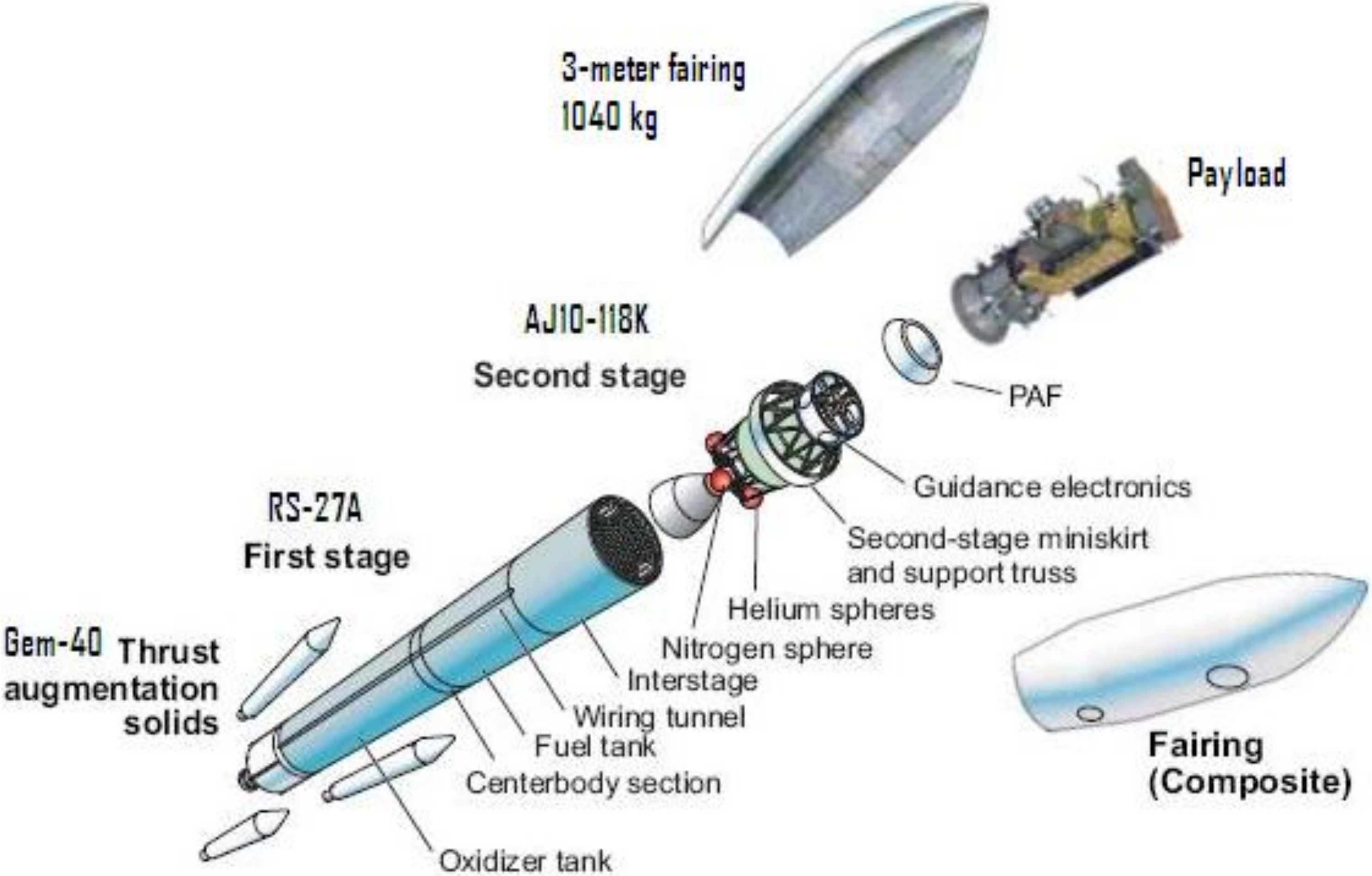
Homework 1

Delta II 7320 Launch Vehicle



A four digit designator has been used to distinguish between Delta configurations since the early 1970s. However, Boeing uses a new designation system for the Delta IV vehicles. The Delta III is currently available in only one configuration, and thus its four digit designator is rarely used. The Delta IV is available in two basic types, Medium and Heavy. These are designated DIV-M and DIV-H. The Medium can be enhanced ("Medium-Plus") with a larger fairing and strap-on boosters. These configurations are designated with a digit for the fairing diameter in meters, and digit for the number of strap-on boosters. Thus, the DIV-M+ (4,2) has a 4-m fairing and two strap-on boosters, while the DIV-M+ (5,4) has a 5-m fairing and four strap-on boosters.

Example

Delta 7925-10

First Digit

First-stage type, engine type, strap-on motor type

0: Long tank, MB-3 engine, Castor II

1: Extended long tank, MB-3 engine, Castor II

2: Extended long tank, RS-27 engine, Castor II

3: Extended long tank, RS-27 engine, Castor IV

4: Extended long tank, MB-3 engine, Castor IVA

5: Extended long tank, RS-27 engine, Castor IVA

6: Extra extended long tank, RS-27 engine, Castor IVA

7: Extra extended long tank, RS-27A engine, GEM-40

8: Delta III shortened first stage, RS-27A engine, GEM-46

Second Digit

Number of strap-on motors (3,4,6, or 9)

Third Digit

Second-stage engine

0: AJ-10-118 (Aerojet)

1: TR-201 (TRW)

2: AJ-10-118K (Aerojet)

3: RL10B-2 (Pratt & Whitney)

Fourth Digit

Third-stage motor

0: No third stage or unspecified

3: TE-364-3

4: TE-364-4

5: PAM-D derivative Star 48B

6: Star 37FM

Dash Number

Fairing type (optional)

None: Standard fairing (9.5 ft for Delta II)

-8: 8-ft fairing

-10: 10-ft composite fairing

-10L: 10-ft stretched composite fairing

Reference material

International Reference Guide to Space Launch Systems, 4th ed., Stephen J. Isakowitz, Joseph P. Hopkins, Jr., and Joshua B. Hopkins, American Institute of Aeronautics and Astronautics, Reston, VA, 2003. ISBN: 1-56347-591-X

Stage 1 Properties

- Boeing Delta II Rocket…Stage 1
  - Sea Level Thrust: 890kN
  - Vacuum Thrust: 1085.8 kN
  - **Nozzle Expansion Ratio: 15.2503:1**
  - **Conical Nozzle, 30.5 deg exit angle**

- Combustion Properties:
  (RS-27A Rocketdyne Engine)
  - Lox/Kerosene, Mixture Ratio: 2.24:1
  - **Chamber Pressure (P_0): 5161.463 kPa**
  - Combustion temperature (T_0): 3455 K
  - $\gamma = 1.2220$
  - $M_W = 21.28$ *kg/kg-mol*

- Propellant Mass: 97.08 Metric Tons
- Stage 1 Launch Mass: 101.8 Metric Tons

Gem 40 Augmentation Rocket Properties (ATK)

- 3 Boosters Total – Ground Lit
  - Sea Level Thrust: 499.20kN
  - Vacuum Thrust: 442.95 kN
  - **Nozzle Expansion Ratio: 10.65:1**
  - **Conical Nozzle, 20 deg exit angle**
- Combustion Properties: (Gem 40)
  - Ap/Aluminum/HTPB
  - **Chamber Pressure (P_0): 5652.66 kPa**
  - Combustion temperature (T_0): 3600 K
  - γ = 1.2000
  - M_W = 28.15 *kg/kg-mol*
- Propellant Mass (Each): 11,765 kg
- Launch Mass: 13,080 kg

Stage II Properties

- Boeing Delta II Rocket…Stage 2 AJ10-118 Aerojet Engine

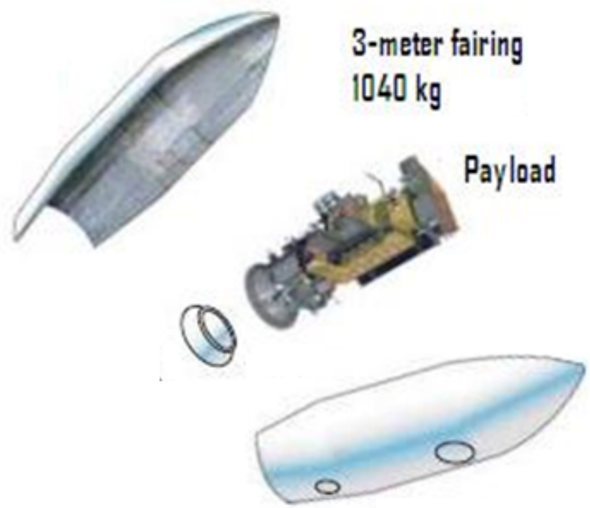
Propellants N_2O_4/Aerozine 50

- Vacuum I_{sp}: 319 seconds
- Vacuum Thrust: 43.657 kN
- Chamber Pressure: 5700 kPa
- Mixture Ratio: 1.8:1
- Nozzle Expansion Ratio: 65:1
- Bell nozzle, exit angle ~ 0 deg.

- Propellant Mass: 6004 kg
- Stage 2 Launch Mass: 6954 kg

Stage III Properties

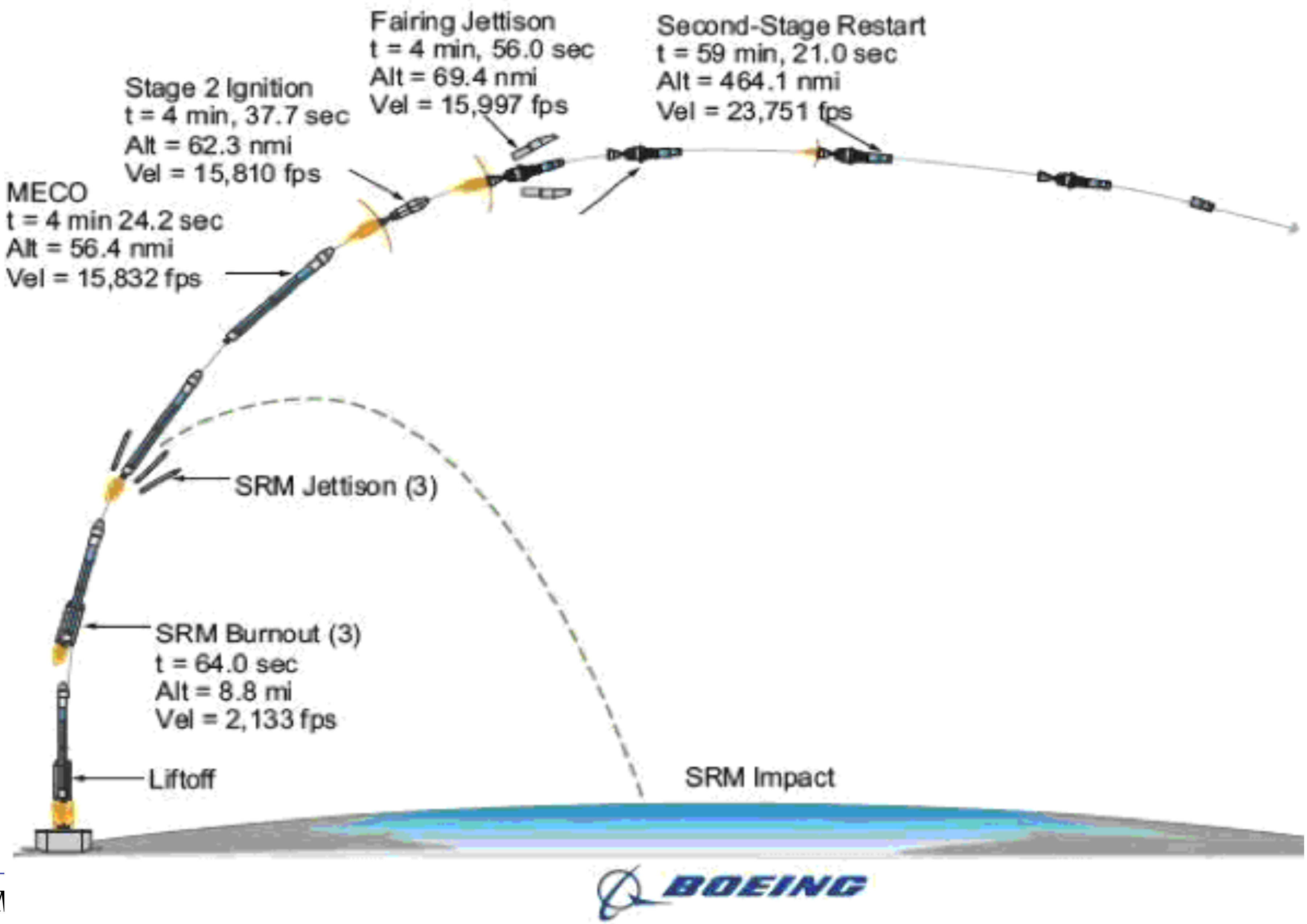
- Payload Inside of 3 meter (10 ft) shroud



- Payload Delivered to Orbit by stages 1-2 (no Kick motor burn)

- Shroud jettisoned prior to reaching orbit
  3-meter Shroud weight ~ 1040 kg

Launch Profile



Problem Objectives (1)

• Estimate Total Payload mass that can be delivered to a 464.1 *nmi* (860.06 km) LEO orbit at inclination 28.7° … KSC Launch Due East

• Assume that all gravity losses occur while stage 1 (RS-27A) is burning and the vehicle flys ***"straight up"*** while Gem 40's are burning and then at ***30 deg pitch angle*** for remainder of RS-27A burn

$$\left[\left(\Delta V\right)_{\substack{gravity \\ loss}}\right]_{stage} \approx \left[\frac{2}{3}\cdot g_{\left(h_{initial}\right)} + \frac{1}{3}\cdot g_{\left(h_{initial}\right)}\right]\cdot \sin\theta \cdot T_{burn}$$

$$..use...g_{(h)} = \frac{\mu}{R^2}..gravity \text{ model}$$

• Assume no gravity losses during stage 2 burn ..

Problem Objectives (2)

• Estimate Total Payload mass that can be delivered to a 464.1 *nmi* (860.06 km) LEO orbit at inclination 28.7° … KSC Launch Due East

• Assume 3% kinematic ***Delta*** V losses due to drag (includes interference from GEM 40 Boosters) During the stage 1 burns

$$\Delta V_{drag} = 0.03 \cdot g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)_{stage}$$

• Assume 1040 kg (2.9 meter) shroud + adapter weight

(not budgeted as part of payload) …. Jettisoned during stage 2 burn

• ***(be sure to account for conical nozzle exit thrust losses)***

Problem Objectives (3)

- 1) Calculate … total required delta V for the mission

    … be sure to include

    a) Required Orbital Velocity
    b) Change in Potential Energy
    c) Local Earth Rotational Velocity

2) Compare required delta V to available delta V ... for each stage

… sure to account for

a) mass changes due to stage separation
b) gravity and drag losses during stage 1 burn
c) shroud jettison 4 min and 56 seconds into burn

You are going to have to iterate the payload weight until
"Available Delta V" = "Required Delta V"

Mission Requirements

- First establish Delta V requirements

Calculate

a) Final Orbital Velocity
b) "Boost Velocity" from earth along direction of launch
(use true local Earth radius at 28.7 deg latitude here)
c) Kinematic Delta V ($V_{orbital}$-V_{boost})
d) Gravitational Potential Delta V
e) Total Delta V

Solution … Mission Delta V Requirements (1)

- First establish Delta V requirements

$$V_{orbit} = \sqrt{\frac{\mu}{\left(R_{earth} + h_{orbit}\right)}} = \left(\frac{3.9860044\times10^{5}}{6371 + 860}\right)^{0.5} = 7.4245 \text{ km/sec}$$

- "Boost" Velocity … along the direction of flight

$$V_{"boost"} = \left(R_{\oplus} + h_{launch}\right)\cdot\Omega_{\oplus}\cdot\cos i_{orbit} =$$

$$\frac{6373.186 + 0}{86164.1} 2\pi \cos\left(\frac{\pi}{180} 28.7\right) = 0.4076 \text{ km/sec}$$

Solution … Mission Delta V Requirements (2)

- "Lift" Delta V… equivalent change in potential energy

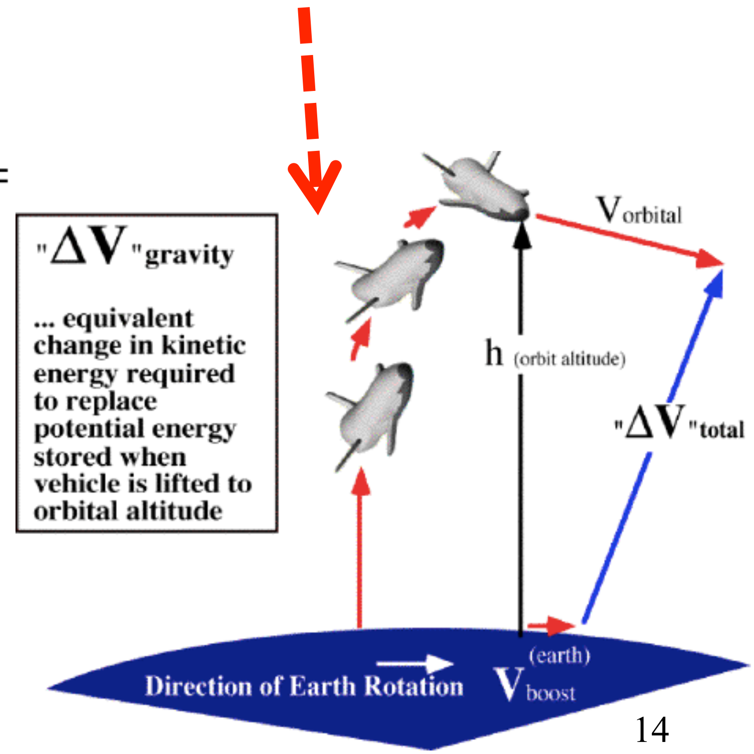
$$\Delta V_{\substack{potential\\energy}} = \sqrt{2\frac{\mu \cdot h_{orbit}}{R_{earth}\cdot\left(R_{earth}+h_{orbit}\right)}} = \left(\frac{2\,(3.9860044\times10^{5})\,860.06}{6373.18565\,(6373.18565+860.06)}\right)^{0.5}$$

$= 3.8566$ km/sec

$$\left(\Delta V_{total}\right)_{required} = \sqrt{\left(V_{orbit}-V_{"boost"}\right)^2+\left(\Delta V_{gravity}\right)^2} =$$

$$\left(\left(7.4233-0.4076\right)^2 + 3.8566^2\right)^{0.5}$$

= 8.0059 km/sec



Calculate Stage 1 Booster properties first

- Need calculations of Mass flow, exit conditions to analyze altitude effects on performance

Model RS-27A Nozzle Areas Next

- Boeing Delta II Rocket…Stage 1
  - Sea Level Thrust: 890kN
  - Vacuum Thrust: 1085.8 kN
  - Nozzle Expansion Ratio: 12:1

$$F_{thrust_{vac}} - F_{thrust_{SL}} = \dot{m}_e V_e + \left(p_e A_e\right) - \left[\dot{m}_e V_e + \left(p_e A_e - p_{SL} A_e\right)\right] = p_{SL} A_e$$

$$\rightarrow A_e = \frac{F_{thrust_{vac}} - F_{thrust_{SL}}}{p_{SL}} = \frac{(1085.8 - 890)}{101.325} = 1.9324 \text{ m}^2$$

1.569 m diameter

$$A^* = \frac{A_{exit}}{Exp.Ratio} = \frac{\frac{1085.9 - 890}{101.325}}{15.2503} = 0.12671 \text{ m}^2$$

0.4018 m diameter

Calculate RS-27A Exit Conditions

$$\frac{A_e}{A^*} = 15.2503 = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{(\gamma-1)}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} =$$

Iterative Solution … → M_{exit}=3.6523

$$T_{exit} = \frac{T_0}{1+\frac{(\gamma-1)}{2}M_{exit}{}^2} = \frac{3455}{\left(1+\frac{1.222-1}{2}3.6523^2\right)} = 1392.77 \text{ K}$$

$$P_{exit} = \frac{P_0}{\left(1+\frac{(\gamma-1)}{2}M_{exit}{}^2\right)^{\frac{\gamma}{\gamma-1}}} = \frac{5161.463}{\left(1+\frac{1.222-1}{2}3.6523^2\right)^{\frac{1.222}{(1.222-1)}}}$$

$$= 34.743 \text{ kPa}$$

Calculate RS-27A Exit Conditions (2)

$$V_{exit} = M_{exit}\sqrt{\gamma R_g T_{exit}} = 3.6523\,(1.222 \cdot 390.715 \cdot 1392.77)^{0.5}$$

$$= 2978.33 \text{ m/sec}$$

- Choking mass flow

$$\dot{m} = A^* \sqrt{\left(\frac{\gamma}{R_g}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \cdot \frac{P_0}{\sqrt{T_0}} =$$

$$0.12671\left(\frac{1.222}{390.715}\left(\frac{2}{1.222+1}\right)^{\frac{1.222+1}{1.222-1}}\right)^{0.5}\frac{5161.464\times 10^3}{3455^{0.5}}$$

$$= 367.447 \text{ kg/sec}$$

-- $R_g = 8314.4126\,/21.28 = 390.715$ J/K-kg

Compute RS-27A $I_{sp's}$

- Boeing Delta II Rocket…RS-27A
  - Lox/Kerosene First Stage
  - Sea Level Thrust: 890kN
  - Vacuum Thrust: 1085.8 kN

- Sea Level

$$I_{sp_{SL}} = \frac{F_{thrust_{SL}}}{g_0 \dot{m}} = \frac{890\times10^3}{9.8067\cdot367.447} = 246.99 \text{ sec}$$

- Vacuum

$$I_{sp_{vac}} = \frac{F_{thrust_{vac}}}{g_0 \dot{m}} = \frac{1085.8\times10^3}{9.8067\cdot367.447} = 301.323 \text{ sec}$$

- Burn Time:

$$T_{burn} = \frac{m_{prop}}{\dot{m}} = \frac{97{,}080\ kg}{367.447\ kg/\text{sec}} = 264.201\ \text{sec}$$

Model Gem 40 Nozzle Areas Next

- Boeing Delta II Rocket…Stage 1 • ***3 x Gem 40)***
  - Sea Level Thrust: 890kN
  - Vacuum Thrust: 1085.8 kN
  - Nozzle Expansion Ratio: 12:1

$$F_{thrust_{vac}} - F_{thrust_{SL}} = \dot{m}_e V_e + \left(p_e A_e\right) - \left[\dot{m}_e V_e + \left(p_e A_e - p_{SL} A_e\right)\right] = p_{SL} A_e$$

$$\rightarrow A_e = \frac{F_{thrust_{vac}} - F_{thrust_{SL}}}{p_{SL}} = \frac{499.2 - \mathbf{442.95}}{101.325}$$

0.8407 *m diameter*

$=0.525\ m^2$

$$A^* = \frac{A_{exit}}{Exp.Ratio} = \frac{\left(\frac{499.2 - \mathbf{442.95}}{101.325}\right)}{10.65} = \mathbf{0.0521}\ m^2$$

0.2576 *m diameter*

Calculate Gem 40 Exit Conditions

$$\frac{A_e}{A^*} = 10.65 = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{(\gamma-1)}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} =$$

Iterative Solution … → $M_{exit}=3.3222$

$$T_{exit} = \frac{T_0}{1+\frac{(\gamma-1)}{2}M_{exit}^{\ 2}} = \frac{3600}{1+\frac{1.2-1}{2}3.3222^2} = 1711.25 \text{ K}$$

$$P_{exit} = \frac{P_0}{\left(1+\frac{(\gamma-1)}{2}M_{exit}^{\ 2}\right)^{\frac{\gamma}{\gamma-1}}} = \frac{5652.66}{\left(1+\frac{1.2-1}{2}3.3222^2\right)^{\frac{(1.2)}{1.2-1}}}$$

$$= 65.211 \text{ kPa}$$

Calculate Gem 40 Exit Conditions (3)

$$V_{exit} = M_{exit}\sqrt{\gamma R_g T_{exit}} = 3.322\,(1.2\cdot 295.361\cdot 1711.25)^{0.5}$$

$$= 2587.35 \text{ m/sec}$$

- Choking mass flow $\dot{m} = A^*\sqrt{\left(\frac{\gamma}{R_g}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\cdot\frac{P_0}{\sqrt{T_0}} =$

$$\mathbf{0.0521}\left(\frac{1.2}{295.361}\left(\frac{2}{1.2+1}\right)^{\frac{1.2+1}{1.2-1}}\right)^{0.5}\frac{\mathbf{5652.66}\times 10^{3}}{3600^{0.5}}$$

$$= \mathbf{185.32} \text{ kg/sec}$$

-- R_g = 8314.4126 /28.15 = 295.361 J/K-kg

Compute Gem-40 $I_{sp's}$

- Boeing Delta II Rocket…Gem-40
  - Lox/Kerosene First Stage
  - Sea Level Thrust: 446kN
  - Vacuum Thrust: 499.2 kN

- Sea Level

$$I_{sp_{SL}} = \frac{F_{thrust_{SL}}}{g_0 \dot{m}} = \frac{442.95 \times 10^3}{9.8067 \cdot 185.32} = 246.41 \text{ sec}$$

- Vacuum

$$I_{sp_{vac}} = \frac{F_{thrust_{vac}}}{g_0 \dot{m}} = \frac{499.2 \times 10^3}{9.8067 \cdot 185.33} = 274.69 \text{ sec}$$

- Burn Time: $T_{burn} = \frac{m_{prop}}{\dot{m}} = \frac{11{,}765_{kg}}{185.32_{kg/\sec}} = 63.49_{\sec}$

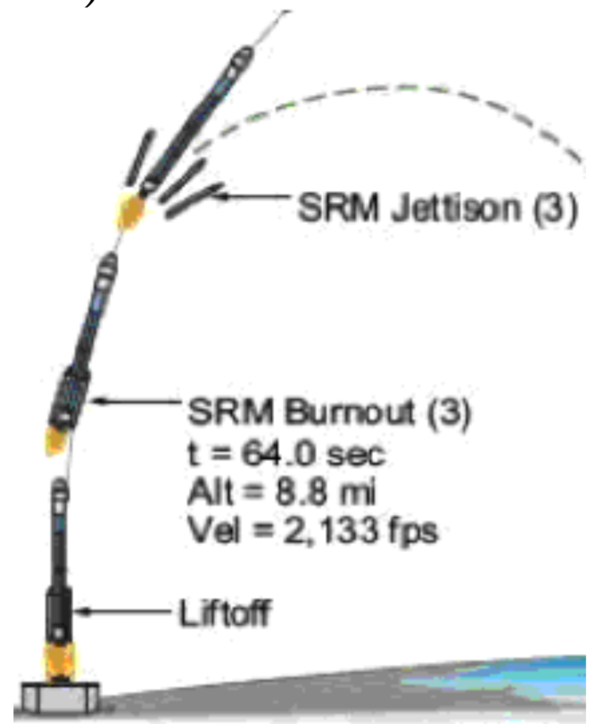
… Stage "1a"

- 3 x Gem 40 + Stage 1 (RS-27A)

-- Gem 40 Burnout Altitude ~ 8.8 *nmi* (16.31 km)

-- Calculate:

i) Total Lift off Thrust
ii) Burn Time for Gem-40(s)
iii) Plot total Thrust profile during Burn "1a" vs Altitude
iv) Total propellant consumed during "stage 1a" burn
vi) Effective Specific Impulse (3 x Gem 40 + RS-27A over operating altitude range
vii) Stage masses at Gem40 burnout



$$Use \rightarrow \left(I_{sp}\right)_{eff} = \frac{2}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\3x\ Gem40}}\right]_{launch} + \frac{1}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\3x\ Gem40}}\right]_{\substack{Gem40\\Burnout}}$$

... Stage "1a" (2)

-- Calculate:

i) Total Lift off Thrust

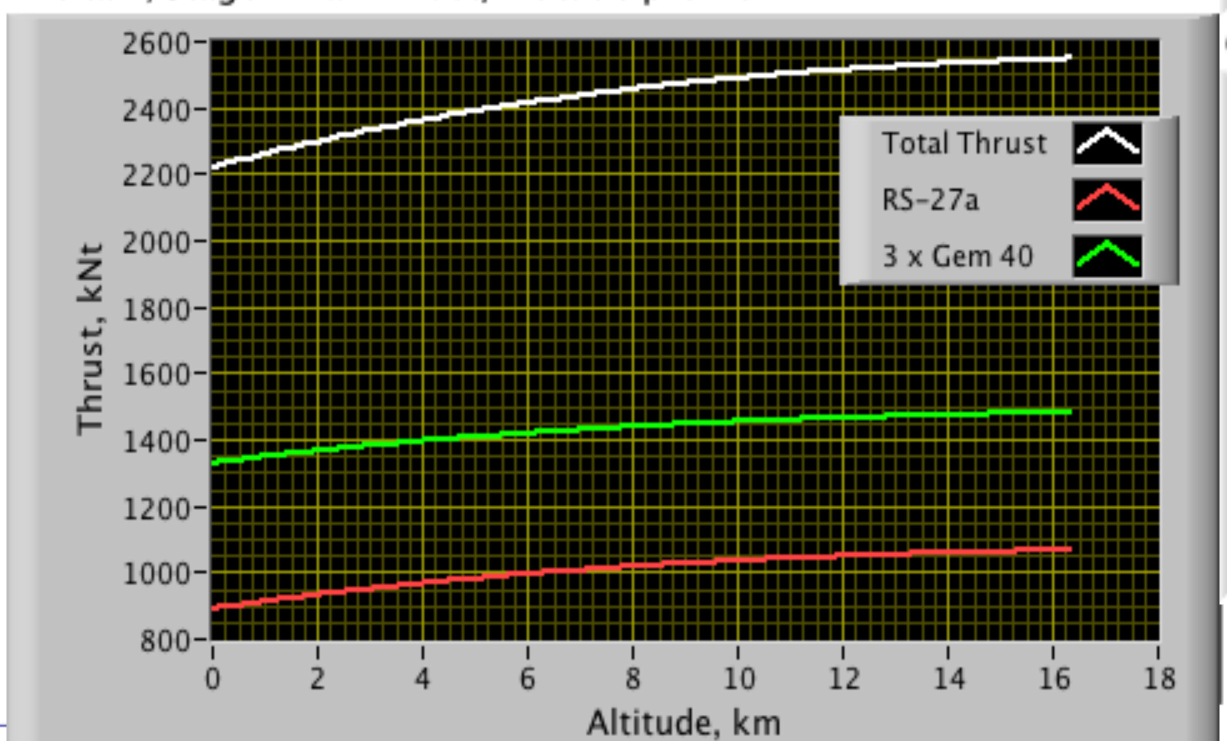
$$F_{liftoff} = 3 \times 442.95 + 890 = 2218.95 \text{ kNt}$$

ii) Burn Time for Gem-40(s)

$$T_{burn} = 63.4865 \text{ sec}$$

iii) Plot total Thrust profile during Burn "1a" vs Altitude

Delta II, Stage "1-a" Thrust/Altitude profile



RS 27A Data

Initial Thrust, kNt: 890	Initial Isp, sec: 246.986
Final Thrust, kNt: 1066.87	Final Isp, sec: 296.069
Burn time, sec: 264.202	mdot, kg/sec: 367.447

Gem 40 Data

Initial Thrust, kNt: 442.95	Initial Isp, sec: 243.737
Final Thrust, kNt: 493.761	Final Isp, sec: 271.696
Burn time, sec: 63.4865	mdot, kg/sec: 185.315

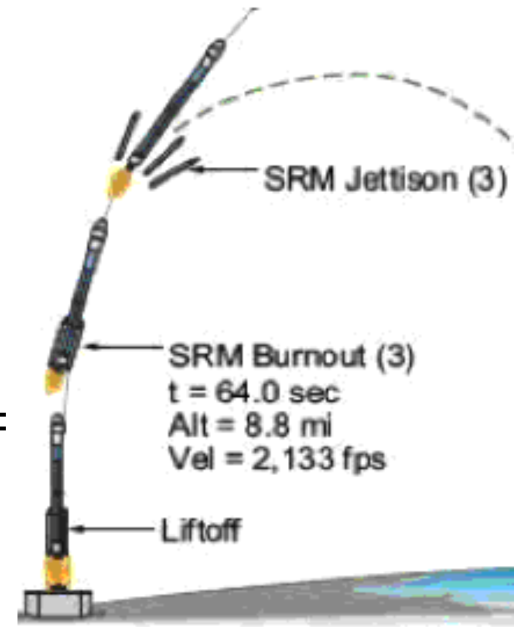
… Stage "1a" (3)

-- Calculate:

iv) Total propellant consumed during "stage 1a" burn

$$\left(M_{prop}\right)_{\substack{stage\\ "1a"}} = \dot{m}_{RS-27A} \cdot \left(T_{burn}\right)_{\substack{stage\\ "1a"}} + 3 \cdot \dot{m}_{Gen40} \cdot \left(T_{burn}\right)_{\substack{stage\\ "1a"}} =$$

$$(367.447 + 3 \cdot \mathbf{185.32})\ \mathbf{63.49} = 58{,}717.1 \text{ kg}$$



Total Stage "1a" Masses

Lift Off: 141,040 kg

Gem 40 Burnout: ***82,417.1*** kg

RS-27A stage after Gem 40 Motor Jettison: ***78,472.1*** kg

Remaining RS-27A stage propellant: ***73,752.1*** kg

… Stage "1a" (4)

vi) Effective Specific Impulse

(3 x Gem 40 + RS-27A over operating altitude range

$$\left(I_{sp}\right)_{launch} = \frac{\left(F_{RS-27A} + 3\cdot F_{Gem40}\right)_{launch}}{g_0\cdot\left(\dot{m}_{RS-27A} + 3\cdot \dot{m}_{Gem40}\right)} = \frac{(890 + 442.95)\,1000}{9.8067\,(367.447 + 3\cdot 185.32)}$$

$$= 245.03 \text{ sec}$$

$$\left(I_{sp}\right)_{\substack{Gem40\\burnout}} = \frac{\left(F_{RS-27A} + 3\cdot F_{Gem40}\right)_{\substack{Gem40\\burnout}}}{g_0\cdot\left(\dot{m}_{RS-27A} + 3\cdot \dot{m}_{Gem40}\right)} = \frac{(1066.87 + 493.76\cdot 3)\,1000}{9.8067\,(367.447 + 3\cdot 184.57)}$$

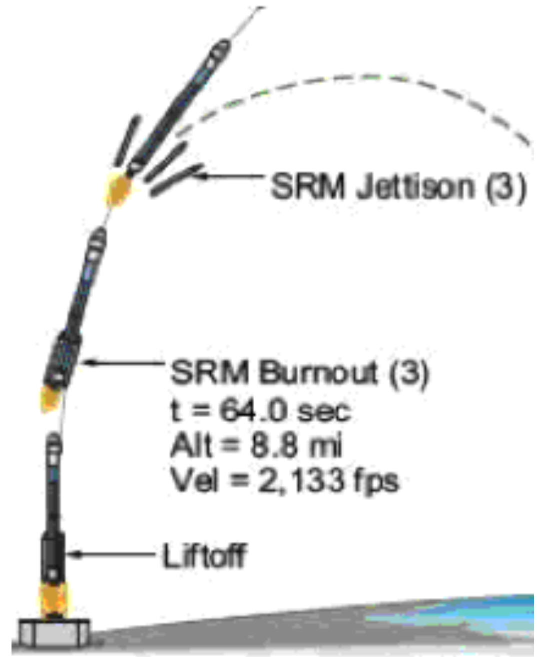
$$= 281.40 \text{ sec}$$

$$\left(I_{sp}\right)_{eff} = \frac{2}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\3x\ Gem40}}\right]_{Launch} + \frac{1}{3}\left[\left(I_{sp}\right)_{\substack{Rs-27A+\\3x\ Gem40}}\right]_{\substack{Gem40\\Burnou}} = \frac{2\cdot 245.03}{3} + \frac{281.40}{3}$$

$$= 257.15 \text{ sec}$$

… Stage "1a" Final summary

Final Summary



Lift-Off Gross Thrust: ***2218.85*** *kNt*

Effective Specific Impulse: ***257.15*** *sec*

Stage 1a Propellant Consumed: ***58,622.9*** *kg*

(Initial) Lift Off Stage Mass: *141,040 kg*

(Final) @Gem-40 Burnout Mass: ***82,417.1*** *kg*

(Stage 1b Initial Mass) RS-27A Stage Mass @ Gem40 Jettison): ***78,472.1*** *kg*

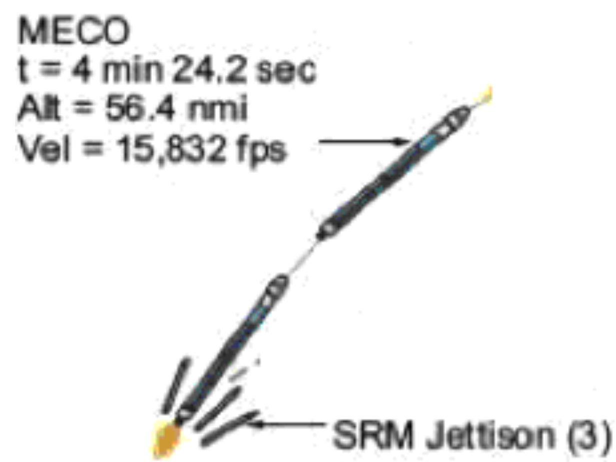
Remaining RS-27A Stage Propellant Mass: ***73,752.1*** *kg*

… Stage "1b"

Stage 1 (RS-27A) burning from Gem 40 Burnout Altitude ~ 8.8 *nmi* (16.31 km) to MECO altitude, 56.4 *nmi* (105.52 km)

-- Calculate:

- ***i) Burn Time from Gem-40(s) burnout to MECO***
- ***ii) Plot thrust profile during "1b" burn vs altitude***
- ***iii) Total propellant consumed during "stage 1b" burn***
- ***iv) Effective*** I_{sp} ***Over Altitude Range (16.31 km to 105.52 km)***



… Stage "1b" (2)

-- Calculate:

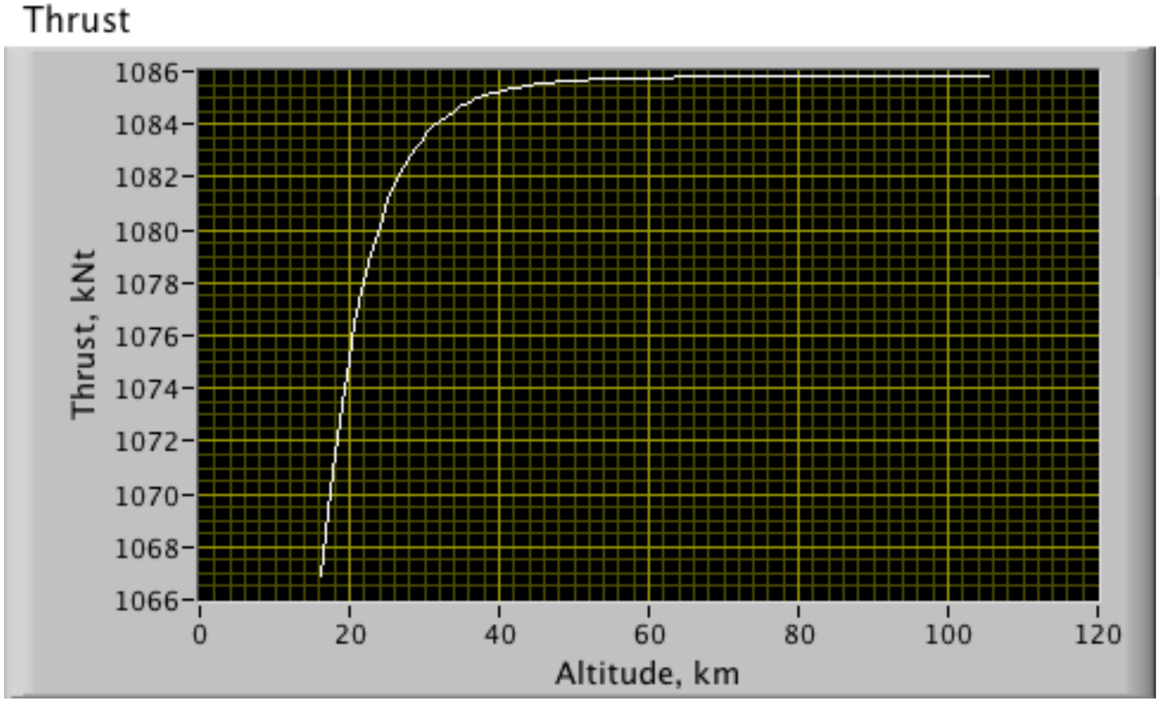
i) Burn Time from Gem-40(s) burnout to MECO

$$(T_{burn})_{stage\ \text{“1b”}} = 4*60+24.2-63.4865 = 200.714 \text{ sec}$$

Or ….. $$(T_{burn})_{stage\ \text{“1b”}} = 97{,}080/367.447 - 11.765/184.57 = 200.715 \text{ sec}$$

"round off" error from ULA/Boeing charts

ii) Plot thrust profile during "1b" burn vs altitude



Initial Thrust, kNt	Initial Isp, sec
1066.87	296.069

Final Thrust, kNt	Final Isp, sec
1085.8	301.323

… Stage "1b" (3)

-- Calculate:

iii) Total propellant consumed during "stage 1b" burn

$$\left(M_{prop}\right)_{\substack{stage\\ "1b"}} = \dot{m}_{RS-27A} \cdot \left(T_{burn}\right)_{\substack{stage\\ "1b"}} = 367.447 \cdot 200.4586$$

$$= \mathit{73{,}752.1} \text{ kg}$$

Total Stage "1b" Masses

RS-27A stage "1b" initial total mass: ***78,472*** *kg*
RS-27A stage "1b" initial propellant mass ***73,752.1*** *kg*
Mass at *RS-27A stage "1b" MECO: 4720.0 kg*

UtahState UNIVERSITY

Mechanical & Aerospace Engineering

… Stage "1b" (4)

-- Calculate:

iv) Effective I_{sp} ***Over Altitude Range (16.31 km to 105.52 km)***

$$\left(I_{sp}\right)_{eff} = \frac{2}{3}\left[\left(I_{sp}\right)_{R2-27A}\right]_{\substack{Gem40 \\ Burnou}} + \frac{1}{3}\left[\left(I_{sp}\right)_{R2-27A}\right]_{\substack{R2-27A \\ MECO}} =$$

Initial Thrust, kNt

1066.87

Final Thrust, kNt

1085.8

$$\frac{1000\left(\frac{2}{3}1066.87 + \frac{1}{3}1085.8\right)}{9.8067 \cdot 367.447} = 297.82 \text{ sec}$$

$$\dot{m}_{R2-27A} = 367.447 \, \frac{kg}{\text{sec}}$$

… Stage "*1b*" Final summary

Final Summary

Initial Thrust: *1066.87 kNt*

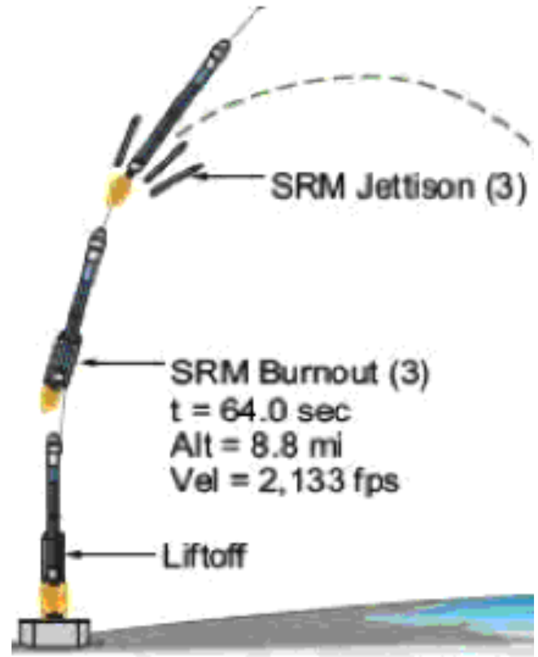
Effective Specific Impulse: *297.82 sec*

Stage "*1b*" Propellant Consumed: ***73,752.1*** *kg*

Initial Stage "*1b*" Mass: ***78,472.1*** *kg*

Final @MECO Stage "*1b*" Mass: *4720.0 kg*

Remaining RS-27A Stage Propellant Mass: *0.0 kg*

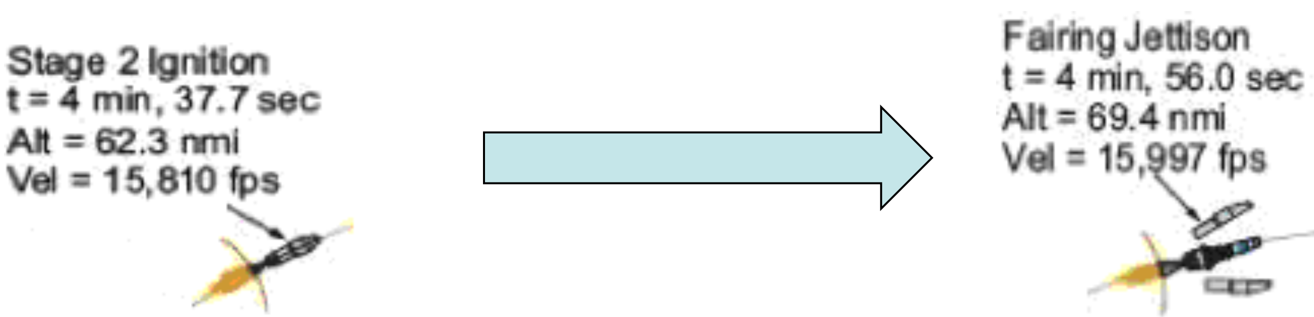


… Stage "2a"

Stage 2 (AJ10-118K Aerojet Engine) burning ignition (4 min 37.7 sec) to fairing jettison (4 min 56 sec) … Altitude ~ 62.3 *nmi* (115.45 km) to 69.4 *nmi* (128.61 km)

-- Calculate:

i) Stage "2a" massflow
ii) Stage "2a" burn time
iii) Total propellant consumed during "stage 2a" burn
iv) Initial Stage "2a" mass
v) Stage 2a "final mass" before shroud jettison
vi) Final Stage "2a" mass after shroud jettison



… Stage "2a" (2)

Stage 2 (AJ10-118K Aerojet Engine) burning ignition (4 min 37.7 sec) to fairing jettison (4 min 56 sec) … Altitude ~ 62.3 *nmi* (115.45 km) to 69.4 *nmi* (128.61 km)

-- Calculate:

i) Stage "2a" massflow

$$\dot{m}_{\text{AJ10-118K}} = \frac{F_{vac}}{g_0\left(I_{sp}\right)_{vac}} = \frac{43.657\times10^3}{9.8067\cdot319}$$ **= 13.955 kg/sec**

ii) Stage "2a" burn time

$(T_{burn})_{stage\ \text{"1b"}}$ = 4*60+56 – 4*60-37.7 = 18.3 sec

iii) Total propellant consumed during "stage 2a" burn

$$\left(M_{prop}\right)_{stage\ \text{"2a"}} = \dot{m}_{\text{AJ10-118K}}\cdot T_{burn} = \frac{43.657\times10^3}{9.8067\cdot319}18.3 = 255.38\ \text{kg}$$

… Stage "2a" (3)

iv) Initial Stage "2a" mass before shroud jettison

- Propellant Mass: *6004 kg*
- Stage 2 Launch Mass: *6954 kg*
- Shroud Mass, *1040 kg*

$$\left(M_{stage"2a"}\right)_{initial} = \left(M_{stage"2a"}\right)_{gross} + M_{shroud} = 6954 + 1040 = 7994 \text{ kg}$$

v) Final Stage "2a" mass before shroud jettison

$$\left(M_{stage"2a"}\right)_{final} = \left(M_{stage"2a"}\right)_{initial} - M_{prop} = 6954 + 1040 - 255.38$$

$$= 7738.62 \text{ kg}$$

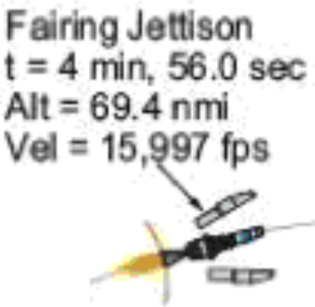
vi) Final Stage "2a" mass after shroud jettison

$$\left(M_{stage"2b"}\right)_{initial} = \left(M_{stage"2a"}\right)_{final} - M_{shroud} = 7738.62 - 1040$$

$$= 6698.62 \text{ kg}$$

… Stage "2*a*" Final summary

Final Summary



Initial Thrust: 43.657 kN

Effective Specific Impulse: *319.0 sec*

Stage "2*a*" Propellant Consumed: 255.38 *kg*

Initial Stage "2*a*" Mass: *7994 kg*

Final before jettison Stage "2*a*" Mass: *7738.62 kg*

Final after jettison Stage "2*a*" Mass: *6698.62 kg*

Remaining Stage 2 Propellant Mass: *5748.62 kg*

… Stage "2b"

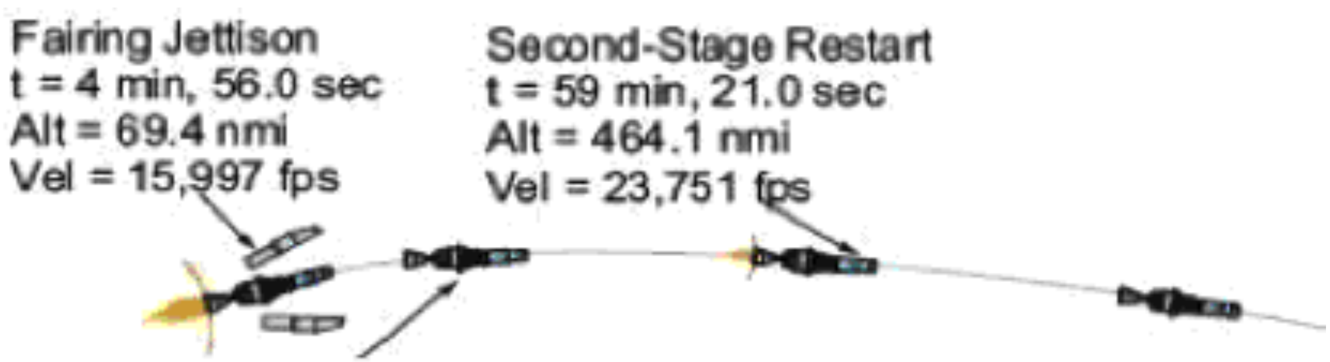
Stage 2 from Fairing Jettison to SECO … assume all propellant is consumed in stage

-- Calculate:

i) Total propellant consumed during "stage 2b" burn

ii) Initial and final masses (excluding payload)

Stage "2b"



… Stage "2b" (2)

Stage 2 from Fairing Jettison to SECO … assume all propellant is consumed in stage

-- Calculate:

i) Total propellant consumed during "stage 2b" burn

$$\left(M_{prop}\right)_{stage"2b"} = \left(M_{\substack{prop\\stage"2b"}}\right)_{total} - \left(M_{prop}\right)_{stage"2a"} = 5748.62\ kg$$

ii) Initial and final masses (excluding payload)

$$(M_{initial})_{Stage"2b"} = 6698.62\ kg$$

$$(M_{initial})_{Stage"2b"} = 6954\ kg - 6004 = 950\ kg$$

… Stage Mass Fractions

For an assumed payload mass …. Calculate

i) Gross $\frac{M_{initial}}{M_{final}}$ for each "stage" … includes stuff each stage lifts

ii) ΔV For each "stage" (include gravity and drag losses .. Where appropriate ***(hint: work backwards from stage "2a")***

iii) Total available ΔV

iv) Compare to $\Delta V_{available}$ *to* $\Delta V_{required}$

………… iterate until you get match

Working Backwards ….

$M_{payload}$ → Delivered payload mass

Stage "2b"

$$\left(\frac{M_{initial}}{M_{final}}\right)_{stage"2b"} = \frac{\left(M_{initial}\right)_{stage"2b"} + M_{payload}}{\left(M_{final}\right)_{stage"2b"} + M_{payload}} = \frac{6698.2_{kg} + M_{payload}}{950_{kg} + M_{payload}}$$

No gravity or drag losses

$$\left(\Delta V\right)_{stage"2b"} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)_{stage"2b"} =$$

$$9.8067_{\frac{m}{\sec^2}} \cdot 319_{\sec} \cdot \ln\left(\frac{6698.2_{kg} + M_{payload}}{950_{kg} + M_{payload}}\right)$$

Working Backwards …. (2)

$M_{payload}$ → Delivered payload mass

Stage "2a"

$$\left(\frac{M_{initial}}{M_{final}}\right)_{stage"2a"} = \frac{\left(M_{initial}\right)_{stage"2a"} + M_{payload}}{\left(M_{final}\right)_{stage"2b"} + M_{shroud} + M_{payload}} = \frac{7994_{kg} + M_{payload}}{7738.62_{kg} + M_{payload}}$$

No gravity or drag losses

$$\left(\Delta V\right)_{stage"2a"} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)_{stage"2a"} =$$

$$9.8067_{\frac{m}{\sec^2}} \cdot 319_{\sec} \cdot \ln\left(\frac{7994_{kg} + M_{payload}}{7738.62_{kg} + M_{payload}}\right)$$

Working Backwards …. (2)

$M_{payload}$ → Delivered payload mass

Stage "1b"

$$\left(\frac{M_{initial}}{M_{final}}\right)_{stage"1b"} = \frac{\left(M_{initial}\right)_{stage"1b"} + \left(M_{initial}\right)_{stage"2a"} + M_{payload}}{\left(M_{final}\right)_{stage"1b"} + \left(M_{initial}\right)_{stage"2a"} + M_{payload}} = \frac{78{,}472.1_{kg} + 7994_{kg} + M_{payload}}{4720_{kg} + 7994_{kg} + M_{payload}}$$

$$\left(\Delta V_{"1b"}\right)_{propulsive} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)_{stage"1b"} = 9.8067_{\frac{m}{\sec^2}} \cdot 297.82_{\sec} \cdot \ln\left(\frac{78{,}472.1_{kg} + 7994_{kg} + M_{payload}}{4720_{kg} + 7994_{kg} + M_{payload}}\right)$$

..Drag Loss … 3% …

$$\left(\Delta V_{"1b"}\right)_{kinematic} = 0.97 \cdot g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)_{stage"1b"} =$$

$$0.97 \cdot 9.8067_{\frac{m}{\sec^2}} \cdot 297.82_{\sec} \cdot \ln\left(\frac{78{,}472.1_{kg} + 7994_{kg} + M_{payload}}{4720_{kg} + 7994_{kg} + M_{payload}}\right)$$

Working Backwards …. (3)

$M_{payload}$ → Delivered payload mass

Stage "1b" ..Gravity Losses… assumed 30 deg pitch angle

$$(\Delta V)_{\substack{gravity\\loss}} = \int_{T_{burn}} g(t)\cdot\sin\theta\cdot dt \rightarrow \textit{use altitude averaged } g_{(h)}$$

$$\left[(\Delta V)_{\substack{gravity\\loss}}\right]_{stage} \approx \left[\frac{2}{3}\cdot g_{(h_{initial})} + \frac{1}{3}\cdot g_{(h_{initial})}\right]\cdot\sin\theta\cdot T_{burn} =$$

$$\left(\frac{2}{3}\frac{3.9860044\times10^{5}}{(6373.2+16.31)^{2}}+\frac{1}{3}\frac{3.9860044\times10^{5}}{(6373.2+105.52)^{2}}\right)200.4586\sin\left(\frac{\pi}{180}30\right)$$

= 0.9697 km/sec

Working Backwards …. (5)

$M_{payload}$ → Delivered payload mass

Stage "1a"

$$\left(\frac{M_{initial}}{M_{final}}\right)_{stage"1a"} = \frac{\left(M_{initial}\right)_{stage"1a"} + \left(M_{initial}\right)_{stage"2a"} + M_{payload}}{\left(M_{final}\right)_{stage"1a"} + \left(M_{initial}\right)_{stage"2a"} + M_{payload}} = \frac{141{,}040_{kg} + 7994_{kg} + M_{payload}}{\mathbf{82{,}417.1}_{kg} + 7994_{kg} + M_{payload}}$$

$$\left(\Delta V_{"1a"}\right)_{propulsive} = g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)_{stage"1b"} = 9.8067_{\frac{m}{\sec^2}} \cdot 258.484_{\sec} \cdot \ln\left(\frac{141{,}040_{kg} + 7994_{kg} + M_{payload}}{\mathbf{82{,}417.1}_{kg} + 7994_{kg} + M_{payload}}\right)$$

..Drag Loss … 3% …

$$\left(\Delta V_{"1a"}\right)_{kinematic} = 0.97 \cdot g_0 \cdot I_{sp} \cdot \ln\left(\frac{M_{initial}}{M_{final}}\right)_{stage"1b"} =$$

$$0.97 \cdot 9.8067_{\frac{m}{\sec^2}} \cdot 258.484_{\sec} \cdot \ln\left(\frac{141{,}040_{kg} + 7994_{kg} + M_{payload}}{\mathbf{82{,}417.1}_{kg} + 7994_{kg} + M_{payload}}\right)$$

Working Backwards …. (6)

$M_{payload} \rightarrow$ Delivered payload mass

Stage "1a" ..Gravity Losses… assumed 90 deg pitch angle

$$(\Delta V)_{\substack{gravity\\loss}} = \int_{T_{burn}} g(t)\cdot\sin\theta\cdot dt \rightarrow \textit{use altitude averaged } g_{(h)}$$

$$\left[(\Delta V)_{\substack{gravity\\loss}}\right]_{stage} \approx \left[\frac{2}{3}\cdot g_{(h_{initial})} + \frac{1}{3}\cdot g_{(h_{initial})}\right]\cdot\sin\theta\cdot T_{burn} =$$

$$\left(\frac{2}{3}\frac{3.9860044\times10^{5}}{(6373.2+0)^{2}} + \frac{1}{3}\frac{3.9860044\times10^{5}}{(6373.2+16.31)^{2}}\right)63.743\sin\left(\frac{\pi}{180}90\right)$$

= 0.6245 km/sec

Total Delta V Summary

$$(\Delta V)_{stage"2b"} = 9.8067_{\frac{m}{\sec^2}} \cdot 319_{\sec} \cdot \ln\left(\frac{6698.2_{kg} + M_{payload}}{950_{kg} + M_{payload}}\right)$$

$$(\Delta V)_{stage"2a"} = 9.8067_{\frac{m}{\sec^2}} \cdot 319_{\sec} \cdot \ln\left(\frac{7994_{kg} + M_{payload}}{7738.62_{kg} + M_{payload}}\right)$$

$$(\Delta V_{"1b"})_{available} = 0.97 \cdot 9.8067_{\frac{m}{\sec^2}} \cdot 297.82_{\sec} \cdot \ln\left(\frac{78{,}472.1_{kg} + 7994_{kg} + M_{payload}}{4720_{kg} + 7994_{kg} + M_{payload}}\right) - 0.9697_{km/\sec}$$

$$(\Delta V_{"1a"})_{available} = 0.97 \cdot 9.8067_{\frac{m}{\sec^2}} \cdot 258.484_{\sec} \cdot \ln\left(\frac{141{,}040_{kg} + 7994_{kg} + M_{payload}}{82{,}417.1_{kg} + 7994_{kg} + M_{payload}}\right) - 0.6245_{\frac{km}{\sec}}$$

Example 1000 kg payload

$$\left(\Delta V\right)_{stage"2b"} = 9.8067\cdot 319 \ln\left(\frac{6698.2+1000}{950+1000}\right) = 4.2957 \text{ km/sec}$$

$$\left(\Delta V\right)_{stage"2a"} = 9.8067\cdot 319 \ln\left(\frac{7994+1000}{7738.62+1000}\right) = 0.0901 \text{ km/sec}$$

$$\left(\Delta V_{"1b"}\right)_{available} = \frac{0.97\cdot 9.8067\cdot 297.82\left(\ln\left(\frac{78377.9+7994+1000}{4720+7994+1000}\right)\right)}{1000} - 0.9697 = 4.2751 \text{ km/sec}$$

$$\left(\Delta V_{"1a"}\right)_{available} = \frac{0.97\cdot 9.8067\cdot 258.484 \ln\left(\frac{141040+7994+1000}{82332.9+7994+1000}\right)}{1000} - 0.6245 = 0.5797 \text{ km/sec}$$

$$Total\ \Delta V_{available} = 4.2957 + 0.0901 + 4.2751 + 0.5797 = 9.2406 \text{ km/sec}$$

Example 1000 kg payload

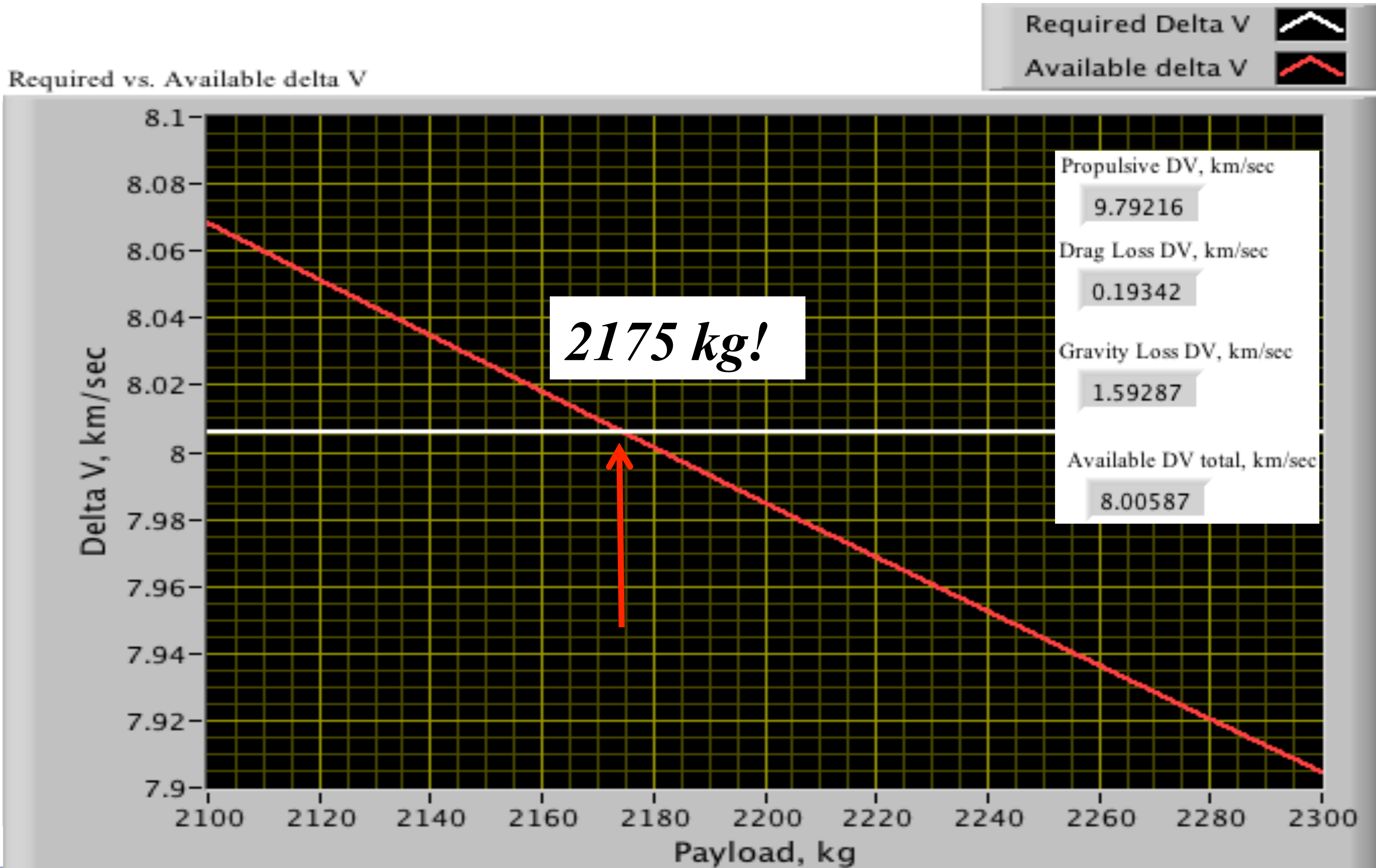
$$Total\ \Delta V_{available} = 9.2406 \text{ km/sec}$$

$$\left(\Delta V_{total}\right)_{required} = \sqrt{\left(V_{orbit} - V_{"boost"}\right)^2 + \left(\Delta V_{gravity}\right)^2} = \boxed{= 8.0059 \text{ km/sec}}$$

Can lift significantly solve problem numerically …

$$\Delta V_{required} = \Delta V_{available} = \left(\Delta V_{"1a"}\right)_{available} + \left(\Delta V_{"1b"}\right)_{available} + \left(\Delta V\right)_{stage"2a"} + \left(\Delta V\right)_{stage"2b"}$$

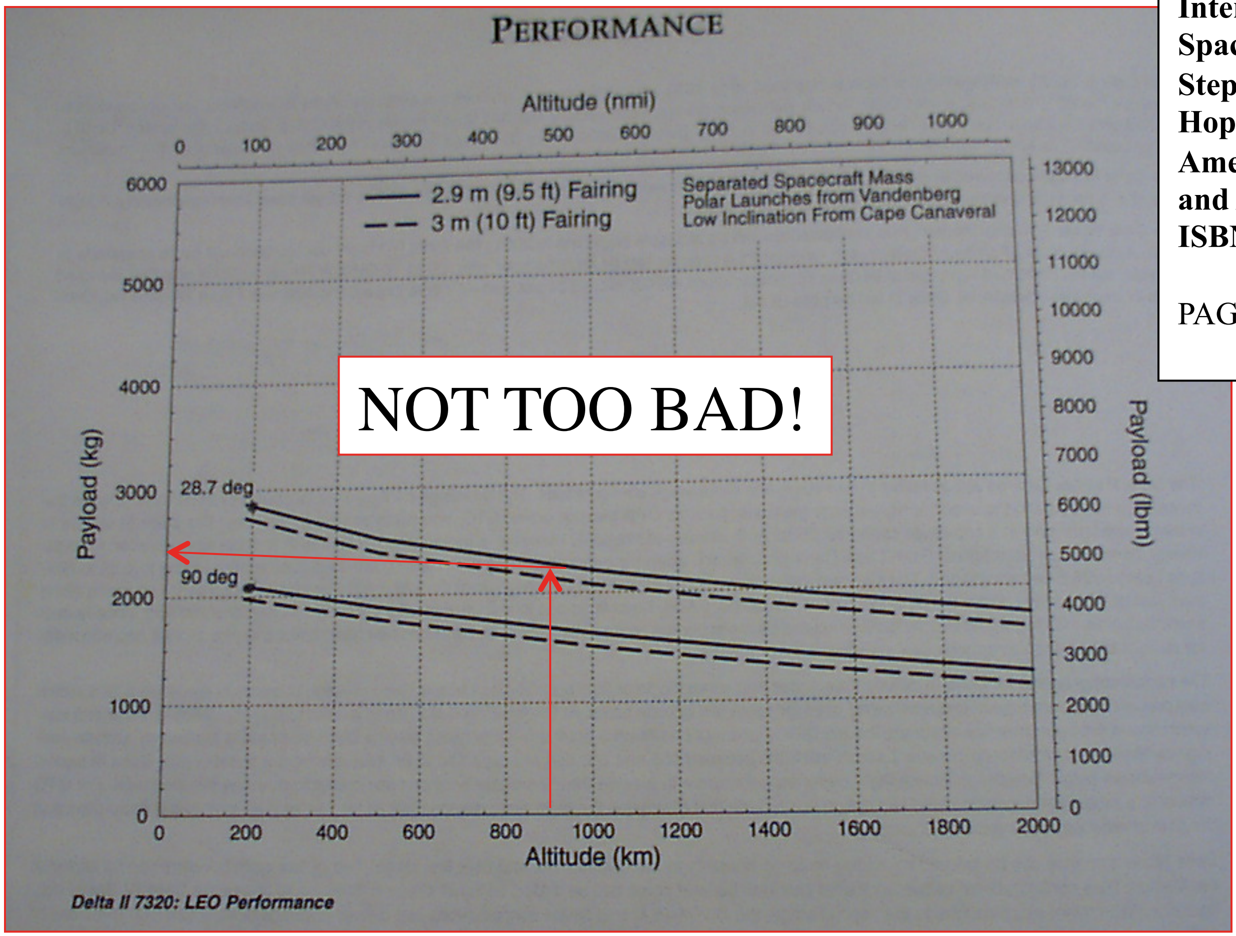
Required versus Available Delta V



UtahState University

Mechanical & Aerospace Engineering

Sanity Check



International Reference Guide to Space Launch Systems, 4th ed., Stephen J. Isakowitz, Joseph P. Hopkins, Jr., and Joshua B. Hopkins, American Institute of Aeronautics and Astronautics, Reston, VA, 2003. ISBN: 1-56347-591-X

PAGE 100