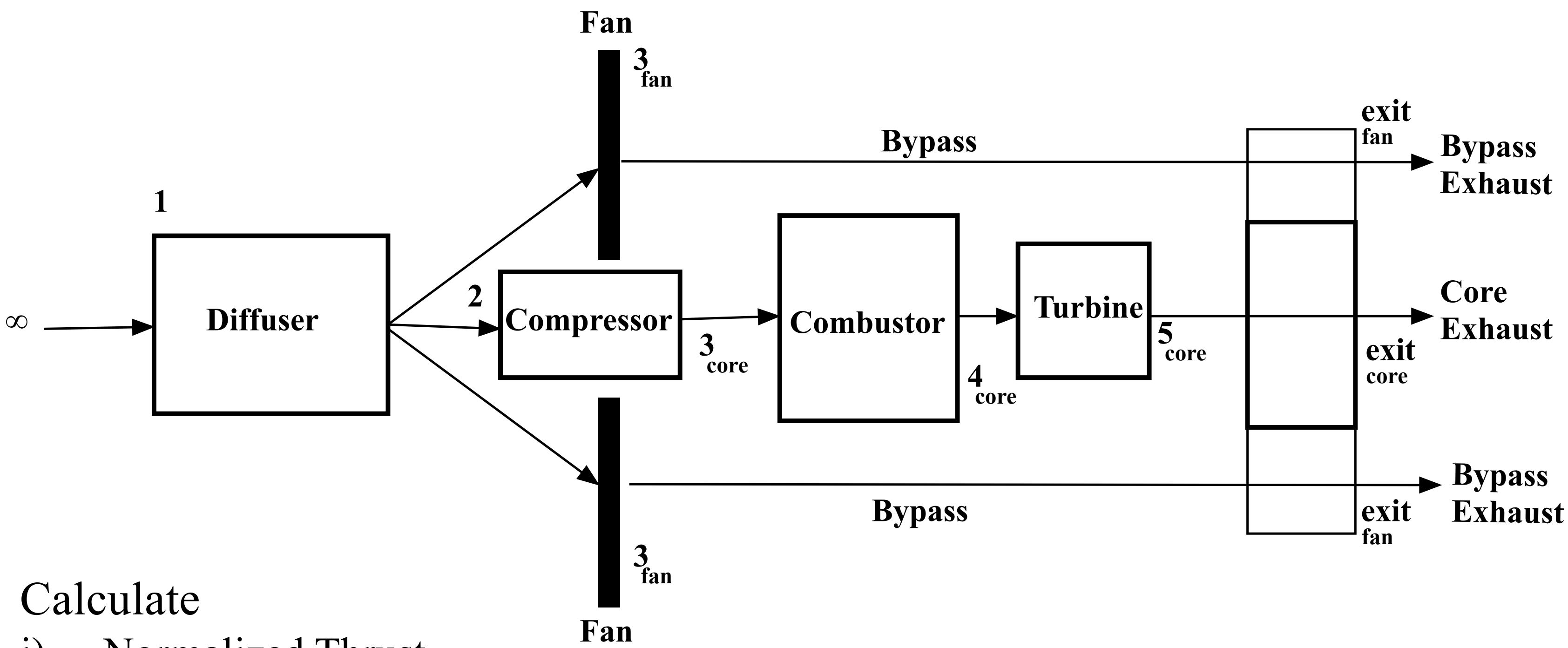
iii) Combustor is 35% efficient → $\eta_b = \frac{\dot{Q}_{3-4}}{\dot{m}_{fuel} \cdot h_f} = \tau_f \cdot \frac{C_p \cdot T_\infty}{h_f}$

iv) Fuel massflow is NOT negligible

v) Mean specific heats, gamma are constant across engine constant → $\gamma \approx 1.4$, $C_p \approx 1004.96_{J/kg-K}$

vi) Fan, Core Nozzle Exits Optimized for Altitude

Homework 6.2 (4)



Calculate

i) Normalized Thrust
ii) % of Thrust delivered by Core Flow
iii) % of Thrust delivered by Bypass Flow
ii) Ratio of Bypass Thrust to Core Thrust
iii) Normalized Specific Impulse
iv) TSFC *lbm/lbf-hr*
v) Bypass Ratio for Optimal Isp
vi) Optimal TSFC
vii) Thermal, Propulsive, and Total Efficiency

Verify Graphical peak is at optimal Bypass ratio

$$\mathbb{T} = \frac{F_{thrust}}{p_\infty \cdot A_\infty}$$

$$\mathbb{I} = \frac{I_{sp} \cdot g_0}{c_\infty}$$

$$TSFC = \frac{1}{g_0 \cdot I_{sp}}$$

TurboFan Efficiencies

- Recall that

Propulsive Efficiency =

$$\eta_{propulsive} = \frac{\dot{W}_p}{\left(K.E._{exit} - K.E._{\infty}\right)}$$

Kinetic energy production rate

Thermal Efficiency = $\rightarrow$

$$\eta_{thermal} = \frac{\left(K.E._{exit} - K.E._{\infty}\right)}{\dot{m}_{fuel} \cdot h_{fuel}}$$

Kinetic energy production rate

Combustion Enthalpy of Fuel

TurboFan Efficiencies (2)

Propulsive Efficiency

$$\eta_{propulsive} = \frac{Thrust\ power}{Rate\ of\ kinetic\ enegy\ added\ to\ engine\ flow} \qquad (8)$$

Turbofan engines have two different streams that called hot stream which comes from the core of the engine and cold stream which passes through fan of the engine. The first expression of propulsive efficiency becomes:

$$\eta_{propulsive} = \left| \begin{matrix} \dfrac{V_\infty \cdot \left(T_{fan} + T_{core}\right)}{\frac{1}{2}\dot{m}_{a_{core}} \cdot \left(\left(\frac{1+f}{f}\right)\cdot V^2_{exit_{core}} - V^2_\infty\right) + \frac{1}{2}\dot{m}_{a_{fan}} \cdot \left(V^2_{exit_{fan}} - V^2_\infty\right)} = \dfrac{V_\infty \cdot \left(T_{fan} + T_{core}\right)}{\frac{1}{2}\dot{m}_{a_{core}} \cdot \left[\left(\frac{1+f}{f}\right)\cdot V^2_{exit_{core}} - V^2_\infty + \beta \cdot \left(V^2_{exit_{fan}} - V^2_\infty\right)\right]} = \\ \dfrac{2 \cdot V_\infty \cdot \left(T_{fan} + T_{core}\right)}{\dot{m}_{a_{core}} \cdot \left[\left(\frac{1+f}{f}\right)\cdot V^2_{exit_{core}} + \beta \cdot V^2_{exit_{fan}} - \left(1+\beta\right)\cdot V^2_\infty\right]} = \dfrac{2\cdot\left(T_{fan} + T_{core}\right)/V_\infty}{\dot{m}_{a_{core}} \cdot \left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2 + \beta \cdot \left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2 - \left(1+\beta\right)\right]} \end{matrix} \right.$$

TurboFan Efficiencies (2)

Thermal Efficiency

- Also, recall from earlier …

$$\eta_{th} = 1 - \frac{\left(1 + \frac{1}{f}\cdot\left(1+\beta\right)\right)\cdot\frac{h_{exit_{core}}}{h_\infty} - 1}{\tau_{c_{core}}\cdot\tau_r\left[\left(1 + \frac{1}{f}\cdot\left(1+\beta\right)\right)\cdot\frac{\tau_\gamma}{\tau_{c_{core}}\cdot\tau_r} - 1\right]} = 1 - \frac{\left(1 + \frac{1}{f}\cdot\left(1+\beta\right)\right)\cdot\frac{\tau_\gamma}{\tau_{c_{core}}\cdot\tau_r} - 1}{\tau_{c_{core}}\cdot\tau_r\left[\left(1 + \frac{1}{f}\cdot\left(1+\beta\right)\right)\cdot\frac{\tau_\gamma}{\tau_{c_{core}}\cdot\tau_r} - 1\right]} = 1 - \frac{1}{\tau_{c_{core}}\cdot\tau_r}$$

- This solution is identical to the turbo jet analysis with the core flow replacing the normal turbine flow path.

- This analysis shows for the ideal (isentropic) fan → $h_{exit_{fan}} = h_\infty$ --> *the* the heat rejected by the fan stream is zero.

- Therefore the thermal efficiency of the ideal turbofan is independent of the parameters of the fan stream.

TurboFan Efficiencies (3)

Propulsive Efficiency

$$\eta_{propulsive} = \frac{2\cdot\left(T_{fan}+T_{core}\right)\Big/\left(\dot{m}_{a_{core}}\cdot V_\infty\right)}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2+\beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2-\left(1+\beta\right)\right]} = \frac{2\cdot\left(T_{fan}+T_{core}\right)\Big/\left(\rho_\infty\cdot A_\infty V_\infty^{\ 2}\right)}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2+\beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2-\left(1+\beta\right)\right]} =$$

$$\frac{2\cdot\frac{R_g\cdot T_\infty}{V_\infty^{\ 2}}\left(\frac{T_{fan}+T_{core}}{p_\infty A_\infty}\right)}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2+\beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2-\left(1+\beta\right)\right]} = \frac{\frac{2}{\gamma}\cdot\frac{\gamma\cdot R_g\cdot T_\infty}{V_\infty^{\ 2}}\left(\frac{T_{fan}+T_{core}}{p_\infty A_\infty}\right)}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2+\beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2-\left(1+\beta\right)\right]} =$$

$$\frac{\frac{2}{\gamma\cdot M_\infty^2}\cdot\left(\frac{T_{fan}+T_{core}}{p_\infty A_\infty}\right)}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2+\beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2-\left(1+\beta\right)\right]} = \frac{\frac{2}{\gamma\cdot M_\infty^2}\cdot\left(\mathbb{T}_{fan}+\mathbb{T}_{core}\right)}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2+\beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2-\left(1+\beta\right)\right]}$$

TurboFan Efficiencies (4)

Propulsive Efficiency

$$\left(\mathbb{T}\right)_{turbofan} = \left(\mathbb{T}_{fan} + \mathbb{T}_{core}\right) = \gamma \cdot M_\infty^2 \left[\left(\frac{1}{1+\beta}\right)\cdot\left(\frac{V_{exit\,core}}{V_\infty} - 1\right) + \left(\frac{\beta}{1+\beta}\right)\cdot\left(\frac{V_{exit\,fan}}{V_\infty} - 1\right)\right] \rightarrow$$

$$\eta_{propulsive} = \frac{\frac{2}{\gamma \cdot M_\infty^2}\cdot\left(\mathbb{T}_{fan} + \mathbb{T}_{core}\right)}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2 + \beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2 - \left(1+\beta\right)\right]} = \frac{2\cdot\left[\left(\frac{1}{1+\beta}\right)\cdot\left(\frac{V_{exit\,core}}{V_\infty} - 1\right) + \left(\frac{\beta}{1+\beta}\right)\cdot\left(\frac{V_{exit\,fan}}{V_\infty} - 1\right)\right]}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2 + \beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2 - \left(1+\beta\right)\right]}$$

Check → for Turbojet

$$\beta = 0$$

$$\left(\frac{1+f}{f}\right) = 1 \quad \rightarrow \eta_{propulsive} = \frac{2\cdot\left(\frac{V_{exit\,core}}{V_\infty} - 1\right)}{\left[\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2 - 1\right]} = \frac{2\cdot\left(\frac{V_{exit\,core}}{V_\infty} - 1\right)}{\left(\frac{V_{exit\,core}}{V_\infty} - 1\right)\cdot\left(\frac{V_{exit\,core}}{V_\infty} + 1\right)} = \frac{2}{\left(1 + \frac{V_{exit\,core}}{V_\infty}\right)} \surd$$

TurboFan Efficiencies (6)

Propulsive Efficiency

$$\eta_{propulsive} = \frac{2\cdot\left[\left(\frac{1}{1+\beta}\right)\cdot\left(\frac{V_{\substack{exit\\core}}}{V_\infty}-1\right)+\left(\frac{\beta}{1+\beta}\right)\cdot\left(\frac{V_{\substack{exit\\fan}}}{V_\infty}-1\right)\right]}{\left[\left(\frac{1+f}{f}\right)\cdot\left(\frac{V_{exit_{core}}}{V_\infty}\right)^2+\beta\cdot\left(\frac{V_{exit_{fan}}}{V_\infty}\right)^2-(1+\beta)\right]}$$

$$\rightarrow\left(\frac{V_{\substack{exit\\core}}}{V_\infty}\right)=\sqrt{\left(\frac{\tau_r\cdot\tau_c\cdot\tau_t-1}{\tau_r-1}\right)}\cdot\sqrt{\frac{\tau_\lambda}{\tau_c\cdot\tau_r}}\rightarrow\left(\frac{V_{\substack{exit\\fan}}}{V_\infty}\right)=\sqrt{\frac{\tau_r\cdot\tau_{c_{fan}}-1}{\tau_r-1}}$$

TurboFan Efficiencies *(7)*

Thermal Efficiency

- Also, recall from earlier …

$$\eta_{th} = 1 - \frac{\left(1+\frac{1}{f}\cdot(1+\beta)\right)\cdot\frac{h_{exit_{core}}}{h_\infty} - 1}{\tau_{c_{core}}\cdot\tau_r\left[\left(1+\frac{1}{f}\cdot(1+\beta)\right)\cdot\frac{\tau_\gamma}{\tau_{c_{core}}\cdot\tau_r} - 1\right]} = 1 - \frac{\left(1+\frac{1}{f}\cdot(1+\beta)\right)\cdot\frac{\tau_\gamma}{\tau_{c_{core}}\cdot\tau_r} - 1}{\tau_{c_{core}}\cdot\tau_r\left[\left(1+\frac{1}{f}\cdot(1+\beta)\right)\cdot\frac{\tau_\gamma}{\tau_{c_{core}}\cdot\tau_r} - 1\right]} = 1 - \frac{1}{\tau_{c_{core}}\cdot\tau_r}$$

- This solution is identical to the turbo jet analysis with the core flow replacing the normal turbine flow path.

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